International Journal of Control, Automation and Systems 16(1) (2018) 150-157
http://dx.doi.org/10.1007/s12555-016-0606-5 ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Missing Output Identification Model Based Recursive Least Squares Algorithm for a Distributed Parameter System

Jing Chen, Bin Jiang*, and Juan Li

Abstract: This paper proposes a recursive least squares algorithm for a distributed parameter system with missing observations. By using the finite difference method, the distributed parameter system can be turned into a lumped parameter system. Then a missing output identification model based recursive least squares algorithm is derived to estimate the unknown parameters of the lumped parameter system. Furthermore, the parameters of the distributed parameter system can be computed by the estimated parameters of the lumped parameter system. The simulation results indicate that the proposed method is effective.

Keywords: Distributed parameter system, finite difference method, missing output identification model, parameter estimation, recursive least squares.

1. INTRODUCTION

Systems can be roughly divided into two classes: the lumped parameter systems (LPSs) and the distributed parameter systems (DPSs). The LPSs are described by difference equations and have finite dimension. The DPSs are described by partial differential equations and are of infinite dimension. Recently, LPSs identification has received much attention, and there exist a lot of identification methods for LPSs [1–3], including the least squares (LS) algorithms [4–6], the stochastic gradient algorithms [7–9] and the iterative algorithms [10–12].

The DPSs are widely existed in engineering practice, e.g., in semiconductor manufacturing, nanotechnology, biotechnology and chemical engineering [13, 14]. System identification is the first step for many applications such as prediction, control and fault tolerant control [15–19]. Unfortunately, the identification methods for LPSs are not suitable for DPSs, because the LPSs identification methods ignore the important features of the spatial response exhibited by the DPSs.

Over the past few decades, several methods have been developed for DPSs identification [20–23], and these methods use proper basis functions to turn DPSs into LPSs. For example, Zill and Cullen proposed a weighted residual method for DPSs, the spatio-temporal variable of the DPSs can be expanded as a set of spatial basis functions [24]. Li and Qi used a Karhunen-Loeve method to turn a DPS into an LPS, then applied an LS algorithm to estimate the unknown parameters of the LPS [25]. However, when use these methods to truncate the infinite dimension to a finite dimension, the selection of dimension is difficult, a small dimension may lead a large identification error, but a large dimension may reduce the computational efficiency. Recently, Chen and Jiang developed a RLS algorithm and an SG algorithm for a two-dimensional system with the assumption that the system is a single-rate system [26].

Dual-rate systems which have different sampled rates of input and output, are widely existed in many engineering applications [27–29]. For example, in polymer reactors, the manipulated variables can be adjusted at relatively fast rate, whereas the composition, density or molecular weight distribution measurements are typically obtained after several minutes of analysis [30]. The lifting technique and the polynomial transformation technique are two methods which are generally used for dual-rate systems identification [31–34]. However, these two methods can only estimate the parameters of the transformed system and can increase the number of the unknown parameters. In order to overcome these difficulties, Chen presented a missing output identification (MOI) method for dual-rate one-dimensional systems, the method can estimate the parameters of the dual-rate systems directly and

Manuscript received September 27, 2016; revised December 26, 2016 and March 2, 2017; accepted May 24, 2017. Recommended by Associate Editor Yongping Pan under the direction of Editor Duk-Sun Shim. This work is supported by the National Natural Science Foundation of China (Nos. 61403165,61374126), the Natural Science Foundation of Jiangsu Province (No. BK20131109), the Natural Science Foundation of Shandong Province (ZR2013FM021), the Post Doctoral Foundation of Jiangsu Province (No. 1501015A) and the Natural Science Fund for Colleges and Universities in Jiangsu Province (No. 16KJB120006).

Jing Chen and Bin Jiang are with the College of Automation Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, P. R. China (e-mails: chenjing1981929@126.com, binjiang@nuaa.edu.cn). Juan Li is with the College of Mechanical and Electrical Engineering, Qingdao Agricultural University, Qingdao 266109, P. R. China (e-mail: lijuan291@sina.com).
* Corresponding author.

keep the number of the unknown parameters unchanged [2]. In this paper, we will extend the MOI method to a dual-rate two-dimensional system.

The contributions of this paper are as follows.

1) Propose a missing output identification model based recursive least squares (MOI-RLS) algorithm for dual-rate two-dimensional systems.
2) The proposed method can estimate the unknown parameters and the missing outputs simultaneously; Furthermore, cannot increase the number of the unknown parameters.
3) Compared with the work in [35], the MOI-RLS method has less computation efforts.

The rest of this paper is organized as follows. Section 2 introduces the DPS and the finite difference method. Section 3 presents an MOI-RLS algorithm for a general dual-rate two-dimensional system. Section 4 provides an illustrative example. Finally, concluding remarks are given in Section 5.

2. PROBLEM FORMULATION

Let "$A =: X$" or "$X := A$" stand for "A is defined as X", the superscript T denote the matrix transpose.

Consider the following DPS,

$$\frac{\partial y(x,t)}{\partial t} = a_1\frac{\partial^2 y(x,t)}{\partial x^2} + a_2\frac{\partial y(x,t)}{\partial x} + a_3 y(x,t) + a_4 u(x,t), \tag{1}$$

where $y(x,t)$ is the system output, $u(x,t)$ is the system input, a_1, a_2, a_3 and a_4 are unknown parameters to be estimated, and with the assumption that $0 \leqslant a_1$, $0 \leqslant a_2 \leqslant L$ and $K \leqslant a_3 \leqslant 0$, L is a positive constant and K is a negative constant, both L and K are known in prior.

Using the finite difference method gives

$$\frac{\partial^2 y(x,t)}{\partial x^2}|_{x_i,t_j} \approx \frac{y_{i+1,j+1} - 2y_{i,j+1} + y_{i-1,j+1}}{\Delta x^2},$$
$$\frac{\partial y(x,t)}{\partial t}|_{x_i,t_j} \approx \frac{y_{i,j+1} - y_{i,j}}{\Delta t},$$
$$\frac{\partial y(x,t)}{\partial x}|_{x_i,t_j} \approx \frac{y_{i+1,j} - y_{i,j}}{\Delta x},$$

where $y_{i,j} = y(x_i,t_j)$ is the output at discretization node (i,j), Δx and Δt are two small intervals at space and time, respectively. By this difference algorithm, derivatives at each discretization node (i,j) are approximated by the difference over a small interval. Then the DPS in (1) can be transformed as follow,

$$\frac{y_{i,j+1} - y_{i,j}}{\Delta t} \approx a_1\frac{y_{i+1,j+1} - 2y_{i,j+1} + y_{i-1,j+1}}{\Delta x^2} + a_2\frac{y_{i+1,j} - y_{i,j}}{\Delta x} + a_3 y_{i,j} + a_4 u_{i,j},$$

and can be simplified as an LPS (or two-dimensional system),

$$(1+2r_1)y_{i,j+1} - r_1 y_{i+1,j+1} - r_1 y_{i-1,j+1} = r_2 y_{i+1,j} + (r_3 - r_2 + 1)y_{i,j} + r_4 u_{i,j}, \tag{2}$$

where $r_1 = a_1\frac{\Delta t}{\Delta x^2}$, $r_2 = a_2\frac{\Delta t}{\Delta x}$, $r_3 = a_3\Delta t$ and $r_4 = a_4\Delta t$.

The finite difference method cannot guarantee that the DPS be approximated by the LPS because of the approximation error. Therefore, in order to keep the finite difference method convergent, Von. Neumann stability analysis is introduced.

Based on Von. Neumann stability analysis, the error equation has the same structure with the LPS [36], and can be written as

$$(1+2r_1)z_{i,j+1} - r_1 z_{i+1,j+1} - r_1 z_{i-1,j+1} = r_2 z_{i+1,j} + (r_3 - r_2 + 1)z_{i,j}, \tag{3}$$

where $z_{i,j+1}$ is the approximation error at $(i, j+1)$, and

$$z_{i,j} = V^j(k)e^{ikx_i}, \quad z_{i,j+1} = V^{j+1}(k)e^{ikx_i},$$
$$z_{i+1,j+1} = V^{j+1}(k)e^{ikx_i}e^{ik\Delta x},$$
$$z_{i-1,j+1} = V^{j+1}(k)e^{ikx_i}e^{-ik\Delta x},$$
$$z_{i+1,j} = V^j(k)e^{ikx_i}e^{ik\Delta x}, \tag{4}$$

in which $V(k)$ is a growth factor, k is the number of waves and e is short for exp. When $|V(k)| \leqslant 1$, the difference algorithm is convergent; when $|V(k)| > 1$, the difference algorithm is divergent.

Rewrite (3) as

$$(1+2r_1)V^{j+1}(k)e^{ikx_i} - r_1 V^{j+1}(k)e^{ikx_i}e^{ik\Delta x} - r_1 V^{j+1}(k)e^{ikx_i}e^{-ik\Delta x} = r_2 V^j(k)e^{ikx_i}e^{ik\Delta x} + (r_3 - r_2 + 1)V^j(k)e^{ikx_i}, \tag{5}$$

and the growth factor $V(k)$ can be expressed as

$$V(k) = \frac{r_2 e^{ik\Delta x} + r_3 - r_2 + 1}{1 + 2r_1 - r_1 e^{ik\Delta x} - r_1 e^{-ik\Delta x}}. \tag{6}$$

Since

$$e^{ik\Delta x} + e^{-ik\Delta x} = 2\cos(k\Delta x),$$
$$e^{ik\Delta x} - e^{-ik\Delta x} = 2\sin(k\Delta x)i,$$
$$1 - \cos(k\Delta x) = 2\sin^2\frac{k\Delta x}{2},$$

we have $e^{ik\Delta x} = \cos(k\Delta x) + \sin(k\Delta x)i$.

Simplifying Equation (6) gives

$$V(k) = \frac{r_2 e^{ik\Delta x} + r_3 - r_2 + 1}{1 + 2r_1 - 2r_1\cos(k\Delta x)} = \frac{r_2\cos(k\Delta x) + r_3 - r_2 + 1 + r_2\sin(k\Delta x)i}{1 + 4r_1\sin^2\frac{k\Delta x}{2}}$$

$$=\frac{r_2\cos(k\Delta x)+r_3-r_2+1}{1+4r_1\sin^2\frac{k\Delta x}{2}}+\frac{r_2\sin(k\Delta x)}{1+4r_1\sin^2\frac{k\Delta x}{2}}i. \tag{7}$$

Let

$$a=\frac{r_2\cos(k\Delta x)+r_3-r_2+1}{1+4r_1\sin^2\frac{k\Delta x}{2}},$$
$$b=\frac{r_2\sin(k\Delta x)}{1+4r_1\sin^2\frac{k\Delta x}{2}}.$$

Then we have

$$\begin{aligned}&|V(k)|^2\\&=a^2+b^2\\&=\frac{(r_2\cos(k\Delta x)+r_3-r_2+1)^2}{(1+4r_1\sin^2\frac{k\Delta x}{2})^2}+\frac{(r_2\sin(k\Delta x))^2}{(1+4r_1\sin^2\frac{k\Delta x}{2})^2}\\&=\frac{4r_2\sin^2(\frac{k\Delta x}{2})(r_2-r_3-1)+(r_3+1)^2}{(1+4r_1\sin^2\frac{k\Delta x}{2})^2}.\end{aligned}$$

Since $r_1=a_1\frac{\Delta t}{\Delta x^2}$, $r_2=a_2\frac{\Delta t}{\Delta x}$, $r_3=a_3\Delta t$, $a_1\geqslant 0$, $a_2\geqslant 0$ and $a_3\leqslant 0$, we can choose two small positive numbers Δt and Δx to keep $r_1\geqslant 0$, $0\leqslant r_2\leqslant\frac{1}{2}$ and $-\frac{1}{2}\leqslant r_3\leqslant 0$. Then we can get

$$\begin{aligned}|V(k)|^2&=\frac{4r_2^2\sin^2(\frac{k\Delta x}{2})(r_2-r_3-1)+(r_3+1)^2}{(1+4r_1\sin^2\frac{k\Delta x}{2})^2}\\&\leqslant\frac{(r_3+1)^2}{(1+4r_1\sin^2\frac{k\Delta x}{2})^2}\leqslant 1.\end{aligned}\tag{8}$$

Here, $r_1\geqslant 0$, $0\leqslant r_2\leqslant\frac{1}{2}$ and $-\frac{1}{2}\leqslant r_3\leqslant 0$ are equivalent to $a_1\geqslant 0$, $L\leqslant\frac{\Delta x}{2\Delta t}$ and $-\frac{1}{2\Delta t}\leqslant K$. Thus, based on Von. Neumann stability analysis and the assumption of a_1, a_2 and a_3, we can conclude that the DPS in (1) can be approximated by the LPS in (2).

Simplifying (2) gets

$$\begin{aligned}y(i+1,j+1)=&a_{0,1}y(i+1,j)+a_{1,0}y(i,j+1)\\&+a_{2,0}y(i-1,j+1)+a_{1,1}y(i,j)\\&+b_{1,1}u(i,j),\end{aligned}\tag{9}$$

where $y(i,j)=y_{i,j}$, $a_{0,1}=-\frac{r_2}{r_1}$, $a_{1,0}=\frac{1+2r_1}{r_1}$, $a_{2,0}=-1$, $a_{1,1}=-\frac{r_3-r_2+1}{r_1}$ and $b_{1,1}=-\frac{r_4}{r_1}$.

Rewrite (9) as

$$\begin{aligned}y(i,j)+y(i-2,j)=&a_{0,1}y(i,j-1)+a_{1,0}y(i-1,j)\\&+a_{1,1}y(i-1,j-1)\\&+b_{1,1}u(i-1,j-1).\end{aligned}\tag{10}$$

3. THE MOI-RLS ALGORITHM

Consider the general dual-rate two-dimensional system as

$$\begin{aligned}y(x,tq)=&a_{0,1}y(x,tq-1)+a_{0,2}y(x,tq-2)+\cdots\\&+a_{0,q}y(x,tq-q)+\cdots+a_{0,n}y(x,tq-n)\\&+a_{1,0}y(x-1,tq)+a_{2,0}y(x-2,tq)+\cdots\\&+a_{m,0}y(x-m,tq)\\&+a_{1,1}y(x-1,tq-1)+a_{1,2}y(x-1,tq-2)\\&+\cdots+a_{1,q}y(x-1,tq-q)+\cdots\\&+a_{1,n}y(x-1,tq-n)+\cdots\\&+a_{m,1}y(x-m,tq-1)+a_{m,2}y(x-m,tq-2)\\&+\cdots+a_{m,q}y(x-m,tq-q)+\cdots\\&+a_{m,n}y(x-m,tq-n)\\&+b_{1,1}u(x-1,tq-1)+v(x,tq),\end{aligned}\tag{11}$$

where $u(x,t)$ and $y(x,t)$ are input and output, respectively. $v(x,t)$ is a stochastic white noise with zero mean. n means that the output at time tq depends on all the outputs from $tq-n$ to $tq-1$, while m means that the output at space x depends on all the outputs from $x-m$ to $x-1$. For the dual-rate sampled data system, the input data $u(x,t)$ is sampled at a quicker rate than the output $y(x,t)$, thus all the input data $\{u(x,t),x=1,2,\cdots,M,t=0,1,2,\cdots\}$ and only the scarce output data $\{y(x,tq),(q\geqslant 2)\}$ are measureable. The intersample outputs $\{y(x,tq+h),h=1,2,\cdots,q-1\}$ are unavailable. M is the total number of the points in the space at time t, as shown in Fig. 1.

Define the parameter vector θ and the information vector $\varphi(x,tq)$ as

$$\begin{aligned}\theta:=&[a_{0,1},a_{0,2},\cdots,a_{0,n},a_{1,0},a_{2,0},\cdots,\\&a_{m,0},a_{1,1},a_{1,2},\cdots,a_{1,n},\cdots,\\&a_{m,1},a_{m,2},\cdots,a_{m,n},b_{1,1}]^{\mathrm{T}}\in\mathbb{R}^p,\\&p=(m+1)(n+1),\end{aligned}\tag{12}$$

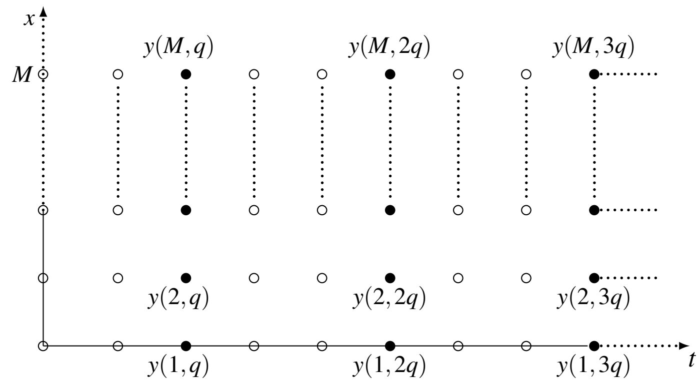
$$\begin{aligned}\varphi(x,tq):=&[y(x,tq-1),y(x,tq-2),\cdots,\\&y(x,tq-q),\cdots,y(x,tq-n),y(x-1,tq),\\&y(x-2,tq),\cdots,y(x-m,tq),\\&y(x-1,tq-1),y(x-1,tq-2),\cdots,\\&y(x-1,tq-q),\cdots,\\&y(x-1,tq-n),\cdots,y(x-m,tq-1),\\&y(x-m,tq-2),\cdots,y(x-m,tq-q),\cdots,\\&y(x-m,tq-n),\\&u(x-1,tq-1)]^{\mathrm{T}}\in\mathbb{R}^p.\end{aligned}\tag{13}$$

Rewrite (11) as an identification model,

$$y(x,tq)=\varphi^{\mathrm{T}}(x,tq)\theta+v(x,tq).$$

Using the following RLS algorithm proposed in [26] to estimate the parameter vector $\theta(x,tq)$:

$$\begin{aligned}\hat\theta(x,tq)=&\hat\theta(x-1,tq-1)+P(X,Tq)P^{-1}(X-1,tq)\\&\times(\hat\theta(x-1,tq)-\hat\theta(x-1,tq-1))\\&+P(X,Tq)\times P^{-1}(x,Tq-1)(\hat\theta(x,tq-1)\end{aligned}$$



•: available observations; ∘: missing observations

Fig. 1. The missing observations pattern

$$-\hat{\theta}(x-1,tq-1))$$
$$+P(X,Tq)\varphi(x,tq)(y(x,tq)$$
$$-\varphi^{\mathrm{T}}(x,tq)\hat{\theta}(x-1,tq-1)), \tag{14}$$

$$P^{-1}(X,Tq)=P^{-1}(0,0)+\sum_{i=1}^{x}\sum_{j=1}^{tq}\varphi(i,j)\varphi^{\mathrm{T}}(i,j),$$
$$P(0,0)=p_0I,\quad p_0=10^6, \tag{15}$$

$$P^{-1}(X-1,tq)=P^{-1}(0,0)+\sum_{i=1}^{x-1}\varphi(i,tq)\varphi^{\mathrm{T}}(i,tq), \tag{16}$$

$$P^{-1}(x,Tq-1)=P^{-1}(0,0)+\sum_{j=1}^{tq-1}\varphi(x,j)\varphi^{\mathrm{T}}(x,j), \tag{17}$$

where $P(\cdot)$ is the covariance matrix, $P^{-1}(\cdot)$ is the inverse matrix of $P(\cdot)$ and I is an identity matrix. Unfortunately, this RLS algorithm is impossible to implement, because the information vector $\varphi(x,tq)$ on the right-hand side of (14) contains the missing outputs $y(x,tq-i), i=1,2\cdots,q-1$. If the system is a one-dimensional system, the lifting technique and the polynomial transformation technique can be used to overcome this difficulty. However, due to the complexity of the polynomial of the output, the lifting technique and the polynomial transformation technique cannot be used for this dual-rate two-dimensional system. In this paper, we use the MOI-RLS algorithm to overcome this difficulty.

The missing outputs are replaced with the outputs of an MOI model,

$$\hat{y}(x,tq-q+h)=\hat{\varphi}^{\mathrm{T}}(x,tq-q+h)\hat{\theta}(x,tq-q),$$
$$x=1,\cdots,M,\ h=1,2,\cdots,q-1,$$
$$\hat{\varphi}(x,tq-q+h)=[\hat{y}(x,tq-q+h-1),$$
$$\hat{y}(x,tq-q+h-2),\cdots,y(x,tq-q),\cdots,$$
$$\hat{y}(x,tq-q+h-n),\hat{y}(x-1,tq-q+h),$$
$$\hat{y}(x-2,tq-q+h),\cdots,\hat{y}(x-m,tq-q+h),$$
$$\hat{y}(x-1,tq-q+h-1),\hat{y}(x-1,tq-q+h-2),\cdots,$$
$$y(x-1,tq-q),\cdots,\hat{y}(x-1,tq-q+h-n),\cdots,$$
$$\hat{y}(x-m,tq-q+h-1),\hat{y}(x-m,tq-q+h-2),\cdots,$$
$$y(x-m,tq-q),\cdots,\hat{y}(x-m,tq-q+h-n),$$
$$u(x-1,tq-q+h-1)]^{\mathrm{T}},$$

where $\hat{y}(x,tq-q+h)$ represents the estimate of $y(x,tq-q+h)$ at $(x,tq-q+h)$, $\hat{\theta}(x,tq-q)$ represents the estimate of θ at $(x,tq-q)$, and $\hat{\varphi}(x,tq-q+h)$ represents the estimate of φ at $(x,tq-q+h)$.

Using the following MOI-RLS algorithm to estimate the parameter vector θ in (12):

$$\hat{\theta}(x,tq)=\hat{\theta}(x-1,tq-q)+P(X,Tq)P^{-1}(X-1,tq)$$
$$\times(\hat{\theta}(x-1,tq)-\hat{\theta}(x-1,tq-q))$$
$$+P(X,Tq)P^{-1}(x,Tq-q)(\hat{\theta}(x,tq-q)$$
$$-\hat{\theta}(x-1,tq-q))$$
$$+P(X,Tq)\hat{\varphi}(x,tq)(y(x,tq)$$
$$-\hat{\varphi}^{\mathrm{T}}(x,tq)\hat{\theta}(x-1,tq-q)), \tag{18}$$

$$\hat{\theta}(x,tq-q+h)=\hat{\theta}(x,tq-q),\ h=1,2,\cdots,q-1, \tag{19}$$

$$P^{-1}(X,Tq)=P^{-1}(0,0)+\sum_{i=1}^{x}\sum_{j=1}^{tq}\hat{\varphi}(i,j)\hat{\varphi}^{\mathrm{T}}(i,j),$$
$$P(0,0)=p_0I,\quad p_0=10^6, \tag{20}$$

$$P^{-1}(X-1,tq)=P^{-1}(0,0)+\sum_{i=1}^{x-1}\hat{\varphi}(i,tq)\hat{\varphi}^{\mathrm{T}}(i,tq), \tag{21}$$

$$P^{-1}(x,Tq-q)=P^{-1}(0,0)+\sum_{j=1}^{tq-q}\hat{\varphi}(x,j)\hat{\varphi}^{\mathrm{T}}(x,j), \tag{22}$$

$$\hat{y}(x,tq-q+h)=\hat{\varphi}^{\mathrm{T}}(x,tq-q+h)\hat{\theta}(x,tq-q), \tag{23}$$
$$\hat{\varphi}(x,tq-q+h)$$
$$=[\hat{y}(x,tq-q+h-1),\ \hat{y}(x,tq-q+h-2),\cdots,$$

$$y(x,tq-q),\ \cdots,\ \hat{y}(x,tq-q+h-n),\ \hat{y}(x-1,tq-q+h),\ \hat{y}(x-2,tq-q+h),\cdots,\ \hat{y}(x-m,tq-q+h),\ \hat{y}(x-1,tq-q+h-1),\ \hat{y}(x-1,tq-q+h-2),\ \cdots,\ y(x-1,tq-q),\ \cdots,\ \hat{y}(x-1,tq-q+h-n),\ \cdots,\ \hat{y}(x-m,tq-q+h-1),\ \hat{y}(x-m,tq-q+h-2),\ \cdots,\ y(x-m,tq-q),\ \cdots,\ \hat{y}(x-m,tq-q+h-n),\ u(x-1,tq-q+h-1)]^{\mathrm{T}}. \quad (24)$$

The steps of computing the parameter estimate $\hat{\theta}(x,tq)$ by the MOI-RLS algorithm are listed in the following.

1) Let $y(-i,-j)=0, v(-i,-j)=0, u(-i,-j)=0,\ i=0,1,2,\cdots,m-1,\ j=0,1,2,\cdots,n-1$ and give a small positive numbers ε.
2) Let $t=1$, $r(0)=1$, and $P(0,0)=p_0 I$ and $\hat{\theta}(0,0)=\mathbf{1}/p_0$ with $\mathbf{1}$ being a column vector whose entries are all unity and $p_0=10^6$.
3) Let $h=1$, $x=1$, collect the input-output data $y(x,tq), u(x,t)$.
4) Compute $\hat{y}(x,tq-q+h)$ by (23).
5) Increase x by 1 and compare x with M, if $x\leqslant M$, form $\hat{\varphi}(x,tq-q+h)$ by (24) and go to step 4; otherwise, go to step 6.
6) Let $x=1$ again, and increase h by 1, if $h\leqslant q-1$, go to step 4; otherwise, go to next step.
7) Form $\hat{\varphi}(x,tq)$ by (24), compute $P^{-1}(X,Tq)$, $P^{-1}(X-1,tq)$ and $P^{-1}(x,Tq-q)$ by (20), (21) and (22), respectively.
8) Update the parameter estimation vector $\hat{\theta}(x,tq)$ by (18).
9) Compare $\hat{\theta}(x,tq)$ and $\hat{\theta}(x-1,tq-q)$: if $\|\hat{\theta}(x,tq)-\hat{\theta}(x-1,tq-q)\|\leqslant\varepsilon$, then terminate the procedure and obtain $\hat{\theta}(x,tq)$; otherwise, increase x by 1 and go to next step.
10) Compare x and M: if $x\leqslant M$, go to step 7; otherwise, increase t by 1 and go to step 3.

Next, we will compute the original parameters of the DPS. Rewrite (10) as

$$\begin{aligned} y(x,tq)+y(x-2,tq)=&a_{0,1}y(x,tq-1)\\ &+a_{1,0}y(x-1,tq)\\ &+a_{1,1}y(x-1,tq-1)\\ &+b_{1,1}u(x-1,tq-1). \end{aligned}$$

Define the parameter vector θ and the information vector $\varphi(x,tq)$ as

$$\theta:=[a_{0,1},a_{1,0},a_{1,1},b_{1,1}]^{\mathrm{T}}, \quad (25)$$

$$\varphi(x,tq):=[y(x,tq-1),y(x-1,tq),\ y(x-1,tq-1),u(x-1,tq-1)]^{\mathrm{T}}. \quad (26)$$

By using the estimated parameters $\{\hat{a}_{0,1},\hat{a}_{1,0},\hat{a}_{1,1},\hat{b}_{1,1}\}$, we can compute $\hat{r}_1=\frac{1}{\hat{a}_{1,0}-2}$, $\hat{r}_2=-\hat{r}_1\hat{a}_{0,1}$, $\hat{r}_3=-\hat{r}_1\hat{a}_{1,1}+\hat{r}_2-1$ and $\hat{r}_4=-\hat{r}_1\hat{b}_{1,1}$. Because Δx and Δt are already known, we can compute $\hat{a}_1=\frac{\hat{r}_1\Delta x^2}{\Delta t}$, $\hat{a}_2=\frac{\hat{r}_2\Delta x}{\Delta t}$, $\hat{a}_3=\frac{\hat{r}_3}{\Delta t}$ and $\hat{a}_4=\frac{\hat{r}_4}{\Delta t}$ by the estimates $\hat{r}_1$, $\hat{r}_2$, $\hat{r}_3$ and $\hat{r}_4$.

4. NUMERICAL RESULTS

Consider the following temperature distributed parameter system of large-scale vertical quench furnace in [37],

$$\begin{aligned} \frac{\partial y(x,t)}{\partial t}=&a_1\frac{\partial^2 y(x,t)}{\partial x^2}+a_2\frac{\partial y(x,t)}{\partial x}\\ &+a_3y(x,t)+a_4u(x,t), \end{aligned}$$

where $y(x,t)$ is the temperature of the large-scale vertical quench furnace and $u(x,t)$ is the current of the heating element, $[a_1,a_2,a_3,a_4]=[0.1,0.4,-5,6]$. Assume $L=0.4$ and $K=-5$. Since $L\leqslant\frac{\Delta x}{2\Delta t}$ and $-\frac{1}{2\Delta t}\leqslant K$, one can choose $\Delta t=0.1$ and $\Delta x=0.1$. Then, we have $r_1=1$, $r_2=0.4$, $r_3=-0.5$ and $r_4=0.6$, and

$$\begin{aligned} \theta&=[a_{0,1},a_{1,0},a_{1,1},b_{1,1}]^{\mathrm{T}}=[-0.4,3,-0.1,-0.6]^{\mathrm{T}},\\ \varphi(x,t)&=[y(x,t-1),y(x-1,t),y(x-1,t-1),\\ &\qquad u(x-1,t-1)]^{\mathrm{T}}. \end{aligned}$$

Then the DPS can be simplified as a two-dimensional system. Assume that the two-dimensional system is a dual-rate system with two different updating periods $q=2$ and $q=3$, then we can get

$$y(x,t)+y(x-2,t)=\varphi^{\mathrm{T}}(x,t)\theta+v(x,t).$$

In simulation, the input $\{u(x,t)\}$ is taken as a persistent excitation signal sequence with zero mean and unit variance, and $\{v(x,t)\}$ is taken as a white noise sequence with zero mean and variance $\sigma^2=0.10^2$. The noise-to-signal ratio is $\delta_{ns}=16.667\%$.

Define the parameter estimation error as

$$\delta:=\|\hat{\theta}-\theta\|/\|\theta\|.$$

Assume $M=50$, then apply the MOI-RLS algorithm to estimate the parameters. The parameter estimates and their errors are shown in Tables 1-2, and the parameter estimation errors δ versus (x,t) are shown in Fig. 2.

From Tables 1-2 and Fig. 2, we can conclude:

1. The parameter estimation errors become smaller and smaller and go to zero with (x,t) increasing.
2. When $q=2$, we can compute the original parameters $[a_1,a_2,a_3,a_4]^{\mathrm{T}}=[0.099,0.399,-5.012,5.992]^{\mathrm{T}}$ by using the parameter estimate $\hat{\theta}(50,50)$.
3. When q becomes larger, the parameter estimation errors also become larger because of too much outputs are missing.

Table 1. The MOI-RLS algorithm estimates and errors ($q = 2$).

(x,t)	$a_{0,1}$	$a_{1,0}$	$a_{1,1}$	$b_{1,1}$	δ (%)
$(5,5)$	−0.33974	4.01138	−0.03762	−0.31474	32.55981
$(10,10)$	−0.40872	2.76224	−0.09443	−0.42255	9.15148
$(15,15)$	−0.39827	2.82413	−0.11600	−0.48578	6.48163
$(20,20)$	−0.39900	3.00628	−0.09972	−0.51248	2.70632
$(30,30)$	−0.40003	3.01096	−0.09859	−0.56634	1.09169
$(40,40)$	−0.40007	3.00411	−0.09937	−0.58447	0.49553
$(50,50)$	−0.40004	3.00037	−0.09995	−0.59977	0.40364
True Values	−0.40000	3.00000	−0.10000	−0.60000	

Table 2. The MOI-RLS algorithm estimates and errors ($q = 3$).

(x,t)	$a_{0,1}$	$a_{1,0}$	$a_{1,1}$	$b_{1,1}$	δ (%)
$(5,5)$	−0.83309	4.92312	−0.48469	0.28315	89.05208
$(10,10)$	−0.32622	3.68315	−0.07661	−0.37172	22.74399
$(15,15)$	−0.45332	2.32576	−0.19272	−0.51357	22.03352
$(20,20)$	−0.40905	2.40601	−0.15711	−0.52564	18.59932
$(30,30)$	−0.39676	2.93020	−0.10138	−0.56119	2.46592
$(40,40)$	−0.40034	2.96097	−0.09805	−0.57887	1.36904
$(50,50)$	−0.40006	2.94766	−0.10002	−0.59576	1.61822
True Values	−0.40000	3.00000	−0.10000	−0.60000	

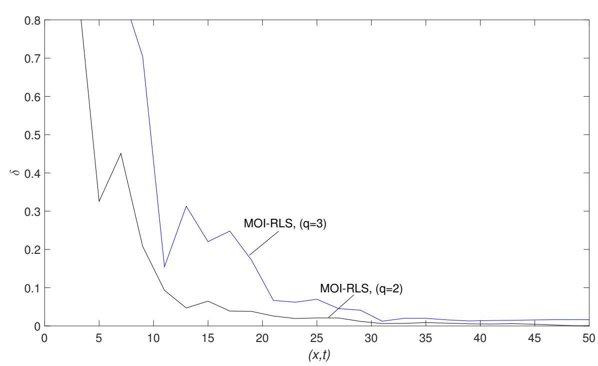


Fig. 2. The parameter estimation errors δ versus (x,t)

5. CONCLUSIONS

This paper proposes an identification method for a distributed parameter system with missing observations. Based on Von. Neumann stability analysis, the distributed parameter system can be simplified as a lumped parameter system. Then a missing output identification model based recursive least squares algorithm is derived to estimate the unknown parameters of the lumped parameter system. Different from the lifting technique and the polynomial transformation technique, the method in this paper can be used for dual-rate two-dimensional system identification, and can estimate the unknown parameters directly. Furthermore, this method can also be extended to fault detection, diagnosis and estimation [38–42].

REFERENCES

[1] P. F. Huang, Z. Y. Lu, and Z. X. Liu, "State estimation and parameter identification method for dual-rate system based on improved Kalman prediction," *International Journal of Control, Automation, and Systems*, vol. 14, no. 4, pp. 986-997, August 2016.

[2] J. Chen, "Several gradient parameter estimation algorithms for dual-rate sampled systems," *Journal of the Franklin Institute*, vol. 351, no. 1, pp. 543-554, January 2014. [click]

[3] J. Chen, Y. Zhang, and R. F. Ding, "Gradient-based parameter estimation for input nonlinear systems with ARMA noises based on the auxiliary model," *Nonlinear Dynamics*, vol. 72, no. 4, pp. 865-871, January 2013. [click]

[4] C. Wang and T. Tang, "Recursive least squares estimation algorithm applied to a class of linear-in-parameters output error moving average systems," *Applied Mathematics Letters*, vol. 29, no. 1, pp. 36-41, January 2014. [click]

[5] P. P. Hu, F. Ding, and J. Sheng, "Auxiliary model based least squares parameter estimation algorithm for feedback nonlinear systems using the hierarchical identification principle," *Journal of the Franklin Institute*, vol. 350, no. 10, pp. 3248-3259, December 2013. [click]

[6] F. Ding, "Combined state and least squares parameter estimation algorithms for dynamic systems," *Applied Mathematical Modelling*, vol. 38, no. 1, pp. 403-412, January 2014. [click]

[7] J. Chen and B. Jiang, "Modified stochastic gradient parameter estimation algorithms for a nonlinear two-variable difference system," *International Journal of Control, Automation, and Systems*, vol. 14, no. 6, pp. 1493-1500, December 2016. [click]

[8] J. Pan, X. Jiang, X. K. Wan, and W. F. Ding, "A filtering based multi-innovation extended stochastic gradient algorithm for multivariable control systems," *International Journal of Control, Automation, and Systems*, vol. 15, no. 3, pp. 1189-1197, June 2017.

[9] Z. Y. Wang, Y. Wang, and Z. C. Ji, "Stochastic gradient algorithm for multi-input multi-output Hammerstein FIRMA-like systems using the data filtering," *Journal of the Franklin Institute*, vol. 352, no. 4, pp. 1440-1454, April 2015. [click]

[10] J. Vörös, "Modeling and identification of systems with backlash," *Automatica*, vol. 46, no. 2, pp. 369-374, February 2010. [click]

[11] J. X. Ma, W. L. Xiong, and F. Ding, "Iterative identification algorithms for input nonlinear output error autoregressive systems," *International Journal of Control, Automation, and Systems*, vol. 14, no. 1, pp. 140-147, January 2016. [click]

[12] X. Y. Ma and F. Ding, "Recursive and iterative least squares parameter estimation algorithms for observability canonical state space systems," *Journal of the Franklin Institute*, vol. 352, no. 1, pp. 248-258, January 2015.

[13] L. Ding, A. Johansson, and T. Gustafsson, "Application of reduced models for robust control and state estimation of a distributed parameter system," *Journal of Process Control*, vol. 19, no. 3, pp. 539-549, March 2009. [click]

[14] M. Patan, "Optimal activation policies for continuous scanning observations in parameter estimation for distributed systems," *International Journal of Systems Science*, vol. 37, no. 11, pp. 763-775, September 2006. [click]

[15] X. W. Li and H. J. Gao, "Reduced-order generalized filtering for linear discrete-time systems with application to channel equalization," *IEEE Transactions on Signal Processing*, vol. 62, no. 13, pp. 3393-3402, May 2014. [click]

[16] L. X. Zhang, S. L. Zhuang, and P. Shi, "Non-weighted quasi-time-dependent H_∞ filtering for switched linear systems with persistent dwell-time," *Automatica*, vol. 54, pp. 201-209, April 2015. [click]

[17] L. G. Wu, R. Yang, P. Shi, and X. Su, "Stability analysis and stabilization of 2-D switched systems under arbitrary and restricted switchings," *Automatica*, vol. 59, pp. 206-215, September 2015. [click]

[18] H. Li and Y. Shi, "Robust distributed model predictive control of constrained continuous-time nonlinear systems: A robustness constraint approach," *IEEE Transactions on Automatic Control*, vol. 59, no. 6, pp. 1673-1678, December 2013. [click]

[19] J. Zhao, B. Jiang, Z. He, and Z. Mao, "Modelling and fault tolerant control for near space vehicles with vertical tail loss," *IET Control Theory and Applications*, vol. 8, no. 9, pp. 718-727, June 2014. [click]

[20] C. P. Lee and H. C. Khatri, "Identification of distributed parameter systems using finite differences," *Journal of Basic Engineering*, vol. 91, no. 2, pp. 239-244, January 1969.

[21] D. Coca and S. A. Billings, "Direct parameter identification of distributed parameter systems," *International Journal of Systems Science*, vol. 31, no. 1, pp. 11-17, November 2000. [click]

[22] C. K. Qi, H. X. Li, X. X. Zhang, X. C. Zhao, S. Y. Li, and F. Gao, "Time/space separation based SVM modeling for nonlinear distributed parameter processes," *Industrial & Engineering Chemistry Research*, vol. 50, no. 1, pp. 332-341, January 2011. [click]

[23] H. X. Li and C. K. Qi, "Modeling of distributed parameter systems for applications-A synthesized review from time-space separation," *Journal of Process Control*, vol. 20, no. 8, pp. 891-901, September 2010. [click]

[24] D. G. Zill and M. R. Cullen, *Differential equations with boundary-value problems*, 5th edn. Brooks/Cole Thomson Learning Pacific Grove CA Australia, 2001.

[25] H. X. Li and C. K. Qi, *Spatio-temporal modeling of nonlinear distributed parameter systems-A time/space separation based approach*, Springer, 2011.

[26] J. Chen and B. Jiang, "Identification methods for two-variable difference systems," *Circuit, Systems and Signal Processing*, vol. 35, no. 8, pp. 3027-3039, August 2016. [click]

[27] D. Q. Wang, H. B. Liu, and F. Ding, "Highly efficient identification methods for dual-rate Hammerstein systems," *IEEE Transactions on Control Systems Technology*, vol. 23, no. 5, pp. 1952-1960, January 2015. [click]

[28] D. Q. Wang, Z. Zhang, and J. Y. Yuan, "Maximum likelihood estimation method for dual-rate Hammerstein systems" *International Journal of Control, Automation, and Systems*, vol. 15, no. 2, pp. 698-705, April 2017. [click]

[29] Y. J. Liu, F. Ding, and Y. Shi, "An efficient hierarchical identification method for general dual-rate sampled-data systems," *Automatica*, vol. 50, no. 3, pp. 962-970, March 2014. [click]

[30] F. Ding and T. Chen, "Combined parameter and output estimation of dual-rate systems using an auxiliary model," *Automatica*, vol. 40, no. 10, pp. 1739-1748, October 2004. [click]

[31] B. Yu, Y. Shi, and H. Huang, "$l_2 - l_\infty$ filtering for multirate systems using lifted models," *Circuit, Systems and Signal Processing*, vol. 27, no. 5, pp. 699-711, October 2008.

[32] M. Sahebsara, T. Chen, and S. L. Shah, "Frequency-domain parameter estimation of general multi-rate systems," *Computers & Chemical Engineering*, vol. 30, no. 5, pp. 838-849, April 2006.

[33] Y. J. Liu, F. Ding, and Y. Shi, "Least squares estimation for a class of non-uniformly sampled systems based on the hierarchical identification principle," *Circuits, Systems and Signal Processing*, vol. 31, no. 6, pp. 1985-2000, December 2012.

[34] J. Chen, L. X. Lv, and R. F. Ding, "Multi-innovation stochastic gradient algorithms for dual-rate sampled systems with preload nonlinearity," *Applied Mathematics Letters*, vol. 26, no. 1, pp. 124-129, January 2013.

[35] M. Ali, S. Chughtai, and H. Werner, "Identification of spatially interconnected systems," *Proceedings of the 48th IEEE Conference on Decision and Control*, pp. 7163-7168, 2009.

[36] L. M. Surhone, M. T. Timpledon, S. F. Marseken, N. Analysis, and F. D. Scheme, *Von Neumann Stability Analysis*, Betascript Publishing, 2010.

[37] S. Y. Yu, Y. B. Cao, and X. Zhou, "Algorithm of parameter identification for temperature distributed parameter system of large-scale vertical quench furnace," *Journal of Central South University*, vol. 39, no. 6, pp. 1285-1290, December 2008.

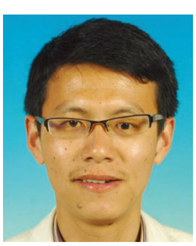
[38] B. Jiang, K. Zhang, and P. Shi, "Integrated fault estimation and accommodation design for discrete-time Takagi-Sugeno fuzzy systems with actuator faults," *IEEE Transactions on Fuzzy Systems*, vol. 19, no. 2, pp. 291-304, April 2011. [click]

[39] B. Jiang, H. Yang, and V. Cocquempot, "Results and perspectives on fault tolerant control for a class of hybrid systems," *International Journal of Control*, vol. 84, no. 2, pp. 396-411, February 2011. [click]

[40] B. Jiang , Q. Shen, and P. Shi, "Neural-networked adaptive tracking control for switched nonlinear pure-feedback systems under arbitrary switching," *Automatica*, vol. 61, pp. 119-125, November 2015. [click]

[41] T. Li, G. Li, and Q. Zhao, "Adaptive fault-tolerant stochastic shape control with application to particle distribution control," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 45, no. 12, pp. 1592-1604, June 2015. [click]

[42] T. Li and W. X. Zheng, "Networked-based generalised H_∞ fault detection filtering for sensor faults," *International Journal of Systems Science*, vol. 46, no. 5, pp. 831-840, May 2013. [click]



Jing Chen received his B.Sc. degree in School of Mathematical Science and M.Sc. degree in School of Information Engineering from Yangzhou University (Yanghzou, China), in 2003 and 2006, respectively, and received his Ph.D. degree in the School of Internet of Things Engineering, Jiangnan University (Wuxi, China) in 2013. He is currently an associate professor in College of Science, Jiangnan University (Wuxi, China). His research interests include Processing Control and system identification.

Bin Jiang received the Ph.D. degree in Automatic Control from Northeastern University (Shenyang, China) in 1995. He had ever been postdoctoral fellow, research fellow, invited professor and visiting professor in Singapore, France, USA and Canada, respectively. Now he is a Chair Professor of Cheung Kong Scholar Program in Ministry of Education and Dean of College of Automation Engineering in Nanjing University of Aeronautics and Astronautics (Nanjing, China). He currently serves as Associate Editor or Editorial Board Member for a number of journals such as IEEE Trans. On Control Systems Technology; IEEE Trans. On Fuzzy Systems; Int. J. Of Control, Automation and Systems; Nonlinear Analysis: Hybrid Systems, etc. He is a senior member of IEEE, Chair of Control Systems Chapter in IEEE Nanjing Section, a member of IFAC Technical Committee on Fault Detection, Supervision, and Safety of Technical Processes. His research interests include intelligent fault diagnosis and fault tolerant control and their applications.

Juan Li received her B.Sc degree in School of Automation Engineering from Shandong University (Jinan, China) in 1994, an M.Sc. degree in School of Electrical Engineering and Information Engineering from Lanzhou Polytechnic University (Lan-zhou, China) in 2000, and a Ph.D. degree in School of Information Science and Engineering from Ocean University of China (Qingdao, China) in 2008. She was a Visiting Professor in School of Automation, Tsinghua University (Beijing, China) from 2010 to 2011 and in the Department of Chemical and Material Engineering, University of Alberta (Edmonton, Canada) from 2015 to 2016, respectively. Her current research interests include fault diagnosis and fault-tolerant control and intelligent detection and control.