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Actuator Fault Detection for Discrete-time Switched Linear Systems with Output Disturbance

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Abstract: In this paper, the problem of actuator fault detection for discrete-time switched systems with output disturbance is investigated. By using the descriptor observer method, an H_∞ fault detection filter is designed to guarantee that the augmented system is admissible and satisfies a prescribed H_∞ performance index. By utilizing switched Lyapunov function approach, a sufficient condition for the admissible of the augmented system is obtained. Based on the obtained results, a desired fault detection filter can be designed. All the results are formulated in the form of linear matrix inequalities. Finally, an example is proposed to illustrate the effectiveness of the developed method.

Keywords: Actuator fault, descriptor observer, fault detection filter, switched system.

1. INTRODUCTION

Switched system belongs to hybrid systems, which is consisted by a number of subsystems and a switching signal specifying the switching between them [1]. In fact, many practical engineering systems can be modeled as switched systems, such as electrical engineering systems, traffic control systems, and chemical process systems and so on [2, 3]. Presently, the study of control and synthesis for switched systems is becoming a hot research topic, and many researchers have devoted themselves to investigating this challenging and meaningful issue. In the past few decades, a lot of achievements have been developed, readers can refer to [4]- [9] and the references therein.

It's well known that practical engineering systems are unavoidable affected by unexpected variations in external surroundings or sudden changes in signals. Due to these accidental reasons, different kinds of malfunctions or imperfect behaviors may appear during normal operations, and these phenomena are called faults. Generally speaking, faults can occur more easily in switched systems than those in non-switched systems for the reason of the large number of subsystems and the arbitrary switching between them. In reality, once an unexpected fault happens in one of the subsystem and is not detected in time, which might bring a big impact on the whole system and even cause potentially catastrophic damage [10]. Therefore, the investigation of fault detection problems for switched systems is of both theoretical and practical importance.

In this context, many theories and techniques have been developed in dealing with fault detection problem ([11]- [17] and references therein). Presently, several popular methods have been employed in dealing with this issue. For examples, [18] proposed a model-based fault detection approach, while the parameter estimation approach was investigated to solve the problem in [19]. [20] considered the fault detection problem by using generalized likelihood method. Among these methods, the model-based method is the most common way, which is to design a fault detection filter or observer generating a residual and compares it with a predefined threshold. When the residual evaluation function has a value larger than the threshold, an alarm is generated. For example, fault detection filter design for linear time-invariant systems was solved in [21]. Fault detection issues for Markovian jump systems and uncertain fuzzy systems were separately studied in [22] and [23].

In reviewing of the development of these theories and techniques for different fault detection system designs, one of the commonly adopted ways in fault detection is to introduce a performance index and formulate the fault detection as an optimization problem [24]. The H_∞ norm of transfer-function matrix from unknown input to residual is accepted as a suitable and effective measure to estimate the influence of the unknown inputs; the H_∞ norm of

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transfer function from fault to residual has been proposed to evaluate the system sensitivity to the faults. In [25], an H_∞ filtering formulation of fault detection has also been presented to solve robust fault detection for uncertain systems. Different from the former scheme, the later one is to make the error between residual and fault (or, more generally, weighted fault) as small as possible, and provides an effective approach to uncertain system fault detection with the aid of an optimization tool [26].

In this paper, the problem of actuator fault detection for discrete-time switched systems with output disturbance is investigated. The advantages can be summarized as the following aspects: firstly, for control input, actuator fault and unknown output disturbance in the discrete-time switched system, a descriptor-based fault detection filter is constructed such that the residual system is admissible with H_∞ performance index; secondly, a sufficient condition for the admissibility of the residual system is obtained; thirdly, compared with other fault detection schemes, descriptor observer approach can not only leads to a simple design procedure, but also can observe disturbance signal. Therefore the descriptor approach doesn't need any prior knowledge of disturbance signal.

The rest of the paper is organized as follows. Section 2 formulates the problem under consideration. Section 3 presents the fault detection filter design for the discrete-time switched system. A numerical example is illustrated in Section 4 to show the usefulness and applicability of the proposed approach, and the paper is concluded in Section 5.

2. PROBLEM STATEMENTS AND PRELIMINARIES

Consider the following discrete-time switched system:

$$x(t+1)=\sum_{i=1}^{N}\delta_i(t)(A_ix(t)+B_iu(t)+F_if(t)), \quad (1)$$

$$y(t)=\sum_{i=1}^{N}\delta_i(t)(C_ix(t)+D_id(t)), \quad (2)$$

where $x(t)\in\mathbb{R}^n$ is the state vector, $u(t)\in\mathbb{R}^m$ is the control input vector, $d(t)\in\mathbb{R}^d$ is the output disturbance vector, $f(t)\in\mathbb{R}^f$ is the fault vector, and $y(t)\in\mathbb{R}^p$ is the measurable output vector. The function $\delta_i(t)$ is the switching signal and assumed be *a priori* unknown, but its instantaneous value is available in real time, where

$$\delta_i(t):Z^+=\{0,1,2\cdots\}\to\{0,1\},\ \sum_{i=1}^{N}\delta_i(t)=1,\ \forall t\in Z^+.$$

A_i, B_i, C_i, D_i, and F_i are constant matrices with appropriate dimensions.

For the purpose of this paper, the following assumptions are given:

- (A_i,C_i) is observable for $i=1,\cdots,N$.
- Matrices D_i $(i=1,\cdots,N)$ is full column rank.

Define the following augmented state and matrices:

$$\bar{x}(t)=\begin{bmatrix} x(t)\\ d(t)\end{bmatrix},\ \bar{E}=\begin{bmatrix} I & 0\\ 0 & 0\end{bmatrix},\ \bar{A}_i=\begin{bmatrix} A_i & 0\\ 0 & -D_i\end{bmatrix},$$
$$\bar{B}_i=\begin{bmatrix} B_i\\ 0\end{bmatrix},\ \bar{D}_i=\begin{bmatrix} 0\\ D_i\end{bmatrix},\ \bar{F}_i=\begin{bmatrix} F_i\\ 0\end{bmatrix},$$
$$\bar{C}_i=\begin{bmatrix} C_i & D_i\end{bmatrix}. \quad (3)$$

Then the system (1)-(2) can be expressed as the following descriptor system:

$$\bar{E}\bar{x}(t+1)=\sum_{i=1}^{N}\delta_i(t)(\bar{A}_i\bar{x}(t)+\bar{B}_iu(t)+\bar{D}_id(t)+\bar{F}_if(t)), \quad (4)$$

$$y(t)=\sum_{i=1}^{N}\delta_i(t)\bar{C}_i\bar{x}(t). \quad (5)$$

For system (4)-(5), we employ the following filter as residual generator:

$$\hat{\bar{x}}(t+1)=\sum_{i=1}^{N}\delta_i(t)(A_{fi}\hat{\bar{x}}(t)+B_{fi}y(t)), \quad (6)$$

$$r(t)=\sum_{i=1}^{N}\delta_i(t)(C_{fi}\hat{\bar{x}}(t)+D_{fi}y(t)), \quad (7)$$

where $\hat{\bar{x}}(t)\in\mathbb{R}^{n+d}$ is the filter's state, $r(t)\in\mathbb{R}^p$ is the so-called residual signal. The matrices A_{fi}, B_{fi}, C_{fi}, and D_{fi} are the filter parameters to be determined.

Remark 1: From the formation of the filter (6)-(7), one can get that the magnitude of the original system's state $x(t)$ and the output disturbance $d(t)$ of system (1)-(2) can be simultaneously estimated.

In order to detect the actuator fault $f(t)$ more efficient and fast, sometimes one is more interested in the fault signal within a certain frequency interval, which can be formulated as the weighted fault $\bar{f}(s)=W_f(s)f(s)$ with $W_f(s)$ being a given stable weighting matrix. One minimal state space realization of $\bar{f}(s)=W_f(s)f(s)$ is supposed to be

$$\tilde{x}(t+1)=A_W\tilde{x}(t)+B_Wf(t), \quad (8)$$
$$\bar{f}(t)=C_W\tilde{x}(t)+D_Wf(t), \quad (9)$$

where $\tilde{x}(t)\in\mathbb{R}^{nw}$ is the state of the weighted fault, $\bar{f}(t)\in\mathbb{R}^f$ is the weighted fault. A_W, B_W, C_W, and D_W are known constant matrices with appropriate dimensions.

Remark 2: As the technique developed in [24], a suitable weighting matrix $W_f(s)$ is introduced to limit the frequency interval of the interested fault, which can definitely improve the fault detection performance of the systems. From another aspect to see, the use of weighted fault $\bar{f}(t)$ is more general than using the original fault $f(t)$, because if we impose $W_f(s)=I$, we can obtain $\bar{f}(t)=f(t)$.

Denoting

$$e(t)=\begin{bmatrix} \bar{x}^T(t) & \hat{\bar{x}}^T(t) & \tilde{x}^T(t) \end{bmatrix}^T, \tag{10}$$

$$\omega(t)=\begin{bmatrix} u^T(t) & d^T(t) & f^T(t) \end{bmatrix}^T, \tag{11}$$

$$r_e(t)=r(t)-\bar{f}(t). \tag{12}$$

Augmenting the model of system (4)-(5) to include the states of (6)-(7) and (8)-(9), the following augmented system can be obtained:

$$\tilde{E}e(t+1)=\sum_{i=1}^{N}\delta_i(t)(A_{ei}e(t)+B_{ei}\omega(t)), \tag{13}$$

$$r_e(t)=\sum_{i=1}^{N}\delta_i(t)(C_{ei}e(t)+D_{ei}\omega(t)), \tag{14}$$

where

$$\tilde{E}=\begin{bmatrix} \bar{E} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix},\ A_{ei}=\begin{bmatrix} \bar{A}_i & 0 & 0 \\ B_{fi}\bar{C}_i & A_{fi} & 0 \\ 0 & 0 & A_W \end{bmatrix},$$

$$B_{ei}=\begin{bmatrix} \bar{B}_i & \bar{D}_i & \bar{F}_i \\ 0 & 0 & 0 \\ 0 & 0 & B_W \end{bmatrix},\ D_{ei}=\begin{bmatrix} 0 & 0 & -D_W \end{bmatrix},$$

$$C_{ei}=\begin{bmatrix} D_{fi}\bar{C}_i & C_{fi} & -C_W \end{bmatrix}. \tag{15}$$

After designing the residual generator (6)-(7), the last step to a successful fault detection is the residual evaluation stage, which needs to construct an residual evaluation function and a threshold. In this paper, the residual evaluation function $J_L(r(t))$ and the threshold J_{th} are selected as

$$J_L(r(t))=\|r(t)\|_2=\left(\sum_{k=k_0}^{k_0+L} r^T(t)r(t)\right)^{\frac{1}{2}}, \tag{16}$$

$$J_{th}=\sup_{u(t)\in l_2,d(t)\in l_2,f(t)=0}\|r(t)\|_2, \tag{17}$$

where k_0 denotes the initial evaluation time instant, L is the evaluation time steps. Based on this, the occurrence of the fault can be successfully detected by comparing $J_L(r(t))$ and J_{th} according to the following regulation:

$$J_L(r(t))>J_{th}\Longrightarrow with\ \ faults\Longrightarrow alarm, \tag{18}$$

$$J_L(r(t))\leq J_{th}\Longrightarrow no\ \ faults. \tag{19}$$

In order to minimize the effect of the disturbance and improve the sensitivity of the residual to the fault, the fault detection filter design can be formulated as an H_∞ filter problem. i.e., the problem to be addressed in this work is expressed as follows: to develop the filter (6)-(7) for the system (4)-(5) such that the augmented system (13)-(14)

- is asymptotically stable when $\omega(t)=0$; and
- under zero-initial condition, the minimum of γ is made small in the feasibility of

$$\sup_{\|\omega(t)\|_2\neq 0}\frac{\|r_e(t)\|_2}{\|\omega(t)\|_2}<\gamma,\quad \gamma>0. \tag{20}$$

For the purpose of this paper, we give the following definitions.

Definition 1 [27]**:** Consider the switched system

$$Ex(t+1)=\sum_{i=1}^{N}\delta_i(t)A_ix(t),$$

- the pair (E,A_i) is said to be regular if $det(sE-A_i)$ is not identically zero for all $i=1,\cdots,N$.
- the pair (E,A_i) is said to be causal if $deg(det(sE-A_i))=rankE$ for all $i=1,\cdots,N$.
- the pair (E,A_i) is said to be regular and causal if every pair (E,A_i) is regular and causal for all $i=1,\cdots,N$.

Definition 2 [28]**:** System (1)-(2) with $u(t)=0$, $d(t)=0$, and $f(t)=0$, is said to be

- regular, causal, if the pair (E,A_i) is regular, causal;
- uniformly asymptotically stable, if for any $\varepsilon>0$, there is a $\vartheta(\varepsilon)>0$ such that for arbitrary switching signal $\delta_i(t)$, $sup\|\phi(s)\|<\vartheta$ implies $\|x(t)\|<\varepsilon$ for all $t\geq 0$, and there is a $\vartheta'(\varepsilon)>0$ such that for arbitrary switching signal $\delta_i(t)$, $sup\|\phi(s)\|<\vartheta'$ implies $x(t)\to 0$ as $t\to\infty$.

Definition 3: System (13)-(14) with $\omega(t)=0$ is said to be uniformly asymptotically stable with γ-disturbance attenuation if system (13)-(14) with $\omega(t)=0$ is regular, causal, uniformly asymptotically stable, and for a given scalar $\gamma>0$, for any disturbance $\omega(t)\in l_2[0,\infty)$, the following H_∞ performance is satisfied:

$$\sum_{t=0}^{\infty} r_e^T(t)r_e(t)\leq\sum_{t=0}^{\infty}\gamma^2\omega^T(t)\omega(t). \tag{21}$$

Remark 3: Regularity and causality of the switched system (13) with $\omega(t)=0$ ensure that for the arbitrary switching signal $\delta_i(t)$, the solution to this system exists and is unique for any compatible initial conditions. However, even if the switched system (13) is regular and causal, it still has finite jumps because of the incompatible initial conditions caused by subsystems switching. These jumps are generally unavoidable [29]. In this paper, for simplicity, we assume that such finite jumps cannot destroy the stability of the switched system.

3. FAULT DETECTION FILTER DESIGN

In this section, the fault detection filter design problem will be solved for the descriptor discrete-time switched systems (4)-(5). The fault detection analysis problem is firstly solved, and then based on this, a full-rank fault detection filter is designed.

3.1. Fault detection analysis

Theorem 1: For a prescribed scalar $\gamma > 0$, the residual system (13)-(14) is admissible with H_∞ performance γ if there exit positive definite matrices P_i, P_j, and matrices R and Q_i for all $i, j = 1, \cdots, N$ such that

$$\begin{bmatrix} \Lambda_i & 0 & C_{ei}^T & A_{ei}^T \\ * & -\gamma^2 I & D_{ei}^T & B_{ei}^T \\ * & * & -I & 0 \\ * & * & * & -P_j^{-1} \end{bmatrix} < 0, \tag{22}$$

where $\Lambda_i = -\tilde{E}^T P_i \tilde{E} + A_{ei}^T R Q_i^T + Q_i R^T A_{ei}$, and R is any matrix with full column which satisfies $\tilde{E}^T R = 0$.

Proof: Firstly, we prove the system (13)-(14) with $\omega(t) = 0$ is admissible. When $\omega(t) = 0$, the system (13)-(14) is written as follows:

$$\tilde{E}e(t+1) = \sum_{i=1}^{N} \delta_i(t) A_{ei} e(t), \tag{23}$$

$$r_e(t) = \sum_{i=1}^{N} \delta_i(t) C_{ei} e(t). \tag{24}$$

Suppose that there exist nonsingular matrices G and H such that

$$G\tilde{E}H = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \ GA_{ei}H = \begin{bmatrix} A_{11ei} & A_{12ei} \\ A_{21ei} & A_{22ei} \end{bmatrix},$$
$$G^{-T} P_i G^{-1} = \begin{bmatrix} P_{11i} & P_{12i} \\ P_{21i} & P_{22i} \end{bmatrix}. \tag{25}$$

Then we denote

$$R = G^T \begin{bmatrix} 0 \\ I \end{bmatrix} N, \ Q_i = H^{-T} \begin{bmatrix} Q_{1i} \\ Q_{2i} \end{bmatrix} N^{-T}. \tag{26}$$

From inequality (22), we have

$$A_{ei}^T P_j A_{ei} - \tilde{E}^T P_i \tilde{E} + A_{ei}^T R Q_i^T + Q_i R^T A_{ei} < 0. \tag{27}$$

Pre- and post-multiplying inequality (27) by H^T and H respectively, we have

$$\begin{bmatrix} \star & \star \\ \star & \Gamma_i \end{bmatrix} < 0, \tag{28}$$

where $\star$ denotes submatrix which is not relevant to our discussion and the form of Γ_i is shown as follows:

$$\begin{aligned} \Gamma_i =& A_{12ei}^T P_{11j} A_{12ei} + A_{22ei}^T P_{21j} A_{12ei} \\ &+ A_{12ei}^T P_{12j} A_{22ei} + A_{22ei}^T P_{22j} A_{22ei} \\ &+ A_{22ei}^T Q_{2i}^T + Q_{2i} A_{22ei} - P_{22i} < 0, \end{aligned} \tag{29}$$

which implies that matrices A_{22ei} are nonsingular. Therefore, under the condition that $\omega(t) = 0$, one can conclude that system (13)-(14) is regular and causal from Definition 1.

Consider the following switched Lyapunov function:

$$V(e(t)) = e^T(t) \tilde{E}^T \left(\sum_{i=1}^{N} \delta_i(t) P_i \right) \tilde{E} e(t). \tag{30}$$

Define $\Delta V(t) = V(t+1) - V(t)$, then along the solution of (13), we have

$$\begin{aligned} \Delta V(t) =& e^T(t+1) \tilde{E}^T \left(\sum_{i=1}^{N} \delta_i(t+1) P_i \right) \tilde{E} e(t+1) \\ &- e^T(t) \tilde{E}^T \left(\sum_{i=1}^{N} \delta_i(t) P_i \right) \tilde{E} e(t) \\ =& e^T(t) \left[\sum_{i=1}^{N} \delta_i(t) A_{ei}^T \left(\sum_{i=1}^{N} \delta_i(t+1) P_i \right) \sum_{i=1}^{N} \delta_i(t) A_{ei} \right. \\ &\left. - \tilde{E}^T \left(\sum_{i=1}^{N} \delta_i(t) P_i \right) \tilde{E} \right] e(t). \end{aligned} \tag{31}$$

For any nonzero state vector $e(t)$ and the particular case $\delta_i(t) = 1$, $\delta_{l \neq i}(t) = 0$, and $\delta_j(t+1) = 1$, $\delta_{l \neq j}(t+1) = 0$, one can get

$$\Delta V(t) = e^T(t) \left(A_{ei}^T P_j A_{ei} - \tilde{E}^T P_i \tilde{E} \right) e(t) \tag{32}$$

Notice that $\tilde{E}^T R = 0$, then

$$\begin{aligned} 0 =& 2e^T(t+1) \tilde{E}^T R \left(\sum_{i=1}^{N} \delta_i(t) Q_i \right) \tilde{E} e(t+1) \\ =& e^T(t) \left[\left(\sum_{i=1}^{N} \delta_i(t) A_{ei}^T \right) R \left(\sum_{i=1}^{N} \delta_i(t) Q_i^T \right) \right. \\ &\left. + \left(\sum_{i=1}^{N} \delta_i(t) Q_i \right) R^T \left(\sum_{i=1}^{N} \delta_i(t) A_{ei} \right) \right] e(t). \end{aligned} \tag{33}$$

Combining the conditions in (32) and (33), one can get

$$\begin{aligned} \Delta V(t) =& e^T(t) \left(A_{ei}^T P_j A_{ei} - \tilde{E}^T P_i \tilde{E} \right. \\ &\left. + A_{ei}^T R Q_i^T + Q_i R^T A_{ei} \right) e(t). \end{aligned} \tag{34}$$

Therefore, from the condition in (22), one can obtain that $\Delta V(t) < 0$. Then we can conclude that the system (13)-(14) is asymptotically stable.

Based on the above discussions, one can get that the system (13)-(14) is admissible. In the following, the γ performance index will be proved.

Define the following performance index:

$$J = \sum_{t=0}^{K-1} [r_e^T(t) r_e(t) - \gamma^2 \omega^T(t) \omega(t)]. \tag{35}$$

Under the zero initial condition, one has

$$J = \sum_{t=0}^{K-1} [r_e^T(t) r_e(t) - \gamma^2 \omega^T(t) \omega(t) + \Delta V(t)] - V(K)$$

$$\leq \sum_{t=0}^{K-1}[r_e^T(t)r_e(t)-\gamma^2\omega^T(t)\omega(t)+\Delta V(t)]. \quad (36)$$

For any nonzero state vector $e(t)$ and the particular case $\delta_i(t)=1$, $\delta_{l\neq i}(t)=0$, and $\delta_j(t+1)=1$, $\delta_{l\neq j}(t+1)=0$, one can get

$$\Delta V(t) \leq \upsilon^T(t)\Theta_{ij}\upsilon(t), \upsilon(t)=\begin{bmatrix} e(t) \\ \omega(t) \end{bmatrix}, \quad (37)$$

where

$$\Theta_{ij}=\begin{bmatrix} \Theta_{11ij} & A_{ei}^TP_jB_{ei}+C_{ei}^TD_{ei} \\ * & B_{ei}^TP_jB_{ei}-\gamma^2I+D_{ei}^TD_{ei} \end{bmatrix},$$
$$\Theta_{11ij}=A_{ei}^TP_jA_{ei}-\tilde{E}^TP_i\tilde{E}+A_{ei}^TRQ_i^T +Q_iR^TA_{ei}+C_{ei}^TC_{ei}. \quad (38)$$

According to Schur complement Lemma and the condition in (22), one can get that $\Theta_{ij}<0$, which implies that $J<0$. Therefore, we obtain $\|r_e(t)\|_2<\gamma\|\omega(t)\|_2$ for any nonzero $\omega(t)\in l_2[0,\infty)$. Thus the proof is completed.

In order to design the fault detection filter more conveniently, we extend Theorem 1 to the following corollary.

Corollary 1: For a prescribed scalar $\gamma>0$, the residual system (13)-(14) is admissible with H_∞ performance γ if there exit positive definite matrices P_i, P_j, and matrices R, Ω and Q_i for all $i,j=1,\cdots,N$ such that

$$\begin{bmatrix} \Lambda_i & 0 & C_{ei}^T & A_{ei}^T\Omega \\ * & -\gamma^2I & D_{ei}^T & B_{ei}^T\Omega \\ * & * & -I & 0 \\ * & * & * & P_j-(\Omega+\Omega^T) \end{bmatrix}<0, \quad (39)$$

where $\Lambda_i=-\tilde{E}^TP_i\tilde{E}+A_{ei}^TRQ_i^T+Q_iR^TA_{ei}$, and R is any matrix with full column which satisfies $\tilde{E}^TR=0$.

Proof: Suppose the LMIs (39) hold. Noting the inequality

$$(P_i-\Omega)^TP_i^{-1}(P_i-\Omega)\geq 0 \quad (40)$$

implies that

$$P_i-(\Omega+\Omega^T)\geq -\Omega^TP_i^{-1}\Omega, \quad (41)$$

which together with (39) yield

$$\begin{bmatrix} \Lambda & 0 & C_{ei}^T & A_{ei}^T\Omega \\ * & -\gamma^2I & D_{ei}^T & B_{ei}^T\Omega \\ * & * & -I & 0 \\ * & * & * & -\Omega^TP_j^{-1}\Omega \end{bmatrix}<0, \quad (42)$$

which pre- and post-multiplying $diag\{I,I,I,\Omega^{-T}\}$ and $diag\{I,I,I,\Omega^{-1}\}$ yields (22), then we conclude the proof.

Remark 4: With the introduction of a new additional matrix Ω, we obtain a sufficient condition in which the matrices P_i and P_j are not involved in any product with matrices A_{ei}, B_{ei}, C_{ei}, and D_{ei}, which makes a fault detection filter design feasible.

3.2. Fault detection filter design

In this subsection, the fault detection filter design problem is investigated based on Corollary 1, i.e., a method will be developed to determine the fault detection filter matrices expressed in (6) and (7), such that the residual system (13)-(14) is asymptotically stable and the performance defined in (20) is guaranteed.

Theorem 2: For prescribed scalars $\gamma>0$ and $\eta>0$, if there exit positive definite matrices P_{11i}, P_{22i}, P_{33i}, P_{11j}, P_{22j}, P_{33j}, and matrices P_{12i}, P_{13i}, P_{23i}, P_{12j}, P_{13j}, P_{23j}, U, L_1, L_2, Q_{1i}, Q_{3i}, $\hat{A}_{fi}$, $\hat{B}_{fi}$, $\hat{C}_{fi}$, $\hat{D}_{fi}$ for any $i,j=1,\cdots,N$ satisfying the following LMIs:

$$\Upsilon_{ij}=\begin{bmatrix} \Upsilon_{11ij} & 0 & \Upsilon_{13ij} & \Upsilon_{14ij} \\ * & -\gamma^2I & \Upsilon_{23ij} & \Upsilon_{24ij} \\ * & * & -I & 0 \\ * & * & * & \Upsilon_{44ij} \end{bmatrix}<0, \quad (43)$$

where

$$\Upsilon_{11ij}=\begin{bmatrix} (1,1) & -\bar{E}^TP_{12i} & -\bar{E}^TP_{13i}+\bar{A}_i^TR_1Q_{3i}^T \\ * & -P_{22i} & -P_{23i} \\ * & * & -P_{33i} \end{bmatrix},$$
$$(1,1)=-\bar{E}^TP_{11i}\bar{E}+\bar{A}_i^TR_1Q_{1i}^T+Q_{1i}R_1^T\bar{A}_i,$$
$$\Upsilon_{13ij}=\begin{bmatrix} \bar{C}_i^T\hat{D}_{fi}^T \\ \hat{C}_{fi}^T \\ -C_W^T \end{bmatrix}, \Upsilon_{23ij}=\begin{bmatrix} 0 \\ 0 \\ -D_W^T \end{bmatrix},$$
$$\Upsilon_{14ij}=\begin{bmatrix} \bar{A}_i^TU^T+\bar{C}_i^T\hat{B}_{fi}^T & \bar{A}_i^TL_1^T+\bar{C}_i^T\hat{B}_{fi}^T & 0 \\ \hat{A}_{fi}^T & \hat{A}_{fi}^T & 0 \\ 0 & 0 & \eta A_W^T \end{bmatrix},$$
$$\Upsilon_{24ij}=\begin{bmatrix} \bar{B}_i^TU^T & \bar{B}_i^TL_1^T & 0 \\ \bar{D}_i^TU^T & \bar{D}_i^TL_1^T & 0 \\ \bar{F}_i^TU^T & \bar{F}_i^TL_1^T & 0 \end{bmatrix},$$
$$\Upsilon_{44ij}=\begin{bmatrix} (2,2) & P_{12j}-(L_2+L_1^T) & P_{13j} \\ * & P_{22j}-(L_2+L_2^T) & P_{23j} \\ * & * & P_{33j}-2\eta I \end{bmatrix},$$
$$(2,2)=P_{11j}-(U+U^T).$$

R_1 is any matrix with full rank and satisfies $\bar{E}^TR_1=0$. Then there exits a fault detection filter in the form of (6)-(7), such that the residual signal system in (13)-(14) is asymptotically stable with H_∞ performance γ. Moreover, if the aforementioned conditions are satisfied, the parameters for the fault detection filter in (6)-(7) can be given by

$$\begin{bmatrix} A_{fi} & B_{fi} \\ C_{fi} & D_{fi} \end{bmatrix}=\begin{bmatrix} X^{-1} & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} \hat{A}_{fi} & \hat{B}_{fi} \\ \hat{C}_{fi} & \hat{D}_{fi} \end{bmatrix} \times\begin{bmatrix} X^{-T}Y^T & 0 \\ 0 & I \end{bmatrix}, \quad (44)$$

where matrices X and Y are any nonsingular matrices satisfying $XY^{-T}X^T=L_2$.

Proof: Based on the condition $\tilde{E}^T R = 0$ in Theorem 1, one can conclude that the matrix R has the following form:

$$R = \begin{bmatrix} R_1^T & 0 & 0 \end{bmatrix}^T. \tag{45}$$

Define the following matrices:

$$\Omega = \begin{bmatrix} \Omega_1 & \Omega_2 & 0 \\ \Omega_3 & \Omega_4 & 0 \\ 0 & 0 & \eta I \end{bmatrix}^T, \; Q_i = \begin{bmatrix} Q_{1i} \\ 0 \\ Q_{3i} \end{bmatrix},$$
$$\Phi = \begin{bmatrix} I & 0 & 0 \\ 0 & \Omega_4^{-T}\Omega_2^T & 0 \\ 0 & 0 & I \end{bmatrix}, \tag{46}$$

where Ω_2 and Ω_4 are nonsingular matrices. Therefore, matrix Φ is invertible.

Performing a congruence transformation to inequality (39) via $diag\{\Phi, I, I, \Phi\}$, and define

$$\hat{A}_{fi} = \Omega_2 A_{fi}\Omega_4^{-T}\Omega_2^T,\; \hat{B}_{fi} = \Omega_2 B_{fi},\; \hat{C}_{fi} = C_{fi}\Omega_4^{-T}\Omega_2^T,$$
$$\hat{D}_{fi} = D_{fi},\; L_1 = \Omega_2\Omega_4^{-1}\Omega_3,\; L_2 = \Omega_2\Omega_4^{-T}\Omega_2^T,$$
$$U = \Omega_1,\; X = \Omega_2,\; Y = \Omega_4,$$
$$\Phi^T P_i \Phi = \begin{bmatrix} P_{11i} & P_{12i} & P_{13i} \\ * & P_{22i} & P_{23i} \\ * & * & P_{33i} \end{bmatrix},$$
$$\Phi^T P_j \Phi = \begin{bmatrix} P_{11j} & P_{12j} & P_{13j} \\ * & P_{22j} & P_{23j} \\ * & * & P_{33j} \end{bmatrix}. \tag{47}$$

Then, Combining the conditions in (46) and (47), one can conclude that the inequality in (43) is equivalent to $\Phi^T \Upsilon_{ij}\Phi < 0$, which clearly guarantees the inequality in (39). Thus, the proof is completed.

4. A NUMERICAL EXAMPLE

In this section, a numerical example is given to illustrate the developed method is feasible. Consider the switched system (1)-(2) with parametric matrices as follows:

$$A_1 = \begin{bmatrix} 0.8 & -0.3 \\ -0.1 & 0.2 \end{bmatrix}, B_1 = \begin{bmatrix} 0.1 \\ -0.2 \end{bmatrix},$$
$$F_1 = \begin{bmatrix} -0.1 \\ 0.09 \end{bmatrix}, C_1 = \begin{bmatrix} 0.1 & -0.7 \end{bmatrix}, D_1 = 0.2,$$
$$A_2 = \begin{bmatrix} 0.2 & -0.4 \\ -0.1 & 0.5 \end{bmatrix}, B_2 = \begin{bmatrix} 0.01 \\ -0.05 \end{bmatrix},$$
$$F_2 = \begin{bmatrix} -0.1 \\ 0.7 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & -2 \end{bmatrix}, D_2 = 0.2.$$

Suppose the fault weighting matrix $A_W = 0.5$, $B_W = 0.25$, $C_W = 1.0$, and $D_W = 0.5$. If we choose the matrix $R_1 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$ and scalar $\eta = 2$ and solve the conditions (43) and (44) in Theorem 2, we can get when the minimum performance index $\gamma = 0.71$, the parametric matrices of the desired fault detection filter parameters are obtained as follows:

$$A_{f1} = \begin{bmatrix} 0.0039 & 0.0010 & 0.0000 \\ 0.0003 & 0.0001 & 0.0000 \\ -0.0000 & -0.0000 & -0.0000 \end{bmatrix},$$
$$B_{f1} = \begin{bmatrix} -1.3687 \\ -0.1102 \\ -0.0000 \end{bmatrix},$$
$$C_{f1} = \begin{bmatrix} -0.0024 & -0.0006 & -0.0000 \end{bmatrix},$$
$$D_{f1} = -0.0221.$$
$$A_{f2} = 10^3 * \begin{bmatrix} 0.7411 & 0.2789 & 0.0000 \\ 0.0312 & -0.3112 & -0.0000 \\ -0.0000 & 0.0000 & 0.0000 \end{bmatrix},$$
$$B_{f2} = \begin{bmatrix} -0.4493 \\ 0.0223 \\ -0.0000 \end{bmatrix},$$
$$C_{f2} = \begin{bmatrix} -0.0024 & -0.0006 & -0.0000 \end{bmatrix},$$
$$D_{f2} = -0.0335.$$

In this paper, the main purpose is to realize fault detection for dscrete-time switched system with sensor output disturbance. For the simulation, the sensor unknown output disturbance is assumed to be random noise with sample time 0.1, as shown in Fig. 1. The arbitrary switched signal is assumed to shown in Fig. 2. The Control input is $u(t) = 0.5cos(t)$. The fault signal is set up as

$$f(t) = \begin{cases} 5, & 20 \le t \le 60, \\ 0, & \text{others}, \end{cases}$$

then the weighted fault signal $\bar{f}(t)$ is described in Fig. 3. We set the initial state is $x(0) = \begin{bmatrix} 0.3 & -0.2 \end{bmatrix}^T$, Figs. 4, 5, 6 seperately depict the tragectories of the state $x(t)$, the state estimation $\hat{x}(t)$ and the measurement output $y(t)$. Figs. 7 and 8 show the response of residual signal $r(t)$ without fault $f(t)$ and under the constant fault $f(t)$. Fig. 9 presents the residual evaluation function $J_L(r(t))$ for both the faulty case (solid line) and fault-free case (dashed line). With a selected threshold $J_{th} = 2.2656$, the simulation results show that $J_L(21.5) = 2.7024 > J_{th}$ for $t = 21.5$, which means that the fault $f(t)$ can be successfully detected two time steps after its occurrance.

5. CONCLUSION

In this paper, fault detection filter design for discrete-time switched systems under actuator fault and sensor disturbance is considered. By constructing a descriptor system, an efficient condition is obtained to guarantee the existence a fault detection filter and such that the augmented systems is asymptotically stable with H_∞ performance. Finally, simulation results have illustrated the proposed methodology and shown the effectiveness of the results.

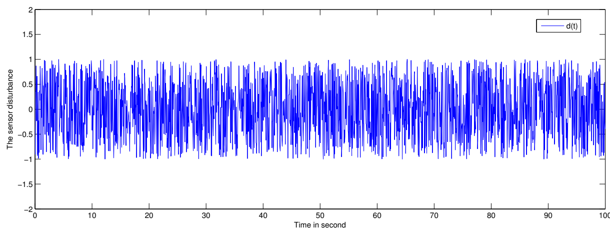

Fig. 1. Unknown output disturbance $d(t)$.

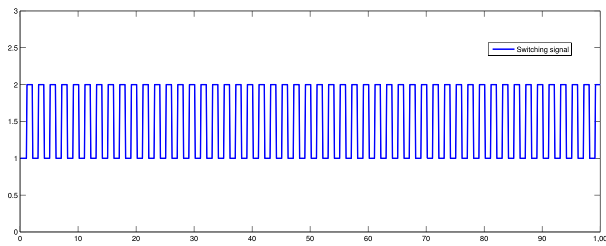

Fig. 2. Switched signal $\delta_i(t)$.

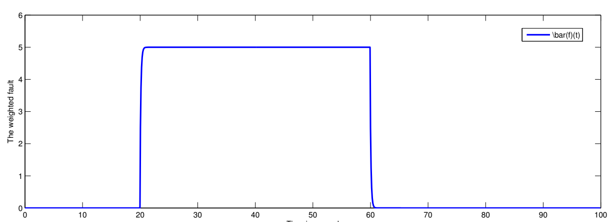

Fig. 3. The weighted fault signal $\bar{f}(t)$.

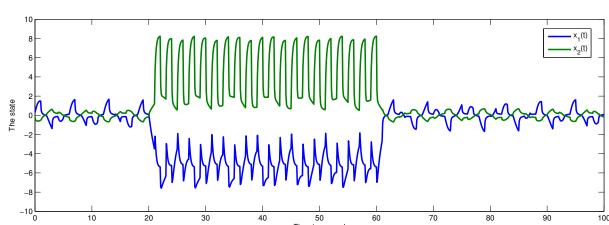

Fig. 4. State vectors $x(t)$.

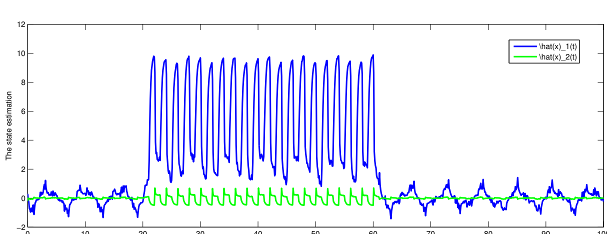

Fig. 5. The estimation of state vectors $\hat{x}(t)$.

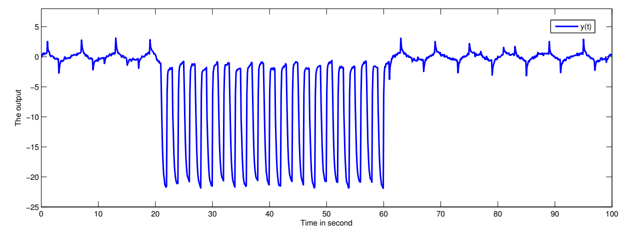

Fig. 6. The output vector $y(t)$.

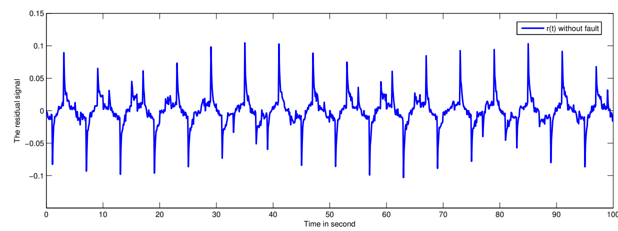

Fig. 7. Residual signal $r(t)$ without fault.

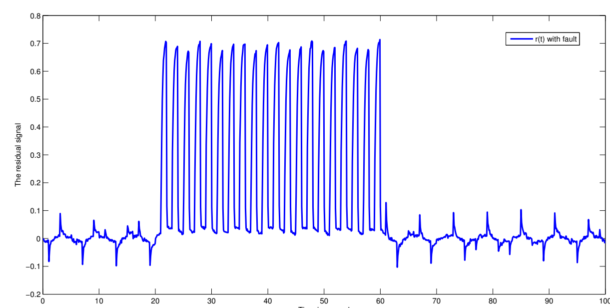

Fig. 8. Residual signal $r(t)$ with fault.

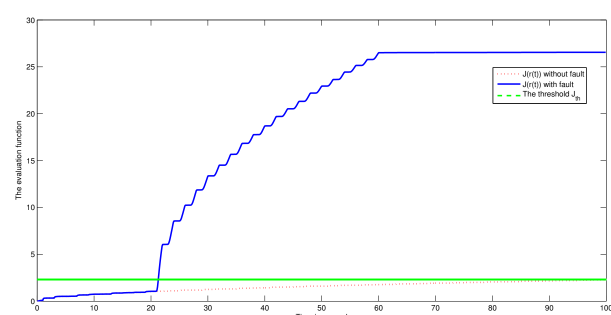

Fig. 9. Evaluation function $J_L(r(t))$.

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