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Coupled Disturbance Reconstruction by Sliding Mode Observer Approach for Nonlinear System

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Abstract: This paper concentrates on the estimation of system states and the reconstruction of disturbances in a class of nonlinear systems considering stateless situation. The disturbances are coupled with time-varying parameters. The sliding mode observer approach is utilized to solve these two issues. Firstly, a descriptor model is presented by transforming the coupled disturbances into the decoupled form. Secondly, the sliding mode observer is designed to estimate the decoupled disturbances and system states of nonlinear systems. The coupled disturbance can be reconstructed hereafter. The detailed methods of designing the observer, together with the sufficient condition to guarantee the existence of the observer are also given. Finally, a simulation and a robot manipulator experiment are provided to examine the validity of the proposed design approach.

Keywords: Descriptor augment, disturbance reconstruction, nonlinear system, sliding mode observer.

1. INTRODUCTION

Disturbance, which can cause not only incorrect performance but also serious damages to the instruments themselves and other equipments, is one of the dangers for the unsecurity of modern industry. An intuitive and necessary measure we can take is to reconstruct or estimate the disturbance. The topics on disturbance estimation, fault estimation (FE) and fault tolerant control (FTC) have drawn the attentions of experts from engineering and academic fields [1, 2]. For example, collision detection and safe reaction for cooperation between manipulator arm and human are investigated [3], the friction compensation for X-Y robot is discussed [4] and real-time implementation of the FTC in benchmark is probed [5]. For the estimation and reconstruction and estimation of disturbance in nonlinear systems, new learning and data processing techniques, like neural networks (NN), data driven and artificial intelligence (AI), are applied dramatically [6–8]. However, the foundations of these methods are long time training, complicated computing, huge data storage and even online dependence. The learning and data processing methods work well for the low speed systems and situations where historic data are available, but might be invalid for those high-speed systems and emergency situations. A new challenge for researchers is how to reconstruct the coupled disturbance in the nonlinear systems by faster and more reliable methods.

We aim at reconstructing the time-varying parameter coupled disturbances of nonlinear systems with the assumptions that the disturbances and part of the system states are unknown. In literatures [9–11], the piecewise methods and fuzzy methods are utilized for the modelling of nonlinear affine systems. In [12], the linear assumption is necessary on analyzing nonlinear system. However, if the necessary elements for the approximation are failed to measure or mix with faults, the piecewise and fuzzy methods on modelling systems would be unavailable to use. Hence, it is of practical value to evaluate the full system states and reconstruct the coupled disturbance by considering the nonlinear models directly. Due to its unsensitivity and adaptivity against uncertainties and faults, in our paper, the sliding mode observer (SMO) approach is used to reconstruct the coupled disturbance and evaluate the full system state.

The sliding mode (SM) ideas have gradually been applied in the control of linear and nonlinear systems [13–15] from the time it was introduced. The SM method has been brought to the new control fields, such as stochastic systems [16, 17], control for spacecraft attitude and

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translation motion [18], uncertain systems for the flying vehicle fault estimation [19], network system control [20] and even FE and FTC for robots and manipulators [21,22]. However, in the current papers about FE by SMO for nonlinear systems [21–23], the full states and the decoupling property are, partly or wholly, necessary for their methods. Besides, less literatures considering the coupled disturbance reconstruction and estimation in the stateless situation for nonlinear systems have been published. Inspired and encouraged by these excellent researchers in [13,24,25], we utilize the descriptor augment method and SMO approach to reconstruct the disturbance which is coupled by time-varying parameters for a class of nonlinear systems.

Since SMO method can not be used to reconstruct the disturbances coupled with the time invariable parameters, the disturbance is technically converted from coupled form into a new decoupled form in our article. Then, a descriptor mathematical model is obtained where the disturbance and system state are put into the new augmented state. Based on the new descriptor mathematical model, the SMO is designed to estimate the full system states and the decoupled disturbance. With the estimations of system states and decoupled disturbance, the coupled disturbance is reconstructed. The main contributions of this article are threefold. Firstly, the nonlinear system is augmented into the descriptor form by treating the disturbance into decoupled form. Secondly, the SMO method is extended to be used to estimate the system states and decoupled disturbances of nonlinear systems. Thirdly, the coupled disturbance of the nonlinear system is reconstructed by SMO with considering the statelessness. Simultaneously, the system states are estimated.

The remainder of this paper is organised as follows: Necessary assumptions and state space model of the control plant are offered in Section 2. Section 3 is consisted of the SMO design, the stability analysis of the error system and the proof process. To examine the validity of the SMO approach proposed in this paper, a mathematical simulation and a two-link experiment example are provided in the Section 4. Section 5 concludes the paper.

2. PRELIMINARIES

2.1. System model

The model of nonlinear system considered the disturbance $d(t) \in \mathbb{R}^d$ is presented as

$$\begin{cases} \dot{x}(t) = Ax(t) + Bg(x,u,t) + B_f f(x,d(t),t), \\ y(t) = Cx(t), \end{cases} \tag{1}$$

where matrices $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times m}, B_f \in \mathbb{R}^{n\times q}, C \in \mathbb{R}^{p\times n}$ are the known system parameters of associated dimensions. Nonlinear time-varying part $g(x,u,t) \in \mathbb{R}^m$ is related to the system state $x \in \mathbb{R}^n$ and control input $u \in \mathbb{R}^r$. The nonlinear time-varying function $f(x,d(t),t) \in \mathbb{R}^q$ depends on the disturbance $d(t) \in \mathbb{R}^d$ and system state $x(t)$. The disturbance $d(t)$ is unmeasurable and coupled with the time-varying parameters. Variable $d(t)$ and nonlinear part $f(x,d(t),t)$ are in one to one mapping relationship when $x(t)$ is given. $y(t) \in \mathbb{R}^p$ means the measurable output, whose parameter matrix $C \in \mathbb{R}^{p\times n}$, $\text{rank}(C) = p \leq n$. $\text{Rank}(CB_f) = q \leq p$. Pair $[A,C]$ is observable.

The following assumptions are made on nonlinearities in (1) and disturbance $d(t)$. For the function $g(x,u,t)$, it assumes there exists $\gamma \geq 0$, such that for any x_1, x_2 [23]

$$\|g(x_2,u,t) - g(x_1,u,t)\| \leq \gamma \|x_2 - x_1\|. \tag{2}$$

Disturbance $d(t)$ and the function $f(x,d,t)$ are bounded

$$\|d(t)\| \leq d_M, \left\|\dot{d}(t)\right\| \leq d_m, \tag{3}$$

and

$$\|f(x,d,t)\| \leq f_M, \left\|\dot{f}(x,d,t)\right\| \leq f_m. \tag{4}$$

In our paper, we need to estimate the full system state $x(t)$ and to reconstruct the coupled disturbance $d(t)$. The estimated state $\hat{x}(t)$ and reconstructed disturbance $\hat{d}(t)$ are of validity if $\hat{x}(t) \to x(t)$, $\hat{d}(t) \to d(t)$ and $f(\hat{x},\hat{d},t) \to f(x,d,t)$, i.e the equilibrium point approximates to zero, gradually. We analyze the asymptotical stability of equilibrium point by Lyapunov approach. Note that the model of this article is of nonlinear form. It's more general compared with the fuzzy and piecewise models.

3. MAIN RESULT

We aim to solve two problems in this paper. The first one is to estimate the full state of the nonlinear system that there exist unknown disturbance inputs. The second one is to reconstruct the coupled disturbance. Therefore, a SMO approach is applied to estimate the full state and decoupled disturbance. After that, the coupled disturbance is reconstructed. To construct the SMO, we rebuild the state space model (1) into a descriptor system by converting the coupled torque disturbance into an equal decoupled form.

3.1. Model transformation

As presented in (1), the disturbance $d(t)$ is coupled with the time-varying parameter related to $x(t)$. It cannot be reconstructed by using the SMO method directly. Thus, some equal transformation techniques should be taken if we want to use the SMO method to reconstruct the coupled disturbance. Instead of considering the time-varying parameters dependent disturbance $d(t)$ directly, the $f(x,d,t)$ is treated as

$$f(x,d,t) = f_d(t). \tag{5}$$

$f_d(t)$ is unknown time-varying disturbance, and not coupled with any time variable parameters. Then (1) can be rewritten as the following form

$$\begin{cases} \dot{x}(t) = Ax(t) + Bg(x,u,t) + B_f f_d(t), \\ y(t) = Cx(t). \end{cases} \tag{6}$$

The disturbance $d(t)$, coupled with the time-varying parameters and state $x(t)$ in $f(x,d,t)$, presented in (1), is changed into the decoupled form $f_d(t)$, as presented in (6), such that we can reconstruct the coupled disturbance after the evaluation of decoupled disturbance $f_d(t)$. The model (6) is in nonlinear form but available for observer design and fault estimation. The coupled disturbance $d(t)$ and the nonlinear part $f(x,d,t)$ are assumed to be bounded as in (3). Then, to estimate the full state and decoupled disturbance, similar in [26,27], we transform model (6) into the descriptor form. We choose an approach, where (6) is augmented into descriptor form (7) by introducing an auxiliary difference equation (8) for the disturbance f_d. The resulting descriptor system is derived

$$\begin{cases} \bar{E}\dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \bar{B}g(x,u,t) + \bar{B}_f \bar{f}(t), \\ y(t) = \bar{C}\bar{x}(t). \end{cases} \tag{7}$$

The auxiliary equation is

$$\dot{f}_d(t) = \Theta f_d(t) - \Theta f_d(t) + \dot{f}_d(t). \tag{8}$$

The parameters in (7) are

$$\bar{E} = \begin{bmatrix} I & B_f\Theta^{-1} \\ 0 & I \end{bmatrix}, \bar{A} = \begin{bmatrix} A & 0 \\ 0 & -\Theta \end{bmatrix},$$

$$\bar{x}(t) = \begin{bmatrix} x(t) \\ f_d(t) \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad \bar{B}_f = \begin{bmatrix} B_f, \\ \Theta \end{bmatrix}$$

$$\Theta > 0, \quad \bar{C} = \begin{bmatrix} C & 0 \end{bmatrix}, \quad \bar{f}(t) = \left[f_d(t) + \Theta^{-1}\dot{f}_d(t)\right].$$

In order to get the descriptor system (7), a new auxiliary disturbance $\bar{f}(t) = f_d(t) + \Theta^{-1}\dot{f}_d(t)$ is introduced. The amplitude of the discontinuous element $u_s(t)$ can be decreased by selecting a proper Θ. In the new descriptor model (7), by technically adding the auxiliary difference equation (8) and transforming the coupled disturbance $d(t)$ into a complete decoupled $f_d(t)$, the decoupled disturbance $f_d(t)$ and the state $x(t)$ are put into the augmented state $\bar{x}(t)$.

With the parameter Θ and the observable pair(A,C), the observability of pair$(\bar{A},\bar{C})$ in descriptor system (7) can be guaranteed. For the convenience of expression, we rebuild the augmented system into the descriptor form. It can also be written into the normal form for the reason that $\bar{E}$ is not singular. Then, the SMO is proposed based on the augmented descriptor form in (7) in subsection 3.2.

3.2. SMO design

For the estimation of the unknown states and disturbance in $\bar{x}(t)$, the following SMO is proposed:

$$\begin{cases} \bar{E}\dot{\hat{\bar{x}}}(t) = \bar{A}\hat{\bar{x}}(t) + \bar{B}g(\hat{x},u,t) + \bar{L}_f u_s(t) + \bar{L}(y - \bar{C}\hat{\bar{x}}(t)), \\ \hat{y}(t) = \bar{C}\hat{\bar{x}}(t). \end{cases} \tag{9}$$

Here $\hat{\bar{x}}(t) = [\hat{x}^T(t)\ \hat{f}_d^T(t)]^T$ is the estimate of $\bar{x}(t)$. The estimated disturbance $\hat{d}(t)$ can be reconstructed as $\hat{d}(t) = f^{-1}(\hat{x}, \hat{f}_d, t)$ if $\hat{f}_d(t)$ approaches $f_d(t)$ well. Matrices $\bar{L}$ and $\bar{L}_f$ are the gains to be designed or selected. Input $u_s(t)$ is the discontinuous control input. How to design or choose $\bar{L}$, $\bar{L}_f$ and $u_s(t)$ are presented in the following.

Using the SMO (9) and the descriptor model (7), we can get the error system

$$\bar{E}\dot{\bar{e}}(t) = (\bar{A} - \bar{L}\bar{C})\bar{e}(t) + \bar{B}\bar{e}_g(\hat{x},u,t) - \bar{L}_f u_s(t) + \bar{B}_f \bar{f}(t), \tag{10}$$

where $\bar{e}(t) = \bar{x}(t) - \hat{\bar{x}}(t) = [e_x^T(t)\ e_f^T(t)]^T$. The other parameters are the same as mentioned in the former subsections. From assumption in 2, a positive parameter γ exists that $\bar{e}_g(\hat{x},x,u,t) = g(x,u,t) - g(\hat{x},u,t) \le \gamma\|e_x(t)\|$. The presented error system will be used to prove that the estimated state $\hat{\bar{x}}(t)$ in SMO (9) approaches the state $\bar{x}(t)$ finally. The validity of the SMO can be demonstrated if the error system (10) is stable. The asymptotic stability analysis for (10) is provided in Subsection 3.3.

The nonlinear system (6) could be obtained by adding $Bg(x,u,t)$ to the linear system $\dot{x}(t) = Ax(t) + B_f f_d(t)$. So, we can extend methods of SMO design from linear systems, presented in [27,28], to the nonlinear dynamics (7). Stability of (10) can be proved by appropriate choice of $\bar{L}$, $\bar{L}_f(t)$ and $u_s(t)$.

The discontinuous function $u_s(t)$ is designed as

$$u_s(t) = \left(f_M + \lambda_{\max}\left(\Theta^{-1}\right) f_m + \zeta\right) \operatorname{sgn}(s(t)), \tag{11}$$

where the gains f_M and f_m have been defined in (4). Positive constant ζ is selected as $\zeta \ll f_M + \lambda_{max}\left(\Theta^{-1}\right) f_m$. As presented in subsection 3.1, the amplitude of the constant gain $\left(f_M + \lambda_{max}\left(\Theta^{-1}\right) f_m\right)$ in function $u_s(t)$ can be minimized via choosing Θ. The switching function $s(t)$ is defined as

$$s(t) = H\bar{C}\bar{e}(t), \tag{12}$$

where $H \in \mathbb{R}^{(n+p)\times p}$ is defined by the bounded constraint

$$\left(H\bar{C}\right)^T = \bar{P}\bar{E}^{-1}\bar{B}_f. \tag{13}$$

The matrix $\bar{P}$ in (13), results from a Lyapunov equation introduced in (10), is provided in the following subsection 3.3 The approach to solve the constraint is presented in subsection 3.4 In [22], the high order SMO method was used, it is a kind of new switching function, and the calculation might be complex and hard to operate in SMO. For the simplicity and easiness, we select a switching function (12). The gain $\bar{L}_f$ for discontinuous function $u_s(t)$ is selected as$\bar{L}_f = \bar{B}_f$. Following the Lemma 2 in [28], Lemma 3 in [27], we design the high gain $\bar{L}$ for the SMO in following Lemma 1 by similar method.

Lemma 1: If the descriptor system (7) is observable, matrix $\bar{E}^{-1}(\bar{A}-\bar{L}\bar{C})$ can be designed to be Hurwitz with one appropriate matrix $\bar{L}$.

Proof: As presented, the matrix $\bar{E}$ is non-singular, and (7) is observable. Thus, a positive number κ exists such that

$$Re[\lambda_i(\bar{E}^{-1}\bar{A})] > -\kappa, \forall i \in \{1,2,\ldots,2n\}, \tag{14}$$

and

$$Re[\lambda_i(-(\kappa I+\bar{E}^{-1}\bar{A}))] < 0, \forall i \in \{1,2,\ldots,2n\}. \tag{15}$$

Further, (15) implies that $[-(\kappa I+\bar{E}^{-1}\bar{A}),\bar{C}]$ is observable. There exists a positive definite matrix $\bar{Q}$ such that

$$-\bar{C}^T\bar{C} = -(\kappa I+\bar{E}^{-1}\bar{A})^T\bar{Q} - \bar{Q}(\kappa I+\bar{E}^{-1}\bar{A}). \tag{16}$$

Selecting $\bar{L}$

$$\bar{L} = \bar{E}\bar{Q}^{-1}\bar{C}^T, \tag{17}$$

then there exists a positive Q such that

$$-\bar{C}^T\bar{C} = \bar{Q}[\kappa I+\bar{E}^{-1}(\bar{A}-\bar{L}\bar{C})] + [\kappa I+\bar{E}^{-1}(\bar{A}-\bar{L}\bar{C})]^T\bar{Q}, \tag{18}$$

thus,

$$Re[\lambda_i(\bar{E}^{-1}(\bar{A}-\bar{L}\bar{C}))] < -\kappa, \forall i \in \{1,2,\ldots,\bar{n}\},$$

i.e., $\bar{E}^{-1}(\bar{A}-\bar{L}\bar{C})$ is Hurwitz. □

Except this method, there also exist other methods to select the high observer gain $\bar{L}$. Similar in [29], together with the Lyapunov function to prove the stability of error system, the observer gain is selected as $\bar{L} = \bar{E}\bar{Q}^{-1}\bar{C}^T$. In our paper the observer gain is selected before proving the stability of error system. With the introduced gain κ, the method we used to select $\bar{L}$ is more conservative. Therefore, in this paper Lemma 1 is introduced to design the high gain for SMO.

3.3. Stability of error system

Now, with the high gain $\bar{L}$ from Lemma 1, discontinuous element $u_s(t)$ from (11) and its gain matrix $\bar{L}_f = \bar{B}_f$, we are able to prove the equilibrium point $\bar{e}(t)$ is asymptotically stable. Thus, $\lim_{t\to+\infty}\hat{\bar{x}}(t) = \bar{x}(t)$, the estimate $\hat{\bar{x}}(t)$ approaches to the augmented state $\bar{x}(t)$ eventually.

The sufficient condition for the stability analysis of error system (10) is as in Theorem 1.

Theorem 1: With the observer gain $\bar{L}$ in Lemma 1, the discontinuous function $u_s(t)$ in (11) and its gain matrix $\bar{L}_f = \bar{B}_f$, the error system (10) is asymptotically stable, if there exist a positive constant $\gamma \geq 0$ and matrices $\bar{P} \geq 0$, H with appropriate dimensions, such that the bounded constraint (13) and the below constraint (19) hold

$$\begin{aligned}&(\bar{P}\bar{E}^{-1}(\bar{A}-\bar{L}\bar{C})+\gamma\bar{P}|\bar{B}|T_e)\\&\quad+(\bar{P}\bar{E}^{-1}(\bar{A}-\bar{L}\bar{C})+\gamma\bar{P}|\bar{B}|T_e)^T < 0,\end{aligned} \tag{19}$$

where $T_e = [I\ 0]$.

Proof: To prove the error system (10) is asymptotically stable, we choose the following Lypunov function

$$V(t) = \bar{e}^T(t)\bar{P}\bar{e}(t). \tag{20}$$

By (10), we obtain that

$$\begin{aligned}\dot{V}(t) =& 2\bar{e}^T(t)\bar{P}\dot{\bar{e}}(t)\\=& 2\bar{e}^T(t)\bar{P}\bar{E}^{-1}\left[(\bar{A}-\bar{L}\bar{C})\bar{e}(t)\right.\\&\left.+\bar{B}\bar{e}_g(\hat{x},x,u,t) - \bar{L}_f u_s(t) + \bar{B}_f\bar{f}(t)\right].\end{aligned} \tag{21}$$

With the equations (12) and (13) we have [27, 28]

$$\begin{aligned}&2\bar{e}^T(t)\bar{P}\bar{E}^{-1}\left[-\bar{L}_f u_s(t) + \bar{B}_f\bar{f}(t)\right]\\&= -2(f_M+\lambda_{\max}(\Theta^{-1})f_m+\zeta)\left|s^T(t)\right| + 2s^T(t)\bar{f}(t)\\&\leq -2(f_M+\lambda_{\max}(\Theta^{-1})f_m+\zeta)\left|s^T(t)\right| + 2\left|s^T(t)\right|\left|\bar{f}(t)\right|\\&\leq 0.\end{aligned} \tag{22}$$

According to $T_e = [I\ 0]$, $\bar{E}^{-1}\bar{B} = \bar{B}, \bar{B} = \left[B^T\ 0\right]^T$, $\bar{P} > 0$ and the assumption in (2), we obtain

$$2\bar{e}^T(t)\bar{P}\bar{E}^{-1}\bar{B}\bar{e}_g(\hat{x},u,t) \leq 2\gamma\bar{P}|\bar{B}|T_e\bar{e}^T(t)\bar{e}(t).$$

Combining (21), (22) and (23), it derives that

$$\dot{V}(t) \leq 2\bar{e}^T(t)(\bar{P}\bar{E}^{-1}(\bar{A}-\bar{L}\bar{C})+\gamma\bar{P}|\bar{B}|T_e)\bar{e}(t). \tag{23}$$

Hence, if the condition (19) is satisfied, $\dot{V}(t) < 0$. The error system (10) is stable, and the effectiveness of SMO (9) can be guaranteed. □

Adapting from the method on processing the error proof in [23], the element $\gamma\bar{P}|\bar{B}|T_e$ is put inside in (19). With the assumption (2), the element $\bar{e}_g(t)$ in (10) is reshaped into the form related to γ and $\bar{e}(t)$. And, it also makes that the high gain $\bar{L}$ in SMO is more conservative, and the $\bar{e}_g(t)$ affects less to the stability analysis.

3.4. Calculation details and design procedure

In the previous subsections, the method to design the SMO is presented. However, there still exists a key constraint (13). Following the ideas on solving this kind of constraint in [27–29], we utilize the LMI approach to solve the constraint (13).

The constraint (13) can be written as

$$\mathrm{Trace}[((H\bar{C})^T-\bar{B}_f^T\bar{E}^{-T}\bar{P}^T)^T((H\bar{C})^T-\bar{B}_f^T\bar{E}^{-T}\bar{P}^T)]=0.$$

Thus there is a parameter $\eta>0$,

$$((H\bar{C})^T-\bar{B}_f^T\bar{E}^{-T}\bar{P}^T)^T((H\bar{C})^T-\bar{B}_f^T\bar{E}^{-T}\bar{P}^T)\leq \eta I, \tag{24}$$

where the parameter η is related to optimization (25)

$$\min \eta. \tag{25}$$

By Schur-complement, (24) is rebuilt as

$$\begin{bmatrix} -\eta I & ((H\bar{C})^T-\bar{B}_f^T\bar{E}^{-T}\bar{P}^T)^T \\ * & -I \end{bmatrix}\leq 0. \tag{26}$$

The bounded constraint is solved by using the LMI equations (25) and (26) together.

The design procedure is summarized as follows:

(i) Augment the nonlinear affine system (1) into the descriptor model as in (7) via selecting a proper matrix Θ. Select $\bar{L}_f=\bar{B}_f$. Choosing a positive parameter κ, the high gain $\bar{L}$ in SMO is designed by Lemma 1.

(ii) With the parameter γ in (2), the stability for error system (10) can be examined by combining the LMI functions in (19), (25) and (26). The gain H can be designed via the LMI equations. Then, with the known parameters f_M and f_m in (4), the discontinuous function $u_s(t)$ and the switching function $s(t)$ can be selected as in (11) and (12) respectively.

(iii) With the estimated full state $\hat{x}(t)$ and the one-to-one mapping relationship of decoupled disturbance $f_d(t)$ and $d(t)$ in function $f(x,d(t),t)$, the reconstruction of coupled disturbance $\hat{d}(t)$ can be calculated as

$$\hat{d}(t)=f^{-1}\left(\hat{x},\hat{f}_d(t),t\right). \tag{27}$$

For the reason of noises, a proper filter can be designed to smooth the disturbance $\hat{d}(t)$.

Remark 1: Instead of considering the piecewise and fuzzy models, the nonlinear system model is considered directly in this paper. The nonlinear part $g(x,u,t)$ in state space model (1) is assumed to be limited as in (2). If the nonlinear element $g(x,u,t)$ is replaced by linear element, the SMO design method and (2) also hold. It means that our study is also appropriate for the linear system.

Remark 2: By treating the coupled disturbance into the decoupled form, the SMO method is applied to reconstruct the time-varying parameter coupled disturbance. On one side the SMO is developed into the coupled disturbance reconstruction on the nonlinear systems. On the other side, compared with the learning and data processing methods, our method demands less computation and storage. Furthermore, the technique to reconstruct the coupled disturbance is potential for engineering applications such as robotic manipulator, flying vehicle, helicopter and space manipulation.

4. EXAMPLE

In this section, we offer a numerical simulation and an experiment to testify the method in our paper.

4.1. Simulation

The parameters of nonlinear system in (1) are as follows:

$$A=\begin{bmatrix} -1.7 & 0 & 3.5 & -1.8 \\ 0 & -2 & 0 & 1 \\ 0 & 0.8 & -1.5 & 0 \\ -3.7 & 0.7 & 0 & -4.9 \end{bmatrix},\ B_f=\begin{bmatrix} 0 & 0.7 \\ 1.2 & 0 \\ 0.1 & 0.7 \\ 0.9 & 0 \end{bmatrix},$$

$$f(x,d,t)=\begin{bmatrix} \frac{1}{\cos\left(2x_1^2\right)+3.54} & 0.4\cos(x_4) \\ 0 & \frac{1}{\sin^2(x_2+x_3)+0.5}+2.4 \end{bmatrix} d(t),$$

$$g(x,u,t)=\begin{bmatrix} \cos(x_1+x_2)+u_1 \\ \sin(x_3)+u_2\frac{1}{\cos(x_2+x_3)+2} \end{bmatrix},$$

$$C=\begin{bmatrix} 0.14 & 1 & 0 & 0 \\ 1.1 & 0 & 0 & 0.5 \\ 0 & 0 & 2 & 0 \end{bmatrix},\quad B=\begin{bmatrix} 1.1 & 0 \\ 0 & -0.5 \\ 0.4 & 0 \\ 0 & -1 \end{bmatrix}$$

$d(t)=[d_1^T(t)\ d_2^T(t)]^T$ is

$$d_1(t)=\begin{cases} 0, & t\in[0,\pi/2) \\ 20\sin(t-\pi/2), & t\in[\pi/2,\pi), \\ 20, & t\in[\pi,5+\pi/2), \\ 0, & t\in[5+\pi/2,+\infty), \end{cases}$$

$$d_2(t)=\begin{cases} 0, & t\in[0,8+\pi/2) \\ -24(t-4-\pi/2)/\pi, & t\in[8+\pi/2,8+\pi), \\ -12, & t\in[8+\pi,13+\pi/2), \\ -12\sin(t-13), & t\in[13+\pi/2,13+\pi), \\ 0, & t\in[13+\pi,+\infty). \end{cases}$$

The initial states of $x(t)$ in (6), $\hat{x}(t)$ and $\hat{f}_d(t)$ in SMO (9) are $x(0)=[1\ -6.5\ 0\ 2.5]^T$, $\hat{x}(0)=[0\ 0\ 0\ 0]^T$, $\hat{f}_d(0)=[0\ 0]^T$. The control input torque $u(t)$ is assumed as in Fig. 1. According to the design procedures, we estimate the state $x(t)$ and reconstruct the disturbance $d(t)$ as follows:

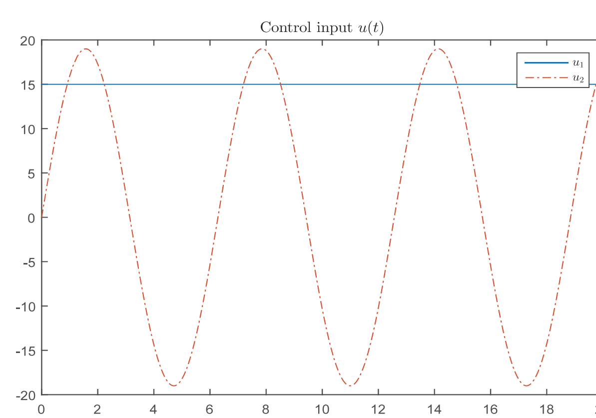

Fig. 1. Simulation: Control input $u(t)$.

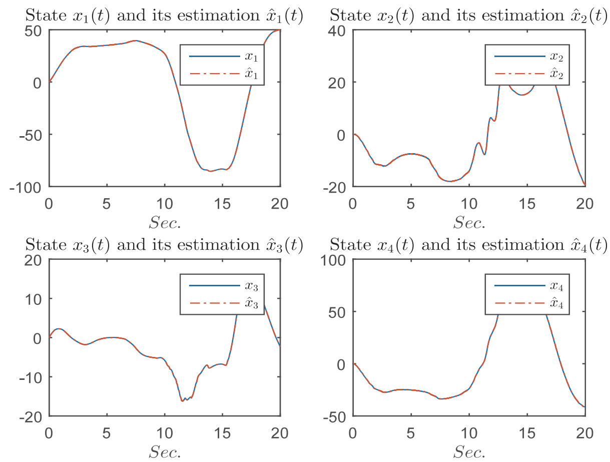

Fig. 2. Simulation: State $x(t)$ and its estimation $\hat{x}(t)$.

(i) Select positive matrix Θ

$$\Theta = \begin{bmatrix} 100 & 2 \\ 2 & 10 \end{bmatrix}.$$

Augment system (1) into (7). Choose $\bar{L}_f = \bar{B}_f$, $\kappa = 0.1$. By (17) and (18) in Lemma 1, we obtain gain $\bar{L}$

$$\bar{L} = 10^4 \times \begin{bmatrix} -0.1778 & 0.6119 & -0.1281 \\ 0.0491 & -0.0890 & 0.0379 \\ 0.0287 & 0.0123 & 0.1816 \\ 0.4409 & -1.2449 & 0.6380 \\ -0.0073 & -0.0037 & -0.0524 \\ 0.3854 & 0.1717 & 2.3247 \end{bmatrix}.$$

(ii) Assume $f_M = 20, f_m = 100, \zeta = 0.1, \gamma = 100$. By (20), (25) and (26), the stability for error system (10) can be examined. Gain H is chosen as

$$H = \begin{bmatrix} 0.0040 & -0.0138 & 0.0063 \\ 0.0004 & -0.0010 & 0.0000 \end{bmatrix}.$$

By (25), parameter η is 5.0768×10^{-11}. $u_s(t)$ and $s(t)$ are designed as in (11) and (12). The sign function $\text{sgn}(s(t))$ in $u_s(t)$ is calculated as $\text{sgn}(s(t)) = s(t)/(\|s(t)\| + \delta)$ via choosing $\delta = 1 \times 10^{-6}$.

(iii) By (27) and $\hat{f}_d(t)$, $\hat{d}(t)$ is reconstructed).

With the simulation results, we give the below analysis and introduction. First, shown in Fig. 2 in the sensorless case, although a portion of the state $x(t)$ is not completely known, the full estimated state variable $\hat{x}(t)$ can be accurately obtained in limited time; Second, with the full estimated state $\hat{x}(t)$, the shapes of the coupled disturbance $d(t)$ can be roughly reconstructed, as in Fig. 3; Third, the SMO approach can be used to estimate the general shape of the disturbances, but not proper for rapid changing disturbances; Fourth, our former research and the simulation result in Fig. 4 has demonstrated that, even being

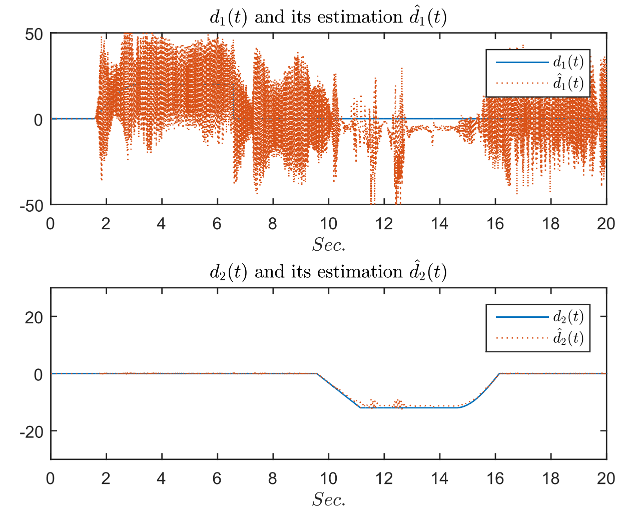

Fig. 3. Simulation: Disturbance $d(t)$ and its estimation $\hat{d}(t)$.

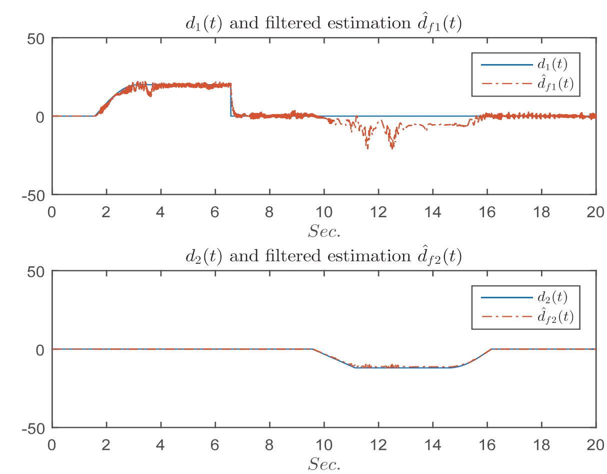

Fig. 4. Simulation: Disturbance $d(t)$ and filtered estimation $\hat{d}_f(t)$.

filtered, for strong disturbance (or high amplitude disturbance), duration 2s-6s of the up figure in Fig. 4, the SMO method works well, however, for the weak disturbance (or low amplitude disturbance), 10s-16s of the up figure in Fig. 4, its applicability is limited.

The estimated disturbance in Fig. 3 is noisy. It cannot be utilized directly in the real engineering. So, a smoother $\hat{d}_f(t) = [\hat{d}_{f1}^T(t)\ \hat{d}_{f2}^T(t)]^T$ in Fig. 4 can be obtained by $\hat{d}_{f1} = \frac{1}{0.05s+1}\hat{d}_1$, $\hat{d}_{f2} = \frac{1}{0.01s+1}\hat{d}_2$. The coupled disturbance can be reconstructed by the SMO method, which is one of breaks compared with [20, 21, 27, 28]. In the simulations of [21, 23], the authors assume the states of nonlinear systems are measured or the disturbance is decoupled with time varying parameters. In our simulation, the full system states don't need to be fully measured. With only part of the nonlinear system being measured, we can estimate the disturbances and system states successfully.

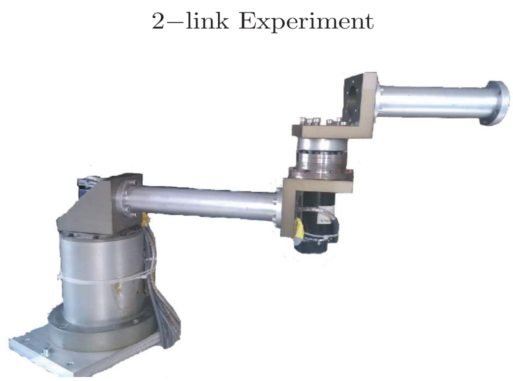

Fig. 5. Experiment.

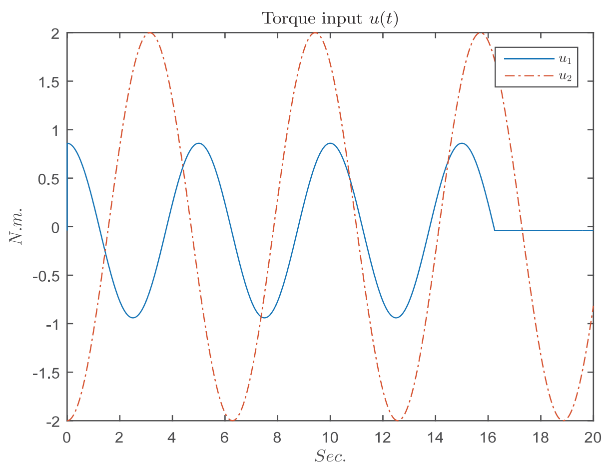

Fig. 6. Experiment: Torque Input $u(t)$.

Their methods cannot be used to reconstruct the coupled disturbance $d(t)$ in 4.1.

4.2. Experiment

In this subsection, we provide a planar 2-DOF manipulator experiment in Fig. 5, whose parameters in (1) are

$$A=\begin{bmatrix}0 & I\\ 0 & 0\end{bmatrix},\quad B_f=\begin{bmatrix}0\\ I_n\end{bmatrix},\quad B=\begin{bmatrix}0\\ I_n\end{bmatrix},$$
$$q=\Upsilon x(t),\quad \Upsilon=[I \quad 0],\quad C=[I_n \quad 0],$$
$$x(t)=[q^T(t) \quad \dot{q}^T(t)]^T,\quad f(x,t)=M^{-1}(\Upsilon x),$$
$$g(x,u,t)=-M^{-1}(\Upsilon x)(C(\Upsilon x,\dot{q})\dot{q}+F\dot{q})+M^{-1}(\Upsilon x)u.$$

Variable $q(t)\in\mathbb{R}^{n\times 1}$ is the vector containing joint angles. Generalized coordinates $\dot{q}(t)\in\mathbb{R}^{n\times 1}$ are the joint velocities. Matrices $M(q), C(q,\dot{q})$ and $F(\dot{q})$ are as follows.

$$M(q)=\begin{bmatrix}0.442+0.0286\cos(q_2) & *\\ 0.0088+0.0143\cos(q_2) & 0.2226\end{bmatrix},$$

$$C(q,\dot{q})=\begin{bmatrix}-0.029\sin(q_2)\dot{q}_2 & -0.014\sin(q_2)\dot{q}_2\\ 0.014\sin(q_2)\dot{q}_1 & 0\end{bmatrix},$$

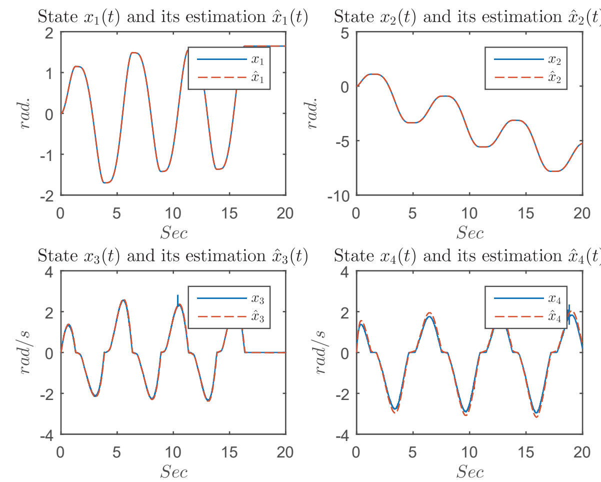

Fig. 7. Experiment: State $x(t)$ and its estimation $\hat{x}(t)$.

$$F(\dot{q})=\begin{bmatrix}2.6\times10^{-4}\dot{q}_1\\ 2.6\times10^{-4}\dot{q}_2\end{bmatrix}.$$

The control input $u(t)$ is in Fig. 6. The initial values of $x(t)$, $\hat{x}(t)$, $d(t)$ and $\hat{d}(t)$ are all zero vectors. The design procedures of SMO are presented as follows.

(i) Select $\Theta=36I_n$. Gain $\bar{L}_f=\bar{B}_f$, $\kappa=0.1$. The high gain $\bar{L}$ is designed as

$$\bar{L}=10^3\times\begin{bmatrix}0.1446 & 0\\ 0 & 0.1446\\ 5.2418 & 0\\ 0 & 5.2418\\ 0.2621 & 0\\ 0 & 0.2621\end{bmatrix}.$$

(ii) Assume $f_M=20, f_m=360, \zeta=0.1, \gamma=100$ and

$$H=\begin{bmatrix}-0.0115 & 0\\ 0 & -0.0115\end{bmatrix}.$$

It obtains the parameter $\eta=6.6282e-11$.

(iii) By (27), $\hat{x}(t)$ and $\hat{f}_d(t)$, we can reconstruct torque disturbance $\hat{d}(t)$ as $\hat{d}(t)=M(\hat{q})\hat{f}_d(t)$.

In Fig. 7, $\dot{q}(t)=[x_3^T(t)\ x_4^T(t)]^T$ is calculated as $\dot{q}(t)=q(t)\frac{s}{0.001s+1}$. The torque disturbance $d(t)$ in Fig. 8 is calculated as $d(t)=\tau(t)-u(t)$. The torque input $\tau(t)$ is computed $\tau(t)=M\ddot{q}+C(q,\dot{q})\dot{q}+F$. The acceleration vector $\ddot{q}(t)$ is obtained by $\ddot{q}(t)=\dot{q}(t)\frac{s}{0.02s+1}$. In Fig. 7 and Fig. 8, the $\hat{x}(t)$ and $\hat{d}(t)$ approach the real state $x(t)$ and $d(t)$ well. The fault of encoder leads to the peaks, about 10.3s and 18.9s, in Fig. 7 and Fig. 8. The disturbance $d(t)$ in this experiment is caused by model uncertainties, static friction and mechanical gears. Apparently, by the scheme proposed in our paper, the reconstructed disturbance $\hat{d}(t)$ and the estimated system state $\hat{x}(t)$ approximate the calculated disturbance $d(t)$ and system state $x(t)$ well. It shows the validity of our method.

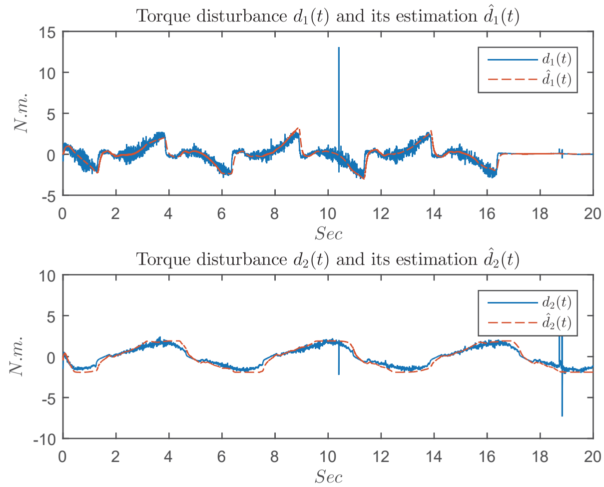

Fig. 8. Experiment: Torque disturbance $d(t)$ and its estimation $\hat{d}(t)$.

5. CONCLUSIONS

The study of this paper focuses mainly on the state estimation and coupled disturbance reconstruction problems of a class of nonlinear systems by SMO method. The nonlinear system model and the sensorless problems are considered. The model transformation is firstly utilized to augment the original nonlinear system into a novel descriptor system, based on which the SMO was designed. With the state estimates in SMO, the sufficient criterium on error system stability is verified. The full system state and coupled disturbance are estimated successfully. At the end of this paper, a numerical simulation and an experiment show the effectiveness of the scheme.

Our future work will focus on the trajectory tracking control, fault tolerant control, fault compensation of nonlinear systems, and their applications in robots, self-driving car and other complicate industrial applications.

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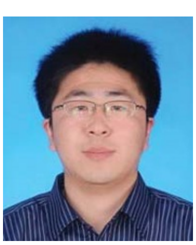

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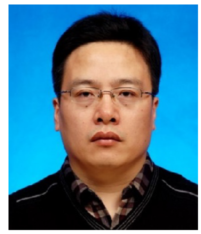

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