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Complex Performance Control Using Sliding Mode Fuzzy Approach for Discrete-Time Nonlinear Systems via T-S Fuzzy Model with Bilinear Consequent Part

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Abstract: In this paper, a stabilization problem for the discrete nonlinear system with external disturbance, multiplicative noises and multiple constraints has been discussed in accordance with the definition of Lyapunov stability. Based on fuzzy modeling approach, the overall fuzzy model of a nonlinear plant is transformed into a class of linear systems. Applying a Sliding Mode Fuzzy Control (SMFC) scheme, the designed controller causes the closed-loop system converging to the sliding surface and achieving the required control performance. For the control performance, the concepts of stability, individual state variance and passivity constraints are introduced for the sliding mode fuzzy control system. To apply convex optimal programming algorithm, some sufficient conditions derived in this paper are reduced to Linear Matrix Inequality (LMI) problem. At last, two simulation examples are proposed to demonstrate the applicability and usefulness of the proposed design method. One of the examples is to discuss the conservatism of this paper. Another is to show that the discrete truck-trailer system controlled by sliding mode fuzzy controller can achieve stability constraints, individual state variance constraints and passivity constraints.

Keywords: Individual state variance constraint and passivity constraint, sliding mode fuzzy control, Takagi-Sugeno fuzzy model.

1. INTRODUCTION

Most of the plants in the industry have severe nonlinearity, which makes the researches for nonlinear control systems possess more practical significance. Model-free fuzzy control technology can provide an effective and simple solution to the control of plants that are complex, nonlinear, uncertain, ill-defined, and have available qualitative knowledge from domain experts for their controller design. Though the model-free method is useful and practical, this control technique lacks the mathematics theory to support it. In recent decades, the famous Takagi-Sugeno (T-S) fuzzy model [1] in which a linear system is adopted as the consequent part of a fuzzy rule has been developed. It is known that the T-S fuzzy model offers an efficient representation of nonlinear behaviors. Applying the T-S fuzzy model, the original nonlinear system can be divided into several local linear subsystems by the membership functions. In this case, the stabilization, observation and controller design of nonlinear systems can be simplified. Therefore, the stability and stabilization methods inspired from the study of linear control technique have been proposed in [2, 3]. An effective active queue management router has been designed for transmission control protocol network via T-S fuzzy model in [4]. Controller synthesis of Hénon map has been investigated via fuzzy approach in [5]. Consensus problem of multi-agent games was discussed in [6] via fuzzy adaptive programming. Impulsive synchronization of chaotic systems was researched via T-S fuzzy model in [7]. Some relaxed stability criteria for T-S fuzzy models have been developed in [8–10]. Observer design method for nonlinear descriptor systems has been developed in [11]. An observer feedback control problem of affine T-S fuzzy models was discussed in [12]. Estimated state feedback control of discrete nonlinear stochastic systems has been investigated in [13]. Robust observer control of T-S fuzzy mode was discussed in [14]. A simple adaptive control for unmanned helicopters has been discussed in [15] by fuzzy approximation. An adaptive control of nonlinear hyperbolic systems was investigated to achieve the predefined performance in [16]. Most of the proposed results in the literature [2–16] are usually formulated as an optimization problem under the Linear Matrix Inequalities (LMIs) constraints. It has been shown that the LMI problem can be successfully solved by the convex optimal programming algorithm [17].

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Besides stability issue, another important requirement for a control system is its robustness. Sliding-mode control has proven to be an effective robust control strategy for incompletely modeled or uncertain systems and has received a considerable amount of research interests, especially in the continuous systems. However, with the wide use of computers, discrete sliding-mode control becomes a hot researching field. Generally speaking, the main idea of the normal SMC scheme is to utilize a discontinuous control law to drive the system trajectories onto a specified sliding surface. And then, the system trajectories are kept moving along the sliding surface toward the origin with desired performance. The survey of sliding mode control can be referred to [18, 19]. Based on the concept of the sliding mode control, some stability issues of practical applications have been investigated, such as inertia wheel pendulum system [20], hybrid vehicle [21] and spacecraft [22, 23]. Sliding-mode control has been extensively studied and successfully applied to control linear systems, nonlinear systems and complex systems with uncertain dynamics, unknown system parameters and external disturbances [24]. For the nonlinear systems, the sliding mode control is also employed to provide good performances for the closed-loop systems [25, 26]. Moreover, sliding mode control has been applied to nonlinear systems by using T-S fuzzy model approach [1]. In [27, 28], a robust adaptive fuzzy sliding mode fuzzy controller design method has been proposed. Sliding mode fuzzy control of nonlinear systems was discussed in [29, 30]. According to the T-S fuzzy models, the sliding mode control has been successfully combined with fuzzy control technique for the nonlinear systems [31–33]. In this paper, we start from this point and consider the problem of T-S fuzzy model-based discrete sliding mode control for the nonlinear systems.

Referring to [34–36], some stability criteria have been proposed to deal with control issue of stochastic systems. Through applying the T-S fuzzy model approach, the stability analysis of discrete-time nonlinear stochastic systems has been discussed. Therefore, several controller design methods have been developed from nonlinear deterministic system to nonlinear stochastic systems, for example, covariance control scheme [37], passivity theory [38], gain-scheduled control [39] and so on. Studying them, a concept of quadratic optimization has been applied to achieve the required performance constraints. However, they can only guarantee that the control system state vector as a whole functions well. A straightforward controller design methodology, which is called as covariance control theory allows the designers to assign state covariance matrix by solving the inverse solution of Lyapunov equation for the closed-loop stochastic systems. Moreover, the covariance matrix is usually expressed as the upper bounds on the steady-state variances of system states. As a result, a large number of control methods have been developed to seek a convenient way to solve the variance constrained control problem. In literature, the Linear Quadratic Gaussian (LQG) control plays an important role in the minimum variance design methods. However, the LQG controllers minimize a linear quadratic performance index without guaranteeing the variance constraints with respect to individual system states. The covariance control theory [30, 31] has provided a more direct methodology for achieving the individual state variance constraints than the LQG control approach. It has been shown that the covariance control approach is capable of solving multi-objective control design problems [40, 41]. This advantage is based on the fact that several control design objectives can be directly related to the steady-state covariance of the closed-loop systems. Furthermore, covariance control methodology for fuzzy systems has been proposed by [42, 43] to constrain state variance. In the past few years, the state variance constrained control based on covariance control approach has received many interests and some results have been reported in the literature [44]. It should be pointed out that most of the available literature regarding covariance control theory has been concerned with linear and nonlinear stochastic systems with the LMI technique.

For studying the multi-objective control design problem, the passivity constraint is also included in the proposed fuzzy controller design process in addition to individual state variance constraint. It is well known that the passive theory uses the behaviors of energy dissipative between input and output of systems. The most important definitions of the passive theory are storage function [45] and power supply [46] for decision the property of passivity. In recent years, the passivity constraint has been considered in the fuzzy controller design for nonlinear systems [3, 9, 38, 47]. In order to accomplish the combined multi-objective controller design, some sufficient conditions are derived based on Lyapunov stability theory. Using the matrix transform technique and Schur complement [17], these sufficient conditions are transformed into Linear Matrix Inequality (LMI) problems. Finally, a numerical example is provided to discuss the conservatism of the proposed design method. Moreover, a control problem of nonlinear discrete truck-trailer system [48, 49] is investigated subject to individual state variance constraint and passivity constraint so as to illustrate the applicability and efficiency of proposed sliding mode fuzzy controller design approach. In future, this work will be extended to deal with stability issues of multi-agent systems [50] or strict-feedback systems [51]. Moreover, the concept of this paper can be merged other control technologies, such as adaptive control [52] or fault-tolerant control [53], to develop more powerful control scheme.

This paper is organized as follows: In Section 2, a T-S fuzzy model with external disturbances and multiplicative noises is used to represent the discrete nonlinear stochastic systems. The definition of sliding surface is also described in this section. In Section 3, reaching conditions and a switching controller are designed to show that the system states will converge to the sliding surface. In Section 4, after reaching the sliding surface, the individual state variance constraints and passivity constraints can be considered. Some sufficient conditions are solved by LMI technique then we can obtain the sliding mode fuzzy controller in the proposed. In Section 5, two numerical examples are considered to demonstrate the applicability and effectiveness of proposed sliding mode fuzzy control method. Lastly, some conclusions are provided in Section 6.

2. SYSTEM DESCRIPTIONS AND PRELIMINARY KNOWLEDGES

A discrete stochastic nonlinear system represented by a T-S model is considered as follows:

Rule i:

IF $z_1(k)$ is M_{i1} and ... and $z_{n_x}(k)$ is M_{in_x}

THEN

$$x(k+1) = \left[\mathbf{A}_1 + \sum_{e=1}^{m} \mathbf{Q}_{ei} v_{ei}(k)\right] x(k) + \mathbf{B}_i u(k) + \mathbf{K}_i w(k), \tag{1a}$$

$$y(k) = \mathbf{C}_i x(k) + \mathbf{T}_i w(k), \tag{1b}$$

where $z_{n_x}(k)$ is premise variable, M_{in_x} is fuzzy set, $i = 1, 2, \cdots, r$ and r is the number of IF-THEN rule, n_x is the premise variable number, $\mathbf{A}_i \in \Re^{n_x \times n_x}$, $\mathbf{B}_i \in \Re^{n_x \times n_u}$, $\mathbf{K}_i \in \Re^{n_x \times n_w}$, $\mathbf{C}_i \in \Re^{n_y \times n_x}$, $\mathbf{T}_i \in \Re^{n_y \times n_w}$ and $\mathbf{Q}_{ei} \in \Re^{n_x \times n_x}$ are constant matrices, $x(k) \in \Re^{n_x}$ is the state vector, $u(k) \in \Re^{n_u}$ is the input vector, $y(k) \in \Re^{n_y}$ is the output vector, $w(k) \in \Re^{n_w}$ is the external disturbance input vector which is assumed as a zero mean Gaussian white noise with intensity $\mathbf{W}$ ($\mathbf{W} > 0$), variance one and satisfies $\|w(k)\| < \delta$, where δ is a positive scalar, $v_{ei}(k) \in \Re$ is assumed as a zero-mean Gaussian white noise. Referring to [39], the property of $E\{v_{ei}(k)x(k)\} = E\{v_{ei}(k)\}E\{x(k)\} = 0$ is assumed that $v_{ei}(k)$ and $x(k)$ are mutually independent where $E\{\bullet\}$ denotes the expected value of $\bullet$. Without loss of generality, it is assumed that the premise variables of the above T-S fuzzy model are measurable.

Given the pair $(x(k), u(k))$, an overall fuzzy model can be described as follows:

$$x(k+1) = \sum_{i=1}^{r} \lambda_i(z(k)) \left\{ \left[\mathbf{A}_i + \sum_{e=1}^{m} \mathbf{Q}_{ei} v_{ei}(k)\right] x(k) + \mathbf{B}_i u(k) + \mathbf{K}_i w(k) \right\}, \tag{2a}$$

$$y(k) = \sum_{i=1}^{r} \lambda_i(z(k)) \{\mathbf{C}_i x(k) + \mathbf{T}_i w(k)\}, \tag{2b}$$

where $\lambda_i(z(k)) = \frac{\omega_i(z(k))}{\sum_{i=1}^{r} \omega_i(z(k))}$, $\omega_i(z(k)) = \prod_{j=1}^{n_x} \mathrm{M}_{ij}(z_j(k))$, $\lambda_i(z(k)) \geq 0$, $\sum_{i=1}^{r} \lambda_i(z(k)) = 1$ and $\mathrm{M}_{ij}(z_j(k))$ is the grade of the membership of $z_j(k)$ in M_{ij}.

In this paper, the following sliding surface function $S(k)$ is selected corresponding to $x(k)$, where $x(k)$ is the solution of (1a).

$$S(k) = \mathbf{J}\left[x(k) - \tilde{\mathbf{A}} x(k-1)\right], \tag{3}$$

where

$$S(k) = \begin{bmatrix} s_1(k) & s_2(k) & \cdots & s_{n_u}(k) \end{bmatrix}^T \tag{4}$$

$$\tilde{\mathbf{A}} = \sum_{i=1}^{r} \sum_{j=1}^{r} \lambda_i(z(k-1)) \lambda_j(z(k-1)) \times \left(\mathbf{A}_i + \mathbf{B}_i \mathbf{G}_j + \sum_{e=1}^{m} \mathbf{Q}_{ei} v_{ei}(k)\right), \tag{5}$$

and $\mathbf{J} \in \Re^{n_u \times n_x}$ is a constant matrix to be designed such that $\mathbf{JB}_i$ is a positive definite matrix.

Remark 1: To choose matrix $\mathbf{J}$, a strict condition $\mathbf{JB}_i = (\mathbf{JB}_i)^T > 0$ is required that may increase the difficulty to determine the sliding surface (3). For a case as $\mathbf{B}_1 = \begin{bmatrix} 0 & 1 \end{bmatrix}$ and $\mathbf{B}_2 = \begin{bmatrix} 0 & -1 \end{bmatrix}$, the matrix $\mathbf{J}$ cannot be designed to satisfy the condition. Thus, the relaxed study for the condition is still an open problem. And, it will be discussed in our future work.

In the following section, a switching controller is designed such that the system states will converge to the sliding surface. After switching control, the individual state variance constraint and passivity constraint can be considered on the sliding surface.

3. REACHING CONDITIONS AND SWITCHING CONTROLLER DESIGN

In this section, the reaching conditions and switching controller design problem is investigated for the sliding mode fuzzy controllers. It is designed such that the trajectory of the closed-loop system is driven onto the sliding surface and the reachability is ensured in finite time. Referring to [54], the sliding mode fuzzy controller can be chosen as follows:

$$u(k) = u_{eq}(k) + u_s(k), \tag{6}$$

where $u_s(k)$ and $u_{eq}(k)$ are respectively switching controller and fuzzy controller. They can be furtherly designed as follows:

$$u_{eq}(k) = \sum_{j=1}^{r} \lambda_j(z(k)) \mathbf{G}_j x(k), \tag{7}$$

$$u_s(k) = \left(\sum_{i=1}^{r} \lambda_i(z(k)) \mathbf{JB}_i\right)^{-1} [S(k) - qTS(k) - \varphi T sgn(S(k)) - \tau sgn(S(k))], \tag{8}$$

where $\mathbf{G}_j \in \Re^{n_u \times n_x}$ is the state feedback gain matrix, T is the sampling interval, $q > 0$, $\varphi > 0$ and $sgn(S(k))$ is the sign function as follows:

$sgn(S(k))$

$$= \left[\begin{array}{cccc} sgn(s_1(k)) & sgn(s_2(k)) & \cdots & sgn(s_{n_u}(k)) \end{array} \right]^T, \tag{9}$$

$$sgn(s_{n_u}(k)) = \begin{cases} 1 & \text{if } s_{n_u}(k) > 0, \\ 0 & \text{if } s_{n_u}(k) = 0, \\ -1 & \text{if } s_{n_u}(k) < 0. \end{cases} \tag{10}$$

Referring to [55], a convenient discrete reaching law (11) can be chosen as follows according to the sliding surface function $S(k)$ defined in (3).

$$S(k+1) - S(k) = -qTS(k) - \varphi T sgn(S(k)). \tag{11}$$

Multiplying the discrete reaching law on the left by $S^T(k)$, one can obtain the following discrete reaching condition (12).

$$\begin{aligned} & S^T(k)[S(k+1) - S(k)] \\ & = -qT\|S(k)\|^2 - \varphi T(|s_1(k)| + |s_2(k)| \\ & \quad + \cdots + |s_{n_u}(k)|) < 0. \end{aligned} \tag{12}$$

Based on the above discrete reaching condition (12), this implies that the states starting from any initial state will move toward the sliding surface and arrive in finite time. The conditions for converging to the sliding surface are introduced in the following theorem.

Theorem 1: For the system (1) controlled by the sliding mode fuzzy controller (6), the states of the system will converge to the sliding surface if the following condition is satisfied.

$$\tau > \left\| \left(\sum_{i=1}^{r} \lambda_i(z(k)) \mathbf{JK}_i \right) \right\| \cdot \|w(k)\|. \tag{13}$$

Proof: Considering (3), one has

$$S(k+1) = \mathbf{J}\left[x(k+1) - \tilde{\mathbf{A}}x(k)\right]. \tag{14}$$

Substituting (2a) and (5) into (14), one can rewrite (14) as

$$\begin{aligned} & S(k+1) \\ & = \mathbf{J}\Bigg\{ \sum_{i=1}^{r} \lambda_i(z(k)) [\mathbf{A}_i x(k) + \mathbf{B}_i u(k) + \mathbf{K}_i w(k) \\ & \quad + \sum_{e=1}^{m} \mathbf{Q}_{ei} v_{ei}(k) x(k) \Bigg] - \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(z(k)) \lambda_j(z(k)) \\ & \quad \times \left(\mathbf{A}_i + \mathbf{B}_i \mathbf{G}_j + \sum_{e=1}^{m} \mathbf{Q}_{ei} v_{ei}(k) \right) x(k) \Bigg\}. \end{aligned} \tag{15}$$

Putting (6) into (15), one can obtain

$$\begin{aligned} & S(k+1) \\ & = \mathbf{J}\Bigg\{ \sum_{i=1}^{r} \lambda_i(z(k)) [\mathbf{A}_i x(k) + \mathbf{B}_i u_{eq}(k) + \mathbf{B}_i u_s(k) \\ & \quad + \mathbf{K}_i w(k) + \sum_{e=1}^{m} \mathbf{Q}_{ei} v_{ei}(k) x(k) \Bigg] - \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(z(k)) \\ & \quad \times \lambda_j(z(k)) \left(\mathbf{A}_i + \mathbf{B}_i \mathbf{G}_j + \sum_{e=1}^{m} \mathbf{Q}_{ei} v_{ei}(k) \right) x(k) \Bigg\} \\ & = \sum_{i=1}^{r} \lambda_i(z(k)) \mathbf{JB}_i \cdot u_s(k) + \sum_{i=1}^{r} \lambda_i(z(k)) \mathbf{JK}_i w(k). \end{aligned} \tag{16}$$

From (8), the following equation can be inferred.

$$\begin{aligned} S(k+1) = & S(k) - qTS(k) - \varphi T sgn(S(k)) \\ & - \tau sgn(S(k)) + \sum_{i=1}^{r} \lambda_i(z(k)) \mathbf{JK}_i w(k). \end{aligned} \tag{17}$$

According to the above described, one can define

$$\begin{aligned} & \Delta S(k) = S(k+1) - S(k) \\ & = -qTS(k) - \varphi T sgn(S(k)) \\ & \quad - \tau sgn(S(k)) + \sum_{i=1}^{r} \lambda_i(z(k)) \mathbf{JK}_i w(k). \end{aligned} \tag{18}$$

Multiplying the left-hand side of (18) by $S^T(k)$, one can get the following inequality.

$$\begin{aligned} & S^T(k)[S(k+1) - S(k)] \\ & = -qT\|S(k)\|^2 - \varphi T(|s_1(k)| + |s_2(k)| \\ & \quad + \cdots + |s_{n_u}(k)|) \\ & \quad - \tau(|s_1(k)| + |s_2(k)| + \cdots + |s_{n_u}(k)|) \\ & \quad + S^T(k) \sum_{i=1}^{r} \lambda_i(z(k)) \mathbf{JK}_i w(k) \\ & \le -\tau(|s_1(k)| + |s_2(k)| + \cdots + |s_{n_u}(k)|) \\ & \quad + S^T(k) \sum_{i=1}^{r} \lambda_i(z(k)) \mathbf{JK}_i w(k) \\ & \le -\tau(|s_1(k)| + |s_2(k)| + \cdots + |s_{n_u}(k)|) \\ & \quad + \|S(k)\| \left\| \sum_{i=1}^{r} \lambda_i(z(k)) \mathbf{JK}_i \right\| \|w(k)\|. \end{aligned} \tag{19}$$

If we removed both $(|s_1(k)| + |s_2(k)| + \cdots + |s_{n_u}(k)|)$ and $\|S(k)\|$ terms in (19), the following inequality is also satisfied due to

$$\begin{aligned} & |s_1(k)| + |s_2(k)| + \cdots + |s_{n_u}(k)| \ge \|S(k)\|, \\ & S^T(k)[S(k+1) - S(k)] \\ & \le -\tau + \left\| \sum_{i=1}^{r} \lambda_i(z(k)) \mathbf{JK}_i \right\| \|w(k)\| \\ & \le 0. \end{aligned} \tag{20}$$

Therefore, if condition (20) holds, the above discrete reaching conditions (12) are satisfied and the states will converge to the sliding surface in finite time. □

In practice, the choice of τ for satisfying the condition (20) of Theorem 1 is an interesting problem. In the following corollary, the way of choosing τ and a modified condition for converging to the sliding surface are introduced.

Corollary 1: For the system (1) controlled by the sliding mode fuzzy controller (6), the states of the system will converge to the sliding surface in finite time if the following condition is satisfied.

$$\tau \geq \delta \cdot \hat{\tau}, \tag{21}$$

where δ is defined in (1), i.e., $\|w(k)\| < \delta$. Besides, $\hat{\tau}$ is defined as follows:

$$\hat{\tau} = \max_{i \in r} \|\mathbf{JK}_i\|. \tag{22}$$

Proof: Considering (13) of Theorem 1, one has

$$\begin{aligned}
&\left\| \left(\sum_{i=1}^{r} \lambda_i(z(k))\,\mathbf{JK}_i \right) \right\| \\
&\leq \|\lambda_1(z(k))\,\mathbf{JK}_1\| + \|\lambda_2(z(k))\,\mathbf{JK}_2\| \\
&\quad + \cdots + \|\lambda_r(z(k))\,\mathbf{JK}_r\| \\
&= \lambda_1(z(k))\,\|\mathbf{JK}_1\| + \lambda_2(z(k))\,\|\mathbf{JK}_2\| \\
&\quad + \cdots + \lambda_r(z(k))\,\|\mathbf{JK}_r\| \\
&\leq \lambda_1(z(k)) \cdot \hat{\tau} + \lambda_2(z(k)) \cdot \hat{\tau} + \cdots + \lambda_r(z(k)) \cdot \hat{\tau} \\
&= \hat{\tau} \cdot \sum_{i=1}^{r} \lambda_i(z(k)) \\
&= \hat{\tau}.
\end{aligned} \tag{23}$$

Thus, it can be found that if the condition (21) is satisfied, then the condition (13) is also satisfied. □

By applying the switching controller (8), the control performance is not suitable while the chattering may exist in the system. In order to alleviate the chattering phenomenon on the sliding mode, some researches have been proposed to smooth the output of switching control input, such as boundary layer control [56, 57], integral sliding mode control [58], second-order sliding-mode control [59] and exponential law [60]. In case of boundary layer control, the switching controller (8) can be replaced by the follows:

$$\begin{aligned}
u_s(k) = {} & \left(\sum_{i=1}^{r} \lambda_i(z(k))\,\mathbf{J}B_i \right)^{-1} [S(k) - qTS(k) \\
& - \varphi T \cdot sat(S(k)/\eta) - \tau \cdot sat(S(k)/\eta)],
\end{aligned} \tag{24}$$

where η is the boundary layer thickness and the $sat(S(k))$ is the saturation function as follows:

$$\begin{aligned}
& sat(S(k)) \\
& = \begin{bmatrix} sat(s_1(k)) & sat(s_2(k)) & \cdots & sat(s_{n_u}(k)) \end{bmatrix}^T,
\end{aligned} \tag{25}$$

$$sat(s_{n_u}(k)) = \begin{cases} sgn(s_{n_u}(k)), & \text{if } |s_{n_u}(k)| > \eta \\ \dfrac{s_{n_u}(k)}{\eta}, & \text{if } |s_{n_u}(k)| \leq \eta. \end{cases} \tag{26}$$

The conditions of switching controller have been developed in Theorem 1 and Corollary 1. In the following section, the individual state variance constraint and passivity constraint are considered in the sliding mode fuzzy controller design process.

4. SLIDING MODE FUZZY CONTROLLER DESIGN

Some sufficient conditions are derived in the following theorem such that the individual state variance constraint and passivity constraint can be achieved when system converges to the sliding surface.

Substituting (6) and (7) into (2a), one can infer the following equation:

$$\begin{aligned}
x(k+1) = {} & \sum_{i=1}^{r} \sum_{j=1}^{r} \lambda_i(z(k))\,\lambda_j(z(k)) \Big\{ \Big(A_i + B_i \mathbf{G}_j \\
& + \sum_{e=1}^{m} \mathbf{Q}_{ei} v_{ei}(k) \Big) x(k) + B_i u_s(k) + \mathbf{K}_i w(k) \Big\}.
\end{aligned} \tag{27}$$

When system states converge to the sliding surface, the switching control input will approach to 0, i.e., $u_s(k) = 0$. Thus, we have

$$\begin{aligned}
x(k+1) = {} & \sum_{i=1}^{r} \sum_{j=1}^{r} \lambda_i(z(k))\,\lambda_j(z(k)) \Big\{ \Big(A_i + B_i \mathbf{G}_j \\
& + \sum_{e=1}^{m} \mathbf{Q}_{ei} v_{ei}(k) \Big) x(k) + \mathbf{K}_i w(k),
\end{aligned} \tag{28a}$$

$$y(k) = \sum_{i=1}^{r} \lambda_i(z(k)) \left\{ C_i x(k) + \mathbf{T}_i w(k) \right\}. \tag{28b}$$

Considering each subsystem of the above T-S fuzzy system (28), the steady state covariance matrix of the state vector $x(k)$ has the following form:

$$\mathbf{X}_i = \mathbf{X}_i^T > \mathbf{0}, \tag{29}$$

where $X_i = \lim_{t \to \infty} E\left\{ x(k) x^T(k) \right\}$ is called the common state covariance matrix for all rules and X_i is the solution of the following Lyapunov equation [37].

$$\begin{aligned}
& (\mathbf{A}_i + \mathbf{B}_i \mathbf{G}_i)\,\mathbf{X}_i\,(\mathbf{A}_i + \mathbf{B}_i \mathbf{G}_i)^{\mathrm{T}} - \mathbf{X}_i + \mathbf{K}_i \mathbf{W} \mathbf{K}_i^{\mathrm{T}} \\
& + \sum_{e=1}^{m} \mathbf{Q}_{ei} \mathbf{X}_i \mathbf{Q}_{ei}^{\mathrm{T}} = 0, \quad i = 1,\ 2,\ \ldots,\ r.
\end{aligned} \tag{30}$$

Definition 1 [37]**:** The individual state variance constraint considered in this paper is defined as follows:

$$[\mathbf{X}_i]_{kk} \leq \sigma_k^2, \tag{31}$$

where $[\cdot]_{kk}$ denotes the kth diagonal element of matrix $[\cdot]$ and σ_k, $k = 1, 2, \cdots, n_x$, denote the root-mean-squared constraints for the variance of system states.

In addition to the above individual state variance constraints (31), the passivity constraint defined in the following definition is also considered in this paper.

Definition 2 [47]**:** The system (28) with external disturbance $w(k)$ and output $y(k)$ is called strictly input passive if there exists a dissipative rate μ and a positive definite matrix $\mathbf{\Lambda} = \mathbf{\Lambda}^{\mathrm{T}} > 0$ such that

$$E\left\{2\sum_{k=0}^{k_p} y^{\mathrm{T}}(k)\mathbf{\Lambda}w(k)\right\} > E\left\{\sum_{k=0}^{k_p} \mu w^{\mathrm{T}}(k)w(k)\right\} \quad (32)$$

for all $k_p > 0$ and $w(k) \neq 0$. The $k_p > 0$ is the terminal time and $\mathbf{\Lambda} \in \Re^{n_y \times n_w}$ is a constant matrix.

Using the proposed design method, the closed-loop system (28) can be stabilized under the assigned individual state variance constraint. Under the constraint, the passivity constraint can be achieved via satisfying inequality (32). The purpose of this paper is to design a sliding-mode fuzzy controller (6) to guarantee the existence of the sliding mode, and then properly choose the feedback gain $\mathbf{G}_i$ so that the individual state variance constraint (31) and passivity constraint (32) can be achieved in the sliding mode.

Theorem 2: If there exists a common positive definite matrix $\bar{\mathbf{X}} > 0$, feedback gains $\mathbf{G}_i$, dissipative rate μ and a positive definite matrix $\mathbf{\Lambda} = \mathbf{\Lambda}^{\mathrm{T}} > 0$ satisfying the following sufficient conditions, then the closed-loop T-S fuzzy system (28) is asymptotically stable, strictly input passive and $[\mathbf{X}_i]_{kk} \leq \sigma_k^2$.

$$\begin{bmatrix} -\bar{\mathbf{X}} + \mathbf{K}_i\mathbf{W}\mathbf{K}_i^{\mathrm{T}} + \sum_{e=1}^{m} \mathbf{Q}_{ei}\mathbf{X}_i\mathbf{Q}_{ei}^{\mathrm{T}} & * \\ \left(\mathbf{A}_i\bar{\mathbf{X}} + \mathbf{B}_i\mathbf{\Theta}_i\right)^T & -\bar{\mathbf{X}} \end{bmatrix} < 0, \quad (33)$$

$$\begin{bmatrix} -\bar{\mathbf{X}} & -\bar{\mathbf{X}}\mathbf{C}_i^{\mathrm{T}}\mathbf{\Lambda} & * & * & \cdots & * \\ -\mathbf{\Lambda}^{\mathrm{T}}\mathbf{C}_i\bar{\mathbf{X}} & \mu\mathbf{I} - \mathbf{T}_i^{\mathrm{T}}\mathbf{\Lambda} - \mathbf{\Lambda}^{\mathrm{T}}\mathbf{T}_i & * & * & \cdots & * \\ \left(\mathbf{A}_i\bar{\mathbf{X}} + \mathbf{B}_i\mathbf{\Theta}_i\right) & \mathbf{K}_i & -\bar{\mathbf{X}} & * & \cdots & * \\ \mathbf{Q}_{1i}\bar{\mathbf{X}} & 0 & 0 & -\frac{\bar{\mathbf{X}}}{m} & \cdots & * \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{Q}_{mi}\bar{\mathbf{X}} & 0 & 0 & 0 & \cdots & -\frac{\bar{\mathbf{X}}}{m} \end{bmatrix} < 0, \quad (34)$$

$$\begin{bmatrix} -\bar{\mathbf{X}} & * \\ \left(\frac{\left(\mathbf{A}_i\bar{\mathbf{X}} + \mathbf{B}_i\mathbf{\Theta}_j\right) + \left(\mathbf{A}_j\bar{\mathbf{X}} + \mathbf{B}_j\mathbf{\Theta}_i\right)}{2}\right) & -\bar{\mathbf{X}} \end{bmatrix} < 0, \text{ for } i < j, \quad (35)$$

$$\bar{\mathbf{X}} - diag\left(\sigma_1^2, \cdots, \sigma_{n_x}^2\right) < 0, \quad (36)$$

where $*$ denotes the transposed elements or matrices for the symmetric position, $\mathbf{\Phi}_{ij} = \mathbf{A}_i + \mathbf{B}_i\mathbf{G}_j$ and $\mathbf{\Theta}_i = \mathbf{G}_i\bar{\mathbf{X}}$.

Proof: We consider a positive define matrix $\bar{\mathbf{X}}$, where $\bar{\mathbf{X}} = \mathbf{P}^{-1}$, satisfying the following inequality:

$$\left(\mathbf{A}_i + \mathbf{B}_i\mathbf{G}_i\right)\bar{\mathbf{X}}\left(\mathbf{A}_i + \mathbf{B}_i\mathbf{G}_i\right)^{\mathrm{T}} - \bar{\mathbf{X}} + \mathbf{K}_i\mathbf{W}\mathbf{K}_i^{\mathrm{T}} + \sum_{e=1}^{m} \mathbf{Q}_{ei}\bar{\mathbf{X}}\mathbf{Q}_{ei}^{\mathrm{T}} < 0. \quad (37)$$

Subtracting (30) from (37), one has

$$\left(\mathbf{A}_i + \mathbf{B}_i\mathbf{G}_i\right)\left(\bar{\mathbf{X}} - \mathbf{X}_i\right)\left(\mathbf{A}_i + \mathbf{B}_i\mathbf{G}_i\right)^{\mathrm{T}} - \left(\bar{\mathbf{X}} - \mathbf{X}_i\right) + \sum_{e=1}^{m} \mathbf{Q}_{ei}\left(\bar{\mathbf{X}} - \mathbf{X}_i\right)\mathbf{Q}_{ei}^{\mathrm{T}} < 0. \quad (38)$$

Referring to [37], it is known that if (33) and (35) are satisfied, one can obtain that the closed-loop system (28) is asymptotically stable. In this case, it can be concluded that $\bar{\mathbf{X}} - \mathbf{X}_i > 0$ via (38). From (36) and $\bar{\mathbf{X}} > \mathbf{X}_i$, one has

$$[\mathbf{X}_i]_{kk} < \left[\bar{\mathbf{X}}\right]_{kk} \leq \sigma_k^2. \quad (39)$$

Thus, the individual state variance constraint (31) is satisfied. Now going back and looking at (37), this inequality can be rewritten from using the Schur complement [17] as follows:

$$\begin{bmatrix} -\bar{\mathbf{X}} + \mathbf{K}_i\mathbf{W}\mathbf{K}_i^{\mathrm{T}} + \sum_{e=1}^{m} \mathbf{Q}_{ei}\bar{\mathbf{X}}\mathbf{Q}_{ei}^{\mathrm{T}} & * \\ \mathbf{\Phi}_{ii}^{\mathrm{T}} & -\bar{\mathbf{X}}^{-1} \end{bmatrix} < 0. \quad (40)$$

However, inequality (40) is not a standard LMI problem. Multiplying (40) on the left-hand and right-hand sides by $diag\left(I, \bar{\mathbf{X}}\right)$, where $\bar{\mathbf{X}} = P^{-1}$, one can obtain (33).

On the other hand, in order to analyze the attenuating performance of the closed-loop T-S fuzzy system (28), a Lyapunov function is chosen as $V(x(k)) = x^{\mathrm{T}}(k)\mathbf{P}x(k)$, where $\mathbf{P}$ is a positive definite matrix. It is known that the passivity theory provides a useful and effective tool to design the controller to achieve the energy constraints for the closed-loop systems. Considering the passivity constraint defined in Definition 2, let us define the following performance function.

$$\begin{aligned} & E\left\{\sum_{k=0}^{k_p} \mu w^{\mathrm{T}}(k)w(k) - \sum_{k=0}^{k_p} 2y^{\mathrm{T}}(k)\mathbf{\Lambda}w(k)\right\} \\ & = E\left\{\sum_{k=0}^{k_p} \left\{\mu w^{\mathrm{T}}(k)w(k) - 2y^{\mathrm{T}}(k)\mathbf{\Lambda}w(k)\right.\right. \\ & \qquad \left.\left. + \Delta V(x(k))\right\} - V\left(x\left(k_q + 1\right)\right)\right\} \\ & \leq E\left\{\sum_{k=0}^{k_p} \left\{\mu w^{\mathrm{T}}(k)w(k) - 2y^{\mathrm{T}}(k)\mathbf{\Lambda}w(k)\right.\right. \\ & \qquad \left.\left. + \Delta V(x(k))\right\}\right\} \\ & = E\left\{\Gamma(x, w, k)\right\}, \end{aligned} \quad (41)$$

where $\Delta V(x(k))$ is the first forward difference of $V(x(k))$ and

$$\Gamma(x, w, k) = \sum_{k=0}^{k_p} \left\{\mu w^{\mathrm{T}}(k)w(k) - 2y^{\mathrm{T}}(k)\mathbf{\Lambda}w(k)\right.$$

$$+\Delta V\left(x(k)\right)\}. \tag{42}$$

Without loss of generality, it is assumed that $V(x(0)) = 0$. It is obvious that

$$\begin{aligned}
&E\{\Delta V(x(k))\}\\
&= E\left\{x^{\mathrm{T}}(k+1)\mathbf{P}x(k+1) - x^{\mathrm{T}}(k)\mathbf{P}x(k)\right\}\\
&= E\left\{\sum_{i=1}^{r}\sum_{k=1}^{r}\lambda_i(z(k))\lambda_k(z(k))\right.\\
&\quad\times\left[\left(\left(\mathbf{\Phi}_{ii}+\sum_{e=1}^{m}\mathbf{Q}_{ei}v_{ei}(k)\right)x(k)+\mathbf{K}_i w(k)\right)^{\mathrm{T}}\right.\\
&\quad\times\mathbf{P}\left(\left(\mathbf{\Phi}_{kk}+\sum_{g=1}^{m}\mathbf{Q}_{gk}v_{gk}(k)\right)x(k)+\mathbf{K}_k w(k)\right)\\
&\quad\left.-x^{\mathrm{T}}(k)\mathbf{P}x(k)\right]+2\sum_{i<j}^{r}\sum_{k<l}^{r}\lambda_i(z(k))\lambda_j(z(k))\lambda_k(z(k))\\
&\quad\times\lambda_l(z(k))\,x^{\mathrm{T}}(k)\left[\left(\frac{\mathbf{\Phi}_{ij}+\mathbf{\Phi}_{ji}}{2}\right)^{\mathrm{T}}\mathbf{P}\left(\frac{\mathbf{\Phi}_{kl}+\mathbf{\Phi}_{lk}}{2}\right)\right.\\
&\quad\left.\left.-\mathbf{P}\right]x(k)\right\}\\
&\le E\left\{\sum_{i=1}^{r}\lambda_i(z(k))\left[\left(\left(\mathbf{\Phi}_{ii}+\sum_{e=1}^{m}\mathbf{Q}_{ei}v_{ei}(k)\right)x(k)\right.\right.\right.\\
&\quad\left.+\mathbf{K}_i w(k)\right)^{\mathrm{T}}\times\mathbf{P}\left(\left(\mathbf{\Phi}_{ii}+\sum_{g=1}^{m}\mathbf{Q}_{gi}v_{gi}(k)\right)x(k)\right.\\
&\quad\left.\left.+\mathbf{K}_i w(k)\right)-x^{\mathrm{T}}(k)\mathbf{P}x(k)\right]\\
&\quad+2\sum_{i<j}^{r}\lambda_i(z(k))\lambda_j(z(k))\\
&\quad\left.\times x^{\mathrm{T}}(k)\left[\left(\frac{\mathbf{\Phi}_{ij}+\mathbf{\Phi}_{ji}}{2}\right)^{\mathrm{T}}\mathbf{P}\left(\frac{\mathbf{\Phi}_{ij}+\mathbf{\Phi}_{ji}}{2}\right)-\mathbf{P}\right]x(k)\right\}\\
&= E\left\{\sum_{i=1}^{r}\lambda_i(z(k))\left[x^{\mathrm{T}}(k)\mathbf{\Phi}_{ii}^{\mathrm{T}}\mathbf{P}\mathbf{\Phi}_{ii}x(k)-x^{\mathrm{T}}(k)\mathbf{P}x(k)\right.\right.\\
&\quad+x^{\mathrm{T}}(k)\mathbf{\Phi}_{ii}^{\mathrm{T}}\mathbf{P}\mathbf{K}_i w(k)+w^{\mathrm{T}}(k)\mathbf{K}_i^{\mathrm{T}}\mathbf{P}\mathbf{\Phi}_{ii}x(k)\\
&\quad+w^{\mathrm{T}}(k)\mathbf{K}_i^{\mathrm{T}}\mathbf{P}\mathbf{K}_i w(k)+\sum_{e=1}^{m}x^{\mathrm{T}}(k)v_{ei}^{\mathrm{T}}(k)\mathbf{Q}_{ei}^{\mathrm{T}}\mathbf{P}\mathbf{\Phi}_{ii}x(k)\\
&\quad+\sum_{g=1}^{m}x^{\mathrm{T}}(k)\mathbf{\Phi}_{ii}^{\mathrm{T}}\mathbf{P}\mathbf{Q}_{gi}v_{gi}(k)x(k)\\
&\quad+\sum_{e=1}^{m}x^{\mathrm{T}}(k)v_{ei}^{\mathrm{T}}(k)\mathbf{Q}_{ei}^{\mathrm{T}}\mathbf{P}\mathbf{K}_i w(k)\\
&\quad+\sum_{g=1}^{m}w^{\mathrm{T}}(k)\mathbf{K}_i^{\mathrm{T}}\mathbf{P}\mathbf{K}_{gi}v_{gi}(k)x(k)\\
&\quad+\sum_{e=1}^{m}\sum_{g=1}^{m}x^{\mathrm{T}}(k)v_{ei}^{\mathrm{T}}(k)\mathbf{Q}_{ei}^{\mathrm{T}}\mathbf{P}\mathbf{Q}_{gi}v_{gi}(k)x(k)\\
&\quad+2\sum_{i<j}^{r}\lambda_i(z(k))\lambda_j(z(k))x^{\mathrm{T}}(k)\\
&\quad\left.\times\left[\left(\frac{\mathbf{\Phi}_{ij}+\mathbf{\Phi}_{ji}}{2}\right)^{\mathrm{T}}\mathbf{P}\left(\frac{\mathbf{\Phi}_{ij}+\mathbf{\Phi}_{ji}}{2}\right)-\mathbf{P}\right]x(k)\right\}.
\end{aligned}\tag{43}$$

Due to $E\{x(k)v_{ei}(k)\} = E\{x(k)\}E\{v_{ei}(k)\} = 0$ and $E\{v_{ei}(k)v_{ei}(k)\} = 1$, one has

$$\begin{aligned}
&E\{\Delta V(x(k))\}\\
&= E\left\{\sum_{i=1}^{r}\lambda_i(z(k))\left[x^{\mathrm{T}}(k)\left(\mathbf{\Phi}_{ii}^{\mathrm{T}}\mathbf{P}\Phi_{ii}-\mathbf{P}\right)x(k)\right.\right.\\
&\quad+x^{\mathrm{T}}(k)\mathbf{\Phi}_{ii}^{\mathrm{T}}\mathbf{P}\mathbf{K}_i w(k)+w^{\mathrm{T}}(k)\mathbf{K}_i^{\mathrm{T}}\mathbf{P}\mathbf{\Phi}_{ii}x(k)\\
&\quad\left.+w^{\mathrm{T}}(k)\mathbf{K}_i^{\mathrm{T}}\mathbf{P}\mathbf{K}_i w(k)+\sum_{e=1}^{m}\sum_{g=1}^{m}x^{T}(k)\mathbf{Q}_{ei}^{\mathrm{T}}\mathbf{P}\mathrm{Q}_{gi}x(k)\right]\\
&\quad+2\sum_{i<j}^{r}\lambda_i(z(k))\lambda_j(z(k))x^{\mathrm{T}}(k)\\
&\quad\left.\times\left[\left(\frac{\mathbf{\Phi}_{ij}+\mathbf{\Phi}_{ji}}{2}\right)^{\mathrm{T}}\mathbf{P}\left(\frac{\mathbf{\Phi}_{ij}+\mathbf{\Phi}_{ji}}{2}\right)-\mathbf{P}\right]x(k)\right\}\\
&\le E\left\{\sum_{i=1}^{r}\lambda_i(z(k))\left[x^{\mathrm{T}}(k)\left(\mathbf{\Phi}_{ii}^{\mathrm{T}}\mathbf{P}\Phi_{ii}-\mathbf{P}\right)x(k)\right.\right.\\
&\quad+x^{\mathrm{T}}(k)\mathbf{\Phi}_{ii}^{\mathrm{T}}\mathbf{P}\mathbf{K}_i w(k)+w^{\mathrm{T}}(k)\mathbf{K}_i^{\mathrm{T}}\mathbf{P}\mathbf{\Phi}_{ii}x(k)\\
&\quad\left.+w^{\mathrm{T}}(k)\mathbf{K}_i^{\mathrm{T}}\mathbf{P}\mathbf{K}_i w(k)+m\sum_{e=1}^{m}x^{\mathrm{T}}(k)\mathbf{Q}_{ei}^{\mathrm{T}}\mathbf{P}\mathbf{Q}_{ei}x(k)\right]\\
&\quad+2\sum_{i<j}^{r}\lambda_i(z(k))\lambda_j(z(k))x^{\mathrm{T}}(k)\\
&\quad\left.\times\left[\left(\frac{\mathbf{\Phi}_{ij}+\mathbf{\Phi}_{ji}}{2}\right)^{\mathrm{T}}\mathbf{P}\left(\frac{\mathbf{\Phi}_{ij}+\mathbf{\Phi}_{ji}}{2}\right)-\mathbf{P}\right]x(k)\right\}.
\end{aligned}\tag{44}$$

Substituting (2b) and (44) into (41), one has

$$\begin{aligned}
&E\{\Gamma(x,w,k)\}\\
&= E\left\{\sum_{i=1}^{r}\lambda_i(z(k))\begin{bmatrix}x(k)\\ w(k)\end{bmatrix}^{\mathrm{T}}\right.\\
&\quad\times\begin{bmatrix}\mathbf{\Phi}_{ii}^{\mathrm{T}}\mathbf{P}\mathbf{\Phi}_{ii}-\mathbf{P}+m\sum_{e=1}^{m}\mathbf{Q}_{ei}^{\mathrm{T}}\mathbf{P}\mathbf{Q}_{ei} & * \\ \mathbf{K}_i^{\mathrm{T}}\mathbf{P}\mathbf{\Phi}_{ii}-\mathbf{\Lambda}^{\mathrm{T}}\mathbf{C}_i & \mathbf{K}_i^{\mathrm{T}}\mathbf{P}\mathbf{K}_i+\mu\mathbf{I}-\mathbf{T}_i^{\mathrm{T}}\mathbf{\Lambda}-\mathbf{\Lambda}^{\mathrm{T}}\mathbf{T}_i\end{bmatrix}\\
&\quad\times\begin{bmatrix}x(k)\\ w(k)\end{bmatrix}+2\sum_{i<j}^{r}\lambda_i(z(k))\lambda_j(z(k))x^{\mathrm{T}}(k)\\
&\quad\left.\times\left[\left(\frac{\mathbf{\Phi}_{ij}+\mathbf{\Phi}_{ji}}{2}\right)^{\mathrm{T}}\mathbf{P}\left(\frac{\mathbf{\Phi}_{ij}+\mathbf{\Phi}_{ji}}{2}\right)-\mathbf{P}\right]x(k)\right\}.
\end{aligned}\tag{45}$$

It is obviously that if the following inequalities (46) and

(47) hold, then one can obtain $\Gamma(x,w,k)<0$.

$$\begin{bmatrix} \mathbf{\Phi}_{ii}^{\mathrm{T}}\mathbf{P}\mathbf{\Phi}_{ii}-\mathbf{P}+m\sum_{e=1}^{m}\mathbf{Q}_{ei}^{\mathrm{T}}\mathbf{P}\mathbf{Q}_{ei} & * \\ \mathbf{K}_i^{\mathrm{T}}\mathbf{P}\mathbf{\Phi}_{ii}-\mathbf{\Lambda}^{\mathrm{T}}\mathbf{C}_i & \mathbf{K}_i^{\mathrm{T}}\mathbf{P}\mathbf{K}_i+\mu\mathbf{I}-\mathbf{T}_i^{\mathrm{T}}\mathbf{\Lambda}-\mathbf{\Lambda}^{\mathrm{T}}\mathbf{T}_i \end{bmatrix}<0, \tag{46}$$

$$\left(\frac{\mathbf{\Phi}_{ij}+\mathbf{\Phi}_{ji}}{2}\right)^{\mathrm{T}}\mathbf{P}\left(\frac{\mathbf{\Phi}_{ij}+\mathbf{\Phi}_{ji}}{2}\right)-\mathbf{P}<0,\ \ i<j. \tag{47}$$

From (41), $\Gamma(x,w,k)<0$ implies

$$E\left\{2\sum_{k=0}^{k_p}y^{\mathrm{T}}(k)\mathbf{\Lambda}w(k)\right\}>E\left\{\sum_{k=0}^{k_p}\mu w^{\mathrm{T}}(k)w(k)\right\} \tag{48}$$

for all nonzero external disturbance. From Definition 2, it can be thus concluded that if (46) and (47) are satisfied, then the closed-loop T-S fuzzy system (28) is strictly input passive.

Because (46) and (47) are not standard LMI problem, one can obtain the following inequality by using Schur complement from (46).

$$\begin{bmatrix} -\mathbf{P}+m\sum_{e=1}^{m}\mathbf{Q}_{ei}^{\mathrm{T}}\mathbf{P}\mathbf{Q}_{ei} & * & * \\ -\mathbf{\Lambda}^{\mathrm{T}}C_i & \mu\mathbf{I}-\mathbf{T}_i^{\mathrm{T}}\mathbf{\Lambda}-\mathbf{\Lambda}^{\mathrm{T}}\mathbf{T}_i & * \\ \mathbf{\Phi}_{ii} & \mathbf{K}_i & -P^{-1} \end{bmatrix}<0 \tag{49}$$

Multiplying the both sides of (49) with $diag\left(\bar{\mathbf{X}},\mathbf{I},\mathbf{I}\right)$, where $\bar{\mathbf{X}}=\mathbf{P}^{-1}$, one can obtain the following inequality.

$$\begin{bmatrix} -\bar{\mathbf{X}}+m\sum_{e=1}^{m}\bar{\mathbf{X}}\mathbf{Q}_{ei}^{\mathrm{T}}\mathbf{P}\mathbf{Q}_{ei}\bar{\mathbf{X}} & * & * \\ -\mathbf{\Lambda}^{\mathrm{T}}\mathbf{C}_i\bar{\mathbf{X}} & \mu I-\mathbf{T}_i^{\mathrm{T}}\mathbf{\Lambda}-\mathbf{\Lambda}^{\mathrm{T}}\mathbf{T}_i & * \\ \mathbf{\Phi}_{ii}\bar{\mathbf{X}} & \mathbf{K}_i & -\bar{\mathbf{X}} \end{bmatrix}<0. \tag{50}$$

Utilizing the Schur complement again, inequality (50) can be converted into LMI form. Similarly, using the Schur complement, (47) can be inferred as follows:

$$\begin{bmatrix} -\mathbf{P} & * \\ \left(\frac{\mathbf{\Phi}_{ij}+\mathbf{\Phi}_{ji}}{2}\right) & -\mathbf{P}^{-1} \end{bmatrix}<0. \tag{51}$$

Multiplying the both sides of (51) with $diag\left(\bar{\mathbf{X}},I\right)$, one can obtain condition (35).

In the above derivation, stability constraints, individual state variance constraints and passivity constraints can be reduced to LMI problems. Therefore, the proposed sliding mode fuzzy controller design problem can be solved by using the LMI technique. □

Theorem 2 provides some sufficient conditions that can be used to design sliding mode fuzzy controller such that the closed-loop system achieves individual state variance constraint and passivity constraint, simultaneously. Concluding the above design method, the effects of the designed parameters in (6) are described in the following remark.

Remark 2: In this paper, the sliding-mode fuzzy controller (6) is consisted of the fuzzy controller (7) and switching controller (8). In (8), $q>0$, $\varphi>0$ and $\mathbf{J}$ are needed to be assigned to establish switching controller. Through Theorem 1 and Corollary 1, the main arm of (8) is to guarantee that the trajectories of the closed-loop system are driven onto sliding surface (3). Besides, gain matrices $\mathbf{G}_j$ are designed to establish fuzzy controller (7) to achieve individual state covariance and passivity constraints.

Remark 3: In this paper, a sliding-mode fuzzy controller design method is proposed to deal with multiple performance issue of discrete nonlinear stochastic systems. With the consideration of stochastic behavior, the issue discussed in this paper is the more general case than [30–33]. Moreover, the considered control issue is also more difficult than [30–33] because the multiple performance constraint is concerned. Besides, a fuzzy controller design method was proposed by [61] for discrete nonlinear stochastic systems with considering the similar multiple performance constraint. However, an extra iteration process is needed to apply the method of [61]. Thus, the application of the proposed design method is easier than [61] by using LMI algorithm.

In the following section, two numerical examples are provided to show the application of proposed sliding mode fuzzy control approach.

5. A NUMERICAL EXAMPLE FOR THE NONLINEAR DISCRETE TRUCK-TRAILER SYSTEM

In this section, an example is provided to compare with the method of [61] to discuss the conservatism of the proposed design method. Another example is to discuss the control problem for a nonlinear discrete truck-trailer system.

Example 1: In order to emphasize the contribution of this paper, the design method in [61] is applied. Referring to [61], the following T-S fuzzy system is introduced.

$$x(k+1)=\sum_{i=1}^{2}\lambda_i(z(k))\left\{\left[\mathbf{A}_i+\sum_{e=1}^{2}\mathbf{Q}_e v_{ei}(k)\right]x(k)+\mathbf{B}_i u(k)+\mathbf{K}_i w(k)\right\}, \tag{52a}$$

$$y(k)=\sum_{i=1}^{2}\lambda_i(z(k))\left\{\mathbf{C}_i x(k)+\mathbf{T}_i w(k)\right\}, \tag{52b}$$

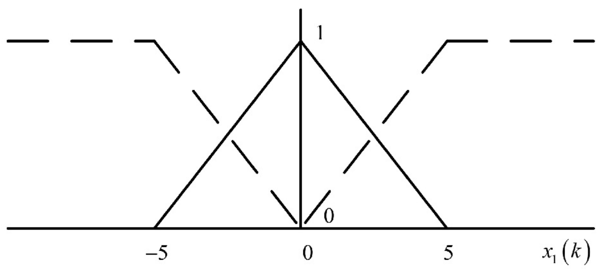

Fig. 1. The membership functions.

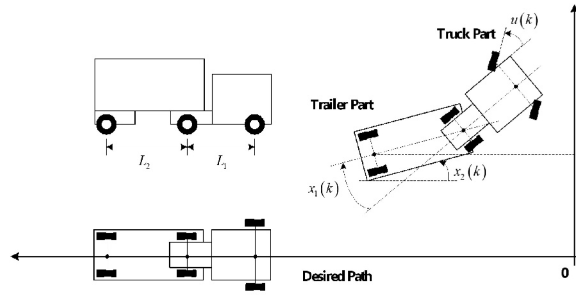

Fig. 2. The model of reversing a truck-trailer system.

where $\mathbf{A}_1 = \begin{bmatrix} 1 & 0.3 \\ 0.6 & -0.5 \end{bmatrix}$, $\mathbf{A}_2 = \begin{bmatrix} 0 & 0.3 \\ 0.6 & -0.5 \end{bmatrix}$, $\mathbf{B}_1 = \mathbf{B}_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\mathbf{Q}_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.05 \end{bmatrix}$, $\mathbf{Q}_2 = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.1 \end{bmatrix}$, $\mathbf{K}_1 = \mathbf{K}_2 = \begin{bmatrix} 0 \\ 0.25 \end{bmatrix}$, $\mathbf{C}_1 = \mathbf{C}_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}$ and $\mathbf{T}_1 = \mathbf{T}_2 = 1$.

The membership function of (52) is given as Fig. 1. In this example, it is only to discuss the conservative issue of the method of [61] and of the proposed method. The minimum of individual state constraint is an index for the comparison. By fixing $\mu = 0.6$ and $\Lambda = 1$, the following feasible solutions of Theorem 2 can be found under the given values as $\sigma_1^2 = 0.1$ and $\sigma_2^2 = 0.1$.

$$\bar{\mathbf{X}} = \begin{bmatrix} 0.0359 & * \\ 0.0153 & 0.0936 \end{bmatrix}, \tag{53a}$$

$$\mathbf{G}_1 = \begin{bmatrix} -0.6472 & -0.5071 \end{bmatrix}, \tag{53b}$$

$$\mathbf{G}_2 = \begin{bmatrix} 0.3528 & -0.5071 \end{bmatrix}. \tag{53c}$$

Besides, With the same $\mu = 0.6$ and $\Lambda = 1$, the minimum of individual state constraint as $\sigma_1^2 = 0.3$ and $\sigma_2^2 = 0.3$ can be obtained by using the method of [61]. It is obviously to find that the individual state constraint obtained by Theorem 2 is smaller than one found by [61]. Therefore, the proposed design method is less conservative than the method of [61] in this example. According to the simulation results, it can be concluded that the proposed design method provides some improvements for [61] in stabilizing the complex nonlinear discrete-time stochastic systems.

Example 2: In this example, the sliding mode fuzzy control problem for a discrete nonlinear truck-trailer system is studied. Referring to [48], the following truck-trailer model is introduced.

$$\begin{aligned} x_1(k+1) = {} & \left(1 - \frac{vel \cdot \Delta t}{L_2}\right) x_1(k) + \frac{vel \cdot \Delta t}{L_1} u(k) \\ & + 0.01 w(k) + 0.003 v_1(k) x_1(k) \\ & + 0.005 v_2(k) x_2(k), \end{aligned} \tag{54a}$$

$$x_2(k+1) = \frac{vel \cdot \Delta t}{L_2} x_1(k) + x_2(k), \tag{54b}$$

$$\begin{aligned} x_3(k+1) = {} & vel \cdot \Delta t \cdot \sin\left(\frac{vel \cdot \Delta t}{2L_2} x_1(k) + x_2(k)\right) \\ & + x_3(k), \end{aligned} \tag{54c}$$

where $x_1(k)$ is the angle difference between truck and trailer, $x_2(k)$ is the angle of trailer, $x_3(k)$ is the vertical position of rear end of trailer, $u(k)$ is the steering angle, $w(k)$ is an external disturbance and $v(k)$ is a multiplicative noise. In (54), disturbance $w(k)$ and multiplicative noise $v(k)$ are employed to represent stochastic behaviors. In this example, the following parameter values are used for simulation:

$$\begin{aligned} & L_1 = 2.8\,(m), \;\; L_2 = 5.5\,(m), \;\; vel = -1.0\,(m/s) \\ & \text{and} \;\; \Delta t = 0.1\,(s), \end{aligned}$$

where L_1 is the length of truck, L_2 is the length of trailer, vel is the constant velocity of backing up, Δt is the sampling time. The control purpose of this example is to back up a truck-trailer along the straight line $(x_3(k) = 0)$ without forward movements as shown in Fig. 1 that is $(x_1(k) \to 0)$, $(x_2(k) \to 0)$ and $(x_3(k) \to 0)$. In order to transform the truck-trailer model into a T-S fuzzy model, we assume that $\frac{vel \cdot \Delta t}{2L_2} x_1(k) + x_2(k)$ is operated between $\pm\pi$. Based on the fuzzy modeling technique, the corresponding T-S fuzzy model of the discrete nonlinear truck-trailer system can be described as follows:

Rule 1:

IF $\frac{vel \cdot \Delta t}{2L_2} x_1(k) + x_2(k)$ is about 0

THEN

$$\begin{aligned} x(k+1) = {} & \left[\mathbf{A}_1 + \sum_{e=1}^{2} \mathbf{Q}_{e1} v_{e1}(k)\right] x(k) + \mathbf{B}_1 u(k) \\ & + \mathbf{K}_1 w(k), \\ y(k) = {} & \mathbf{C}_1 x(k) + \mathbf{T}_1 w(k) \end{aligned} \tag{55a}$$

Rule 2:

IF $\frac{vel \cdot \Delta t}{2L_2} x_1(k) + x_2(k)$ is about $\pm\pi$

THEN

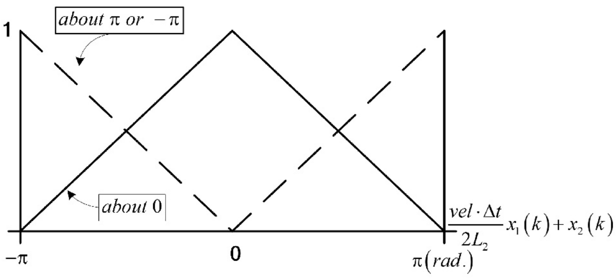

Fig. 3. The membership function of $\frac{vel\cdot\Delta t}{2L_2}x_1(k)+x_2(k)$.

$$x(k+1)=\left[\mathbf{A}_2+\sum_{e=1}^{2}\mathbf{Q}_{e2}v_{e2}(k)\right]x(k)+\mathbf{B}_2u(k)+\mathbf{K}_2w(k),$$
$$y(k)=\mathbf{C}_2x(k)+\mathbf{T}_2w(k) \tag{55b}$$

where $\|w(k)\|<1$, $\mathbf{A}_1=\begin{bmatrix}1-\frac{vel\cdot\Delta t}{L_2} & 0 & 0\\ \frac{vel\cdot\Delta t}{L_2} & 1 & 0\\ \frac{vel^2\cdot\Delta t^2}{L_2} & vel\cdot\Delta t & 1\end{bmatrix}$, $\mathbf{A}_2=\begin{bmatrix}1-\frac{vel\cdot\Delta t}{L_2} & 0 & 0\\ \frac{vel\cdot\Delta t}{L_2} & 1 & 0\\ \frac{\varepsilon\cdot vel^2\cdot\Delta t^2}{L_2} & \varepsilon\cdot vel\cdot\Delta t & 1\end{bmatrix}$, $\mathbf{Q}_{11}=\mathbf{Q}_{12}=\begin{bmatrix}0.003 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$, $\mathbf{K}_1=\mathbf{K}_2=\begin{bmatrix}0.01\\ 0\\ 0\end{bmatrix}$, $\mathbf{Q}_{21}=\mathbf{Q}_{22}=\mathbf{Q}_{23}=\begin{bmatrix}0 & 0.005 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$, $\mathbf{C}_1=\mathbf{C}_2=\begin{bmatrix}1 & 0 & 1\end{bmatrix}$, $\mathbf{B}_1=B_2=\begin{bmatrix}\frac{vel\cdot\Delta t}{L_1}\\ 0\\ 0\end{bmatrix}$, $\mathbf{T}_1=\mathbf{T}_2=1$ and $\varepsilon=10^{-2}/\pi$.

The membership function for $\frac{vel\cdot\Delta t}{2L_2}x_1(k)+x_2(k)$ is given in Fig. 3.

For the analyzing and designing, let us select the supply rate $\mu=0.6$, $\Lambda=1$ and assign the variance constraints as $\sigma_1^2=0.3$, $\sigma_2^2=0.2$ and $\sigma_3^2=0.5$. By solving the sufficient conditions of Theorem 2 via LMI algorithm of [17], the matrix $\bar{\mathbf{X}}$ and $\mathbf{G}_i$ can be obtained as follows:

$$\bar{\mathbf{X}}=\begin{bmatrix}0.2865 & 0.0132 & -0.0005\\ 0.0132 & 0.0026 & 0.001\\ -0.0005 & 0.001 & 0.0009\end{bmatrix}, \tag{56}$$

$$\mathbf{G}_1=\begin{bmatrix}33.2495 & -282.4629 & 288.8263\end{bmatrix}, \tag{57a}$$

$$\mathbf{G}_2=\begin{bmatrix}32.9746 & -253.354 & 288.3833\end{bmatrix}. \tag{57b}$$

Besides, let us choose $\mathbf{J}=\begin{bmatrix}-1 & 0 & 0\end{bmatrix}$, then one can obtain $\hat{\tau}=0.01$ via (22). From Corollary 1, the parameter τ can be assigned as $\tau=0.1$ because $\tau\geq\delta\cdot\hat{\tau}$, where $\delta=1$ and $\hat{\tau}=0.01$. Thus, the sliding mode fuzzy controller can be obtained as follows:

$$u(k)=\sum_{i=1}^{2}\lambda_i(z(k))\mathbf{G}_ix(k)+\left(\sum_{i=1}^{2}\lambda_i(z(k))\mathbf{J}B_i\right)^{-1}$$

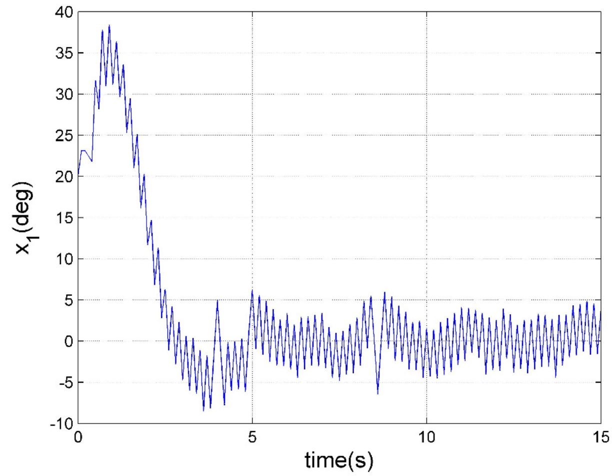

Fig. 4. The responses of state $x_1(k)$ (no boundary layer).

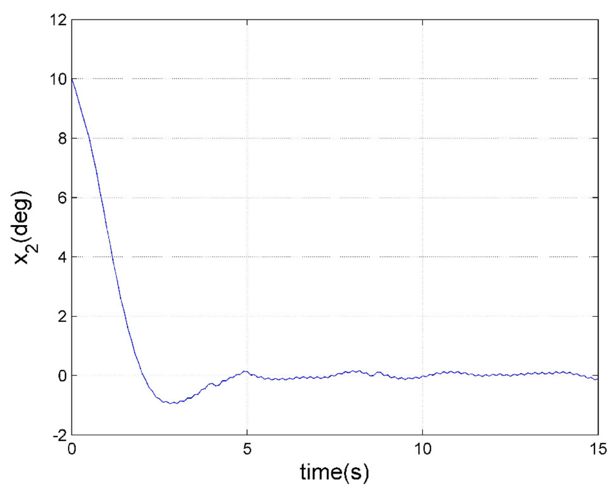

Fig. 5. The responses of state $x_2(k)$ (no boundary layer).

$$\times\left[S(k)-qTS(k)-\varphi Tsgn(S(k))-\tau sgn(S(k))\right], \tag{58}$$

where $q=0.01$, $\varphi=0.01$, $T=0.1$, $\tau=0.1$. Besides, $\mathbf{G}_1$ and $\mathbf{G}_2$ are stated in (57a) and (57b), respectively. Applying the sliding mode fuzzy controller (58) to system (54), the simulation responses of states and sliding surface function are shown in Figs. 4-7 with initial condition $x(0)=\begin{bmatrix}20^\circ & 10^\circ & 0.15\end{bmatrix}^{\mathrm{T}}$ and 15 seconds of simulation time.

Looking at the Figs. 4-7, the control performance is not smooth while the chattering may exist in the system. To tackle these problems considering advantages of the boundary layer approach [55,56], we can change the sliding mode fuzzy controller (58) into the following equation by setting the boundary layer thickness $\eta=0.15$ and show

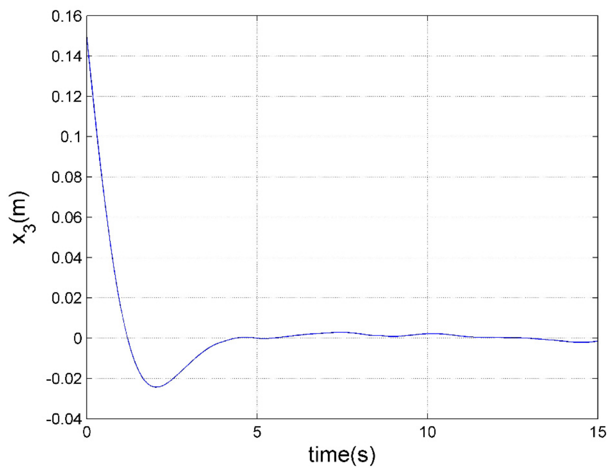

Fig. 6. The responses of state $x_3(k)$ (no boundary layer).

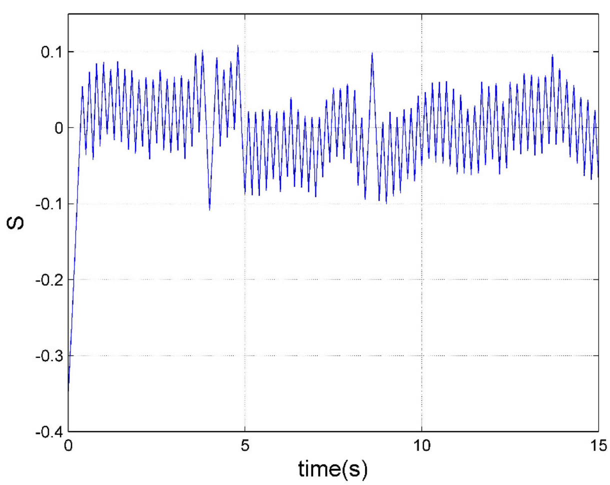

Fig. 7. The responses of $S(k)$ (no boundary layer).

the simulation responses of states in Figs. 8-11.

$$u(k)=\sum_{i=1}^{2}\lambda_i(z(k))\mathbf{G}_i x(k)+\left(\sum_{i=1}^{2}\lambda_i(z(k))\mathbf{J}B_i\right)^{-1} \times [S(k)-qTS(k)-\varphi T\cdot sat(S(k)) -\tau\cdot sat(S(k))]. \tag{59}$$

From the simulation results, the variance of states can also be calculated in the following.

$$var(x_1)=0.0293\leq\sigma_1^2, \tag{60a}$$

$$var(x_2)=0.0014\leq\sigma_2^2, \tag{60b}$$

$$var(x_3)=5.8614\times10^{-4}\leq\sigma_3^2, \tag{60c}$$

where $\sigma_1^2=0.3$, $\sigma_2^2=0.2$ and $\sigma_3^2=0.5$. It is obvious that the state variances described in (60) satisfy the individual state variance constraint (31). Besides, the effect of the external disturbance on the proposed system can be

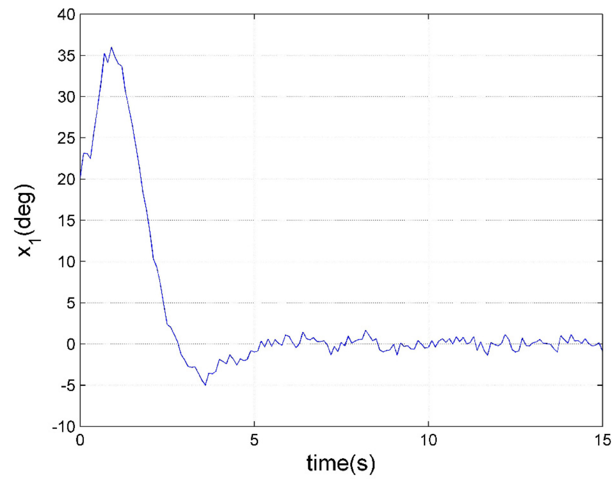

Fig. 8. The responses of state $x_1(k)$ (boundary layer thickness is 0.15).

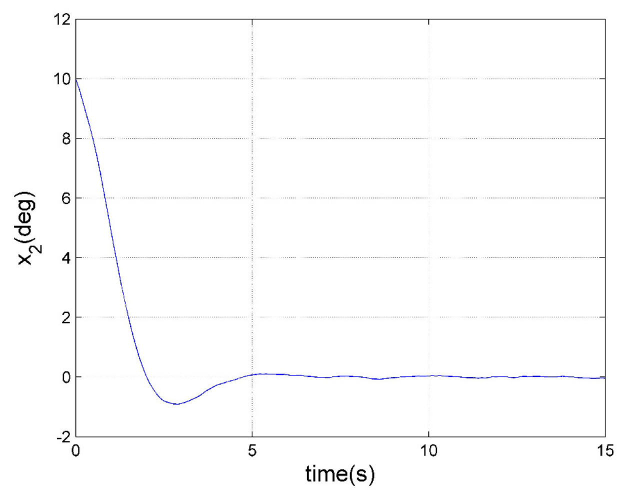

Fig. 9. The responses of state $x_2(k)$ (boundary layer thickness is 0.15).

criticized as follows:

$$\frac{E\left\{2\sum_{k=0}^{k_p}y^T(k)\mathbf{\Lambda}w(k)\right\}}{E\left\{\sum_{k=0}^{k_p}w^T(k)w(k)\right\}}=1.984, \tag{61}$$

where $\mathbf{\Lambda}=1$ for the passivity constraint defined in Definition 2. The ratio value of (61) is bigger than the determined dissipation rate $\mu=0.6$, thus one can find that the condition (32) of Definition 2 is satisfied.

Remark 4: From Figs. 4-11, the chattering phenomenon on the sliding-mode control extremely affects control performance in short and long responses. In Figs. 4-7, the vibration in responses is caused by the chattering phenomenon of switching signal. Through substituting saturation function $sat(S(k))$ for $sgn(S(k))$, the chattering phenomenon in Figs. 4-7 can be eliminated.

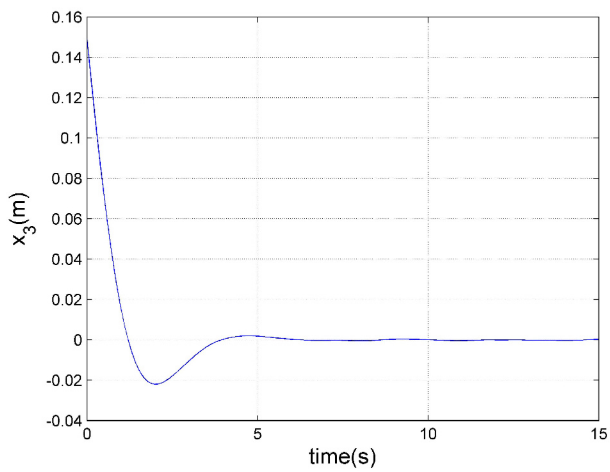

Fig. 10. The responses of state $x_3(k)$ (boundary layer thickness is 0.15).

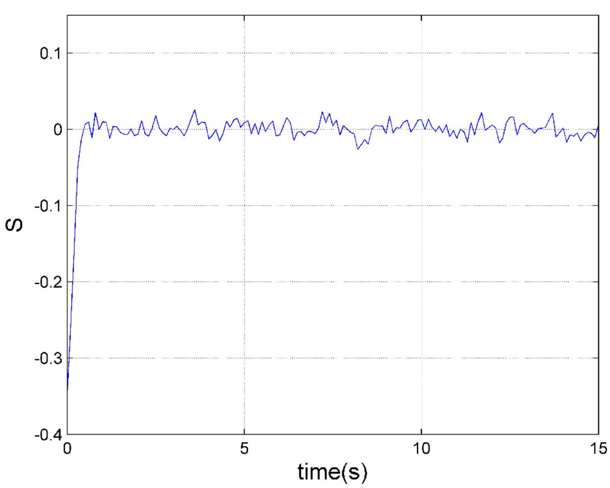

Fig. 11. The responses of $S(k)$ (boundary layer thickness is 0.15).

And, the state responses with free chattering phenomenon are shown in Figs. 8-11.

Therefore, the closed-loop discrete nonlinear truck-trailer system can achieve stability constraint, individual state variance constraints and strictly input passivity constraint by the proposed sliding mode fuzzy controller.

6. CONCLUSIONS

The problem of sliding mode fuzzy control for discrete nonlinear systems with external disturbance, multiplicative noises and multiple constraints is considered. In order to combine sliding mode control method and T-S fuzzy model, we establish the T-S fuzzy model for the system firstly. Then, a switching function is chosen and a switching controller is designed such that the states will converge to the sliding surface in the finite time. Secondly, Using Lyapunov stability theory, upper bound covariance control theory, and passivity theory, some sufficient conditions are derived to find parallel distributed compensation based fuzzy controllers. Thirdly, the LMI algorithm is employed in this paper to solve these sufficient conditions. Finally, the simulation results are proposed to show that the multiple constraints of the discrete nonlinear truck-trailer system can be achieved via the designed sliding mode fuzzy controller. In future, an open problem as choosing matrix **J** will be discussed in future work. Besides, the proposed design method will be extended to deal with stability issues of complex dynamic systems is future work. Moreover, a more powerful controller design method will be considered by merging other control technologies.

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