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Stochastic Stability for a Class of Discrete-time Switched Neural Networks with Stochastic Noise and Time-varying Mixed Delays

Ying Cui, Yurong Liu*, Wenbing Zhang, and Fuad E. Alsaadi

Abstract: In this paper, stochastic stability is analyzed for a class of discrete-time switched neural networks, in which time-varying mixed delays and stochastic noise are considered. Specifically, benefitting from the triple summation term included in a new Lyapunov functional, time-varying distributed delays are tackled and a criterion of decay estimation for a non-switched neural network is firstly obtained. Subsequently, in view of average dwell time methodology and stochastic analysis, several sufficient conditions are obtained to ensure that the stochastic stability problem is solvable. Furthermore, the derived sufficient conditions reflect that the decay rate of the considered neural networks has a close relationship with average dwell time, upper and lower bounds of delays and intensity of stochastic noise. Finally, validity of the inferred conclusions is given by a simulated example.

Keywords: Discrete-time switched neural networks (DSNNs), stochastic stability, stochastic noise, time-varying mixed delays.

1. INTRODUCTION

During the past decades, neural networks have been promisingly applied in a wide range of fields, including sequence recognition [1], signal processing [2] and the high-capacity associative memories [3], etc.. To a great extent, various designs and applications of neural networks depend on their dynamical behaviours. Thereby, much attention has been paid to investigate dynamical problems for neural networks, and numerous research results concerning such subject have been acquired. For instance, literatures [4–7] have investigated intensively synchronization and stability issues for a variety of neural networks. To be more specify, [4] has applied time-varying Lyapunov functional technique and convex combination approach to investigate how to achieve synchronization and stabilization by means of impulsive controller. In [5], the authors have applied nonsmooth analysis to fulfill finite time synchronization for two types of switched coupled neural networks.

In the actual neural networks, time delays usually take place in the storage and transmission of information. Meanwhile, multifarious axon sizes and lengths comprise a mass of parallel pathways such that neural networks have the spatial property. Such a property of neural networks generates that dynamical temporal behaviors have been affected by some period of time as well as a certain past instant. Generally, on account of the occurring ways of time delays, one can classify time delays as two types: discrete and distributed. By reviewing past researches, some mathematical methods have been used to investigate dynamical properties of time-delayed systems, including matrix measure [8], Lyapunov functional [9], and so on. Meanwhile, there are some results devoting to reducing conservatism, see, e.g., [10–12]. Especially in recent years, mixed time-delays have been garnering an increasing interest, and there have been some research results reported in relevant literatures, e.g., [13–17].

On the other side, synaptic propagation of the real neural networks is a noise process. Such a noise process is formed by random fluctuation due to the release of neurotransmitter or others probable factors [18]. The noise disturbance might give rise to instability and other poor performances of neural networks. Subsequently, it is natural to consider that time delays and stochastic noise may have the simultaneous impact on performances of neural networks. Up to present, for neural networks limited to these two phenomena, there have been many studies on

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Ying Cui is with the Department of Mathematics, Yangzhou University, Yangzhou 225002, and also with the Department of Mathematics, Fuyang Normal College, Fuyang 236032, China(e-mail: cuiying0328@163.com). Yurong Liu is with the Department of Mathematics, Yangzhou University, Yangzhou 225002, China, and also with the Communication Systems and Networks (CSN) Research Group, Faculty of Engineering, King Abdulaziz University, Jeddah 21589, Saudi Arabia(e-mail: yrliu@yzu.edu.cn). Wenbing Zhang is with the Department of Mathematics, Yangzhou University, Yangzhou 225002, China (e-mail: zwb850506@126.com). Fuad E. Alsaadi is with the Communication Systems and Networks (CSN) Research Group, Faculty of Engineering, King Abdulaziz University, Jeddah 21589, Saudi Arabia (e-mail: 445625145@qq.com).
* Corresponding author.

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all sorts of dynamical properties, such as stochastic stability [19], finite-time stabilization [20], exponential input-to-state stability [6], and so forth.

As is widely recognized, neural networks may bring about sudden changes in structures or parameters due to various realistic phenomena, such as abrupt failures, sudden environmental changes, mutation of interconnections [21]. Such an abrupt phenomenon can be suitably modeled by the switched neural networks. As such, significance of studies on switched neural networks has been highlighted. Recently, several analyses and designs of the switched neural networks have been extensively explored, see, e.g., [22–25]. When investigating switched systems, average dwell time approach that was firstly introduced by [26] is a very popular method. Owing to its simplicity and efficiency, there have been lots of research results by utilizing such approach to explore dynamical behaviors of switched systems, see, e.g., [17, 27–29]. For instance, in view of average dwell time and delay partitioning methods, exponential stability and synchronization of DSNNs have been investigated in [29]. [17] has derived delay-dependent conditions such that DSNNs with various activation functions can reach passivity. Unfortunately, by the observation of a multitude of literatures, little research has dealt with DSNNs influenced by stochastic noise and time-varying mixed delays. Actually, the real neural networks could be closely approximated by nonlinear systems subjected to time-varying mixed delays, stochastic noise, and a finite number of modes. Therefore, from theoretical and practical viewpoints, it is desirable for such a neural network to make the investigation of stochastic stability issue.

Inspired by the aforementioned above, this paper will apply some techniques to tackle stability issue of DSNNs restricted to stochastic noise and time-varying mixed delays. Specifically, to deal with time-varying distributed delays, which has been overlooked by recent publications, e.g. [11, 12, 17, 19, 30], we construct the triple summation term included in a new Lyapunov functional. Moreover, stochastic analysis is applied to deal with stochastic noise presented in the considered DSNNs. Main contributions are listed in three aspects: (1) the considered neural network model is simultaneously subjected to switching signal, time-varying mixed delays and stochastic noise; (2) to handle time-varying mixed delays and a finite number of modes, a novel mode-dependent Lyapunov functional is set up; (3) the established sufficient conditions illustrate the relationships among the decay rate, dwell time on the average, upper and lower bounds of time-varying mixed delays, and intensity of stochastic noise.

The following arranges the remainder of this paper. Section 2 presents stochastic stability issue for the addressed DSNNs model influenced by switching signal, time-varying mixed delays and stochastic noise. In Section 3, delay-dependent conditions of stability for DSNNs are derived. Section 4 demonstrates usefulness of the inferred theoretical results by a numerical example. At last, Section 5 gives some conclusions.

Notations: Symbol $\mathbb{R}^n$ denotes n-dimensional Euclidean space with Euclidean norm $\|\cdot\|$, and matrix transposition is designated by superscript "T". For any symmetric matrix A, $A \leq (<) 0$ indicates A is negative semi-definite (definite). Notations $\lambda_M(\cdot)$ and $\lambda_m(\cdot)$, respectively, stand for greatest and least eigenvalue of symmetric matrix. Notation $\mathbb{Z}_{\geq s}$ represents $\{k \in \mathbb{Z}_+ | k \geq s\}$.

2. PROBLEM FORMULATION

Consider DSNNs model subjected to time-varying mixed delays and stochastic noise as follows:

$$\begin{cases} \mathrm{x}(k+1) = A_{\pi(k)}\mathrm{x}(k) + B_{\pi(k)} f(\mathrm{x}(k)) \\ \qquad + C^{(1)}_{\pi(k)} g(\mathrm{x}(k-\tau_1(k))) \\ \qquad + C^{(2)}_{\pi(k)} \sum_{i=1}^{\tau_2(k)} h(\mathrm{x}(k-i)) \\ \qquad + \varsigma(k, \mathrm{x}(k))\mathcal{B}(k), \\ \mathrm{x}(\ell) = \phi(\ell), \quad k_0 - r \leq \ell \leq k_0, \end{cases} \tag{1}$$

where the state vector of n neurons is expressed by $\mathrm{x}(k) = (\mathrm{x}_1(k), \mathrm{x}_2(k), \ldots, \mathrm{x}_n(k))^T \in \mathbb{R}^n$, and its initial condition is $\phi(\ell)$; $A_{\pi(k)} = \mathrm{diag}\left\{a_{1,\pi(k)}, a_{2,\pi(k)}, \ldots, a_{n,\pi(k)}\right\}$ denotes self-feedback matrix, where $|a_{j,\pi(k)}| < 1$; $B_{\pi(k)}, C^{(1)}_{\pi(k)}$ and $C^{(2)}_{\pi(k)}$ are connection weighted matrices; nonlinear neuron activation functions are f, g and $h : \mathbb{R}^n \to \mathbb{R}^n$; the noise intensity function $\varsigma : \mathbb{Z}_{\geq k_0} \times \mathbb{R}^n \longrightarrow \mathbb{R}^n$ is a Borel measurable n-dimension vector function; $\{\mathcal{B}(k) | \, k \in \mathbb{Z}_{\geq k_0}\}$ represents one-dimension Wiener process, in which $\mathbf{E}[\mathcal{B}(k)] = 0$ and $\mathbf{E}[\mathcal{B}^2(k)] = 1$; $\tau_1(k)$ and $\tau_2(k)$, respectively, describe discrete and distributed delays. Denote $r = \max\{\tau_{1,M}, \tau_{2,M}\}$ with $\tau_{1,m} \leq \tau_1(k) \leq \tau_{1,M}$ and $\tau_{2,m} \leq \tau_2(k) \leq \tau_{2,M}$.

The switching signal is denoted by $\pi : \mathbb{Z}_{\geq k_0} \to \mathcal{M} = \{1, 2, \ldots, m_0\}$, where m_0 is a positive integer. Given integer k, let $k_0 < k_1 < \ldots < k_t$, $(t \in \mathbb{Z}_{\geq 1})$ denote switching instants of $\pi(s)$ for $k_0 \leq s < k$, and switching sequence is designated by $\{(\pi(k_0), k_0),\, (\pi(k_1), k_1), \ldots, (\pi(k_t), t_t), \ldots\}$, which means that $\pi(s) = \pi(k_j) \in \mathcal{M}$ when $k_j \leq s < k_{j+1}$.

Remark 1: Lately, dynamical performances of the switched time-delayed systems have been discussed in recent literatures, for example [12,19,29,30]. To the best our knowledge, the existing results only consider discrete delays or constant distributed delays. However, the real neural networks can be more closely approximated by nonlinear switched systems with time-varying mixed delays. Due to mathematical difficulty, time-varying mixed delays have been neglected in some recent works, in this paper, we shall construct a new Lyapunov functional including a triple summation to tackle the time-varying distributed delays of the addressed DSNNs (1).

Now, some assumptions are made on activation functions $f(\cdot)$, $g(\cdot)$, $h(\cdot)$ and noise intensity function $\varsigma(\cdot)$.

Assumption 1: Functions $f(\cdot)$, $g(\cdot)$ and $h(\cdot)$ vanish at the origin and are bounded.

Remark 2: Under Assumption 1, it is obvious that zero is the equilibrium point of system (1). In the paper, we shall investigate stability problem of zero equilibrium point for system (1).

Assumption 2 [13]**:** The neuron activation functions meet with the following conditions

$$\gamma_j^- \leq \frac{f_j(u_1)-f_j(u_2)}{u_1-u_2} \leq \gamma_j^+,$$
$$\omega_j^- \leq \frac{g_j(u_1)-g_j(u_2)}{u_1-u_2} \leq \omega_j^+,$$
$$\xi_j^- \leq \frac{h_j(u_1)-h_j(u_2)}{u_1-u_2} \leq \xi_j^+.$$

for any $1 \leq j \leq n$ and $u_1 \neq u_2$, where γ_j^-, γ_j^+, $\omega_j^-, \omega_j^+, \xi_j^-$ and ξ_j^+ are constant scalars.

Hereafter, we designate bounds about activation functions by the following matrices:

$$\Gamma_1 = \mathrm{diag}\{\gamma_1^-\gamma_1^+, \gamma_2^-\gamma_2^+, \ldots, \gamma_n^-\gamma_n^+\},$$
$$\Gamma_2 = \mathrm{diag}\left\{\frac{\gamma_1^-+\gamma_1^+}{2}, \frac{\gamma_2^-+\gamma_2^+}{2}, \ldots, \frac{\gamma_n^-+\gamma_n^+}{2}\right\},$$
$$\Omega_1 = \mathrm{diag}\{\omega_1^-\omega_1^+, \omega_2^-\omega_2^+, \ldots, \omega_n^-\omega_n^+\},$$
$$\Omega_2 = \mathrm{diag}\left\{\frac{\omega_1^-+\omega_1^+}{2}, \frac{\omega_2^-+\omega_2^+}{2}, \ldots, \frac{\omega_n^-+\omega_n^+}{2}\right\},$$
$$\Xi_1 = \mathrm{diag}\{\xi_1^-\xi_1^+, \xi_2^-\xi_2^+, \ldots, \xi_n^-\xi_n^+\},$$
$$\Xi_2 = \mathrm{diag}\left\{\frac{\xi_1^-+\xi_1^+}{2}, \frac{\xi_2^-+\xi_2^+}{2}, \ldots, \frac{\xi_n^-+\xi_n^+}{2}\right\}.$$

Remark 3: Assumption 2 was firstly introduced in [13]. It gives the upper and lower bounds of activation functions that could be negative, zero and positive. Hence, the usual Lipschitz-type and sigmoid-type can be generalized by this assumption. Meanwhile, with the help of this assumption, the conservatism of the theoretical results can be reduced, and one can refer to [13] for more details.

Assumption 3: The vector-valued function $\varsigma(\cdot)$ satisfies $\|\varsigma(k,\mathrm{x})\| \leq \rho\|\mathrm{x}\|$, for all $k \in \mathbb{Z}_{\geq k_0}$, where constant $\rho > 0$.

Definition 1: For any solution x of (1) with initial condition ϕ, we call neural networks (1) exponentially stable in mean square, if there are constants $\lambda \in (0,\ 1)$ and $K > 0$ such that

$$\mathbf{E}\left[\|\mathrm{x}(k)\|^2\right] \leq K\lambda^{k-k_0} \sup_{k_0-r\leq \ell \leq k_0} \mathbf{E}\left[\|\phi(\ell)\|^2\right] \tag{2}$$

holds for any $k \in \mathbb{Z}_{\geq k_0}$.

Remark 4: For any nonzero initial condition ϕ, if the solution of system (1) is $\mathrm{x}(k) = 0$ for $k \in \mathbb{Z}_{\geq k_0}$, then (2) is obviously valid. In the sequel, we shall investigate stability problem for system (1) when $\mathrm{x}(k) \neq 0$ for $k \in \mathbb{Z}_{\geq k_0}$. In this case, Assumption 2 is true for $u_1 \neq u_2$, and will be used to reduce the conservatism of the theoretical results.

Definition 2 [31]**:** On the interval $[k_0, k)$, the switching number of switching signal π is denoted by $N_\pi(k,k_0)$. If $N_\pi(k,k_0) \leq \frac{k-k_0}{T_0} + N_0$ is valid for $N_0 \geq 0$ and $T_0 > 0$, then we say T_0 the average dwell time and N_0 the chatter bound, respectively. For simplicity, we shall take $N_0 = 0$ in this paper.

In what follows, stochastic stability for DSNNs (1) will be explored. By considering a novel Lyapunov functional and employing average dwell time methodology, we shall present sufficient conditions, warranting exponential stability for DSNNs (1). The derived sufficient conditions will interpret the correlation among decay rate of DSNNs (1), dwell time on the average, upper and lower bounds of time-varying mixed delays, and intensity of stochastic noise.

3. MAIN RESULTS AND PROOFS

We are ready to begin with some lemmas to prepare the main results.

Lemma 1 [22]**:** If constant matrices L_1, L_2 and L_3 satisfy $L_1 = L_1^T$ and $L_2 > 0$, then

$$L_1 + L_3^T L_2^{-1} L_3 < 0$$

if and only if

$$\begin{bmatrix} L_1 & L_3^T \\ L_3 & -L_2 \end{bmatrix} < 0.$$

Lemma 2 (Discrete Jensen Inequality) [32]**:** Suppose matrix W is positive semi-definite, then the inequality

$$\left(\sum_{j=s_1}^{s_2} \alpha_j\right)^T W \left(\sum_{j=s_1}^{s_2} \alpha_j\right) \leq (s_2 - s_1 + 1) \sum_{j=s_1}^{s_2} \alpha_j^T W \alpha_j$$

holds, where $s_1, s_2 \in \mathbb{Z}_{\geq 1}$ and $s_2 \geq s_1$.

Lemma 3 [22]**:** Let $\mathcal{C} = \mathrm{diag}\{c_1, c_2, \ldots, c_n\} \geq 0$. If nonlinear function $\mathcal{F}(\ell) = (\bar{f}_1(\ell_1), \bar{f}_2(\ell_2), \ldots, \bar{f}_n(\ell_n))^T$ is continuous satisfying

$$u_k^- \leq \frac{\bar{f}_k(s)}{s} \leq u_k^+, \quad s \in \mathbb{R}\backslash\{0\},\ 1 \leq k \leq n,$$

with constants u_k^- and u_k^+, then

$$\begin{bmatrix} \ell \\ \mathcal{F}(\ell) \end{bmatrix}^T \begin{bmatrix} \mathcal{C}U_1 & -\mathcal{C}U_2 \\ -\mathcal{C}U_2 & \mathcal{C} \end{bmatrix} \begin{bmatrix} \ell \\ \mathcal{F}(\ell) \end{bmatrix} \leq 0,$$

or

$$\mathcal{F}^T(\ell)\mathcal{C}\mathcal{F}(\ell) + \ell^T\mathcal{C}U_1\ell - 2\ell^T\mathcal{C}U_2\mathcal{F}(\ell) \leq 0,$$

where

$$U_1 = \text{diag}\{u_1^- u_1^+, u_2^- u_2^+, \ldots, u_n^- u_n^+\},$$

and

$$U_2 = \text{diag}\left\{\frac{u_1^- + u_1^+}{2}, \frac{u_2^- + u_2^+}{2}, \ldots, \frac{u_n^- + u_n^+}{2}\right\}.$$

Firstly, consider the following non-switched case subjected to stochastic noise and time-varying mixed delays:

$$\begin{cases} \mathrm{x}(k+1) = A\mathrm{x}(k) + Bf(\mathrm{x}(k)) + C^{(1)}g(\mathrm{x}(k-\tau_1(k))) \\ \qquad + C^{(2)}\sum_{i=1}^{\tau_2(k)} h(\mathrm{x}(k-i)) + \varsigma(k,\mathrm{x}(k))\mathcal{B}(k), \\ \mathrm{x}(\ell) = \phi(\ell), \ k_0 - r \le \ell \le k_0. \end{cases} \tag{3}$$

Right now, we intend to state and prove the next lemma, which plays an important part in stability analysis.

Lemma 4: Suppose non-switched neural networks (3) satisfies Assumptions 1-3, for the given scalar $0 < \alpha < 1$, if there are number $\delta^* > 0$, matrices $Q > 0$, $R > 0$, $S > 0$ and diagnal matrices $\Sigma > 0, \Upsilon > 0, \Lambda > 0$ satisfying the following inequalities:

$$Q < \delta^* I, \tag{4}$$

$$\Phi = \begin{bmatrix} \Phi_{11} & * & * & * & * & * & * \\ \Gamma_2\Sigma & -\Sigma & * & * & * & * & * \\ \Omega_2\Upsilon & 0 & \Phi_{33} & * & * & * & * \\ 0 & 0 & 0 & \Phi_{44} & * & * & * \\ \Xi_2\Lambda & 0 & 0 & 0 & \Phi_{55} & * & * \\ 0 & 0 & 0 & 0 & 0 & \Phi_{66} & * \\ QA & QB & 0 & QC^{(1)} & 0 & QC^{(2)} & -Q \end{bmatrix} < 0, \tag{5}$$

where

$$\Phi_{11} = \rho^2\delta^* I - \alpha Q - \Sigma\Gamma_1 - \Upsilon\Omega_1 - \Lambda\Xi_1, \tag{6}$$

$$\Phi_{33} = (1 + \tau_{1,M} - \tau_{1,m})R - \Upsilon, \tag{7}$$

$$\Phi_{44} = -\alpha^{\tau_{1,M}} Q, \tag{8}$$

$$\Phi_{55} = \left[\tau_{2,m} + \frac{1}{2}(\tau_{2,M} - \tau_{2,m})(\tau_{2,M} + \tau_{2,m} - 1)\right]S - \Lambda, \tag{9}$$

$$\Phi_{66} = -\frac{1}{\tau_{2,M}} S, \tag{10}$$

then the inequality

$$\mathbf{E}[V(\mathcal{X}_k, k)] \le \alpha^{k-k_0}\mathbf{E}[V(\mathcal{X}_{k_0}, k_0)], \ k \in \mathbb{Z}_{\ge k_0}, \tag{11}$$

is true for any trajectory of (3), where

$$\mathcal{X}_k = \left[\mathrm{x}^T(k) \ \mathrm{x}^T(k-1) \ \ldots \ \mathrm{x}^T(k-r)\right]^T.$$

Proof: Introduce Lyapunov functional defined by:

$$V(\mathcal{X}_k, k) = \sum_{j=1}^{5} V_j(\mathcal{X}_k, k), \tag{12}$$

where

$$V_1(\mathcal{X}_k, k) = \mathrm{x}^T(k)Q\mathrm{x}(k), \tag{13}$$

$$V_2(\mathcal{X}_k, k) = \sum_{i=k-\tau_1(k)}^{k-1} \alpha^{k-1-i} g^T(\mathrm{x}(i))Rg(\mathrm{x}(i)), \tag{14}$$

$$V_3(\mathcal{X}_k, k) = \sum_{d=\tau_{1,m}}^{\tau_{1,M}-1}\sum_{i=k-d}^{k-1} \alpha^{k-1-i} g^T(\mathrm{x}(i))Rg(\mathrm{x}(i)), \tag{15}$$

$$V_4(\mathcal{X}_k, k) = \sum_{d=1}^{\tau_2(k)}\sum_{i=k-d}^{k-1} \alpha^{k-1-i} h^T(\mathrm{x}(i))Sh(\mathrm{x}(i)), \tag{16}$$

$$V_5(\mathcal{X}_k, k) = \sum_{\theta=\tau_{2,m}+1}^{\tau_{2,M}}\sum_{d=1}^{\theta-1}\sum_{i=k-d}^{k-1} \alpha^{k-1-i} h^T(\mathrm{x}(i))Sh(\mathrm{x}(i)). \tag{17}$$

For presentation convenience, denote

$$\mathcal{A} = \begin{bmatrix} A & B & 0 & C^{(1)} & 0 & C^{(2)} \end{bmatrix},$$

$$\vartheta(k) = \Big[\mathrm{x}^T(k) \ f^T(\mathrm{x}(k)) \ g^T(\mathrm{x}(k)) \ g^T(\mathrm{x}(k-\tau_1(k))) \ h^T(\mathrm{x}(k)) \sum_{i=1}^{\tau_2(k)} h^T(\mathrm{x}(k-i))\Big]^T,$$

$$\triangle V_j(k) = V_j(\mathcal{X}_{k+1}, k) - \alpha V_j(\mathcal{X}_k, k), \ \text{for } 1 \le j \le 5,$$

and then we conduct the following computation of Lyapunov functional (12) for system (3):

$$\begin{aligned} &\mathbf{E}[\triangle V_1(k)] \\ &= \mathbf{E}\big[\vartheta^T(k)\mathcal{A}^T Q\mathcal{A}\vartheta(k) - \alpha\mathrm{x}^T(k)Q\mathrm{x}(k) \\ &\quad + \varsigma^T(k,\mathrm{x}(k))Q\varsigma(k,\mathrm{x}(k))\big] \\ &\le \mathbf{E}\big[\vartheta^T(k)\mathcal{A}^T Q\mathcal{A}\vartheta(k) - \alpha\mathrm{x}^T(k)Q\mathrm{x}(k) \\ &\quad + \rho^2\delta^*\mathrm{x}^T(k)\mathrm{x}(k)\big], \end{aligned} \tag{18}$$

$$\begin{aligned} &\mathbf{E}[\triangle V_2(k)] \\ &= \mathbf{E}\Bigg[g^T(\mathrm{x}(k))Rg(\mathrm{x}(k)) \\ &\quad - \alpha^{\tau_1(k)} g^T(\mathrm{x}(k-\tau_1(k)))Rg(\mathrm{x}(k-\tau_1(k))) \\ &\quad + \sum_{i=k+1-\tau_1(k+1)}^{k-1} \alpha^{k-i} g^T(\mathrm{x}(i))Rg(\mathrm{x}(i)) \\ &\quad - \sum_{i=k+1-\tau_1(k)}^{k-1} \alpha^{k-i} g^T(\mathrm{x}(i))Rg(\mathrm{x}(i))\Bigg] \\ &\le \mathbf{E}\Bigg[g^T(\mathrm{x}(k))Rg(\mathrm{x}(k)) \\ &\quad - \alpha^{\tau_{1,M}} g^T(\mathrm{x}(k-\tau_1(k)))Rg(\mathrm{x}(k-\tau_1(k))) \end{aligned}$$

$$
\begin{aligned}
&+\sum_{i=k+1-\tau_{1,M}}^{k-1}\alpha^{k-i}g^T(\mathrm{x}(i))Rg(\mathrm{x}(i))\\
&-\sum_{i=k+1-\tau_{1,m}}^{k-1}\alpha^{k-i}g^T(\mathrm{x}(i))Rg(\mathrm{x}(i))\Bigg]\\
=&\,\mathbf{E}\Bigg[g^T(\mathrm{x}(k))Rg(\mathrm{x}(k))\\
&-\alpha^{\tau_{1,M}}g^T(\mathrm{x}(k-\tau_1(k)))Rg(\mathrm{x}(k-\tau_1(k)))\\
&+\sum_{i=k+1-\tau_{1,M}}^{k-\tau_{1,m}}\alpha^{k-i}g^T(\mathrm{x}(i))Rg(\mathrm{x}(i))\Bigg], \qquad (19)
\end{aligned}
$$

$$
\begin{aligned}
&\mathbf{E}[\triangle V_3(k)]\\
=&\,\mathbf{E}\Bigg[\sum_{d=\tau_{1,m}}^{\tau_{1,M}-1}\big[g^T(\mathrm{x}(i))Rg(\mathrm{x}(i))-\alpha^d g^T(\mathrm{x}(k-d))\\
&\times Rg(\mathrm{x}(k-d))\big]\\
=&\,\mathbf{E}\Big[(\tau_{1,M}-\tau_{1,m})g^T(\mathrm{x}(k))Rg(\mathrm{x}(k))\\
&-\sum_{d=\tau_{1,m}}^{\tau_{1,M}-1}\alpha^d g^T(\mathrm{x}(k-d))Rg(\mathrm{x}(k-d))\Bigg], \qquad (20)
\end{aligned}
$$

$$
\begin{aligned}
&\mathbf{E}[\triangle V_4(k)]\\
=&\,\mathbf{E}\Bigg[\sum_{d=1}^{\tau(k+1)}\sum_{i=k+1-d}^{k}\alpha^{k-i}h^T(\mathrm{x}(i))Sh(\mathrm{x}(i))\\
&-\sum_{d=1}^{\tau_2(k)}\sum_{i=k-d}^{k-1}\alpha^{k-i}h^T(\mathrm{x}(i))Sh(\mathrm{x}(i))\Bigg]\\
\le&\,\mathbf{E}\Bigg[\sum_{d=1}^{\tau_{2,M}}\sum_{i=k+1-d}^{k}\alpha^{k-i}h^T(\mathrm{x}(i))Sh(\mathrm{x}(i))\\
&-\sum_{d=1}^{\tau_{2,m}}\sum_{i=k+1-d}^{k-1}\alpha^{k-i}h^T(\mathrm{x}(i))Sh(\mathrm{x}(i))\\
&-\sum_{d=1}^{\tau_2(k)}\alpha^d h^T(\mathrm{x}(k-d))Sh(\mathrm{x}(k-d))\Bigg]\\
=&\,\mathbf{E}\Bigg[\sum_{d=1}^{\tau_{2,M}}\Big[h^T(\mathrm{x}(k))Qh(\mathrm{x}(k))\\
&+\sum_{i=k+1-d}^{k-1}\alpha^{k-i}h^T(\mathrm{x}(i))Sh(\mathrm{x}(i))\Big]\\
&-\sum_{d=1}^{\tau_{2,m}}\sum_{i=k+1-d}^{k-1}\alpha^{k-i}h^T(\mathrm{x}(i))Sh(\mathrm{x}(i))\\
&-\sum_{d=1}^{\tau_2(k)}\alpha^d h^T(\mathrm{x}(k-d))Sh(\mathrm{x}(k-d))\Bigg]\\
\le&\,\mathbf{E}\Bigg[\tau_{2,M}h^T(\mathrm{x}(k))Sh(\mathrm{x}(k))\\
&+\sum_{d=\tau_{2,m}+1}^{\tau_{2,M}}\sum_{i=k+1-d}^{k-1}\alpha^{k-i}h^T(\mathrm{x}(i))Sh(\mathrm{x}(i))\\
&-\alpha^{\tau_{2,M}}\sum_{d=1}^{\tau_2(k)}h^T(\mathrm{x}(k-d))Sh(\mathrm{x}(k-d))\Bigg], \qquad (21)
\end{aligned}
$$

$$
\begin{aligned}
&\mathbf{E}[\triangle V_5(k)]\\
=&\,\mathbf{E}\Bigg[\sum_{\theta=\tau_{2,m}+1}^{\tau_{2,M}}\sum_{d=1}^{\theta-1}\sum_{i=k+1-d}^{k}\alpha^{k-i}h^T(\mathrm{x}(i))Sh(\mathrm{x}(i))\\
&-\sum_{\theta=\tau_{2,m}+1}^{\tau_{2,M}}\sum_{d=1}^{\theta-1}\sum_{i=k-d}^{k-1}\alpha^{k-i}h^T(\mathrm{x}(i))Sh(\mathrm{x}(i))\Bigg]\\
=&\,\mathbf{E}\Bigg[\frac{1}{2}(\tau_{2,M}-\tau_{2,m})(\tau_{2,M}+\tau_{2,m}-1)h^T(\mathrm{x}(k))Sh(\mathrm{x}(k))\\
&-\sum_{\theta=\tau_{2,m}+1}^{\tau_{2,M}}\sum_{d=1}^{\theta-1}\alpha^d h^T(\mathrm{x}(k-d))Sh(\mathrm{x}(k-d))\Bigg]. \qquad (22)
\end{aligned}
$$

Hence, we deduce from (18)-(22) that

$$
\begin{aligned}
&\mathbf{E}[V(\mathcal{X}_{k+1},k+1)-\alpha V(\mathcal{X}_k,k)]\\
=&\sum_{j=1}^{5}\mathbf{E}[\triangle V_j(k)]\\
\le&\,\mathbf{E}\Bigg[\vartheta^T(k)\mathcal{A}^T Q\mathcal{A}\vartheta(k)-\alpha\mathrm{x}^T(k)Q\mathrm{x}(k)\\
&+\delta^*\rho^2\mathrm{x}^T(k)Q\mathrm{x}(k)+g^T(\mathrm{x}(k))Rg(\mathrm{x}(k))\\
&-\alpha^{\tau_{1,M}}g^T(\mathrm{x}(k-\tau_1(k)))Rg(\mathrm{x}(k-\tau_1(k)))\\
&+(\tau_{1,M}-\tau_{1,m})g^T(\mathrm{x}(k))Rg(\mathrm{x}(k))\\
&+\tau_{2,M}h^T(\mathrm{x}(k))Sh(\mathrm{x}(k))\\
&-\alpha^{\tau_{2,M}}\sum_{d=1}^{\tau_2(k)}h^T(\mathrm{x}(k-d))Sh(\mathrm{x}(k-d))\\
&+\frac{1}{2}(\tau_{2,M}-\tau_{2,m})(\tau_{2,M}+\tau_{2,m}-1)\\
&\times h^T(\mathrm{x}(k))Sh(\mathrm{x}(k))\Bigg]. \qquad (23)
\end{aligned}
$$

Assumption 2 and Lemma 3 indicate that

$$
\begin{aligned}
&f^T(\mathrm{x}(k))\Sigma f(\mathrm{x}(k))+\mathrm{x}^T(k)\Sigma\Gamma_1\mathrm{x}(k)\\
&\quad-2\mathrm{x}^T(k)\Sigma\Gamma_2 f(\mathrm{x}(k))\le 0, \qquad (24)\\
&g^T(\mathrm{x}(k))\Upsilon g(\mathrm{x}(k))+\mathrm{x}^T(k)\Upsilon\Omega_1\mathrm{x}(k)\\
&\quad-2\mathrm{x}^T(k)\Upsilon\Omega_2 g(\mathrm{x}(k))\le 0, \qquad (25)\\
&h^T(\mathrm{x}(k))\Lambda h(\mathrm{x}(k))+\mathrm{x}^T(k)\Lambda\Xi_1\mathrm{x}(k)\\
&\quad-2\mathrm{x}^T(k)\Lambda\Xi_2 h(\mathrm{x}(k))\le 0. \qquad (26)
\end{aligned}
$$

Meanwhile, one can derive from Lemma 2 that

$$
\begin{aligned}
&-\sum_{d=1}^{\tau_2(k)}h^T(\mathrm{x}(k-d))Sh(\mathrm{x}(k-d))\\
&\le-\frac{1}{\tau_2(k)}\left(\sum_{d=1}^{\tau_2(k)}h(\mathrm{x}(k-d))\right)^T S\left(\sum_{d=1}^{\tau_2(k)}h(\mathrm{x}(k-d))\right)
\end{aligned}
$$

By applying (24)-(27) into (23), we obtain

$$\mathbf{E}[V(\mathcal{X}_{k+1},k+1)-\alpha V(\mathcal{X}_k,k)] \\ \leq \mathbf{E}\left[\vartheta^T(k)\mathcal{A}^TQ\mathcal{A}\vartheta(k)+\vartheta^T(k)\Phi_1\vartheta(k)\right], \quad (27)$$

where

$$\Phi_1 = \begin{bmatrix} \Phi_{11} & * & * & * & * & * \\ \Gamma_2\Sigma & -\Sigma & * & * & * & * \\ \Omega_2\Upsilon & 0 & \Phi_{33} & * & * & * \\ 0 & 0 & 0 & -\alpha^{\tau_{1,M}}Q & * & * \\ \Xi_2\Lambda & 0 & 0 & 0 & \Phi_{55} & * \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{\tau_{2,M}}S \end{bmatrix} < 0,$$

with Φ_{11}, Φ_{33} and Φ_{55} defined in (6) and (7), respectively.

According to Lemma 1, it is easy to see that $\Phi_1+\mathcal{A}^TQ\mathcal{A}<0$ is equivalent to $\Phi<0$, and then due to condition (5) we have

$$\mathbf{E}[V(\mathcal{X}_{k+1},k+1)-\alpha V(\mathcal{X}_k,k)] \leq 0,$$

which implies

$$\mathbf{E}[V(\mathcal{X}_{k+1},k+1)] \leq \alpha\mathbf{E}[V(\mathcal{X}_k,k)].$$

Based on the above inequality, it is obtained that

$$\mathbf{E}[V(\mathcal{X}_k,k)] \leq \alpha^{k-k_0}\mathbf{E}[V(\mathcal{X}_{k_0},k_0)],$$

for all $k\in\mathbb{Z}_{\geq k_0}$. □

Remark 5: On account of the novel Lyapunov functional (12), we obtain a criterion of decay estimation for the mentioned non-switched neural networks (3). It is clear that exponential stability in mean square of system (3) is reached provided that the LMIs (4) and (5) are feasible. Besides, the LMIs (4) and (5) show the effect of time-varying delays and stochastic noise intensity on the decay rate of system (3). For instance, if α is given, then the smaller intension of stochastic noise, upper bound of $\tau_1(k)$ and $\tau_2(k)$ are, the more feasible the solutions are. Meanwhile, small difference of upper and lower bounds of time-delays makes the feasible solutions easier to find.

Remark 6: Lemma 4 presents exponential stability in mean square of system (3) without switching signal. However, as illustrated in literature [33], for some switching rules, the switched systems would exhibit unstable performance even if every subsystem is asymptotically stable. Hence, we should also consider the change of switching rule apart from dynamical temporal behavior of each subsystem. The following theorem will explore stability of the addressed DSNNs (1).

Next, with the aid of Lemma 4 and average dwell time methodology, a novel mode-dependent Lyapunov functional is constructed in order to investigate stability problem for system (1).

Theorem 1: Given $0<\alpha<1$. Under Assumptions 1-3, if there are constants $\mu\geq 1$, $\delta_i^*>0$, matrices $Q_i>0, R_i>0, S_i>0$, for $i\in\mathcal{M}$, diagnal matrices $\Sigma>0, \Upsilon>0, \Lambda>0$ satisfying the following inequalities:

$$T_0 \geq T_0^* = -\frac{\ln\mu}{\ln\alpha}, \quad (28)$$

$$Q_i \leq \mu Q_j,\ R_i \leq \mu R_j,\ S_i \leq \mu S_j, \quad (29)$$

$$Q_i < \delta_i^* I, \quad (30)$$

$$\Phi_i = \begin{bmatrix} \Phi_{11}^i & * & * & * & * & * & * \\ \Gamma_2\Sigma & -\Sigma & * & * & * & * & * \\ \Omega_2\Upsilon & 0 & \Phi_{33}^i & * & * & * & * \\ 0 & 0 & 0 & \Phi_{44}^i & * & * & * \\ \Xi_2\Lambda & 0 & 0 & 0 & \Phi_{55}^i & * & * \\ 0 & 0 & 0 & 0 & 0 & \Phi_{66}^i & * \\ Q_iA_i & Q_iB_i & 0 & Q_iC_i^{(1)} & 0 & \Phi_{76}^i & -Q_i \end{bmatrix} < 0, \quad (31)$$

for all $i,\ j\in\mathcal{M}$, where

$$\Phi_{11}^i = \rho^2\delta_i^*I-\alpha Q_i-\Sigma\Gamma_1-\Upsilon\Omega_1-\Lambda\Xi_1, \quad (32)$$

$$\Phi_{33}^i = (1+\tau_{1,M}-\tau_{1,m})R_i-\Upsilon, \quad (33)$$

$$\Phi_{44}^i = -\alpha^{\tau_{1,M}}Q_i, \quad (34)$$

$$\Phi_{55}^i = \left[\tau_{2,m}+\frac{1}{2}(\tau_{2,M}-\tau_{2,m})(\tau_{2,M}+\tau_{2,m}-1)\right]S-\Lambda, \quad (35)$$

$$\Phi_{66}^i = -\frac{1}{\tau_{2,M}}S_i,\ \Phi_{76}^i = Q_iC_i^{(2)}, \quad (36)$$

then system (1) is mean square exponential stability with a decay rate $\lambda=\mu^{\frac{1}{T_0}}\alpha$.

Proof: To analyze the stability, we can utilize the mode-dependent Lyapunov functional:

$$V_{\pi(k)}(k) := V_{\pi(k)}(\mathcal{X}_k,k) = \sum_{s=1}^{5} V_{s,\pi(k)}(\mathcal{X}_k,k), \quad (37)$$

where

$$V_{1,\pi(k)}(\mathcal{X}_k,k) = \mathrm{x}^T(k)Q_{\pi(k)}\mathrm{x}(k), \quad (38)$$

$$V_{2,\pi(k)}(\mathcal{X}_k,k) = \sum_{i=k-\tau_1(k)}^{k-1}\alpha^{k-1-i}g^T(\mathrm{x}(i))R_{\pi(k)}g(\mathrm{x}(i)), \quad (39)$$

$$V_{3,\pi(k)}(\mathcal{X}_k,k) = \sum_{d=\tau_{1,m}}^{\tau_{1,M}-1}\sum_{i=k-d}^{k-1}\alpha^{k-1-i}g^T(\mathrm{x}(i))R_{\pi(k)}g(\mathrm{x}(i)), \quad (40)$$

$$V_{4,\pi(k)}(\mathcal{X}_k,k) = \sum_{d=1}^{\tau_2(k)}\sum_{i=k-d}^{k-1}\alpha^{k-1-i}h^T(\mathrm{x}(i))S_{\pi(k)}h(\mathrm{x}(i)), \quad (41)$$

$$V_{5,\pi(k)}(\mathcal{X}_k,k) = \sum_{\theta=\tau_{2,m}+1}^{\tau_{2,M}}\sum_{d=1}^{\theta-1}\sum_{i=k-d}^{k-1}\alpha^{k-1-i}h^T(\mathrm{x}(i))$$

$$\times S_{\pi(k)}h(\mathrm{x}(i)). \tag{42}$$

When $k \in [k_t, k_{t+1})$, employing Lemma 4 together with (29), we have

$$\begin{aligned}\mathbf{E}[V_{\pi(k)}(k)] \leq& \alpha^{k-k_t}\mathbf{E}[V_{\pi(k_t)}(k_t)]\\ \leq& \mu\alpha^{k-k_t}\mathbf{E}[V_{\pi(k_{t-1})}(k_t)]\\ \leq& \mu\alpha^{k-k_{t-1}}\mathbf{E}[V_{\pi(k_{t-1})}(k_{t-1})]\\ \leq& \mu^{N_\pi(k,k_0)}\alpha^{k-k_0}\mathbf{E}[V_{\pi(k_0)}(k_0)]\\ \leq& \left(\mu^{\frac{1}{T_0}}\alpha\right)^{k-k_0}\mathbf{E}[V_{\pi(k_0)}(k_0)]. \end{aligned} \tag{43}$$

Additionally, let

$$\omega = \max_{1\leq i\leq n}[|\omega_i^-|, |\omega_i^+|], \quad \xi = \max_{1\leq i\leq n}[|\xi_i^-|, |\xi_i^+|].$$

Then, owing to (37), it is implied that

$$\begin{aligned}&\mathbf{E}[V_{\pi(k)}(k)] \geq \beta_1\mathbf{E}[\|\mathrm{x}(k)\|^2],\\ &\mathbf{E}[V_{\pi(k_0)}(k_0)] \leq \beta_2 \sup_{k_0-r\leq\ell\leq k_0}\mathbf{E}[\|\phi(\ell)\|^2],\end{aligned}$$

Thus, (43) implies

$$\mathbf{E}[\|\mathrm{x}(k)\|^2] \leq \frac{\beta_2}{\beta_1}\left(\mu^{\frac{1}{T_0}}\alpha\right)^{k-k_0}\sup_{k_0-r\leq\ell\leq k_0}\mathbf{E}[\|\phi(\ell)\|^2], \tag{44}$$

where

$$\begin{aligned}\beta_1 =& \min_{i\in\mathcal{M}}\lambda_m(Q_i),\\ \beta_2 =& \max_{i\in\mathcal{M}}\lambda_M(Q_i) + [\tau_{1,M} + \frac{1}{2}(\tau_{1,M}-\tau_{1,m})\\ &\times(\tau_{1,M}+\tau_{1,m}-1)]\omega^2\max_{i\in\mathcal{M}}\lambda_M(R_i) + \frac{1}{2}\tau_{2,M}\\ &\times(\tau_{2,M}+1)(\tau_{2,M}-\tau_{2,m}+1)\xi^2\max_{i\in\mathcal{M}}\lambda_M(S_i).\end{aligned}$$

Hence, according to Definition 1, system (1) achieves the exponential stability in mean square with $K = \frac{\beta_2}{\beta_1}$ and $\lambda = \mu^{\frac{1}{T_0}}\alpha$, where $K > 0$ and $0 < \lambda < 1$. □

Remark 7: Recently, dynamical behaviors of switched time-delayed neural networks have been explored, e.g. [17, 29]. However, time-varying distributed delays have been overlooked in [17, 29]. In Theorem 1, time-varying distributed delays in system (1) are tackled by the triple summation term (42) of the novel Lyapunov functional.

Remark 8: Theorem 1 indicates that stochastic stability of system (1) can be achieved under the conditions of mean square stable subsystems together with slowly switching signal. The condition $T_0 \geq T_0^* = -\frac{\ln\mu}{\ln\alpha}$ gives the minimal dwell time on the average, ensuring DSNNs (1) is stochastically stable. Additionally, it is found that the better stochastic performance of every subsystem is, the larger minimal average dwell time is. We shall show this point in the numerical example.

Remark 9: The decay rate of the addressed neural networks (1) satisfies $\lambda = \mu^{\frac{1}{T_0}}\alpha > \alpha$, which means that stability performance of every subsystem is better than that of the total switched systems. Particularly, when $T_0 \to \infty$, we have $\lambda \to \alpha$, which is equal to the decay rate of every subsystem.

Remark 10: It can be seen from (28) that the lower bound T_0^* of average dwell time is monotonic increasing in α and μ. Accordingly, to obtain some small value T_0^*, we should choose α and μ as small as possible under the feasibility of the LMIs (29)-(31). On the other hand, large α and μ is beneficial to the feasibility of the LMIs (29)-(31). Thus, to obtain the minimal value of T_0^*, it is necessary to find the minimal values of α and μ such that the LMIs (29)-(31) are feasible. Specifically, large initial values α and μ are firstly chosen to ensure that (29)-(31) are solvable, and then, according to some step length, α and μ are gradually tuned under the feasibility of the LMIs (29)-(31).

Remark 11: Based on mode-dependent Lyapunov functional technique, this paper has investigated the stochastic stability problem for a class of general neural networks. In Theorem 1, sufficient conditions of exponential stability in mean square for the considered DSNNs are presented in terms of the feasibility of a set of LMIs and the lower bound of average dwell time. When increasing the number of neurons, it is obvious that the size of LMIs (29)-(31) also grows. Therefore, a huge computation burden is inevitable. Currently, workstations need spend an hour on tackling the problems for a thousand design variables [15], but the LMI optimization has become the ever-increasing research topic. So, we expect that the corresponding computational process will be accelerated in the future.

4. A NUMERICAL EXAMPLE

The effectiveness of the inferred theoretical results can be verified by a numerical simulation example of this section.

We consider the DSNNs (1) with two subsystems denoted by S_1 and S_2. The corresponding system parameters are listed as follows:

$$\begin{aligned}&A_1 = \begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix}, \ B_1 = \begin{bmatrix}0.2 & -0.5\\ 0.2 & -0.1\end{bmatrix},\\ &C_1^{(1)} = \begin{bmatrix}0.1 & -0.1\\ 0.2 & -0.2\end{bmatrix}, \ C_1^{(2)} = \begin{bmatrix}0.2 & -0.2\\ 0 & 0.2\end{bmatrix},\\ &A_2 = \begin{bmatrix}0.3 & 0\\ 0 & 0.2\end{bmatrix}, \ B_2 = \begin{bmatrix}0.1 & -0.2\\ 0.1 & -0.1\end{bmatrix},\\ &C_2^{(1)} = \begin{bmatrix}0.2 & 0.1\\ 0.2 & -0.2\end{bmatrix}, \ C_2^{(2)} = \begin{bmatrix}0.2 & -0.1\\ 0.1 & 0.2\end{bmatrix},\\ &6 \leq \tau_1(k) \leq 8, 1 \leq \tau_2(k) \leq 2, \rho = 0.2, \mu = 1.2.\end{aligned}$$

Table 1. Decay Rate of Subsystem α and ADT T_0^*

α	0.98	0.95	0.92	0.89
T_0^*	9.6246	3.5545	2.1886	1.5645

Consider activation functions satisfying Assumption 2:

$$f_1(u) = g_1(u) = h_1(u) = \tanh(-0.6u),$$
$$f_2(u) = g_2(u) = h_2(u) = \tanh(0.4u),$$

then one can see that

$$\Gamma_1 = \Omega_1 = \Xi_1 = 0, \Gamma_2 = \Omega_2 = \Xi_2 = \begin{bmatrix} -0.3 & 0 \\ 0 & 0.2 \end{bmatrix}.$$

Based on the above parameters, we can solve LMIs (29)-(31) by means of Matlab LMI toolbox. The following Table II describes that the calculated values of ADT T_0^* for different values of decay rate α. It means that the better stochastic performance of every subsystem is, the larger minimal average dwell time is.

Particularly, when $\alpha = 0.89$, from Theorem 1, we can get $T_0^* = 1.5645$ and feasible solutions which are listed:

$$\lambda_1^* = 49.0170, \ Q_1 = \begin{bmatrix} 38.0266 & -5.8645 \\ -5.8645 & 24.2283 \end{bmatrix},$$
$$R_1 = \begin{bmatrix} 12.3823 & 3.0318 \\ 3.0318 & 13.0191 \end{bmatrix},$$
$$S_1 = \begin{bmatrix} 13.2423 & -0.0140 \\ -0.0140 & 15.4770 \end{bmatrix},$$
$$\lambda_2^* = 44.7605, \ Q_2 = \begin{bmatrix} 42.0794 & -6.3184 \\ -6.3184 & 22.4979 \end{bmatrix},$$
$$R_2 = \begin{bmatrix} 12.2330 & 3.6678 \\ 3.6678 & 13.0650 \end{bmatrix},$$
$$S_2 = \begin{bmatrix} 12.7416 & 0.2330 \\ 0.2330 & 15.1172 \end{bmatrix},$$
$$\Sigma = \begin{bmatrix} 13.6881 & 0 \\ 0 & 72.0085 \end{bmatrix},$$
$$\Upsilon = \begin{bmatrix} 73.6985 & 0 \\ 0 & 70.4054 \end{bmatrix},$$
$$\Lambda = \begin{bmatrix} 50.5695 & 0 \\ 0 & 56.1754 \end{bmatrix}.$$

Suppose $k_0 = 0$, and the subsystem S_1 is firstly activated. For any initial condition ϕ, Fig. 2 depicts the phase portrait of system (1) with $\alpha = 0.89$, $T_0 = 2 > T_0^*$, while Fig. 3 depicts with $\alpha = 0.95$, $T_0 = 5 > T_0^*$. These two figures all show that system (1) with given parameters is stochastically stable.

5. CONCLUSIONS

In this paper,exploration on stochastic stability has been made for a class of generally switched neural net-

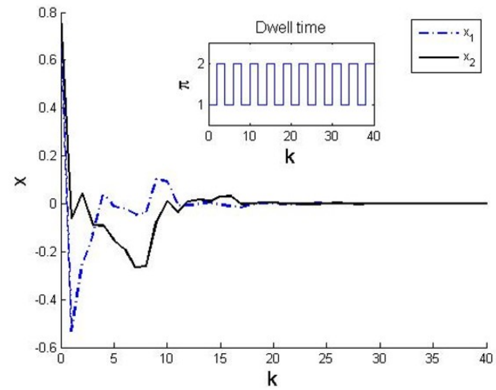

Fig. 1. System state evolution with $\alpha = 0.89$, $T_0 = 2$.

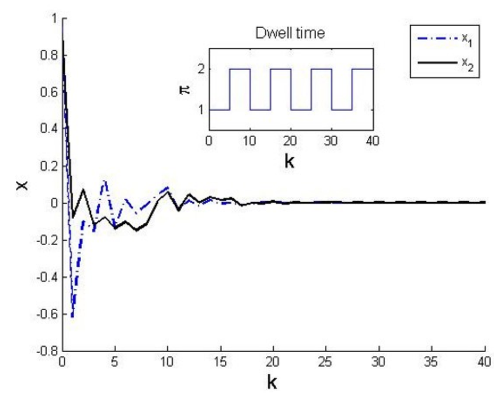

Fig. 2. System state evolution with $\alpha = 0.95$, $T_0 = 5$.

works. The considered model simultaneously incorporates stochastic noise, switching signal, and time-varying mixed delays. The establishment of sufficient conditions for stochastic stability has been dependent on the new Lyapunov functional in combination with average dwell time methodology. The obtained sufficient conditions can reflect the relationship among the decay rate, dwell time on the average, performance of subsystems, upper and lower bounds of time-delays and intensity of noise. Some interesting research topics in future, including the stabilization problem of switched systems not only for more general nonlinear subsystem [34, 35] but also for some network-induced phenomena [14, 36].

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Ying Cui received the M.S. degree in 2007 from Wuhan University, Hubei, China. Now she is pursuing the Ph.D. degree in Mathematics at Yangzhou University, Jiangsu, China. Her research interests include hybrid systems, dynamics of complex networks.

Yurong Liu was born in China in 1964. He received his B.Sc. degree in Mathematics from Suzhou University, Suzhou, China, in 1986, the M.Sc. degree in Applied Mathematics from Nanjing University of Science and Technology, Nanjing, China, in 1989, and the Ph.D. degree in Applied Mathematics from Suzhou University, Suzhou, China, in 2001. Dr. Liu is currently a professor with the Department of Mathematics at Yangzhou University, China. He also serves as an Associate Editor of Neurocomputing. So far, he has published more than 50 papers in refereed international journals. His current interests include stochastic control, neural networks, complex networks, nonlinear dynamics, time-delay systems, multi-agent systems, and chaotic dynamics.

Wenbing Zhang received the M.S. degree in applied mathematics from Yangzhou University, Jiangsu, China, and the Ph.D. degree in pattern recognition and intelligence systems from Donghua University, Shanghai, China, in 2009 and 2012, respectively. He was a Research Associate with The Hong Kong Polytechnic University, Kowloon, Hong Kong, from 2012 to 2013. From July 2014 to Aug 2014, he was a DAAD fellow with the Potsdam Institute for Climate Impact Research, Potsdam, Germany. He is currently an associated professor with the Department of Mathematics, Yangzhou University. His current research interests include synchronization/consensus, networked control systems, and genetic regulatory networks. Dr. Zhang is a very active reviewer for many international journals.

Fuad E. Alsaadi received the BS and MSc degrees in Electronic and Communication from King AbdulAziz University, Jeddah, Saudi Arabia, in 1996 and 2002. He then received the PhD degree in Optical Wireless Communication Systems from the University of Leeds, Leeds, UK, in 2011. Between 1996 and 2005, he worked in Jeddah as a communication instructor in the College of Electronics & Communication. He was a lecturer in the Faculty of Engineering in King AbdulAziz University, Jeddah, Saudi Arabia in 2005. He is currently an assistant professor of the Electrical and Computer Engineering Department within the Faculty of Engineering, King Abdulaziz University, Jeddah, Saudi Arabia. He published widely in the top IEEE communications conferences and journals and has received the Carter award, University of Leeds for the best PhD. He has research interests in optical systems and networks, signal processing, synchronization and systems design.