

# Adaptive Neural Dynamic Surface Control for a General Class of Stochastic Nonlinear Systems with Time Delays and Input Dead-zone

Wen-Jie Si\*, Xun-De Dong, and Fei-Fei Yang

**Abstract:** This paper investigates adaptive tracking control for a more general class of stochastic nonlinear time-delay systems with unknown input dead-zone. For the considered system, the drift and diffusion terms contain time-delay state variables. In control design, Lyapunov-Krasovskii functionals are employed to handle unknown time-delay terms. Then, unknown nonlinear functions are approximated by RBF neural networks, and the dynamic surface control (DSC) technique is utilized to avoid the problem of explosion of complexity. At last, based on the Lyapunov stability theory, a robust adaptive controller is designed to guarantee that all closed-loop signals are bounded in probability and the tracking error converges to a small neighborhood of the origin. The simulation example is presented to further show the effectiveness of the proposed approach.

**Keywords:** Dynamic surface control, input dead-zone, neural adaptive control, stochastic nonlinear systems, unknown time delays.

## 1. INTRODUCTION

Stochastic disturbance and time delay appear in many real systems, which may degrade system performance and even cause instability [1]. To ensure the system stability and control performance, the control design for stochastic nonlinear systems with time delays needs to be studied.

Many studies have focused on the control of complex systems [2, 3]. However, these results were only suitable for nonlinear systems in which the nonlinearities were known or can be linearly parameterized. For the systems with unknown nonlinear dynamics, both fuzzy logic systems (FLSs) and neural networks (NNs) have been proved to be useful in control design [4–6]. In [7, 8], the control of the time-delay systems was presented via fuzzy logic. Hybrid feedback feedforward was presented based on neural learning control in [9, 10]. The fuzzy  $H_\infty$  control was presented in [11], and [12] designed an interval type-2 controller for discrete-time fuzzy systems. Stochastic nonlinear systems were considered in [13]. Recently, many approximation-based control has also been developed to deal with the control problem for stochastic nonlinear time-delay systems [14–16]. The above-mentioned full-state feedback approaches in [14–16] require that all system states are available. When states were unmeasurable, an adaptive NN output-feedback controller was developed in [17] for a class of stochastic nonlinear strict-feedback systems with time delay. An observer-based adaptive con-

trol scheme was proposed in [18] for nonlinear stochastic systems with full-state time delays.

However, the above-mentioned results do not consider the effect of input nonlinearity. In [19, 20], the robust adaptive control methods were used for nonlinear systems with parametric uncertainties subject to the input dead-zone, and the systems must satisfy linear parameterized condition. Recently, when the knowledge of system functions is unavailable, in order to handle unknown nonlinear systems with input dead-zone, many adaptive controllers have been proposed by [21–24]. In [25], adaptive fuzzy backstepping output feedback tracking control was presented for multi-input and multi-output (MIMO) stochastic nonlinear systems. The problem of adaptive decentralized NN control was investigated in [26] for large-scale stochastic nonlinear time-delay systems with input dead-zone.

During the traditional backstepping design procedure, repeated differentiations of virtual functions cause the ‘explosion of complexity’. Recently, to overcome this problem, a dynamic surface control (DSC) method has been developed in [27]. Using the DSC technique, [28] proposed a decentralized adaptive controller for a class of large-scale nonlinear time-delay systems. The work in [29] considered an adaptive fuzzy output-feedback backstepping control approach. The dynamic surface control was incorporated to stochastic nonlinear systems in [30]. In [31, 32], the composite learning was achieved via the

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DSC technique. Furthermore, when the states were unmeasured, in [33], the problem of adaptive neural DSC method was addressed for nonstrict-feedback stochastic nonlinear systems.

Motivated by these observations, we investigate the problem of adaptive neural DSC for a more general class of stochastic nonlinear systems with time delays and input dead-zone. The drift and diffusion terms are the functions of time-delay states and current states, which makes the control design more difficult. The proposed controller can guarantee the boundedness of the closed-loop system in probability.

The main contributions lie in the following:

- 1) A general class of stochastic nonlinear time-delay systems is investigated, in which the drift and diffusion terms are dependent on the states and time-delay states. A Lyapunov-Krasovskii functional is introduced to deal with the unknown time delays. The variable separation technique and the neural networks approximation are effectively employed to design an adaptive controller.
- 2) By using the DSC technique, the designed control scheme can overcome the defect of 'explosion of complexity'. In addition, the norm of the unknown weight vector itself is estimated in this paper. Therefore, the proposed controller can reduce the number of learning parameters, and reduce the computational burden.

The rest of the paper is organized as follows. In Section 2, the preliminaries and problem formulation are provided. Section 3 presents an adaptive neural controller, and the stability analysis is provided. Section 4 shows the simulation example. Finally, this paper is concluded in Section 5.

## 2. PRELIMINARY KNOWLEDGE AND SYSTEM FORMULATION

### 2.1. Preliminaries

Consider the following stochastic nonlinear system

$$dx = f(x,t)dt + h(x,t)d\omega, \forall x \in \mathbf{R}^n, \quad (1)$$

where  $x \in \mathbf{R}^n$  is the system state vector,  $f: \mathbf{R}^n \times \mathbf{R}^+ \rightarrow \mathbf{R}^n$ ,  $h: \mathbf{R}^n \times \mathbf{R}^+ \rightarrow \mathbf{R}^{n \times r}$  are locally Lipschitz.  $\omega$  is an  $r$ -dimensional independent standard Brownian motion defined on the complete probability space  $(\Omega, F, \{F_t\}_{t \geq 0}, P)$  with  $\Omega$  being a sample space,  $F$  being  $\sigma$ -field,  $\{F_t\}_{t \geq 0}$  being a filtration, and  $P$  being a probability measure.

**Definition 1** [34]: For any given positive function  $V(x,t) \in C^2$ , associated with system (1), the differential operator  $L$  is defined as:

$$LV = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f + \frac{1}{2} Tr \{ h^T \frac{\partial^2 V}{\partial x^2} h \}, \quad (2)$$

where  $Tr(A)$  denotes  $A$  trace.

**Lemma 1** (Young's inequality) [35]: For  $\forall (x,y) \in \mathbf{R}^2$ , the following inequality holds:

$$xy \leq \frac{\varepsilon^p}{p} |x|^p + \frac{1}{q\varepsilon^q} |y|^q, \quad (3)$$

where  $\varepsilon > 0$ ,  $p > 1$ ,  $q > 1$ ,  $(p-1)(q-1) = 1$ .

**Lemma 2** [36]: For  $1 \leq i \leq n$  and  $t_i > 0$ , define the set  $\Lambda$  given by  $\Lambda := \{z_i \mid |z_i| \leq 0.8814t_i\}$ . Then, for  $z_i \notin \Lambda$ , the inequality  $1 - 4 \tanh^4(\frac{z_i}{t_i}) < 0$  is satisfied.

### 2.2. System representation

Consider the following stochastic nonlinear time-delay systems

$$\begin{cases} dx_i = (x_{i+1} + \phi_i(\bar{x}_i) + f_i(t, \bar{x}_i(t), \bar{x}_i(t - \tau_i)))dt \\ \quad + h_i^T(t, \bar{x}_i(t), \bar{x}_i(t - \tau_i))d\omega, \\ \quad \quad \quad i = 1, 2, \dots, n-1, \\ dx_n = (u + \phi_n(x) + f_n(t, x(t), x(t - \tau_n)))dt \\ \quad + h_n^T(t, x(t), x(t - \tau_n))d\omega, \\ u = D(v), \\ y(t) = x_1(t), \end{cases} \quad (4)$$

where  $x_i \in \mathbf{R}$  ( $i = 1, \dots, n$ ),  $u \in R$  and  $y \in R$  are the system state variable, system input and output, respectively.  $\bar{x}_i := [x_1, \dots, x_i]^T$ ,  $x = \bar{x}_n := [x_1, \dots, x_n]^T$ .  $\phi_i(\cdot)$  are unknown smooth nonlinear function.  $f_i(\cdot)$  and  $h_i(\cdot)$  are unknown smooth functions.  $\omega$  is defined in (1).  $\tau_i$  is the unknown constant time delay term.  $u(v) \in R$  is the system input and the output of the dead zone, and the dead-zone characteristic is described as  $D(v)$  with  $v$  being the input.

The main goal is to present an adaptive controller for the system (4) such that  $y$  can track the desired signal  $y_d(t)$ , and all closed-loop signals remain bounded.

**Assumption 1:** The desired signal  $y_d(t)$  and its  $n$ th order  $y_d^n(t)$  are continuous and bounded.

**Assumption 2** [37, 38]: For nonlinear functions  $f_i$  and  $h_i$ , there exist nonnegative smooth functions  $f_{i1}$ ,  $f_{i2}$ ,  $h_{i1}$  and  $h_{i2}$  such that

$$\begin{aligned} |f_i(\bar{x}_i, \bar{x}_i(t - \tau_i))| &\leq f_{i1}(\bar{x}) + f_{i2}(\bar{x}(t - \tau_i)), \\ |h_i(\bar{x}_i, \bar{x}_i(t - \tau_i))| &\leq h_{i1}(\bar{x}) + h_{i2}(\bar{x}(t - \tau_i)). \end{aligned} \quad (5)$$

According to Assumption (2), let  $p \geq 1$ , and based on [39], the following inequalities are obtained as

$$\begin{aligned} |f_i(\bar{x}, \bar{x}(t - \tau_i))|^p &\leq 2^{p-1} (f_{i1}^p(\bar{x}) + f_{i2}^p(\bar{x}(t - \tau_i))), \\ |h_i(\bar{x}, \bar{x}(t - \tau_i))|^p &\leq 2^{p-1} (h_{i1}^p(\bar{x}) + h_{i2}^p(\bar{x}(t - \tau_i))). \end{aligned} \quad (6)$$

### 2.3. Dead-zone characteristic

The dead-zone nonlinearity of the actuator can be described as follows [40]:

$$u = D(v) = \begin{cases} h_r(v - b_r), & v \geq b_r, \\ 0, & b_l < v < b_r, \\ h_l(v - b_l), & v \leq b_l, \end{cases} \quad (7)$$

where  $h_r(\cdot), h_l(\cdot)$  are unknown smooth functions.

According to [40], the dead-zone nonlinearity can be rewritten as

$$u = D(v) = \mathcal{E}(v)v + d(v), \quad (8)$$

where  $D_{\min} \leq \mathcal{E} \leq D_{\max}$ ,  $\|d(v)\| \leq d^*$ .

#### 2.4. RBF neural networks

In this paper, RBF NNs will be used to model any continuous function  $f(Z) : R^q \rightarrow R$  over a compact set  $\Omega_Z \subset R^q$  for given arbitrary accuracy  $\varepsilon^* > 0$  as follows

$$f(Z) = W^{*T}S(Z) + \varepsilon(Z), \forall Z \in \Omega_Z, \quad (9)$$

where  $W = [w_1, w_2, \dots, w_l]^T \in R^l$  is the neural weight vector with  $l > 1$  being the NN node number.  $S(Z) = [s_1(Z), s_2(Z), \dots, s_l(Z)]^T$  is the basis function vector with  $s_i(Z) = \exp[-(Z - \xi_i)^T(Z - \xi_i)/\eta^2]$ , where  $\xi_i = [\xi_{i1}, \xi_{i2}, \dots, \xi_{iq}]^T$  is the center of the receptive field and  $\eta$  is the width of Gaussian function<sup>[41]</sup>.  $\hat{W}$  denotes the estimate of  $W^*$ , with  $\tilde{W} = \hat{W} - W^*$ .

### 3. ADAPTIVE CONTROLLER DESIGN

In this section, an adaptive control design and stability analysis will be developed. To simplify the notations,  $f_i = f_i(t, \bar{x}_i(t), \bar{x}_i(t - \tau_i))$ ,  $h_i = h_i(t, \bar{x}_i(t), \bar{x}_i(t - \tau_i))$ . For the time-delay functions,  $h_i(x(t - \tau_i))$  is denoted as  $h_i(\tau_i)$  and  $f_i(x(t - \tau_i))$  is denoted as  $f_i(\tau_i)$ .

The following coordinate transformation is used.

$$\begin{aligned} z_1 &= x_1 - y_d, z_i = x_i - \alpha_{i-1,f}, \\ e_i &= \alpha_{i-1,f} - \alpha_{i-1}, i = 2, \dots, n, \end{aligned} \quad (10)$$

where  $\alpha_{i-1}$  is the virtual control law, which will be developed later.  $e_i$  is the first filter error.  $\alpha_{i-1,f}$  is the filtered virtual control, we pass  $\alpha_{i-1}$  through the following first-order filter

$$\beta_i \dot{\alpha}_{i-1,f} + \alpha_{i-1,f} = \alpha_{i-1}, \quad (11)$$

where  $\beta_i$  is a time constant.

**Step 1:** Define  $z_1 = x_1 - y_d$ , and its differential is

$$\dot{z}_1 = (z_2 + e_2 + \alpha_1 + \phi_1 + f_1 - \dot{y}_d)dt + h_1^T d\omega. \quad (12)$$

To obtain the filtered virtual control  $\alpha_{1f}$ , we pass  $\alpha_1$  through a first-order filter,  $\beta_2 \dot{\alpha}_{1f} + \alpha_{1f} = \alpha_1$ , where  $\beta_2$  is a time constant.

Define the filter output error  $e_2 = \alpha_{1f} - \alpha_1$ . One has  $\dot{\alpha}_{1f} = -\frac{e_2}{\beta_2}$  and its differential is

$$de_2 = \left(-\frac{e_2}{\beta_2} - l\alpha_1\right)dt + \frac{\partial \alpha_1}{\partial y} h_1^T d\omega, \quad (13)$$

where

$$\begin{aligned} l\alpha_1 &= \frac{\partial \alpha_1}{\partial x_1}(x_2 + \phi_1 + f_1) + \frac{\partial \alpha_1}{\partial \theta_1} \dot{\theta}_1 \\ &+ \frac{1}{2} \frac{\partial^2 \alpha_1}{\partial y^2} h_1^T h_1 + \frac{\partial \alpha_1}{\partial y_d} \dot{y}_d + \frac{\partial \alpha_1}{\partial \dot{y}_d} \ddot{y}_d. \end{aligned} \quad (14)$$

Choose the following Lyapunov function candidate

$$V_1 = \frac{1}{4}z_1^4 + \frac{1}{4}e_2^4 + \frac{1}{2r_1}\tilde{\theta}_1^2 + V_{Q1}, \quad (15)$$

where  $V_{Q1}$  is the Lyapunov-Krasovskii function in the following form

$$V_{Q1} = \exp(-\pi_1 t) \int_{t-\tau_1}^t \exp(\pi_1 s) \Psi_1(x_1(s)) ds. \quad (16)$$

$\dot{V}_{Q1}$  is given by

$$\begin{aligned} \dot{V}_{Q1} &= -\pi_1 V_{Q1} + \Psi_1(x_1(t)) \\ &- \exp(-\pi_1 \tau_1) \Psi_1(x_1(t - \tau_1)), \end{aligned} \quad (17)$$

where  $\pi_1$  is a positive constant and  $\Psi_1$  is given later.

$$\begin{aligned} LV_1 &= z_1^3(z_2 + e_2 + \alpha_1 + \phi_1 + f_1 - \dot{y}_d) \\ &+ \frac{3}{2}z_1^2 h_1^T h_1 + e_2^3 \left(-\frac{e_2}{\beta_2} - l\alpha_1\right) \\ &+ \frac{3}{2}e_2^2 \left\| \frac{\partial \alpha_1}{\partial y} h_1^T \right\|^2 + \frac{\tilde{\theta}_1}{r_1} \dot{\theta}_1 - \pi_1 V_{Q1} \\ &+ \Psi_1(x_1) - e^{-\pi_1 \tau_1} \Psi_1(x_1(t - \tau_1)). \end{aligned} \quad (18)$$

We choose  $\Psi_1(x_1(t - \tau_1))$  as

$$\begin{aligned} \Psi_1(x_1(t - \tau_1)) &= 4\exp(\pi_1 \tau_1) f_{12}^4(x_1(t - \tau_1)) \\ &+ 14\exp(\pi_1 \tau_1) h_{12}^4(x_1(t - \tau_1)). \end{aligned} \quad (19)$$

Define a new function  $M_2$

$$\begin{aligned} -M_2 &= -\left(\frac{\partial \alpha_1}{\partial x_1}(x_2 + \phi_1) + \frac{\partial \alpha_1}{\partial \theta_1} \dot{\theta}_1\right. \\ &\left. + \frac{\partial \alpha_1}{\partial y_d} \dot{y}_d + \frac{\partial \alpha_1}{\partial \dot{y}_d} \ddot{y}_d\right). \end{aligned} \quad (20)$$

Obviously,  $M_2$  is a smooth function, which has its maximum denoted by  $B_2^*$

$$-e_2^3 M_2 \leq \frac{3}{4}e_2^4 + \frac{1}{4}B_2^{*4}. \quad (21)$$

Define the following functions

$$\begin{aligned} H_1(x_1) &= \Psi_1(x_1) + 2h_{11}^4(x_1) + 2f_{11}^4(x_1(t)) \\ &+ 6h_{11}^4(x_1) + \frac{3}{4}e_2^4 \left(\frac{\partial \alpha_1}{\partial y}\right)^4 + 6h_{11}^4(x_1) \\ &+ \frac{3}{4}e_2^4 \left(\frac{\partial \alpha_1}{\partial y}\right)^{\frac{4}{3}} + 2f_{11}^4(x_1(t)) + \frac{1}{4}e_2^6 \left(\frac{\partial^2 \alpha_1}{\partial y^2}\right)^2. \end{aligned} \quad (22)$$

Now, choose the virtual control law  $\alpha_1$  as

$$\alpha_1 = -k_1 z_1 - \frac{b_1}{2a_1^2} \hat{\theta}_1 S^T(Z_1) S(Z_1) z_1^3 + \dot{y}_d, \quad (23)$$

where  $k_1$  and  $a_1$  are positive constants with the parameter  $b_1$  being specified later.

Define the adaptive law as follows:

$$\dot{\hat{\theta}}_1 = \frac{r_1 b_1}{2a_1^2} z_1^6 S_1^T(Z_1) S_1(Z_1) - \sigma_1 \hat{\theta}_1, \quad (24)$$

where  $\sigma_1$  is a positive constant.

$$\begin{aligned} LV_1 \leq & z_1^3 (\alpha_1 + \phi_1 - \dot{y}_d + 3z_1 \\ & + \frac{H_1(x_1)}{z_1^3}) + e_2^3 (-\frac{e_2}{\beta_2}) + \frac{1}{4} z_2^4 \\ & + \frac{1}{4} B_2^{*4} + e_2^4 + \frac{\tilde{\theta}_1}{r_1} \dot{\hat{\theta}}_1 - \pi_1 V_{Q1}. \end{aligned} \quad (25)$$

Define the unknown function  $F_1(x_1)$ :

$$F_1(x_1) = \phi_1 + \frac{4}{z_1^3} \tanh^4\left(\frac{z_1}{l_1}\right) H_1, \quad (26)$$

where  $l_1$  is a positive parameter.

It is shown in (25) the unknown term  $\frac{H_1}{z_1^3}$  is not well defined at  $z_1 = 0$ . The hyperbolic tangent function  $\tanh(\frac{z_1}{l_1})$  is introduced here.

A RBF NN  $W_1^{*T} S_1$  is employed to approximate  $F_1$

$$F_1 = W_1^{*T} S_1(Z_1) + \varepsilon_1(Z_1), |\varepsilon_1(Z_1)| \leq \varepsilon_1^*, \quad (27)$$

where  $Z_1 = [x_1, \hat{\theta}_1, y_d]^T$  and  $\varepsilon_1(Z_1)$  is the approximation error.

The following inequality holds:

$$z_1^3 F_1 \leq \frac{b_1}{2a_1^2} z_1^6 \theta_1^* S_1^T S_1 + \frac{1}{2} a_1^2 + \frac{3}{4} z_1^4 + \frac{1}{4} \varepsilon_1^{*4}, \quad (28)$$

where  $\|W_1^*\|^2 = b_1 \theta_1^*$ . Considering  $\tilde{\theta}_1 = \hat{\theta}_1 - \theta_1^*$ , one has:

$$-\frac{\sigma_1}{r_1} \tilde{\theta}_1 \hat{\theta}_1 \leq -\frac{\sigma_1 \tilde{\theta}_1^2}{2r_1} + \frac{\sigma_1 \theta_1^{*2}}{2r_1}. \quad (29)$$

Considering (29), one yields

$$\begin{aligned} LV_1 \leq & - (k_1 - 3\frac{3}{4}) z_1^4 - (\frac{1}{\beta_2} - 1) e_2^4 + \frac{1}{4} z_2^4 \\ & - \frac{\sigma_1 \tilde{\theta}_1^2}{2r_1} + \frac{\sigma_1 \theta_1^{*2}}{2r_1} + \frac{1}{2} a_1^2 + \frac{1}{4} \varepsilon_1^{*4} + \frac{1}{4} B_2^{*4} \\ & - \pi_1 V_{Q1} + (1 - 4 \tanh^4(\frac{z_1}{l_1})) H_1. \end{aligned} \quad (30)$$

**Step i** ( $2 \leq i \leq n-1$ ): At this step, similar to (30), we can obtain a result as

$$V_{i-1} = V_1 + \sum_{j=1}^{i-1} (\frac{1}{4} z_j^4 + \frac{1}{4} e_{j+1}^4 + \frac{1}{2r_j} \tilde{\theta}_j^2 + V_{Qj}). \quad (31)$$

We have

$$\begin{aligned} LV_{i-1} \leq & - (k_1 - 3\frac{3}{4}) z_1^4 - \sum_{j=2}^{i-1} (k_j - 4) z_j^4 \\ & - \sum_{j=2}^i (\frac{1}{\beta_j} - 1) e_j^4 + \frac{1}{4} z_i^4 + \sum_{j=2}^i \frac{1}{4} B_i^{*4} \\ & - \sum_{j=1}^{i-1} \frac{\sigma_j \tilde{\theta}_j^2}{2r_j} + \sum_{j=1}^{i-1} (\frac{\sigma_j \theta_j^{*2}}{2r_j} + \frac{1}{2} a_j^2 + \frac{1}{4} \varepsilon_j^{*4}) \\ & + \sum_{j=1}^{i-1} (-\pi_j V_{Qj} + (1 - 4 \tanh^4(\frac{z_j}{l_j})) H_j). \end{aligned} \quad (32)$$

In the following, we will prove that (32) holds for  $i$ th Lyapunov function defined as follows:

$$V_i = V_{i-1} + \frac{1}{4} z_i^4 + \frac{1}{4} e_{i+1}^4 + \frac{1}{2r_i} \tilde{\theta}_i^2 + V_{Qi}, \quad (33)$$

where  $V_{Qi}$  is the Lyapunov-Krasovskii function given as follows:

$$V_{Qi} = \exp(-\pi_i t) \int_{t-\tau_i}^t \exp(\pi_i s) \Psi_i(\bar{x}_i(s)) ds. \quad (34)$$

Then, its time derivative is

$$\begin{aligned} \dot{V}_{Qi} = & -\pi_i V_{Qi} + \Psi_i(\bar{x}_i(t)) \\ & - \exp(-\pi_i \tau_i) \Psi_i(\bar{x}_i(t - \tau_i)), \end{aligned} \quad (35)$$

where  $\pi_i$  is a positive constant, the positive function  $\Psi_i$  is defined later to cancel the time-delay terms.

A similar procedure is recursively employed for each step  $i$ , define  $z_i = x_i - \alpha_{i-1,f}$ , with the first-order filter  $\alpha_{i-1,f}$  being defined as the  $(i-1)$ th step. Let the virtual control law  $\alpha_{i-1}$  pass through it with time constant  $\beta_i$ , i.e.  $\beta_i \dot{\alpha}_{i-1,f} + \alpha_{i-1,f} = \alpha_{i-1}$ ,  $\alpha_{i-1,f}(0) = \alpha_{i-1}(0)$ , one can obtain that

$$dz_i = (z_{i+1} + e_{i+1} + \alpha_i + \phi_i + f_i + \frac{e_i}{\beta_i}) dt + h_i d\omega. \quad (36)$$

Define the output of this filter as  $e_{i+1} = \alpha_{i,f} - \alpha_i$ . It yields that  $\dot{\alpha}_{i-1,f} = -e_i/\beta_i$ . Its differential is

$$de_{i+1} = (-e_{i+1}/\beta_{i+1} - l\alpha_i) dt - \sum_{j=1}^i \frac{\partial \alpha_i}{\partial x_j} h_j^T d\omega, \quad (37)$$

where

$$\begin{aligned} l\alpha_i = & \sum_{j=1}^i \frac{\partial \alpha_i}{\partial x_j} (x_{j+1} + \phi_j + f_j) \\ & + \sum_{j=1}^i \frac{\partial \alpha_i}{\partial \theta_j} \dot{\theta}_j + \sum_{j=0}^i \frac{\partial \alpha_i}{\partial y_d^{(j)}} y_d^{(j+1)} \\ & + \frac{1}{2} \sum_{p,q=1}^i \frac{\partial^2 \alpha_i}{\partial x_p \partial x_q} h_p^T h_q + \sum_{j=1}^{i-1} \frac{\partial \alpha_i}{\partial \alpha_{i,f}} \dot{\alpha}_{i,f}. \end{aligned} \quad (38)$$

Define  $\Psi_i(\bar{x}_i(t - \tau_i))$  in (35) as

$$\begin{aligned} \Psi_i(\bar{x}_i(\tau_i)) &= \sum_{j=1}^i 8 \exp(\pi_i \tau_i) \|h_{j2}(x_j(\tau_j))\|^4 \\ &\quad + \sum_{j=1}^i 2 \exp(\pi_i \tau_i) \|f_{j2}(x_j(\tau_j))\|^4 \\ &\quad + 2 \exp(\pi_i \tau_i) f_{i2}^4(\bar{x}_i(\tau_i)) \\ &\quad + 6 \exp(\pi_i \tau_i) h_{i2}^4(\bar{x}_i(\tau_i)). \end{aligned} \quad (39)$$

For  $i = 2, \dots, n-1$ , define the following functions

$$\begin{aligned} -M_{i+1} &= -\left( \sum_{j=1}^i \frac{\partial \alpha_i}{\partial x_j} (x_{j+1} + \phi_j) + \sum_{j=1}^i \frac{\partial \alpha_i}{\partial \theta_j} \dot{\theta}_j \right. \\ &\quad \left. + \sum_{j=0}^i \frac{\partial \alpha_i}{\partial y_d^{(j)}} y_d^{(j+1)} + \sum_{j=1}^{i-1} \frac{\partial \alpha_i}{\partial \alpha_{i,f}} \dot{\alpha}_{i,f} \right), \end{aligned} \quad (40)$$

where  $M_{i+1}$  is a smooth functions, which has its maximum denoted by  $B_{i+1}^*$ .

$$-e_{i+1}^3 M_{i+1} \leq \frac{3}{4} e_{i+1}^4 + \frac{1}{4} B_{i+1}^{*4}. \quad (41)$$

Define the following nonlinear function  $H_i$  as

$$\begin{aligned} H_i(\bar{x}_i) &= \Psi_i(\bar{x}_i) + \frac{3}{4} e_{i+1}^4 + 2f_{i1}^4(\bar{x}_i(t)) + 6h_{i1}^4(\bar{x}_i) \\ &\quad + \sum_{j=1}^i \frac{3}{4} i^2 e_{i+1}^4 \left( \frac{\partial \alpha_i}{\partial x_j} \right)^4 + \sum_{j=1}^i 6 \|h_{j1}(x_j(t))\|^4 \\ &\quad + \sum_{j=1}^i \frac{3}{4} e_{i+1}^4 \frac{\partial \alpha_i}{\partial x_j} + \sum_{j=1}^i 2f_{j1}^4(x_j(t)) \\ &\quad + \sum_{j=1}^i \sum_{k=1}^i \frac{1}{4} |e_{i+1}^6| \left| \frac{\partial^2 \alpha_i}{\partial x_j \partial x_k} \right|^2. \end{aligned} \quad (42)$$

Design the virtual control function as

$$\alpha_i = -k_i z_i - \frac{b_i}{2a_i^2} \hat{\theta}_i S_i^T(Z_i) S_i(Z_i) z_i^3 - \frac{e_i}{\beta_i}, \quad (43)$$

where  $k_i$  and  $a_i$  are positive constants with  $b_i$  being a design parameter defined later.

$$\begin{aligned} LV_i &\leq LV_{i-1} + z_i^3 (-k_i - 3) z_i + \phi_i \\ &\quad + \frac{H_i(x_i)}{z_i^3} - \left( \frac{1}{\omega_{i+1}} + 1 \right) e_{i+1}^4 + \frac{1}{4} B_{i+1}^{*4} \\ &\quad + \frac{1}{4} z_{i+1}^4 + \frac{\tilde{\theta}_1}{r_i} \dot{\theta}_1 - \pi_i V_{Q_i}. \end{aligned} \quad (44)$$

Similar to Step 1, the unknown term  $\frac{4}{z_i^3} \tanh^4\left(\frac{z_i}{l_i}\right) H_i$  can be approximated by RBF neural network.

Define the unknown nonlinear function  $F_i(\bar{x}_i)$ :

$$F_i(\bar{x}_i) = \frac{4}{z_i^3} \tanh^4\left(\frac{z_i}{l_i}\right) H_i + \phi_i, \quad (45)$$

where  $l_i$  a positive design parameter.

A RBF NN  $W_i^{*T} S_i$  is employed to approximate  $F_i$

$$F_i = W_i^{*T} S_i(Z_i) + \varepsilon_i(Z_i), |\varepsilon_i(Z_i)| \leq \varepsilon_i^*, \quad (46)$$

where  $Z_i = [x_1, \dots, x_i, \hat{\theta}_1, \dots, \hat{\theta}_1, \alpha_{i,f}]^T$ ,  $\varepsilon_i(Z_i)$  is the approximation error.

The following inequality holds:

$$z_i^3 F_i \leq \frac{b_i}{2a_i^2} z_i^6 \theta_i^* S_i^T S_i + \frac{1}{2} a_i^2 + \frac{3}{4} z_i^4 + \frac{1}{4} \varepsilon_i^{*4}, \quad (47)$$

where  $\|W_i^*\|^2 = b_i \theta_i^*$ .

The adaptive law:

$$\dot{\hat{\theta}}_i = \frac{r_i b_i}{2a_i^2} z_i^6 S_i^T(Z_i) S_i(Z_i) - \sigma_i \hat{\theta}_i. \quad (48)$$

Considering  $\tilde{\theta}_i = \hat{\theta}_i - \theta_i^*$ , one has:

$$-\frac{\sigma_i}{r_i} \tilde{\theta}_i \hat{\theta}_i \leq -\frac{\sigma_i \tilde{\theta}_i^2}{2r_i} + \frac{\sigma_i \theta_i^{*2}}{2r_i}. \quad (49)$$

Then, one has

$$\begin{aligned} LV_i &\leq -\left(k_i - 3\frac{3}{4}\right) z_i^4 - \sum_{j=2}^i (k_j - 4) z_j^4 \\ &\quad - \sum_{j=1}^i \left( \frac{1}{\beta_{j+1}} - 1 \right) e_{j+1}^4 + \frac{1}{4} z_{i+1}^4 - \sum_{j=1}^i \frac{\sigma_j \tilde{\theta}_j^2}{2r_j} \\ &\quad + \sum_{j=1}^i \frac{1}{4} B_{j+1}^{*4} + \sum_{j=1}^i \left( \frac{\sigma_j \theta_j^{*2}}{2r_j} + \frac{1}{2} a_j^2 + \frac{1}{4} \varepsilon_j^{*4} \right) \\ &\quad + \sum_{j=1}^i \left( -\pi_j V_{Q_j} + (1 - 4 \tanh^4\left(\frac{z_j}{l_j}\right)) H_j \right). \end{aligned} \quad (50)$$

**Step n:** This is the final step, the actual control  $u(v)$  will be given. Define  $z_n = x_n - \alpha_{n-1,f}$ , and by Itô formula, we have

$$dz_n = (\mathcal{E}(v)v + d(v) + \phi_n(x) + f_n + \frac{e_n}{\beta_n}) dt + h_n d\omega. \quad (51)$$

Take the stochastic Lyapunov function

$$V_n = V_{n-1} + \frac{1}{4} z_n^4 + \frac{1}{2r_n} \tilde{\theta}_n^2 + V_{Q_n}, \quad (52)$$

where the Lyapunov-Krasovskii function  $V_{Q_n}$  is defined as

$$V_{Q_n} = \exp(-\pi_n t) \int_{t-\tau_n}^t \exp(\pi_n s) \Psi_n(x(s)) ds, \quad (53)$$

and its derivative is

$$\begin{aligned} \dot{V}_{Q_n} &= -\pi_n V_{Q_n} + \Psi_n(\bar{x}_n(t)) \\ &\quad - \exp(-\pi_n \tau_n) \Psi_n(\bar{x}_n(t - \tau_n)), \end{aligned} \quad (54)$$

where  $\pi_n$  is a positive constant and  $\Psi_n$  is defined later.

Then, by the Itô differentiation formula, it yields

$$LV_n = LV_{n-1} + z_n^3 \left( \Xi(v)v + d(v) + \phi_n(x) + f_n + \frac{e_n}{\beta_n} \right) + \frac{3}{2} z_n^2 h_n^T h_n + \frac{1}{r_n} \tilde{\theta}_n \dot{\hat{\theta}}_n + \dot{V}_{Qn}, \quad (55)$$

$$z_n^3 f_n \leq \frac{3}{4} z_n^4 + 2f_{n1}^4(\bar{x}_n(t)) + 2f_{n2}^4(\bar{x}_n(t - \tau_n)), \quad (56)$$

$$\frac{3}{2} z_n^2 h_n^T h_n \leq \frac{3}{4} z_n^4 + 6h_{n1}^4(\bar{x}_n) + 6h_{n2}^4(\bar{x}_n(t - \tau_n)). \quad (57)$$

The actual controller  $v$  is designed as

$$v = \frac{1}{g_v} \left( -k_n z_n - \frac{b_n}{2a_n^2} \hat{\theta}_n S^T(Z_n) S(Z_n) z_n^3 - \frac{e_n}{\beta_n} \right), \quad (58)$$

where  $g_v = D_{\min}$ ,  $k_n$ ,  $b_n$  and  $a_n$  are positive constants, and the adaptive law

$$\dot{\hat{\theta}}_n = \frac{r_n b_n}{2a_n^2} z_n^6 S^T(Z_n) S(Z_n) - \sigma_n \hat{\theta}_n. \quad (59)$$

Considering  $\tilde{\theta}_n = \hat{\theta}_n - \theta_n^*$ , one has:

$$-\frac{\sigma_n}{r_n} \tilde{\theta}_n \hat{\theta}_n \leq -\frac{\sigma_n \tilde{\theta}_n^2}{2r_n} + \frac{\sigma_n \theta_n^{*2}}{2r_n}. \quad (60)$$

The following inequality holds

$$z_n^3 d(v) = \frac{3}{4} z_n^4 + \frac{1}{4} d^{*4}. \quad (61)$$

The function  $\Psi_n(\bar{x}_n(t - \tau_n))$  in (53) is defined as

$$\Psi_n(\bar{x}_n(\tau_n)) = 2 \exp(\pi_n \tau_n) f_{n2}^4(\bar{x}_n(\tau_n)) + 6 \exp(\pi_n \tau_n) h_{n2}^4(\bar{x}_n(\tau_n)). \quad (62)$$

Define the following the nonlinear function  $H_n$  as

$$H_n = \Psi_n(\bar{x}_n) + 2f_{n1}^4(\bar{x}_n(t)) + 6h_{n1}^4(\bar{x}_n), \quad (63)$$

$$LV_n = LV_{n-1} + z_n^3 \left( -(k_n + \frac{9}{4}) z_n + \phi_n(x) + \frac{H_n}{z_n^3} \right) + \frac{1}{4} d^{*4} - \pi_n V_{Qn} + \frac{1}{r_n} \tilde{\theta}_n \dot{\hat{\theta}}_n. \quad (64)$$

Similarly, the unknown term  $\frac{4}{z_n^3} \tanh^4(\frac{z_n}{l_n}) H_n$  can be approximated by neural network.

Define the unknown nonlinear function  $F_n$

$$F_n(\bar{x}_n) = \phi_n(x) + \frac{4}{z_n^3} \tanh^4\left(\frac{z_n}{l_n}\right) H_n. \quad (65)$$

The RBFNN can be used to approximate  $F_n$  as

$$F_n = W_n^{*T} S_n(Z_n) + \varepsilon_n(Z_n), |\varepsilon_n(Z_n)| \leq \varepsilon_n^*, \quad (66)$$

where  $Z_n = [x_1, \dots, x_n, \hat{\theta}_1, \dots, \hat{\theta}_n, \alpha_{nf}]^T$ ,  $\varepsilon_n(Z_n)$  is the approximation error.

The following inequality holds:

$$z_n^3 F_n \leq \frac{b_n}{2a_n^2} z_n^6 \theta_n^* S_n^T S_n + \frac{1}{2} a_n^2 + \frac{3}{4} z_n^4 + \frac{1}{4} \varepsilon_n^{*4}, \quad (67)$$

where  $\|W_n^*\|^2 = b_n \theta_n^*$ .

Substituting the above inequality into (64), we have

$$LV_n \leq LV_{n-1} - (k_n - 3) z_n^4 + \frac{1}{2} a_n^2 + \frac{1}{4} \varepsilon_n^{*4} + \frac{1}{4} d^{*4} - \pi_n V_{Qn} - \frac{\sigma_n \tilde{\theta}_n^2}{2r_n} + \frac{\sigma_n \theta_n^{*2}}{2r_n} + (1 - 4 \tanh^4\left(\frac{z_n}{l_n}\right)) H_n. \quad (68)$$

Based on (50), (68) and Itô differentiation rule, the following inequality can be obtained

$$LV_n \leq -\left(k_1 - 3\frac{3}{4}\right) z_1^4 - \sum_{i=2}^{n-1} (k_i - 4) z_i^4 - \left(k_n - 3\frac{1}{4}\right) z_n^4 - \sum_{i=2}^n \left(\frac{1}{\beta_i} - 1\right) e_i^4 + \sum_{i=2}^n \frac{1}{4} B_i^{*4} + \frac{1}{4} d^{*4} + \sum_{i=1}^n \left(-\frac{\sigma_i \tilde{\theta}_i^2}{2r_i} - \pi_i V_{Qi}\right) + \sum_{i=1}^n \left(\frac{\sigma_i \theta_i^{*2}}{2r_i} + \frac{1}{2} a_i^2\right) + \frac{1}{4} \varepsilon_i^{*4} + \sum_{i=1}^n \left(1 - 4 \tanh^4\left(\frac{z_i}{l_i}\right)\right) H_i. \quad (69)$$

Given  $c_i > 0$  ( $i = 1, \dots, n$ ),  $l_{i+1} > 0$  ( $i = 1, \dots, n-1$ ), such that

$$c_1 = k_1 - 15/4, c_i = k_i - 4, i = 2, \dots, n-1,$$

$$c_n = k_n - 13/4, l_{i+1} = \frac{1}{\beta_{i+1}} - 1, i = 1, \dots, n-1.$$

Then, one has

$$LV \leq -\alpha_0 V + \beta + \sum_{i=1}^n \left(1 - 4 \tanh^4\left(\frac{z_i}{l_i}\right)\right) H_i, \quad (70)$$

where  $\alpha_0 = \min\{4c_i, 4l_i, \sigma_i, \pi_i\}$ ,  $i = 1, \dots, n$ ,  $\beta = \sum_{i=1}^n \left(\frac{1}{2} a_i^2 + \frac{1}{4} \varepsilon_i^{*4} + \frac{\sigma_i \theta_i^{*2}}{2r_i}\right) + \sum_{i=2}^n \frac{1}{4} B_i^{*4} + \frac{1}{4} d^{*4}$ . For the last term of (70), the further discussion will be given.

**Theorem 1:** Consider the closed-loop system consisting of plant (4), the controller (58) together with the virtual control signals (23), (43) and adaptive laws (24), (48), (59) under Assumptions 1, 2. For bounded initial conditions, there exist suitable design parameters  $c_i$ ,  $l_i$ ,  $\sigma_i$ ,  $\pi_i$  such that all closed-loop signals remain bounded in probability, and the tracking errors can be arbitrarily small.

**Proof:** For the last term in (70), it is obvious that it can be rewritten as follows

$$\begin{aligned} & \sum_{i=1}^n \left(1 - 4 \tanh^4\left(\frac{z_i}{l_i}\right)\right) H_i \\ &= \sum_{z_i \in \Lambda_i} \left(1 - 4 \tanh^4\left(\frac{z_i}{l_i}\right)\right) H_i + \sum_{z_i \notin \Lambda_i} \left(1 - 4 \tanh^4\left(\frac{z_i}{l_i}\right)\right) H_i \\ &= \xi_I + \xi_J, \end{aligned} \quad (71)$$

where

$$\begin{aligned}\xi_I &= \sum_{z_i \in \Lambda_i} (1 - 4 \tanh^4(\frac{z_i}{l_i})) H_i \\ \xi_J &= \sum_{z_i \notin \Lambda_i} (1 - 4 \tanh^4(\frac{z_i}{l_i})) H_i.\end{aligned}\quad (72)$$

For  $z_i \notin \Lambda_i$ , by Lemma 2 and  $H_i \geq 0$ , we can get that  $\xi_J \leq 0$ . For  $z_i \in \Lambda_i$ ,  $|z_i| \leq 0.8814v_i$  with  $l_i$  being a positive constant,  $z_i$  is bounded and  $\xi_I$  is also bounded.

Furthermore, there exists a positive constant  $\beta^*$ , such that  $|\beta + \xi_I| < \beta^*$ . Thus, (70) is rearranged as

$$LV \leq -\alpha_0 V + \beta^*. \quad (73)$$

Thus, from Lemma 2 and (73), all the signals of the closed-loop system are bounded.  $\square$

#### 4. SIMULATION RESULTS

Consider the following second-order stochastic time-delay system.

$$\begin{cases} dx_1 = (x_2 + x_1 e^{(-0.5x_1)} \\ \quad + x_1^2(t - \tau_1) \cos(x_1 x_1(t - \tau_1))) dt \\ \quad + \frac{x_1^2}{1 + x_1^2(t - \tau_1)} d\omega, \\ dx_2 = (u(v) + x_1 x_2^2 + \frac{x_1(t - \tau_1)x_2(t - \tau_2)}{1 + x_1^2 + x_2^2}) dt \\ \quad + 0.6 \sin(x_1^2 + x_2^2) x_1(t - \tau_2) x_2(t - \tau_2) d\omega, \end{cases} \quad (74)$$

where  $\dot{\omega}$  is chosen as the one-dimensional Gaussian white noise with zero mean and variance 1.  $\tau_1$  and  $\tau_2$  are time-delay terms, with  $\tau_1 = 3$ ,  $\tau_2 = 1$ .  $u$  is defined as follows:

$$u = D(v) = \begin{cases} 1.2(v - 1.3), & v \geq 1.3, \\ 0, & -0.6 < v < 1.3, \\ 1.5(v + 0.6), & v \leq -0.6. \end{cases} \quad (75)$$

The system output  $y$  tracks the ideal reference signal  $y_d = 0.8 \sin(t)$ .

The initial conditions  $[x_1(0), x_2(0)]^T = [0.3, 0.1]^T$ ,  $[\hat{\theta}_1(0), \hat{\theta}_2(0)]^T = [0, 0]^T$ .

In the simulation, the design parameters are taken as follows as:  $k_1 = 30$ ,  $k_2 = 20$ ,  $r_1 = 1$ ,  $r_2 = 1.2$ ,  $a_1 = 2$ ,  $a_2 = 2$ ,  $b_1 = 1$ ,  $b_2 = 1$ ,  $\sigma_1 = 0.1$ ,  $\sigma_2 = 0.3$ . Base on the control design, we construct the RBF NNs  $\hat{W}_1^T S_1(Z_1)$  using  $5^3$  nodes, with the width  $\eta_1 = 0.95$  and  $\hat{W}_2^T S_2(Z_2)$  using  $3^5$  nodes, with the width  $\eta_2 = 1.49$

The simulation results are shown in Figs. 1-4. From the simulation results, all the signals in the closed-loop system are bounded.

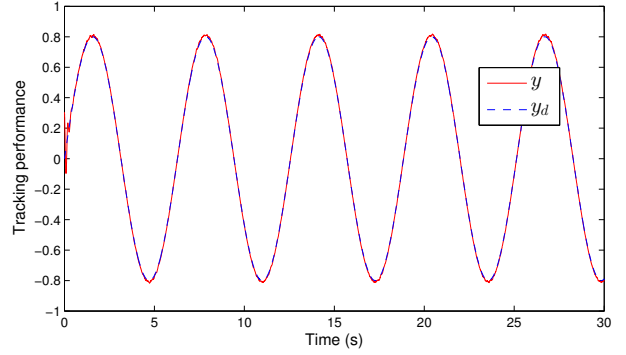


Fig. 1. The system output  $y$  and the reference signal  $y_d$ .

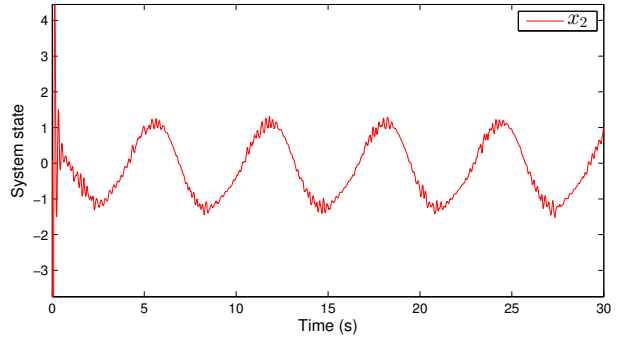


Fig. 2. The System state variable  $x_2$ .

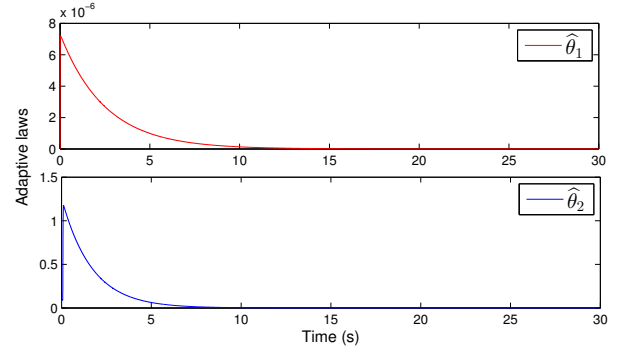


Fig. 3. The Adaptive parameters  $\hat{\theta}_1$  and  $\hat{\theta}_2$ .

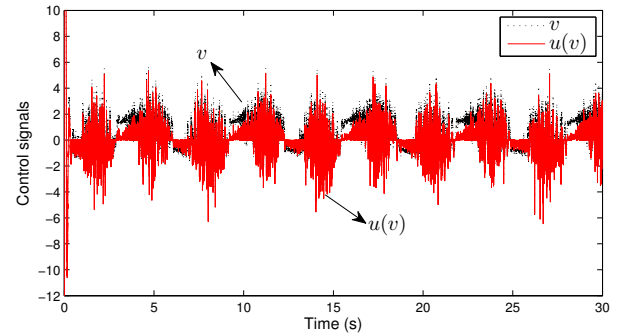


Fig. 4. The input  $v$  and output  $u$  of the dead zone.

## 5. CONCLUSIONS

In this study, the adaptive neural DSC has been presented for a more general class of stochastic nonlinear time-delayed systems with unknown input dead-zone. The drift and diffusion terms of the controlled system are dependent on the states and time-delay variables. It has been shown that the proposed controller can guarantee that all signals in the closed-loop systems are bounded in probability.

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