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Static Output Feedback Stabilization of a Class of Switched Linear Systems with State Constraints

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Abstract: This paper will research the problem of static output feedback (SOF) stabilization of state-constrained switched linear systems via an improved average dwell time method (ADT). Firstly, an improved ADT method is adopted to establish sufficient conditions for SOF of the state-constrained switched linear systems in the form of matrix inequality. It has been shown that this method is less conservative than traditional ADT, which in view of different decay rates of a Lyapunov function related to an active subsystem on the basis of whether the saturations occur or not. Then, a new iterative algorithm is designed to solve the matrix inequality and a SOF controller can be added. In the iterative linear matrix inequality (ILMI) algorithm, it is important not only to overcome the typical bilinear matrix inequality (BMI) problem of SOF, but also to solve the non-convex problem caused by state constraints. Finally, the availability and the applicability of the proposed method is shown by the application of a boost converter.

Keywords: Average dwell time, state constraints, static output feedback, switched systems.

1. INTRODUCTION

As it is known, the switched system is one of the important hybrid systems, which consists of a collection of subsystems and a switching law [1–4]. Under the action of a switching rule, subsystems turn to become activated. Recently, switched systems have been widely applied to practical fields, such as power station control, auto pilot control design, vehicle control, etc. As is known, state constraints are inevitable in practical systems, therefore, researching the switched systems with state constraints has significant value [5, 6].

The static output feedback control has a definite physical meaning, which is easy to be measured. Owing to these advantages, the static output feedback control has received plenty of attention in practical engineering [7]-[8]. However, there are some difficulties in designing SOF. Firstly, it is possible no SOF matrix exists [9]. It is well known that the SOF gain design is a non-convex problem [10]. Secondly, the state constraints increase the difficulty of designing a SOF controller due to the occurrence of saturations.

Recently, some papers have researched on switched linear systems and switched linear systems with state constraints. [11–13] researched stability and stabilization problem of the switched linear systems with unstable systems. [14] researched on state feedback stabilization problem of the switched linear systems. [15] investigated the state and fault estimation problem for linear continuous-time switched systems with simultaneous disturbances, sensor and actuator faults, which adopts two types of observer approaches to solve design issue. [16] researched the state feedback problem of a class switched system with state constraints and designed the state feedback controller via the ADT method. In the study of switched systems, ADT is one of the most effective method to research on stabilization, which has been widely used [17, 18]. However, the traditional ADT can not be suitable for the linear state constrained switched systems. Owing to the existence of state constraints, it is necessary to consider whether or not saturations occur. A saturation dependent ADT is introduced to dispose the state constrained switched systems. In addition, it is possible to note that the switching rules in [14–18] all rely on the state information of systems. Owing to the fact that it is not possible to completely obtain the state, the design of controller through the state feedback is difficult to implement in practice. Thus, the design of the switching rules and output feedback controllers that do not depend on state information is significant. In addition, it is known that there are no research papers about the SOF problem of the state constrained switched systems, which inspired our present

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research.

It is known that output-feedback control has been widely researched now [19, 20]. [21] proposes a fuzzy adaptive output-feedback stabilization control method for non-strict feedback uncertain switched nonlinear systems. [22] and [23] have researched on adaptive fuzzy output-feedback control for nonlinear systems with constraints. The design of the SOF controller is a typical bilinear matrix inequality (BMI) problem [24,25]. To design the ADT controllers, the original BMI problem is transformed into a LMI problem, and then analysing and dealing with the LMI problem, which is called the ILMI method. [26] introduces an additional constraint, which transforms the BMI constraint into a LMI constraint. This paper has adopted the ILMI method, which is introduced by [26], to transform a BMI problem into a LMI problem, which overcome the typical BMI problem of SOF. In addition, the occurrence of state constraints causes a non-convex problem [27]. So it is necessary to introduce a method to solve this puzzle. This idea was adopted to propose an iterative LMI algorithm for the purpose of verifying the sufficient conditions of proposed theorem. Moreover, the saturation dependent ADT method is adopted with which to study the stabilization problem of SOF and design SOF controller, which have not been reported.

The main contributions of this paper are shown in the following aspects. Firstly, sufficient conditions for the SOF stabilization for state constrained switched linear systems are derived first. We design two steps iterative LMI algorithm not only to overcome the typical bilinear matrix inequality (BMI) problem of SOF, but also to solve the non-convex problem caused by state constraints. By transforming a BMI problem to a LMI problem, the SOF controller design problem can be converted to a certain constrained problem expressed by LMIs, guaranteeing the asymptotical stability of the closed-loop switched system. Secondly, a saturation dependent ADT is introduced to dispose SOF problem of the state constrained switched systems. Compared to the traditional ADT method, this approach proved to have fewer negatives.

Our note is organized as follows. In the next section, we present the problem formulation and preliminaries. Main results are addressed in section 3, including sufficient conditions concerning SOF controller designing and an iterative LMI algorithm. In section 4, the application of a boost converter is given to demonstrate the applicability of our result. Finally, concluding the paper in the last section.

Notations: In this note, the notations are used as follows: $\mathbb{R}^n$ represents the n-dimensional Euclidean space, and $\mathbb{Z}^+$ denotes the set of nonnegative integers. The notation $\|\cdot\|$ refers to the Euclidean vector norm. $\mathscr{K}_\infty$ functions denote the space of continuously differentiable functions which is greater than zero and continuous increasing. $\mathcal{C}^1$ denotes the space of continuously differentiable functions.

2. FORMULATION AND PRELIMINARIES

Consider a continuous-time switched linear systems with state constraints as follows:

$$\begin{aligned} \dot{x}(t) &= h(A_{\sigma(t)}x(t) + B_{\sigma(t)}u_{\sigma(t)}(t)), \\ y(t) &= C_{\sigma(t)}x(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u_{\sigma(t)}(t) \in \mathbb{R}^p$ denotes the control input, and $y(t) \in \mathbb{R}^m$ denotes the measured output. A_p, B_p, C_p are constant matrices with appropriate dimensions, $\sigma(t)$ denotes the p^{th} subsystem with $\sigma(t) = p \in \mathcal{M} = \{1, 2, \ldots, N\}$. Moreover, for a switching sequence $0 < t_1 < \ldots < t_i < \ldots$, symbol t_i denotes the moment of the ith switching.When $t \in (t_{i-1}, t_i)$, we say the $\sigma(t_{i-1})^{th}$ subsystem is active.

In this paper, we focus on designing SOF:

$$u_{\sigma(t)}(t) = K_{\sigma(t)}y(t) \tag{2}$$

to ensure stability of the closed loop switched system:

$$\dot{x}(t) = h((A_{\sigma(t)} + B_{\sigma(t)}K_{\sigma(t)}C_{\sigma(t)})x(t)). \tag{3}$$

The saturation function $h(\cdot)$ is introduced as follows:

$$h(A(t)) = [h_1(A_1(t)),\ \ldots,\ h_n(A_n(t))]^T \tag{4}$$

with, for each $i \in \mathcal{N}$,

$$h_i(A_i(t)) = \begin{cases} 0, & \text{if } |A_i(t)| = 1\ ,\ A_i(t)x_i(t) > 0, \\ A_i(t), & \text{otherwise}, \end{cases} \tag{5}$$

where matrix $A = [A_1^T, \ldots, A_n^T]^T \in \mathbb{R}^{n\times n} \in \mathbb{D}^n$.

There are serval methods to solve the saturation function $h(\cdot)$, we handle the saturations by transform it into the vertex of a convex hull.

Symbol L_n indicates the set of $n \times n$ diagonal matrices. The diagonal elements of L_n are 0 or 1. Suppose that every element of L_n is marked as L_s, $L_s^- = I - L_s$, $s \in \{1, 2, \ldots, 2^{n-1}, 2^n\}$.Thus, we get

$$h(Ax) \in \text{co}\{L_s(Ax) + L_s^- G\}. \tag{6}$$

Note that, the diagonal elements of row diagonally dominant matrix G is negative.

The following lemmas and definitions are essential to derive the main results in this paper

Definition 1 [28]**:** For each $0 \le t_1 \le t_2$ and a switching law $\sigma(t)$, let $N_{\sigma(t)}(t_2, t_1)$ indicate the numbers of $\sigma(t)$ in the interval (t_1, t_2). Then $\sigma(t)$ owns an ADT τ_a, if there exists two positive numbers N_0 and τ_a, such that

$$N_{\sigma(t)}(t_2, t_1) \le N_0 + \frac{t_2 - t_1}{\tau_a},\ \forall t_2 \ge t_1 \ge 0. \tag{7}$$

Remark 1: Definition 1 means that if there is a positive number τ_a meets the condition of which the ADT between any two consecutive switching is no smaller than the constant τ_a , we say τ_a has the ADT performance.

Lemma 1 [29]**:** Consider the continuous-time switched linear system with state constraint

$$\dot{x}(t) = f_{\sigma(t)}(x(t)),\ x(t) \in \Omega \subset \mathbb{R}^n,\ \sigma(t) = p \in \mathcal{P},\quad (8)$$

with $f_p(0) = 0$, $\forall p \in \mathcal{P}$. Suppose that Ω contains all. β, α and μ are given constants which meet the conditions $\alpha > \beta > 0$, $\mu \geq 1$. Suppose that there exist some positive definite functions $V_p(x(t)) : \Omega \to \mathbb{R}$, for all $p \in \mathcal{P}$, such that

$$\varphi_1(\|x(t)\|) \leq V_p(x(t)) \leq \varphi_2(\|x(t)\|),\ \forall x(t) \in \Omega,\quad (9)$$

$$\dot{V}_p(x(t)) \leq \begin{cases} -\alpha V_p(x(t)),\ \ \forall t \in \mathbb{T}_f[t_i,t_{i+1}), \\ -\beta V_p(x(t),\ \ \forall t \in \mathbb{T}_s[t_i,t_{i+1}), \end{cases}\quad (10)$$

and $\forall(\sigma(t_i) = p,\ \sigma(t_i^-) = q) \in \mathcal{P} \times \mathcal{P},\ p \neq q$

$$V_p(x(t_i)) \leq \mu V_q(x(t_i)),\ \forall x(t) \in \Omega,\quad (11)$$

where φ_1, φ_2 are some class $\mathcal{K}$ functions, then the switched system with state constraint (8) is GUAS for any switching signal with ADT

$$\tau_a \geq \tau_a^* = \frac{\ln\mu}{\zeta},\ \ \frac{\mathcal{T}_f(t_0,t)}{\mathcal{T}_s(t_0,t)} \geq \frac{\zeta-\beta}{\alpha-\zeta} > 0.\quad (12)$$

Lemma 2 (Schur complements)**:** Given the symmetric matrix $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$, the following statements are equivalent:

1) $A < 0$;
2) $A_{11} < 0, A_{22} - A_{12}^T A_{11}^{-1} A_{12} < 0$;
3) $A_{22} < 0, A_{11} - A_{12} A_{22}^{-1} A_{12}^T < 0$.

3. MAIN RESULTS

In this section, our main work is to give sufficient conditions for existence of SOF gains for switched linear system with state constraints. Next, we will give the main theorem which establish the stability condition of closed-loop system (3) via SOF control rule.

Theorem 1: Consider the linear switched system with state constraints:

$$\dot{x}(t) = h((A_{\sigma(t)} + B_{\sigma(t)} K_{\sigma(t)} C_{\sigma(t)})x(t)),\quad (13)$$

where $\sigma(t) = p \in \mathcal{M}$ which is given constant, and let $\mu \geq 1$, $\alpha > \beta > 0$. $\forall(p,\ q) \in \mathcal{M} \times \mathcal{M}$, $p \neq q$, $L_s \neq I$. Suppose that there exist positive definite matrices $P_p > 0$ and row diagonally dominant matrices G_p, where $|g_{ii}| > \sum_{j=1, j\neq i}^{n} |g_{ij}|$ and $g_{ii} < 0$, $\forall i \in \mathcal{N}$, satisfying the following inequality:

$$\begin{bmatrix} A_p^T P_p + P_p A_p + \alpha P_p - \mathscr{X} & (B_p^T P_p + K_p C_p)^T \\ * & -I \end{bmatrix} < 0,\quad (14)$$

$$\begin{bmatrix} \mathscr{Y} + \beta P_p + X_p^T L_s B_p B_p^T L_s^T X_p & \mathscr{Z} \\ * & -I \end{bmatrix} < 0,\quad (15)$$

$$P_p - \mu_p P_q,\quad (16)$$

where

$$\mathscr{X} = X_p^T B_p B_p^T P_p + P_p B_p B_p^T X_p^T - X_p^T B_p B_p^T X_p,$$
$$\mathscr{Y} = He(P_p L_s A_p + P_p L_s{}^- G_p - X_p^T L_s B_p B_p^T L_s^T P_p),$$
$$\mathscr{Z} = (B_p^T L_s{}^T P_p + K_p C_p)^T,$$

therefore, the linear state constrained switched system (13) is GUAS with ADT satisfying

$$\tau_a \geq \tau_a^* = \frac{\ln\mu}{\zeta},\ \ \frac{\mathcal{T}_f(t_0,t)}{\mathcal{T}_s(t_0,t)} \geq \frac{\zeta-\beta}{\alpha-\zeta} > 0.\quad (17)$$

In addition, we can obtain the SOF gain K_p.

Proof: First, consider the following Lyapunov function:

$$V_p(x(t)) = x^T(t) P_p x(t),\ p \in \mathcal{P}.\quad (18)$$

When t belongs to the non-saturated zone, $t \in \mathbb{E}_p(t_i,t_{i+1})$, $i \in \mathbb{Z}^+$. According to (10), we obtain,

$$\begin{aligned} &\dot{V}_p(x(t)) + \alpha_p V_p(x(t)) \\ &= x(t)^T[(A_p + B_p K_p C_p)^T P_p + P_p(A_p + B_p K_p C_p) \\ &\quad + \alpha P_p]x(t) \\ &\leq 0, \end{aligned}$$

which is equivalent to

$$(A_p + B_p K_p C_p)^T P_p + P_p(A_p + B_p K_p C_p) + \alpha P_p < 0.$$

Accordingly, we have,

$$\begin{aligned} &A_p^T P_p + P_p A_p + \alpha P_p - P_p B_p B_p^T P_p \\ &\quad + (B_p^T P_p + K_p C_p)^T (B_p^T P_p + K_p C_p) < 0. \end{aligned}$$

Owing to $P_p B_p B_p^T P_p$ exists, the above inequality can not solved by LMI. To overcome this difficulty, we introduce an additional variable X_p, which has the same dimension to P_p. Because $(X_p - P_p)^T B_p B_p T(X_p - P_p) \geq 0$ is always established, we get

$$X_p^T B_p B_p^T P_p + P_p B_p B_p^T X_p^T - X_p^T B_p B_p^T X_p \leq P_p B_p B_p^T P_p.$$

Then we obtain,

$$\begin{aligned} &A_p^T P_p + P_p A_p + \alpha P_p - X_p^T B_p B_p^T P_p - P_p B_p B_p^T X_p^T \\ &\quad + X_p^T B_p B_p^T X_p + (B_p^T P_p + K_p C_p)^T (B_p^T P_p + K_p C_p) \\ &< 0. \end{aligned}$$

Applying Schur complements, we get,

$$\begin{bmatrix} A_p^T P_p + P_p A_p + \alpha P_p - \mathscr{X} & (B_p^T P_p + K_p C_p)^T \\ * & -I \end{bmatrix} < 0. \tag{19}$$

Thus, (14) is proved.

When t belongs to the saturated zone, we can prove the saturated case which is similar to the non-saturated condition. we handle the saturations through replace $A_p + B_p K_p C_p$ by $L_s A_p + L_s B_p K_p C_p + L_s^- G_p$, we obtain,

$$\begin{aligned} &(L_s A_p + L_s B_p K_p C_p + L_s^- G_p)^T P_p \\ &\quad + P_p (L_s A_p + L_s B_p K_p C_p + L_s^- G_p) + \beta P_p \\ &< 0. \end{aligned}$$

Accordingly, we can get,

$$\begin{aligned} &He(P_p L_s A_p + P_p L_s^- G_p) + \beta P_p - P_p L_s B_p B_p^T L_s^T P_p \\ &\quad + (B_p^T L_s^T P_p + K_p C_p)^T (B_p^T L_s^T P_p + K_p C_p) < 0, \end{aligned}$$

We introduce an additional variable X_p, which has the same dimension to P_p. Because $(X_p - P_p)^T L_s^T B_p B_p T L_s (X_p - P_p) \geq 0$, is always established, we have,

$$\begin{aligned} &X_p^T L_s B_p B_p^T L_s^T P_p + P_p L_s B_p B_p^T L_s^T X_p^T - X_p^T L_s B_p B_p^T L_s^T X_p \\ &\leq P_p L_s B_p B_p^T L_s^T P_p, \end{aligned}$$

Then we get,

$$\begin{aligned} &He(P_p L_s A_p + P_p L_s^- G_p - X_p^T L_s B_p B_p^T L_s^T P_p) \\ &\quad + \beta P_p + X_p^T L_s B_p B_p^T L_s^- X_p \\ &\quad + (B_p^T L_s^T P_p + K_p C_p)^T (B_p^T L_s^T P_p + K_p C_p) \\ &< 0. \end{aligned}$$

Applying Schur complements, we obtain:

$$\begin{bmatrix} \mathscr{Y} + \beta P_p + X_p^T L_s B_p B_p^T L_s^T X_p & \mathscr{Z} \\ * & -I \end{bmatrix} < 0. \tag{20}$$

Thus, (15) is proved.

Next, according to (11), we can achieve

$$P_p - \mu_p P_q < 0\ . \tag{21}$$

Hence, if (14), (15), and (16) are solvable, we can get the SOF gains K_p , which completes the proof. □

Remark 2: Theorem 1 gives the sufficient conditions for static output feedback stabilization of the switched systems with state constraints. We employ proposed method, which in view of different decay rates of a Lyapunov function related to an active subsystem on the basis of whether the saturations occur or not. It can result in a smaller ADT, which has less conservative, than the traditional ADT method.

Remark 3: GUAS refers to the stability operating on the unit hypercube $\mathbb{D}^n$, rather than the usual $\mathbb{R}^n$. In other words, by GUAS of the origin we mean that the origin is locally uniformly asymptotically stable (LUAS) within $\mathbb{D}^n$, instead of the usual $\mathbb{R}^n$, being the domain of attraction.

Theorem 1 is established under the condition that the inequality is solvable. Next, we will an iterative LMI algorithm which we design to achieve the SOF gain K_p.

We note that K_p and P_p in (14), (15) can be solved by an iterative approach, if X_p is fixed in (14) and (15). Next, we will address how to find suitable X_p and introduce this iterative LMI algorithm.

Step 1: We choose suitable $Q_p > 0$, and solve P_p from the following equation:

$$A_p{}^T P_p + P_p A_p - P_p B_p B_p^T P_p + Q_p = 0.$$

Set $k = 1$, $X_{kp} = P_p$.

Step 2: Using X_{kp} which are solved in step 1,to solve the following LMI optimization problem for P_p, K_p and η_k:

OP1: Minimize η_k be limited by the following constraints:

$$\begin{bmatrix} A_p^T P_{kp} + P_{kp} A_p + \alpha P_{kp} - \mathscr{V} - \eta_k P_{kp} & \mathscr{H} \\ * & -I \end{bmatrix} < 0, \tag{22}$$

$$P_{kp} < \mu_p P_{kq}, \tag{23}$$

where

$$\begin{aligned} &\mathscr{V} = X_k p^T B_p B_p^T P_{kp} + P_{kp} B_p B_p^T X_k p^T - X_k p^T B_p B_p^T X_k p, \\ &\mathscr{H} = (B_p^T P_{kp} + K_p C_p)^T. \end{aligned}$$

We define $\eta_k{}^*$ as the minimized value of η_k.

Step 3: If $\eta_k{}^* \leq 0$, set $r = 0$ and let $X_p = X_{kp}$, go to Step 7. Otherwise, go to next step.

Step 4: Using $\eta_k{}^*$ which are solved in Step 3, to solve the following optimization problem for P_p and K_p:

OP2: Minimize trace(P_{kp}) be limited by (**??**) and (23). We define P_{kp}^* as the minimized value of trace(P_{kp}).

Step 5: If $\|X_k p - P_{kp}^*\| < \delta$, a prescribed tolerance, go to the next step. Otherwise, set $k = k+1$, go to Step 2.

Step 6: The system may have no stable SOF gain. Stop.

Step 7: Using P_p and K_p which are solved in Step 3, to solve the following LMI optimization problem for G_p and ζ:

$$\begin{aligned} &\inf_{G_p} \quad \zeta \\ &\mathbf{s.t.} \begin{bmatrix} \mathscr{U} + \beta P_p + X_p^T L_s B_p B_p^T L_s^T X_p - \zeta P_p & \mathscr{W} \\ * & -I \end{bmatrix} < 0, \\ &T_i G_p U_{ij} < 0,\ \ s \in \mathcal{S},\ p \in \mathcal{P},\ i \in \mathbb{N},\ j \in \mathbb{M}, \end{aligned} \tag{24}$$

where

$$\begin{aligned} &\mathscr{U} = He(P_p L_s A_p + P_p L_s{}^- G_p - X_p^T L_s B_p B_p^T L_s^T P_p), \\ &\mathscr{W} = (B_p^T L_s{}^T P_p + K_p C_p)^T. \end{aligned}$$

If $r=0$ and $\zeta \le 0$, go to Step 9. If $r>0$, $\zeta \le 0$ or $\zeta \ne \zeta_r$, go to Step 9. Otherwise, set $r=r+1$, $\zeta_r=\zeta$, go to the Step 8.

Step 8: Using G_p which is solved in Step 2, to solve the following LMI optimization problem for K_p, P_p, and ζ:

$$\inf_{P_p>0} \quad \zeta$$
$$\text{s.t.} \begin{bmatrix} A_p^T P_{kp} + P_{kp} A_p + \alpha P_{kp} - \mathscr{V} - \zeta P_p & \mathscr{H} \\ * & -I \end{bmatrix} < 0,$$
$$\begin{bmatrix} \mathscr{U} + \beta P_p + X_p^T L_s B_p B_p^T L_s^T X_p - \zeta P_p & \mathscr{W} \\ * & -I \end{bmatrix} < 0,$$
$$s \in \mathcal{S},\ p \in \mathcal{P}.$$

If $\zeta \le 0$ or $\zeta \ne \zeta_r$, and go to Step 9. Otherwise, set $r = r+1$, $\zeta_r=\zeta$, go to Step 7.

Step 9: If $\zeta \le 0$, we can get stable SOF gain K_p. Otherwise, we can not achieve the conclusion. The algorithm may be repeated from Step 1.

Remark 4: In our propose iterative LMI algorithm, we not merely need to overcome the typical bilinear matrix inequality (BMI) problem of SOF, but also to solve the non-convex problem caused by state constraints. The problem which we need to work out is complex. Thus, we put forward two steps iterative LMI algorithm to simplify this problem. First, we find suitable P_p through iterative algorithm. The P_p we have solved is the initial value which we use to achieve the suitable K_p in the second step. Just as the iterative algorithm we design, we can find the inequality is solvable and show Theorem 1 is established.

4. EXAMPLE

In this section, a Pulse-Width-Modulation (PWM) driven boost converter is given to illustrate the practicability of the proposed method for the switched linear system with state constraints. As can be seen from Fig. 1, $e_s(t)$ denotes the source voltage, C represents the capacitance, load resistance is denoted by R, L denotes the inductance, and the switch $s(t)$, which is controlled by a PWM device, can switch at most once in each period.

The differential equations for the boost converter are as follows:

$$\dot{e}_C(\tau) = -\frac{1}{RC} e_C(\tau) + (1-s(\tau))\frac{1}{C_1} i_L(\tau), \tag{25}$$

$$\dot{i}_L(\tau) = -(1-s(\tau))\frac{1}{L_1} e_C(\tau) + s(\tau)\frac{1}{L_1} e_s(\tau), \tag{26}$$

where $\tau = \frac{t}{T}$, $L_1 = \frac{L}{T}$ and $C_1 = \frac{C}{T}$.

Then, let $x = [e_C, i_L]^T$, so (25)-(26) can be formulated by:

$$\dot{x} = A_\sigma x + B_\sigma u_\sigma,\ \sigma \in \{1,2\}. \tag{27}$$

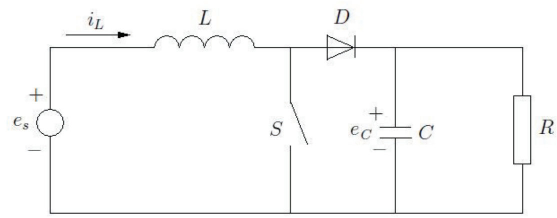

Fig. 1. Boost converter.

where

$$A_1 = \begin{bmatrix} 0 & \frac{1}{C_1} \\ -\frac{1}{L_1} & 0 \end{bmatrix},\quad B_1 = \begin{bmatrix} -\frac{1}{RC_1} \\ 0 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0 & \frac{1}{C_2} \\ -\frac{1}{L_2} & 0 \end{bmatrix},\quad B_2 = \begin{bmatrix} -\frac{1}{RC_2} \\ 0 \end{bmatrix}.$$

So, we can give the matrices in (27) as follows:

$$A_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},\qquad B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0 & 1.2 \\ 1.2 & 0 \end{bmatrix},\quad B_2 = \begin{bmatrix} 1.1 \\ 0 \end{bmatrix}.$$

Correspondingly, other system matrices are shown as follows:

$$C_1 = \begin{bmatrix} 1 & 1 \end{bmatrix},\quad C_2 = \begin{bmatrix} 1.1 & 1.2 \end{bmatrix}.$$

We set the parameters $\alpha = 0.1$, $\beta = 0.05$, $\mu = 1.1$, and chose suitable parameter $\zeta = 0.08$. Then, we obtain $\frac{\mathcal{T}_f(t_0,t)}{\mathcal{T}_s(t_0,t)} \ge 1.5$. When the ADT switching signal satisfying $\tau_a \ge 1.1914$, the switched system (25) is SOF stable. Hence, we set $\tau_a = 1.5$. By solving the inequalities in Theorem 1, P_p and stable SOF gain K_p are obtained, which is presented as follows:

$$P_1 = \begin{bmatrix} 3.3015 & 2.8887 \\ 2.8887 & 5.3953 \end{bmatrix},\ K_1 = \begin{bmatrix} -3.1356 \end{bmatrix},$$
$$P_2 = \begin{bmatrix} 3.1869 & 2.8352 \\ 2.8352 & 5.4030 \end{bmatrix},\ K_2 = \begin{bmatrix} -2.7414 \end{bmatrix},$$

In addition, in order to illustrate the effectiveness of the proposed ADT method, we present the results of both switching signal and state response for system (25). Fig. 2 shows the switching signal with $\tau_a \ge 1.1914$. Fig. 3 shows the state trajectory over $0-30$ under a ADT switching signal with $\tau_a = 1.5$ for the system (25) with state constraints. From Fig. 4, it is easy to see the state trajectory of the nonsaturated system (25). It can be concluded from Fig. 3 and Fig. 4 wether saturation existing or not, the system (25) stabilized by the designed SOF controllers under the proposed ADT switching signal. In addition, compared to Fig. 4, we can find the state trajectory in Fig. 3 is confined to a specific zone owing to saturation existing.

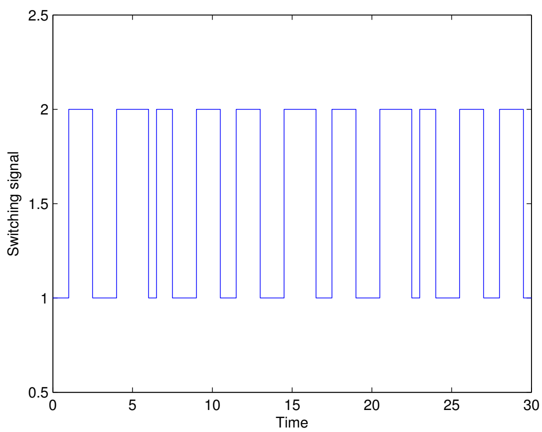

Fig. 2. $\tau_a = 1.5$.

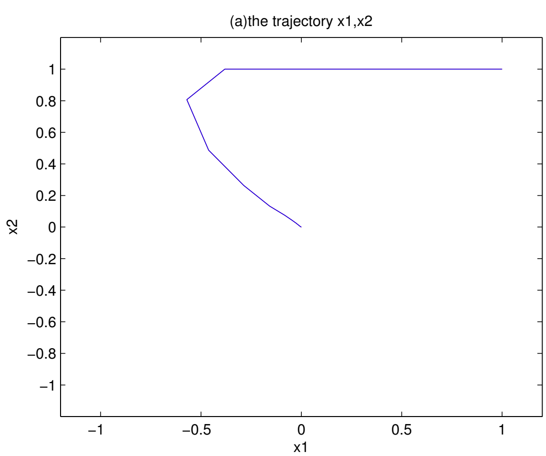

Fig. 3. Saturated trajectory of system (25).

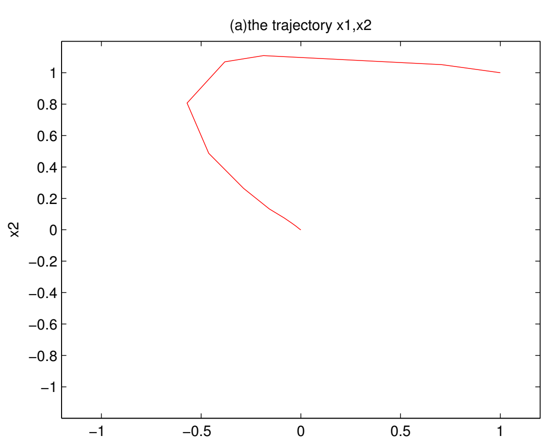

Fig. 4. Non-saturated trajectory of system (25).

5. CONCLUSION

The SOF stability and the stabilization problems for the switched linear system with state constraints is discussed in this paper. Using the proposed ADT method, we establish sufficient conditions for SOF stabilization. It has been demonstrated that this method is less conservative than traditional ADT, which in view of different decay rates of a Lyapunov function related to an active subsystem on the basis of whether the saturations occur or not. Then, we put forward two steps iterative LMI algorithm to simplify non-convex problem, which is efficient. Using proposed iterative LMI algorithm, we obtain SOF controller K_p. Finally, the application of a boost converter is given to demonstrate the effectiveness of the proposed method. Our future work is to extend the results in this paper to switched linear systems and time-delay systems with state constraints.

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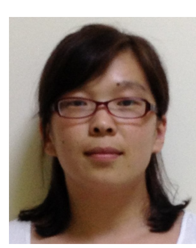

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