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Global Finite-time Stabilization for a Class of Switched Nonlinear Systems via Output Feedback

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Abstract: This paper addresses the problem of global finite-time stabilization for a class of uncertain switched nonlinear systems via output feedback under arbitrary switchings. Based on the adding a power integrator approach, we design a homogeneous observer and controller for the nominal switched system without the perturbing nonlinearities. Then, a scaling gain is introduced into the proposed output feedback stabilizer to implement global finite-time stability of the closed-loop system. In addition, the proposed approach can be also extended to a class of switched nonlinear systems with upper-triangular growth condition. Two examples are given to illustrate the effectiveness of the proposed method.

Keywords: Domination approach, finite-time, output feedback, switched systems.

1. INTRODUCTION

The problem of global output feedback control for switched nonlinear systems is one of the most important problems in the field of nonlinear system control. Compared with the results achieved for non-switched nonlinear systems, for example, the works [1, 2] in the past several decades, there are fewer results available on designing nonlinear observers and output feedback controllers for switched nonlinear systems. In this paper, we will consider the problem of global finite-time stabilization via output feedback for the following class of switched nonlinear systems described by

$$\begin{aligned} &\dot{z}_i = z_{i+1}^{p_i} + f_{i,\sigma(t)}(t,z,u), \quad i = 1,\cdots,n-1 \\ &\dot{z}_n = u^{p_n} + f_{n,\sigma(t)}(t,z,u), \\ &y = z_1, \end{aligned} \tag{1}$$

where $z = (z_1,\cdots,z_n)^T \in \mathbb{R}^n$, $u \in \mathbb{R}$, and $y \in \mathbb{R}$ are the system state, control input and output, respectively. $\sigma(t)$ is the switching signal taking its values in a finite set $M = \{1,\cdots,m\}$ and m is the number of subsystems. For $k \in M$ and $i = 1,\cdots,n$, $f_{i,k}(t,z,u) : \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}$ are uncertain continuous function with $f_{i,k}(t,0,0) = 0$, and $p_i \in \mathbb{R}^{\geq 1}_{odd} \triangleq \{\zeta \in \mathbb{R} \mid \zeta \geq 1 \text{ and } \zeta \text{ is a ratio of odd integers}\}$. Moreover, it is assumed that the state of system (1) does not jump at the switching instants, that is, the trajectory $z(t)$ is everywhere continuous.

It is well known that switched systems are an important class of hybrid systems, which have achieved considerable attention in [3–15], and the references therein. The main concerns in study of switched systems are the issues of stability and stabilization. Many methods have been proposed in the study of switched systems, for example, common Lyapunov function, single Lyapunov function, multiple Lyapunov functions, and so on. The work [16] investigated output feedback control problem for a class of switched nonlinear systems with parameter uncertainty by using single Lyapunov function method. The existence of a common Lyapunov function for all subsystems was shown in [6] to be a necessary and sufficient condition for a switched system to be asymptotically stable under arbitrary switchings. However, a common Lyapunov function may be usually difficult to find for switched nonlinear systems. Therefore, attention is transformed to switched nonlinear systems with special structures, which plays an important role in the stabilization of switched nonlinear systems [3, 4, 6].

On the other hand, due to fast convergence and good performance on robustness and disturbance rejection, many researchers have focused on the problem of finite-time stability, such as the works [17, 19–21] in recent years. The works [18, 19] provided the Lyapunov theory for finite-time stability of nonlinear systems. The work [21] presented a design method for global finite-time state feedback stabilization of a class of nonlinear systems. Compared with the state feedback case, there are fewer results dealing with output feedback finite-time stabilization. The work [22] discussed the finite-time stabilization for a double integrator system. While the work [23] studied the problem of global finite-time stabilization

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by dynamic output feedback for a class of nonlinear systems. A recent work [24] discussed global finite-time output feedback stabilization for a class of non-triangular nonlinear systems. A global finite-time observer with Lipschitz nonlinearity was proposed in [25]. However, the above results are concerned with non-switching case. In switched case, Nonlinear control for power integrator triangular systems was investigated in [26] , and the work [27] considered the problem of global finite-time output feedback stabilization for system (1) with the strict power order restriction ($p_i \equiv 1, i = 1, \cdots, n$) under linear growth condition. While global finite-time stabilisation problem for a class of switched nonlinear systems with ($p_i \equiv 1, i = 1, \cdots, n$) was addressed in [28], and subsequently the work [29] studied the problem of global finite-time *state feedback* stabilization of system (1) in *strict feedback* form under linear growth condition. Immediately, one may raise the following question: *Can it be possible to relax the power order restriction and how can we construct a finite-time output feedback controller for switched high-order nonlinear systems (1) under homogeneous growth condition?*

Motivated by the homogeneous domination approach described in [30], we will resolve the problem of global *finite-time output feedback* stabilization for a class of switched high-order nonlinear systems under arbitrary switchings. The main contributions of the paper are highlighted as follows: (i) We extend the methodology previously developed for non-switched nonlinear systems to switched nonlinear systems. (ii) To overcome unmeasurable states, a homogeneous observer is designed by the homogeneous theory. (iii) To handle uncertain nonlinearities, a scaling gain is introduced, and this makes it more intricate to construct a common Lyapunov function for all subsystems. (iv) Lower-triangular homogeneous growth conditions are extended to upper-triangular homogeneous growth conditions.

Notations: The following notations will be used throughout the paper. $\mathbb{R}^+$ denotes the set of all nonnegative real numbers, $\mathbb{R}^n$ denotes the real n-dimensional space. C^i denotes the set of all functions with continuous ith partial derivatives. $\mathcal{K}$ denotes the set of all functions: $\mathbb{R}^+ \to \mathbb{R}^+$, which are continuous, strictly increasing and vanishing at zero.

2. PRELIMINARIES

Definition 1 [19]**:** Consider the following autonomous system

$$\dot{x} = f(x), \quad \text{with } f(0) = 0,\ x \in D,\ x(0) = x_0, \tag{2}$$

where $f : D \to \mathbb{R}^n$ is continuous on an open neighborhood $D \subseteq \mathbb{R}^n$ of the origin. The zero solution of (2) is *finite-time convergent* if there are an open neighborhood $U \subseteq D$ of the origin and a function $T : U \backslash \{0\} \to (0, \infty)$, such that $\forall x_0 \in U$, the solution trajectory $x(t, x_0)$ of (2) starting from the initial point $x_0 \in U \backslash \{0\}$ is well-defined and unique in forward time for $t \in \big[0, T(x_0)\big)$, and $\lim_{t \to T(x_0)} x(t, x_0) = 0$. Then, $T(x_0)$ is called the settling time. The zero solution of (2) is *finite-time stable* if it is Lyapunov stable and finite-time convergent. When $U = D = \mathbb{R}^n$, the zero solution is said to be *globally finite-time stable*.

Definition 2 [31]**:** For real numbers $r_i > 0$, $i = 1, \cdots, n$ and fixed coordinates $(x_1, \cdots, x_n) \in \mathbb{R}^n$:

- the dilation $\Delta_\varepsilon(x)$ is defined by $\Delta_\varepsilon(x) = (\varepsilon^{r_1} x_1, \cdots, \varepsilon^{r_n} x_n)$, $\forall \varepsilon > 0$, with r_i being called as the weights of the coordinates. For simplicity, we define dilation weight $\Delta = (r_1, \cdots, r_n)$.
- a function $V \in C(\mathbb{R}^n, \mathbb{R}^n)$ is said to be homogeneous of degree τ if there is a real number $\tau \geq 0$ such that

$$V(\Delta_\varepsilon(x)) = \varepsilon^\tau V(x_1, \cdots, x_n),\ \forall x \in \mathbb{R}^n \setminus \{0\}.$$

- a homogeneous $p-$norm is defined as $\|x\|_{\Delta,p} = \big(|x_i|^{p/r_i}\big)^{1/p}, \forall x \in \mathbb{R}^n$, for a constant $p \geq 1$. For simplicity, we choose $p = 2$ and write $\|x\|_\Delta$ for $\|x\|_{\Delta,2}$.

Theorem 1 [19]**:** For system (2), the following statements hold:

(i) If there exists a C^1 function $V : D \to \mathbb{R}$, a class $\mathcal{K}$ function $\alpha(\cdot)$, real numbers $\rho > 0$ and $\lambda \in (0, 1)$, and an open neighborhood $U \subseteq D$ of the origin such that

$$V(0) = 0,\ \alpha(\|x\|) \leq V(x),\ \dot{V}(x) \leq -\rho V(x)^\lambda,\ x \in U, \tag{3}$$

then the zero solution $x(t) = 0$ of system (2) is finite-time stable.

(ii) If $U = D = \mathbb{R}^n$ and (3) holds, then the zero solution $x(t) = 0$ of system (2) is globally finite-time stable.

Lemma 1 [31]**:** Suppose $V : \mathbb{R}^n \to \mathbb{R}$ is a homogeneous function of degree τ with respect to Δ. Then, the following conclusions hold:

(i) $\partial V / \partial x_i$ is homogeneous of degree $\tau - r_i$ with r_i being the homogeneous weight of x_i;

(ii) there is a constant $\bar{c}$ such that $V(x) \leq \bar{c}\|x\|_\Delta^\tau$. Moreover, if $V(x)$ is positive definite, then $\underline{c}\|x\|_\Delta^\tau \leq V(x)$ with $\underline{c} > 0$ being a constant.

3. MAIN RESULTS

At first, we design an output feedback stabilizer for the nominal system

$$\dot{x}_i = x_{i+1}^{p_i},\ i = 1, \cdots, n-1,\ \dot{x}_n = v^{p_n},\ y = x_1, \tag{4}$$

where $x = (x_1, \cdots, x_n)^T \in \mathbb{R}^n$, $v \in \mathbb{R}$, $y \in \mathbb{R}$ are the system state, control input and output, respectively.

According to the approach proposed in [32], we construct a homogenous output feedback stabilizer for system (4), which is given in the following lemma.

Lemma 2: For any constant $\tau \in \left(-\frac{1}{1+\sum_{s=1}^{n-1}(p_1\cdots p_s)}, 0\right)$, there exist constants $l_i > 0, i = 1,\cdots,n-1$ and $\beta_j > 0, j = 1,\cdots,n$, such that the following homogeneous output feedback stabilizer

$$\begin{aligned}
&\dot{\eta}_2 = -l_1\hat{x}_2^{p_1}, \quad \hat{x}_2 = (\eta_2 + l_1 x_1)^{\frac{r_2}{r_1}},\\
&\dot{\eta}_i = -l_{i-1}\hat{x}_i^{p_{i-1}}, \quad \hat{x}_i = (\eta_i + l_{i-1}\hat{x}_{i-1})^{\frac{r_i}{r_{i-1}}},\\
&\ i = 3,\cdots,n
\end{aligned} \tag{5}$$

$$\begin{aligned}
v = &-\beta_n\big(\hat{x}_n^{\frac{1}{r_n}} + \beta_{n-1}^{\frac{1}{r_n}}(\hat{x}_{n-1}^{\frac{1}{r_{n-1}}} + \cdots + \beta_2^{\frac{1}{r_3}}(\hat{x}_2^{\frac{1}{r_2}}\\
&+ \beta_1^{\frac{1}{r_2}} x_1)\cdots)\big)^{r_{n+1}}
\end{aligned} \tag{6}$$

with r_i's (For simplicity, let $\tau = -\frac{q}{p}$ with q being a positive even integer and p being a positive odd integer.) defined as

$$r_1 = 1,\ r_{i+1} = \frac{r_i + \tau}{p_i},\ i = 1,\cdots,n \tag{7}$$

renders system (4) globally finite-time stable.

Proof: By choosing a Lyapunov function V as

$$\begin{aligned}
V =& \sum_{i=1}^{n}\int_{x_i^*}^{x_i}(s^{\frac{1}{r_i}} - x_i^{*\frac{1}{r_i}})^{2-r_i}ds\\
&+ \sum_{i=2}^{n}\int_{(\eta_i+l_{i-1}x_{i-1})^{\frac{2-r_{i-1}}{r_{i-1}}}}^{x_i^{\frac{2-r_{i-1}}{r_i}}}\big(s^{\frac{r_{i-1}}{2-r_{i-1}}} - (\eta_i + l_{i-1}x_{i-1})\big)ds,
\end{aligned} \tag{8}$$

and a set of virtual controllers $x_1^*,\cdots,x_n^*$ defined by

$$\begin{aligned}
&x_1^* = 0, \quad \xi_1 = x_1^{\frac{1}{r_1}} - x_1^{*\frac{1}{r_1}},\\
&x_i^* = -\beta_{i-1}\xi_{i-1}^{r_i}, \quad \xi_i = x_i^{\frac{1}{r_i}} - x_i^{*\frac{1}{r_i}},\ \ i = 2,\cdots,n,
\end{aligned} \tag{9}$$

one can obtain the global finite-time stabilization result for (4), whose proof is similar to the one [32, Theorem1] with some modifications. For the sake of space, the detailed proof is omitted here.

From the construction of V, it can be verified that the Lyapunov function V is positive definite and proper with respect to

$$X := [x_1,\cdots,x_n,\eta_2,\cdots,\eta_n]^T. \tag{10}$$

Under the denoting X, the whole system (4)- (5)-(6) can be written as

$$\begin{aligned}
\dot{X} &= F(X)\\
&= \left(x_2^{p_1},\cdots,x_n^{p_{n-1}},v^{p_n},f_{n+1},\cdots,f_{2n-1}\right)^T,
\end{aligned} \tag{11}$$

where $f_{n+1} = \dot{\eta}_2, f_{n+2} = \dot{\eta}_3,\cdots,f_{2n-1} = \dot{\eta}_n$.

By choosing the dilation weight Δ as

$$\Delta = (\underbrace{r_1,\cdots,r_n}_{for\ x_1,\cdots,x_n},\underbrace{r_1,\cdots,r_{n-1}}_{for\ \eta_2,\cdots,\eta_n}), \tag{12}$$

we can obtain that

$$\begin{aligned}
V(\Delta_\varepsilon(X)) =& \sum_{i=1}^{n}\int_{\varepsilon^{r_i}x_i^*}^{\varepsilon^{r_i}x_i}(s^{\frac{1}{r_i}} - \varepsilon x_i^{*\frac{1}{r_i}})^{2-r_i}ds\\
&+ \sum_{i=2}^{n}\int_{(\varepsilon^{r_{i-1}}\eta_i+l_{i-1}\varepsilon^{r_{i-1}}x_{i-1})^{\frac{2-r_{i-1}}{r_{i-1}}}}^{(\varepsilon^{r_i}x_i)^{\frac{2-r_{i-1}}{r_i}}}\big(s^{\frac{r_{i-1}}{2-r_{i-1}}}\\
&- (\varepsilon^{r_{i-1}}\eta_i + l_{i-1}\varepsilon^{r_{i-1}}x_{i-1})\big)ds\\
=& \sum_{i=1}^{n}\int_{x_i^*}^{x_i}(\varepsilon\lambda_1^{\frac{1}{r_i}} - \varepsilon x_i^{*\frac{1}{r_i}})^{2-r_i}\varepsilon^{r_i}d\lambda_1\\
&+ \sum_{i=2}^{n}\int_{(\eta_i+l_{i-1}x_{i-1})^{\frac{2-r_{i-1}}{r_{i-1}}}}^{x_i^{\frac{2-r_{i-1}}{r_i}}}\big(\varepsilon^{r_{i-1}}\lambda_2^{\frac{r_{i-1}}{2-r_{i-1}}}\\
&- (\varepsilon^{r_{i-1}}\eta_i + l_{i-1}\varepsilon^{r_{i-1}}x_{i-1})\big)\varepsilon^{2-r_{i-1}}d\lambda_2\\
=& \varepsilon^2 V(X)
\end{aligned} \tag{13}$$

is homogeneous of degree 2 with $\lambda_1 = \varepsilon^{-r_i}s$ and $\lambda_2 = \varepsilon^{r_i-2}s$.

Since the whole system (11) is globally finite-time stable, there is a constant $C_1 > 0$ such that

$$\dot{V}(X) = \frac{\partial V}{\partial X}F(X) \le -C_1\|X\|_\Delta^{2+\tau}. \tag{14}$$

Combined with the observer and controller established in Lemma 2, we are ready to use the homogeneous domination approach to achieve the global finite-time stabilization for system (1) under the following assumption.

Assumption 1: There exist constants $c_k > 0$ and $\tau \in \left(-\frac{1}{1+\sum_{s=1}^{n-1}(p_1\cdots p_s)}, 0\right)$ such that for $k \in M$ and $i = 1,\cdots,n$

$$|f_{i,k}(t,z,u)| \le c_k\big(|z_1|^{\frac{r_i+\tau}{r_1}} + |z_2|^{\frac{r_i+\tau}{r_2}} + \cdots + |z_i|^{\frac{r_i+\tau}{r_i}}\big), \tag{15}$$

where r_i's are defined in (7).

Remark 1: In non-switched case, i.e., $k \in M = \{1\}$, the condition (15) doesn't satisfy the structure and growth conditions or the output feedback form in [33, 34]. As we all know, system (1) satisfying Assumption 1 represents an important class of nonlinear systems.

Theorem 2: Under Assumption 1, the problem of global finite-time stabilization via output feedback can be solved for switched nonlinear system (1).

Proof: The construction of the finite-time output feedback stabilizer for (1) is accomplished by introducing a scaling gain into the output feedback observer (5) and controller (6), respectively. Then, the global finite-time stability of the closed-loop system under arbitrary switchings can be guaranteed by an appropriate choice of the scaling gain.

Before proceeding, we introduce the change of coordinates

$$x_i = \frac{z_i}{L^{q_i}},\ i = 1,\cdots,n, \text{ and } v = \frac{u}{L^{q_{n+1}}}, \tag{16}$$

where $q_1 = 0$, $q_{i+1} = \frac{1+q_i}{p_i}$, and $L \geq 1$ is a constant to be determined later.

Under the change of coordinates, system (1) can be rewritten as

$$\begin{aligned} \dot{x}_i &= Lx_{i+1}^{p_i} + \frac{f_{i,k}(\cdot)}{L^{q_i}}, \quad i = 1, \cdots, n-1 \\ \dot{x}_n &= Lv^{p_n} + \frac{f_{n,k}(\cdot)}{L^{q_n}}, \\ y &= x_1. \end{aligned} \tag{17}$$

Next, we construct a homogeneous observer with the parameter L

$$\begin{aligned} \dot{\eta}_2 &= -Ll_1\hat{x}_2^{p_1}, \quad \hat{x}_2 = (\eta_2 + l_1 x_1)^{\frac{r_2}{r_1}} \\ \dot{\eta}_i &= -Ll_{i-1}\hat{x}_i^{p_{i-1}}, \quad \hat{x}_i = (\eta_i + l_{i-1}\hat{x}_{i-1})^{\frac{r_i}{r_{i-1}}}, \\ i &= 3, \cdots, n, \end{aligned} \tag{18}$$

where $l_i > 0, i = 1, \cdots, n-1$, are the gains in Lemma 2.

In addition, we design u using the same construction of (6), that is,

$$\begin{aligned} u = &-L^{q_{n+1}}\beta_n\big(\hat{x}_n^{\frac{1}{r_n}} + \beta_{n-1}^{\frac{1}{r_n}}(\hat{x}_{n-1}^{\frac{1}{r_{n-1}}} + \cdots + \beta_2^{\frac{1}{r_3}}(\hat{x}_2^{\frac{1}{r_2}} \\ &+ \beta_1^{\frac{1}{r_2}} x_1)\cdots)\big)^{r_{n+1}}. \end{aligned} \tag{19}$$

With the help of the notation (11), the closed-loop system (17)-(18)-(19) becomes

$$\begin{aligned} \dot{X} =& LF(X) \\ &+ \big(\frac{f_{1,k}(\cdot)}{L^{q_1}}, \frac{f_{2,k}(\cdot)}{L^{q_2}}, \cdots, \frac{f_{n,k}(\cdot)}{L^{q_n}}, 0, \cdots, 0\big)^T. \end{aligned} \tag{20}$$

By choosing the same Lyapunov function $V(X)$ for system (20), one has

$$\begin{aligned} \dot{V}(X) =& L\frac{\partial V(X)}{\partial X}F(X) + \frac{\partial V(X)}{\partial X}\big(\frac{f_{1,k}(\cdot)}{L^{q_1}}, \frac{f_{2,k}(\cdot)}{L^{q_2}}, \\ &\cdots, \frac{f_{n,k}(\cdot)}{L^{q_n}}, 0, \cdots, 0\big)^T \\ \leq& -LC_1\|X\|_\Delta^{2+\tau} + \sum_{i=1}^{n}\big|\frac{\partial V(X)}{\partial X_i}\frac{f_{i,k}(\cdot)}{L^{q_i}}\big|. \end{aligned} \tag{21}$$

Noting that $r_{i+1}p_i = r_i + \tau$, we can obtain that

$$\frac{q_2}{r_2} = \frac{1}{\tau+1}, \quad \frac{q_3}{r_3} = \frac{1+p_1}{(1+p_1)\tau+1}, \tag{22}$$

and recursively

$$\frac{q_i}{r_i} = \frac{1+p_1+\cdots+p_1p_2\cdots p_{i-2})}{(1+p_1+\cdots+p_1p_2\cdots p_{i-2})\tau+1}. \tag{23}$$

Due to $\tau \in \big(-\frac{1}{1+\sum_{s=1}^{n-1}(p_1\cdots p_s)}, 0\big)$, it follows from (22) and (23) that

$$1 < \frac{q_2}{r_2} < \frac{q_3}{r_3} < \cdots < \frac{q_i}{r_i}. \tag{24}$$

This together with (15) and the fact $L \geq 1$, leads to

$$\begin{aligned} &\big|\frac{f_{i,k}(\cdot)}{L^{q_i}}\big| \\ \leq& \frac{c_k}{L^{q_i}}\big(|L^{q_1}x_1|^{\frac{r_i+\tau}{r_1}} + |L^{q_2}x_2|^{\frac{r_i+\tau}{r_2}} + \cdots + |L^{q_i}x_i|^{\frac{r_i+\tau}{r_i}}\big) \\ \leq& c_kL^{q_i\frac{r_i+\tau}{r_i}-q_i}(|x_1|^{\frac{r_i+\tau}{r_1}} + |x_2|^{\frac{r_i+\tau}{r_2}} + \cdots + |x_i|^{\frac{r_i+\tau}{r_i}}) \\ \leq& c_k(|x_1|^{\frac{r_i+\tau}{r_1}} + |x_2|^{\frac{r_i+\tau}{r_2}} + \cdots + |x_i|^{\frac{r_i+\tau}{r_i}}), \end{aligned} \tag{25}$$

where the last inequality holds owing to $\tau < 0$.

By Lemma 1, $\partial V(X)/\partial X_i$ is homogeneous of degree $2 - r_i$. Thus, it can be verified that

$$\big|\frac{\partial V(X)}{\partial X_i}\big|\big(|x_1|^{\frac{r_i+\tau}{r_1}} + |x_2|^{\frac{r_i+\tau}{r_2}} + \cdots + |x_i|^{\frac{r_i+\tau}{r_i}}\big) \tag{26}$$

is homogeneous of degree $2+\tau$.

With (25) and (26) in mind, one can find a constant $\rho_{i,k} > 0$ such that for $i = 1, \cdots, n$ and $k \in M$

$$\big|\frac{\partial V(X)}{\partial X_i}\frac{f_{i,k}(\cdot)}{L^{q_i}}\big| \leq \rho_{i,k}\|X\|_\Delta^{2+\tau}. \tag{27}$$

Substituting (27) into (21) yields

$$\dot{V}(X) \leq -L\big(C_1 - L^{-1}\max_{k\in M}\{\sum_{i=1}^{n}\rho_{i,k}\}\big)\|X\|_\Delta^{2+\tau}. \tag{28}$$

Therefore, we can choose the scaling gain L as

$$L \geq \frac{1}{C_1}\max_{k\in M}\{\sum_{i=1}^{n}\rho_{i,k}\} \tag{29}$$

such that the right-hand side of (28) is negative definite.

Since $V(X)$ is homogeneous of degree 2 with respect to Δ, there is a constant $\bar{c}_1 > 0$ such that

$$V(X) \leq \bar{c}_1\|X\|_\Delta^2. \tag{30}$$

Combining (28) and (30), one has

$$\dot{V}(X) + k_1V(X)^{\frac{2+\tau}{2}} \leq 0 \tag{31}$$

for a constant $k_1 > 0$, which indicates that the closed-loop system (1)-(18)-(19) is globally finite-time stable under arbitrary switchings.

Remark 2: In the control design procedure, if we choose individual virtual controllers x_{ik}^* $i = 1, \cdots, n, k \in M$ for the kth subsystem, we will obtain $\xi_{ik} = x_i^{1/r_i} - x_{ik}^{*1/r_i}$ instead of $\xi_i = x_i^{1/r_i} - x_i^{*1/r_i}$ in (9) and $\dot{\eta}_{ik} = -l_{i-1,k}\hat{x}_i^{p_{i-1}}$, $\hat{x}_i = (\eta_{ik} + l_{i-1,k}\hat{x}_{i-1})^{r_i/r_{i-1}}$ instead of $\dot{\eta}_i = -l_{i-1}\hat{x}_i^{p_{i-1}}$, $\hat{x}_i = (\eta_i + l_{i-1}\hat{x}_{i-1})^{r_i/r_{i-1}}$ in (5), $i = 2, \cdots, n$, $k \in M$, which leads to different coordinate transformations and observers for different subsystems. This results in the difficulty of stability analysis for the closed-loop system. Therefore, this paper attempts to address the output feedback control problem of system (1) by the common Lyapunov function approach.

Finally, an example is given to illustrate the effectiveness of the proposed method.

Example 1: Consider the following switched nonlinear system:

$$\begin{aligned}&\dot{z}_1 = z_2^{\frac{5}{3}} + f_{1,\sigma(t)}(t,z,u),\\&\dot{z}_2 = u + f_{2,\sigma(t)}(t,z,u),\\&y = z_1,\end{aligned} \tag{32}$$

where $\sigma(t) : [0,\infty) \to M = \{1,2\}$, $f_{1,1} = 0.1z_1^{\frac{13}{15}} \sin z_1$, $f_{1,2} = 0.5z_1^{\frac{13}{15}}$, $f_{2,1} = 0$, $f_{2,2} = z_2^{\frac{29}{39}} \sin z_1$. By choosing $\tau = -\frac{2}{15}$, $r_1 = 1$, and $q_1 = 0$, we can obtain $r_2 = \frac{13}{25}$, $r_3 = \frac{29}{75}$ and $q_2 = \frac{3}{5}$, $q_3 = \frac{8}{5}$. It is clear that Assumption 1 is satisfied.

With the help of (16), system (32) becomes

$$\begin{aligned}&\dot{x}_1 = Lx_2^{\frac{5}{3}} + \frac{f_{1,\sigma(t)}}{L^{q_1}},\\&\dot{x}_2 = Lv + \frac{f_{2,\sigma(t)}}{L^{q_2}},\\&y = x_1.\end{aligned} \tag{33}$$

By using Theorem 2, we design the following homogenous observer and output feedback controller

$$\begin{aligned}&\dot{\eta}_2 = -Ll_1\hat{x}_2^{\frac{5}{3}}, \quad \hat{x}_2 = (\eta_2 + l_1x_1)^{\frac{13}{25}},\\&u = -L^{\frac{8}{5}}\beta_2(\hat{x}_2^{\frac{25}{13}} + \beta_1^{\frac{25}{13}}x_1)^{\frac{29}{75}}\end{aligned} \tag{34}$$

to globally finite-time stabilize system (32) under arbitrary switchings.

In the simulation, the gains are chosen as $\beta_1 = 1.25$, $\beta_2 = 0.98$, $l_1 = 7.8$, $L = 2$, which are sufficient to give satisfactory simulation results shown in Fig.1 with the initial condition $(x_1(0), x_2(0), \eta_2(0))^T = (0.5, -0.3, 0)^T$.

Remark 3: It is worth emphasizing that the works [28, 29] focus on state feedback stabilization problem, but this paper considers output feedback stabilization problem. In addition, the system under consideration in [17, 24] are non-switched nonlinear system while that of this paper is switched nonlinear system. Therefore, the control methods in [17, 24, 28, 29] cannot be applied to system (1), such as Example 3.1.

4. EXTENSION AND DISCUSSION

In the preceding discussion, it is assumed that the nonlinear terms satisfy lower-triangular homogeneous growth conditions. In this part, we extend Assumption 1 to the following upper-triangular form.

Assumption 2: There exist constants $d_k > 0, k \in M$ such that $i = 1, \cdots, n-1$

$$|f_{i,k}(t,z,u)| \le d_k\Big(\sum_{j=i+2}^{n} |z_j|^{\frac{r_i+\tau}{r_j}} + |u|^{\frac{r_i+\tau}{r_{n+1}}}\Big). \tag{35}$$

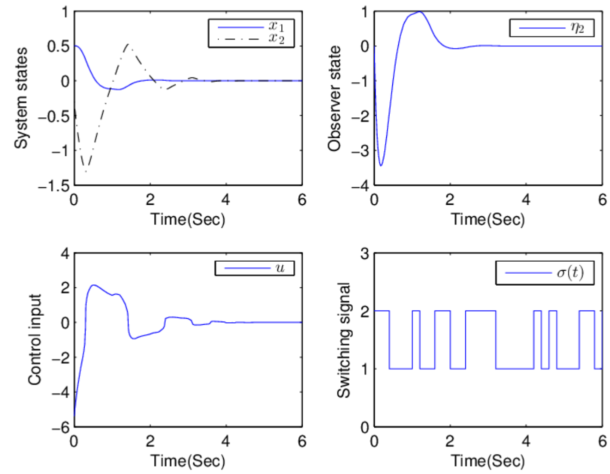

Fig. 1. The response of the closed-loop system (32)-(34).

Theorem 3: Under Assumption 2, the problem of global finite-time stabilization via output feedback is solved for switched nonlinear system (1).

Proof: To start with, we introduce the change of coordinates

$$x_i = \frac{z_i}{\varepsilon^{q_i}},\ i = 1, \cdots, n, \text{ and } v = \frac{u}{\varepsilon^{q_{n+1}}}, \tag{36}$$

where $0 < \varepsilon < 1$ is a constant to be determined later. Under the coordinates (36), system (1) becomes

$$\begin{aligned}&\dot{x}_i = \varepsilon x_{i+1}^{p_i} + \frac{f_{i,k}(\cdot)}{\varepsilon^{q_i}},\ i = 1, \cdots, n-1,\\&\dot{x}_n = \varepsilon v^{p_n},\\&y = x_1.\end{aligned} \tag{37}$$

Similar to (18), one can design a homogenous observer with the parameter ε

$$\begin{aligned}&\dot{\eta}_2 = -\varepsilon l_1\hat{x}_2^{p_1}, \quad \hat{x}_2 = (\eta_2 + l_1x_1)^{\frac{r_2}{r_1}},\\&\dot{\eta}_i = -\varepsilon l_{i-1}\hat{x}_i^{p_{i-1}}, \quad \hat{x}_i = (\eta_i + l_{i-1}\hat{x}_{i-1})^{\frac{r_i}{r_{i-1}}}\\&i = 3, \cdots, n,\end{aligned} \tag{38}$$

where $l_i > 0, i = 1, \cdots, n-1$, are the gains in Lemma 2.

Moreover, the controller u is designed by the same construction of (6), i.e.,

$$\begin{aligned}u =& -\varepsilon^{q_{n+1}}\beta_n\big(\hat{x}_n^{\frac{1}{r_n}} + \beta_{n-1}^{\frac{1}{r_n}}(\hat{x}_{n-1}^{\frac{1}{r_{n-1}}} + \cdots + \beta_2^{\frac{1}{r_3}}(\hat{x}_2^{\frac{1}{r_2}}\\&+ \beta_1^{\frac{1}{r_2}}x_1)\cdots)\big)^{r_{n+1}}.\end{aligned} \tag{39}$$

By the notation (11), the whole system (37)-(38)-(39) can be rewritten as

$$\dot{X} = \varepsilon F(X) + \Big(\frac{f_{1,k}(\cdot)}{\varepsilon^{q_1}}, \frac{f_{2,k}(\cdot)}{\varepsilon^{q_2}}, \cdots, \frac{f_{n-1,k}(\cdot)}{\varepsilon^{q_{n-1}}}, 0, \cdots, 0\Big)^T. \tag{40}$$

By using the same Lyapunov function $V(X)$ for system (40), one has

$$\begin{aligned}\dot{V}(X) =& \varepsilon\frac{\partial V(X)}{\partial X}F(X)+\frac{\partial V(X)}{\partial X}\Big(\frac{f_{1,k}(\cdot)}{\varepsilon^{q_1}},\\ &\frac{f_{2,k}(\cdot)}{\varepsilon^{q_2}},\cdots,\frac{f_{n-1,k}(\cdot)}{\varepsilon^{q_{n-1}}},0,\cdots,0\Big)^T\\ \le& -\varepsilon C_1\|X\|_\Delta^{2+\tau}+\sum_{i=1}^{n-1}\Big|\frac{\partial V(X)}{\partial X_i}\frac{f_{i,k}(\cdot)}{\varepsilon^{q_i}}\Big|. \end{aligned}\tag{41}$$

From the definitions of r_i and q_i, it can be deduced that for $j = i+2,\cdots,n+1$,

$$\begin{aligned}&\frac{q_j}{r_j}(r_i+\tau)-q_i\\ =&\frac{q_j\tau+\frac{(1+p_1+\cdots+p_1\cdots p_{j-2})-(1+p_1+\cdots+p_1\cdots p_{i-2})}{(p_1\cdots p_{i-1})(p_1\cdots p_{j-1})}}{q_j\tau+\frac{1}{p_1\cdots p_{j-1}}}\\ >&1.\end{aligned}\tag{42}$$

Due to $0<\varepsilon<1$, one has

$$\begin{aligned}&\Big|\frac{f_{i,k}(\cdot)}{\varepsilon^{q_i}}\Big|\\ \le&\frac{d_k}{\varepsilon^{q_i}}\Big(\sum_{j=i+2}^{n}|\varepsilon^{q_j}x_j|^{\frac{r_i+\tau}{r_j}}+|\varepsilon^{q_{n+1}}v|^{\frac{r_i+\tau}{r_{n+1}}}\Big)\\ \le& d_k\Big(\sum_{j=i+2}^{n}\varepsilon^{\frac{q_j}{r_j}(r_i+\tau)-q_i}|x_j|^{\frac{r_i+\tau}{r_j}}+\varepsilon^{\frac{q_{n+1}}{r_{n+1}}(r_i+\tau)-q_i}|v|^{\frac{r_i+\tau}{r_{n+1}}}\Big)\\ \le& d_k\varepsilon^{1+\mu}\Big(|x_{i+2}|^{\frac{r_i+\tau}{r_{i+2}}}+\cdots+|x_n|^{\frac{r_i+\tau}{r_n}}+|v|^{\frac{r_i+\tau}{r_{n+1}}}\Big),\end{aligned}\tag{43}$$

where $\mu=\min\{\frac{q_j}{r_j}(r_i+\tau)-q_i-1\}>0$ for $i=1,\cdots,n-1$, $j=i+2,\cdots,n+1$.

Similar to (26), the term

$$\Big|\frac{\partial V(X)}{\partial X_i}\Big|\Big(|x_{i+2}|^{\frac{r_i+\tau}{r_{i+2}}}+\cdots+|x_n|^{\frac{r_i+\tau}{r_n}}+|v|^{\frac{r_i+\tau}{r_{n+1}}}\Big)\tag{44}$$

is homogeneous of degree $2+\tau$. Thus, we can find a constant $\rho_{i,k}>0$ such that for $i=1,\cdots,n-1$ and $k\in M$

$$\Big|\frac{\partial V(X)}{\partial X_i}\frac{f_{i,k}(\cdot)}{\varepsilon^{q_i}}\Big|\le\rho_{i,k}\varepsilon^{1+\mu}\|X\|_\Delta^{2+\tau}.\tag{45}$$

Substituting (45) into (41) yields

$$\dot{V}(X)\le-\varepsilon\big(C_1-\varepsilon^{\mu}\max_{k\in M}\{\sum_{i=1}^{n-1}\rho_{i,k}\}\big)\|X\|_\Delta^{2+\tau}.\tag{46}$$

Obviously, a small enough parameter ε is chosen satisfying $\varepsilon<\Big(\frac{C_1}{\max_{k\in M}\{\sum_{i=1}^{n-1}\rho_{i,k}\}}\Big)^{1/\mu}$ and $0<\varepsilon<1$ such that the right-hand side of (46) is negative definite. It can be deduced from (46) that the whole system (1)-(38)-(39) is globally finite-time stable under arbitrary switchings.

In the end, we provide an example to demonstrate the effectiveness of the proposed control scheme.

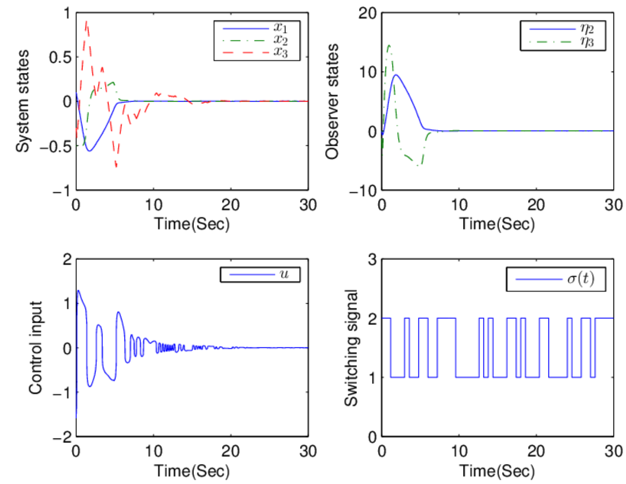

Fig. 2. The response of the closed-loop system (47)-(48).

Example 2: Consider the switched upper-triangular nonlinear system

$$\begin{aligned}\dot{x}_1&=x_2+f_{1,\sigma(t)}(t,x_3,u),\\ \dot{x}_2&=x_3^3+f_{2,\sigma(t)}(t,u),\\ \dot{x}_3&=u,\\ y&=x_1,\end{aligned}\tag{47}$$

where $\sigma(t):[0,\infty)\to M=\{1,2\}$, $f_{1,1}=0.1x_3^{\frac{29}{9}}$, $f_{1,2}=0.2u^{\frac{29}{7}}(\sin(x_3))^3$, $f_{2,1}=u(\sin(u))^5$, $f_{2,2}=0$. Assumption 2 holds with $r_1=1$, $r_2=\frac{29}{31}$, $r_3=\frac{9}{31}$, $r_4=\frac{7}{31}$, $\tau=-\frac{2}{31}$ and $q_1=0$, $q_2=1$, $q_3=\frac{2}{3}$, $q_4=\frac{5}{3}$.

By applying Theorem 3, a homogenous observer and output feedback controller are designed as follows

$$\begin{aligned}\dot{\eta}_2&=-\varepsilon l_1\hat{x}_2,\quad \hat{x}_2=(\eta_2+l_1x_1)^{\frac{29}{31}},\\ \dot{\eta}_3&=-\varepsilon l_2\hat{x}_3^3,\quad \hat{x}_3=(\eta_3+l_2\hat{x}_2)^{\frac{9}{29}},\\ u&=-\varepsilon^{\frac{5}{3}}\beta_3(\hat{x}_3^{\frac{31}{9}}+\beta_2^{\frac{31}{9}}(\hat{x}_2^{\frac{31}{29}}+\beta_1^{\frac{31}{29}}x_1))^{\frac{7}{31}}\end{aligned}\tag{48}$$

to globally finite-time stabilize system (32) under arbitrary switchings.

In the simulation, the gains are chosen as $\beta_1=1$, $\beta_2=1.3$, $\beta_3=3$, $l_1=17$, $l_2=13$, and $\varepsilon=0.43$. With the initial condition $(x_1(0),x_2(0),x_3(0),\eta_2(0),\eta_3(0))^T=(0.1,-0.5,-0.3,0,0)^T$, the simulation results are shown in Fig.2, which demonstrate the effectiveness of the proposed method.

5. CONCLUSION

This paper has discussed global finite-time stabilization problem for a class of uncertain switched nonlinear systems via output feedback. First, we explicitly constructed a homogeneous observer and controller for the nominal system. Then, we introduced an adjustable scaling gain into the output feedback stabilizer to dominate the nonlinearities. It can be indicated that an appropriate choice of the scaling gain will enable us to achieve global finite-time stability of the whole system under arbitrary switchings. In addition, the proposed approach can be also extended to a class of switched nonlinear systems with upper-triangular growth condition. In the future, we will focus on global adaptive finite-time output-feedback control for a class of high-order switched nonlinear systems under weaker conditions.

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