International Journal of Control, Automation and Systems 16(1) (2018) 120-128
http://dx.doi.org/10.1007/s12555-016-0427-6
ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Finite-time H_∞ Control of Cascade Nonlinear Switched Systems under State-dependent Switching

Qingyu Su* and Xiaolong Jia

Abstract: In this paper, we investigate the finite-time H_∞ control problem for the class of cascade nonlinear switched systems consisting of two parts which are also switched systems using a state-dependent switching method. The state feedback controller and the switching law are designed respectively, which guarantees that the corresponding closed-loop system is finite-time bounded and has a prescribed H_∞ performance index; the corresponding sliding motion problems are also considered. Sufficient conditions for the solvability of the problem are obtained. A numerical example is given to demonstrate the validity of the proposed approach.

Keywords: Cascade nonlinear switched systems, finite-time H_∞ control, state-dependent switching method, sliding motion.

1. INTRODUCTION

Generally speaking, a switched system is a dynamical system in which switching plays a dominant role. More specifically, a switched dynamical system is a two-level hybrid system. The lower level is governed by a set of subsystems described by differential or difference equations, while a coordinator orchestrates the switching among the subsystems with respect to the upper level [1]. [2] examines switched systems from a control-theoretic perspective, focusing on stability analysis and control synthesis of systems that combine continuous dynamics with switching events. The increasing attention paid to such systems is mainly due to the many practical systems, such as power electronics, networked haptic systems, mobile robotics, and aerospace systems, that can be modeled by means of switched systems.

Up to now, most of the existing literature on the stability of switched systems concentrates on Lyapunov asymptotic stability, which is defined over an infinite-time interval [3–12]. More recently, many studies of nonlinear control methods have been put forward [13–15]. However, in many practical cases, the transient behaviour of one system is concerned more with a fixed finite-time interval. In recent years, numerous contributions have been made in this field, especially in the case of non-switched systems; see example, [16, 17]. Investigations into the finite-time stability of switched systems have been also launched [18–20]. In [21], some new results regarding global finite-time stabilization for a class of switched strict-feedback nonlinear systems, whose subsystems have chained integrators with the powers of positive odd rational numbers, were presented.

H_∞ control theory has become a powerful tool for solving robust stabilization or disturbance attenuation problems. H_∞ control theory for linear systems is well established and has been generalised for nonlinear systems [22]. However, results concerning the H_∞ control problem for switched systems are relatively rare. H_∞ control problems for both switched linear and nonlinear systems are studied in [23] by using a dwell time approach. [24, 25] deal with this problem using the common Lyapunov function method and the single Lyapunov function method. The problem of robust H_∞ control for the class of switched nonlinear systems with parameter uncertainty is studied by using single Lyapunov function method in [26]. However, most of the above results deal with H_∞ control problems over an infinite-time interval. Very few results for finite-time H_∞ control problems of cascade nonlinear switched systems have appeared, which partly motivates our present work.

In this paper, motivated by the above analysis, we address the finite-time H_∞ control problem for the class of cascade nonlinear switched systems. Specifically, based on a multiple Lyapunov-like function method and a mode-dependent switching strategy, a new, less conservative linear matrix inequalities (LMIs)-based approach is carried out. Then, we design a set of state-feedback controllers

Manuscript received July 18, 2016; revised Janurary 13, 2017 and April 23, 2017; accepted May 23, 2017. Recommended by Associate Editor Atsushi Satoh under the direction of Editor Yoshito Ohta. This work was supported by the National Natural Science Foundation of China (61403075, 61503071), Natural Science Foundation of Jilin Province (20140520060JH), and Science Research of Education Department of Jilin Province (201693, JJKH20170106KJ).

Qingyu Su and Xiaolong Jia are with the School of Automation Engineering, Northeast Electric Power Univeristy, Jilin, Jilin, 132012, China (e-mails: suqingyu@neepu.edu.cn, ncepujxlong@yeah.net).
* Corresponding author.

and a class of switching signals with the largest region function strategy by the obtained method, and the results show that the closed-loop system is finite-time bounded with H_∞ performance γ.

The remainder of the paper is organized as follows. In Section 2, some definitions, assumptions and problem formulations are presented. By using the state-dependent switching approach, sufficient conditions that can guarantee finite-time bounded of cascade nonlinear switched systems with a prescribed H_∞ performance are proposed in Section 3. Then, an example is presented to illustrate the efficiency of the proposed method in Section 4. Conclusions are given in Section 5.

Notation: The notations used in this paper are standard. Let $\mathbb{R}$, $\mathbb{R}^+$, and $\mathbb{Z}^+$ denote the field of real numbers, the set of non-negative reals, and the set of non-negative integers, respectively. The notation $P > 0$ means that P is a real symmetric and positive definite matrix; the symbol "$*$" within a matrix represents the symmetric term of the matrix; the super script "T" stands for matrix transposition; $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ refer to, respectively, the n-dimensional Euclidean space and the set of all $m\times n$ real matrices; $\|\cdot\|$ refers to the Euclidean vector norm; I and 0 represent the identity matrix and a zero matrix, respectively; $diag\{\cdots\}$ stands for a block-diagonal matrix. For a square matrix P, $\lambda_{min}(P)$ and $\lambda_{max}(P)$ denote the minimum and maximum eigenvalues of matrix P, respectively.

2. PROBLEM STATEMENT

Consider the following cascade nonlinear switched systems:

$$\begin{cases} \dot{x}_1(t) = A_{1\sigma(t)}x_1(t) + A_{2\sigma(t)}x_2(t) + B_{\sigma(t)}u_{\sigma(t)} \\ \qquad\quad + G_{\sigma(t)}\omega(t), \\ \dot{x}_2(t) = f_{2\sigma(t)}(x_2(t)), \\ z(t) = C_{\sigma(t)}x_1(t) + D_{\sigma(t)}u_{\sigma(t)} + H_{\sigma(t)}\omega(t), \end{cases} \tag{1}$$

where $x(t) = \left[x_1^T(t),\ x_2^T(t)\right]^T \in \mathbb{R}^n$ is the state, $x_1(t) \in \mathbb{R}^{n-d}$, $x_2(t) \in \mathbb{R}^d$. $u(t) \in \mathbb{R}^m$ is the control input, $z(t) \in \mathbb{R}^q$ is the controlled output, $\omega(t) \in \mathbb{R}^r$ is the exogenous disturbance and satisfies Assumption 2, $\sigma(t) : [0,\infty) \to \Theta = \{1,2,\ldots,N\}$ is the switching signal which is a piecewise constant function depending on time t in this paper. A_{1i}, A_{2i}, B_i, G_i, C_i, D_i and H_i are constant real matrices for $i \in \Theta$. $f_{2i}(x_2)$ is a smooth and continuous function. In this paper, we assume B_i, $i \in \Theta$, are full column rank.

Then the switched state feedback controller

$$u_{\sigma(t)} = k_{\sigma(t)}x_1(t) \tag{2}$$

is designed to solve the H_∞ control problem for the switched system (1). Then, the closed-loop switched systems with mode-dependent state-feedback controllers can be rewritten as

$$\begin{cases} \dot{x}_1(t) = \widetilde{A}_{1\sigma(t)}x_1(t) + A_{2\sigma(t)}x_2(t) + G_{\sigma(t)}\omega(t), \\ \dot{x}_2(t) = f_{2\sigma(t)}(x_2(t)), \\ z(t) = \widetilde{C}_{\sigma(t)}x_1(t) + H_{\sigma(t)}\omega(t), \end{cases} \tag{3}$$

where $\widetilde{A}_{1\sigma(t)} = A_{1\sigma(t)} + B_{\sigma(t)}K_{\sigma(t)}$, $\widetilde{C}_{\sigma(t)} = C_{\sigma(t)} + D_{\sigma(t)}K_{\sigma(t)}$.

The following assumptions for cascade nonlinear switched system (1) are introduced.

Assumption 1 (Du *et al.* [27])**:** The trajectory $x(t)$ is everywhere continuous, i.e., the state of the cascade nonlinear switched system does not jump at the switching instants.

Assumption 2 (Liu and Zhao [28])**:** The exogenous disturbance $\omega(t)$ is time-varying and satisfies the constraint $\int_0^{T_f} \omega^T(t)\omega(t)dt \le d,\ d \ge 0$.

For the purpose of obtaining a Lyapunov-like function for each of the switched subsystems, the entire state space $\mathbb{R}^n$ should be divided into several parts, indicated as Ω_i. To this end, it is essential that these areas Ω_i should cover the entire state space, in other words, the following coverage characteristics was established:

$$\bigcup_{i=1}^{m} \Omega_i = \mathbb{R}^n. \tag{4}$$

In brief, we hypothesize that each area Ω_i has the following forms of expression:

$$\Omega_i = \{x \in \mathbb{R}^n \mid x^T Q_i x \ge 0\},\ i \in \Theta, \tag{5}$$

while $Q_i \in R^{n\times n}$ is on behalf of a symmetric matrix, $i \in \Theta$.

In [29], the lemma 1 was proposed which gives a sufficient condition for the coverage characteristics.

Lemma 1 (Liu *et al.* [30])**:** If for any $x \in \mathbb{R}^n$

$$\sum_{i=1}^{m} \theta_i x^T Q_i x \ge 0, \tag{6}$$

while $\theta_i \ge 0$, $i \in \Theta$, in that case $\bigcup_{i=1}^m \Omega_i = \mathbb{R}^n$.

The switching strategy is defined in terms of the following formula:

$$\sigma(x) = arg\Big(\max\nolimits_{i\in\Theta} x^T Q_i x\Big). \tag{7}$$

The areas $\Omega_{i,j}$ denote where the switching signal switches from subsystem i to j, with the following expression

$$\begin{aligned} \Omega_{i,j} =& \Omega_i \cap \Omega_j \\ =& \{x \in \mathbb{R}^n \mid x^T Q_i x = x^T Q_j x\},\ i,j \in \Theta. \end{aligned} \tag{8}$$

For the sake of geting a precisely defined switched system, these areas should meet two characteristics, which are the coverage characteristic (4) and the switching characteristic $\Omega_{i,j} \subseteq cl\Omega_i \cap cl\Omega_j$, while cl represents the closure of a set.

For system (1) we now define the following piecewise Lyapunov functional candidate as:

$$V_i(x) = x^T \overline{P}_i^{-1} x. \tag{9}$$

On the basis of [29], sufficiently use the $\mathcal{S}$-procedure, such that

$$\overline{P}_i^{-1} - \overline{P}_j^{-1} + \eta_{i,j}(Q_i - Q_j) = 0, \tag{10}$$

while $\eta_{i,j} > 0$.

Remark 1: As it is shown in (7), the state-dependent switching strategy orchestrates the switching among the subsystems with respect to the states. Hence the switching instants are not needed to be given in advance, which is different from the average dwell time method.

To proceed further, some related definitions are recalled as follows.

Definition 1 (Liu *et al.* [30] Finite-Time Stabilizable, FTS)**:** Given a positive definite matrix R three positive constants c_1, c_2, T_f, with $c_1 < c_2$, and a switching signal σ, the cascade nonlinear switched system (1) with control input (2) and $\omega(t) \equiv 0$ is said to be finite-time stabilizable with respect to $(c_1, c_2, T_f, R, \sigma)$, if $x_0^T R x_0 < c_1 \Rightarrow x^T(t) R x(t) < c_2$, $\forall t \in [0, T_f]$.

Definition 2 (Liu *et al.* [30] Finite-Time Bounded, FTB)**:** Given a positive definite matrix R three positive constants c_1, c_2, T_f, with $c_1 < c_2$, and a switching signal σ, the cascade nonlinear switched system (3) is said to be finite-time bounded with respect to $(c_1, c_2, d, T_f, R, \sigma)$, if $x_0^T R x_0 < c_1 \Rightarrow x^T(t) R x(t) < c_2$, $\forall t \in [0, T_f]$, $\int_0^{T_f} \omega^T(t)\omega(t)dt \le d$.

Remark 2: In the existing literature [30] of switched systems on finite-time control, it is commonly considered the continuous-time switched linear systems. In this paper, we will study cascade nonlinear switched systems, which are more general and practical.

Definition 3 (Liu and Zhao [28] Finite-Time H_∞ Performance)**:** Given a positive definite matrix R, two positive constants c_2 and T_f, and a switching signal σ, the cascade nonlinear switched system (1) with $u(t) \equiv 0$ is said to have finite-time H_∞ performance with respect to $(0, c_2, d, T_f, \gamma, R, \sigma)$, if the system is finite time bounded and the following inequality is satisfied:

$$\int_0^{T_f} z^T(t) z(t) dt < \gamma^2 \int_0^{T_f} \omega^T(t)\omega(t) dt, \tag{11}$$

where $\gamma > 0$ is a prescribed scalar and $\omega(t)$ satisfies the Assumption 2.

Definition 4 (Liu and Zhao [28] Finite-Time H_∞ Control)**:** The cascade nonlinear switched system (1) is said to be finite time stabilizable with H_∞ disturbance attenuation level γ, if there exists a control input $u(t)$, $\forall t \in [0,\ T_f]$, such that:

i) The corresponding closed-loop system is finite-time bounded.

ii) Under the zero-initial condition, the controlled output $z(t)$ satisfies inequality (11)

Remark 3: Most of the existing studies with respect to stability of switched systems are performed on the conventional Lyapunov asymptotic stability, which is defined over the infinite-time interval. In this paper, however, the transient behavior of one system is concerned, in which the state must hold below a prescribed upper bound and larger values are not permitted. A Lyapunov asymptotically stable switched system may not be finite-time stable if its states exceed the prescribed bounds during the fixed time interval.

Remark 4: We emphasize that whether the exogenous disturbance is taken into account is the major distinction between FTS and FTB. Throughout this paper, we will adopt the state-dependent switching strategy and design a set of state feedback controllers, under the control law cascade nonlinear switched system (1) is finite-time bounded and guarantee a prescribed H_∞ performance.

The aim of this paper is to present the sufficient criteria and the admissible switching signals under which cascade nonlinear switched system (1) is finite-time bounded with H_∞ disturbance attenuation level γ.

3. MAIN RESULTS

In this section, we investigate the robust H_∞ control problem for system (1) via switched state feedback controllers. The sufficient conditions which can guarantee that the corresponding closed-loop system is finite-time bounded with a prescribed H_∞ performance are derived.

Theorem 1: Consider the cascade nonlinear switched system (1). For any $i, j \in \Theta$, $i \neq j$, let $\overline{P}_i = R^{-\frac{1}{2}} P_i R^{-\frac{1}{2}}$, $P_i = diag\{P_{1i}, P_{2i}\}$, and suppose there exist matrices $P_i > 0$, $Q_i = Q_i^T$, $X_i \in \mathbb{R}^{m\times(n-d)}$, $M_i \in \mathbb{R}^{m\times m}$ with appropriate dimensions, and $\alpha > 0$, $\gamma > 0$, $\theta_i > 0$, $\eta_{ij} > 0$ are some properly chosen constants, such that

$$\begin{bmatrix} \Xi_i & A_{2i}\overline{P}_{2i} & G_i & \overline{P}_{1i}C_i^T + X_i^T D_i^T \\ * & -\alpha\lambda_3 I & 0 & 0 \\ * & * & -\gamma^2 I & H_i^T \\ * & * & * & -I \end{bmatrix} < 0, \tag{12}$$

$$\overline{P}_i^{-1} - \overline{P}_j^{-1} + \eta_{ij}(Q_i - Q_j) = 0, \tag{13}$$

$$\theta_1 Q_1 + \cdots + \theta_m Q_m \ge 0, \tag{14}$$

$$\frac{c_1}{\lambda_2} + \gamma^2 d < \frac{c_2}{\lambda_1} e^{-\alpha T_f}, \tag{15}$$

where

$$\Xi_i = A_{1i}\overline{P}_{1i} + \overline{P}_{1i}A_{1i}^T - \alpha\lambda_3 I + B_i X_i + X_i^T B_i^T.$$

If the switching signals satisfying

$$\sigma(x) = arg\Big(max_{i\in\Theta}\, x^T(t)Q_i x(t)\Big) \tag{16}$$

then switched system (1) with control input $u_i(t) = K_i x_1(t) = X_i\overline{P}_{1i}^{-1}x_1(t)$ and exogenous disturbance $\omega(t) \neq 0$ is finite-time bounded with H_∞ performance γ for any state-dependent switching signal (16) with respect to $(0, c_2, T_f, d, \gamma, R, \sigma(x))$, where $\lambda_1 = \max_{\forall i\in\Theta}(\lambda_{max}(P_i))$, $\lambda_2 = \min_{\forall i\in\Theta}(\lambda_{min}(P_i))$, $\lambda_3 = \min_{\forall i\in\Theta}(\lambda_{min}(\overline{P}_i^{-1}))$.

Proof: Choose (9) as the Lyapunov function. Then, we have

$$\begin{aligned} V_i(x) &= x^T\overline{P}_i^{-1}x = [x_1^T\ x_2^T]\begin{bmatrix} \overline{P}_{1i}^{-1} & 0 \\ 0 & \overline{P}_{2i}^{-1}\end{bmatrix}\begin{bmatrix} x_1 \\ x_2\end{bmatrix} \\ &= V_{1i}(x_1(t)) + J(x_2(t)), \end{aligned} \tag{17}$$

where $V_{1i}(x_1(t)) = x_1^T(t)\overline{P}_{1i}^{-1}x_1(t)$, $J(x_2(t)) = x_2^T(t)\overline{P}_{2i}^{-1}x_2(t)$, $\forall i \in \Theta$. Then $V(x(t))$ is used to measure the energy in the region Ω_i.

Case 1: No sliding motion occurs. Assume $\sigma(x(t_k^-)) = j$ and $\sigma(x(t_k)) = i$. When $t \in [t_k, t_{k+1})$, the derivative of $V(x)$ along the trajectories of subsystem i yields

$$\begin{aligned} \dot{V}(x) \leqslant\, & x_1^T(\tilde{A}_{1i}^T\overline{P}_{1i}^{-1} + \overline{P}_{1i}^{-1}\tilde{A}_{1i})x_1 + x_1^T\overline{P}_{1i}^{-1}A_{2i}x_2 \\ & + x_2^T A_{2i}^T\overline{P}_{1i}^{-1}x_1 + x_1^T\overline{P}_{1i}^{-1}G_i\omega + \omega^T G_i^T\overline{P}_{1i}^{-1}x_1 \\ & + \frac{\partial J}{\partial x_2}f_{2i}(x_2) \\ \leqslant\, & \eta^T(t)\begin{bmatrix} \tilde{A}_{1i}^T\overline{P}_{1i}^{-1} + \overline{P}_{1i}^{-1}\tilde{A}_{1i} & \overline{P}_{1i}^{-1}A_{2i} & \overline{P}_{1i}^{-1}G_i \\ * & 0 & 0 \\ * & * & 0\end{bmatrix}\eta(t) \\ & + \frac{\partial J}{\partial x_2}f_{2i}(x_2), \end{aligned} \tag{18}$$

where $\eta(t) = [x_1^T(t), x_2^T(t), \omega^T(t)]^T$.

Considering $X_i = K_i\overline{P}_{1i}$, from Eq. (12), we obtain

$$\begin{aligned} &\begin{bmatrix} \bar{\phi}_{11} & A_{2i}\overline{P}_{2i} & G_i & \overline{P}_{1i}\widetilde{C}_i^T \\ * & -\alpha\overline{P}_{1i} & 0 & 0 \\ * & * & -\gamma^2 I & H_i^T \\ * & * & * & -I\end{bmatrix} \\ &= \begin{bmatrix} \hat{\phi}_{11} & A_{2i}\overline{P}_{2i} & G_i & \overline{P}_{1i}\widetilde{C}_i^T \\ * & -\alpha\overline{P}_{1i} & 0 & 0 \\ * & * & -\gamma^2 I & H_i^T \\ * & * & * & -I\end{bmatrix} < 0 \end{aligned} \tag{19}$$

with

$$\bar{\phi}_{11} \triangleq A_{1i}\overline{P}_{1i} + \overline{P}_{1i}A_{1i}^T + \overline{P}_{1i}K_i^T B_i^T + B_i K_i\overline{P}_{1i} - \alpha\overline{P}_{1i},$$

$$\hat{\phi}_{11} \triangleq \widetilde{A}_{1i}\overline{P}_{1i} + \overline{P}_{1i}\widetilde{A}_{1i}^T - \alpha\overline{P}_{1i}.$$

In view of $z(t) = \widetilde{C}_{\sigma(t)}x_1(t) + H_{\sigma(t)}\omega(t)$, thus, from (19), one obtain

$$\begin{aligned} \dot{V}(x) <\, & \zeta^T(t)\begin{bmatrix} \alpha\overline{P}_{1i}^{-1} & 0 & 0 & -\widetilde{C}_i^T \\ * & \alpha\overline{P}_{2i}^{-1} & 0 & 0 \\ * & * & \gamma^2 I & -H_i^T \\ * & * & * & I\end{bmatrix}\zeta(t) \\ <\, & \alpha V_{1i}(x_1(t)) + \alpha J(x_2(t)) + \gamma^2\omega^T(t)\omega(t) \\ & - 2z^T(t)z(t) + z^T(t)z(t) \\ <\, & \alpha V_i(x(t)) + \gamma^2\omega^T(t)\omega(t) - z^T(t)z(t) \\ <\, & \alpha V_i(x(t)) + \gamma^2\omega^T(t)\omega(t), \end{aligned} \tag{20}$$

where $\zeta(t) = [x_1^T(t)\ x_2^T(t)\ \omega^T(t)\ z^T(t)]^T$.

According to Assumption 2, we can obtain that

$$\int_0^{T_f}\omega^T(s)\omega(s)ds \leqslant d,\ d \geqslant 0. \tag{21}$$

Integrating (20) from t_k to t gives

$$\begin{aligned} V(x) <\, & e^{\alpha(t-t_k)}V_{\sigma(x(t_k))}(x(t_k)) \\ & + \gamma^2\int_{t_k}^t e^{\alpha(t-s)}\omega^T(s)\omega(s)ds. \end{aligned} \tag{22}$$

According to Assumption 1, we have $x_1(t_k^-) = x_1(t_k)$. Thus, by (13), we can obtain

$$V_{\sigma(x(t_k^-))}(x(t_k^-)) \geqslant V_{\sigma(x(t_k))}(x(t_k)). \tag{23}$$

Combining (21), (22) and (23), we have

$$\begin{aligned} V(x(t)) <\, & e^{\alpha t}V_{\sigma(x(0))}(x(0)) \\ & + \gamma^2\int_0^t e^{\alpha(t-s)}\omega^T(s)\omega(s)ds \\ \leqslant\, & e^{\alpha T_f}V_{\sigma(x(0))}(x(0)) \\ & + \gamma^2 e^{\alpha T_f}\int_0^{T_f}\omega^T(s)\omega(s)ds \\ \leqslant\, & e^{\alpha T_f}(V_{\sigma(x(0))}(x(0)) + \gamma^2 d). \end{aligned} \tag{24}$$

On the other hand, considering $\overline{P}_i = R^{-\frac{1}{2}}P_i R^{-\frac{1}{2}}$, $V(x(t))$ satisfies

$$\begin{aligned} V(x(t)) &= x^T(t)\overline{P}_{\sigma(x(t))}^{-1}x(t) = x^T(t)R^{\frac{1}{2}}P_{\sigma(x(t))}^{-1}R^{\frac{1}{2}}x(t) \\ &\geqslant \lambda_{min}(P_{\sigma(x(t))}^{-1})x^T(t)Rx(t) \\ &\geqslant \frac{1}{\lambda_{max}(P_{\sigma(x(t))})}x^T(t)Rx(t) \\ &\geqslant \frac{1}{\lambda_1}x^T(t)Rx(t). \end{aligned} \tag{25}$$

In addition, in the view of $V_{\sigma(x(0))}(x(0)) = x^T(0)\overline{P}_{\sigma(x(0))}^{-1}x(0)$, we can get

$$V_{\sigma(x(0))}(x(0)) = x^T(0)\overline{P}_{\sigma(x(0))}^{-1}x(0)$$

$$
\begin{aligned}
&= x^T(0)R^{\frac{1}{2}}P^{-1}_{\sigma(x(0))}R^{\frac{1}{2}}x(0)\\
&\leqslant \lambda_{max}(P^{-1}_{\sigma(x(0))})x^T(0)Rx(0)\\
&\leqslant \frac{1}{\lambda_{min}(P_{\sigma(x(0))})}x^T(0)Rx(0)\\
&\leqslant \frac{1}{\lambda_2}x^T(0)Rx(0)\\
&\leqslant \frac{c_1}{\lambda_2}. \qquad (26)
\end{aligned}
$$

From (15), by combining (24)-(26), we get

$$x^T(t)Rx(t) < \lambda_1 e^{\alpha T_f}(\frac{c_1}{\lambda_2}+\gamma^2 d) < c_2. \qquad (27)$$

According to Definition 1, cascade nonlinear switched system (1) is finite-time bounded in Case 1.

Next, under zero initial condition, we establish the weighted H_∞ performance with the state-dependent switching. Then, from condition (12), we have

$$
\begin{aligned}
\dot{V}_{\sigma(t)}(x(t)) <& \alpha x^T(t)\overline{P}^{-1}_{\sigma(t)}x(t)\\
&+\gamma^2\omega^T(t)\omega(t)-z^T(t)z(t). \qquad (28)
\end{aligned}
$$

By integrating (28) for any $t\in[t_k,t_{k+1})$, it holds that

$$
\begin{aligned}
V_{\sigma(t)} <& e^{\alpha(t-t_k)}V_{\sigma(t_k)}(x(t_k))\\
&+\int_{t_k}^{t} e^{\alpha(t-s)}(\gamma^2\omega^T(s)\omega(s)-z^T(s)z(s))ds. \qquad (29)
\end{aligned}
$$

From (23), we attain

$$
\begin{aligned}
V_{\sigma(t)} <& e^{\alpha t}V_{\sigma(0)}(x(0))\\
&+\int_0^t e^{\alpha(t-s)}(\gamma^2\omega^T(s)\omega(s)-z^T(s)z(s))ds\\
<& \int_0^t e^{\alpha(t-s)}(\gamma^2\omega^T(s)\omega(s)-z^T(s)z(s))ds. \qquad (30)
\end{aligned}
$$

Considering $V_{\sigma(t)}(x(t))\geq 0$, we obtain

$$\int_0^t e^{\alpha(t-s)}z^T(s)z(s)ds < \gamma^2\int_0^t e^{\alpha(t-s)}\omega^T(s)\omega(s)ds. \qquad (31)$$

Both sides is multiplied by $e^{-\alpha(t-s)}$, then we have

$$\int_0^t z^T(s)z(s)ds < \gamma^2\int_0^t \omega^T(s)\omega(s)ds. \qquad (32)$$

By setting $t=T_f$, we get

$$\int_0^{T_f} z^T(s)z(s)ds < \gamma^2\int_0^{T_f}\omega^T(s)\omega(s)ds. \qquad (33)$$

On the basis of Definition 3, we can concluded that cascade nonlinear switched system (1) is finite-time bounded with H_∞ performance γ for any state-dependent switching signal (16) in Case 1.

Case 2: When sliding motion occurs. It will be proved that with the given conditions, the similar conclusion can also be obtained in the cases that the sliding motion occur. The defined motions occur at states that fulfil $\max_{i\in\Theta}x^TQ_ix=\max_{j\in\Theta}x^TQ_jx$, which are states where the switching signal switchings occur. As previously described, We define the surface that the sliding motions may appear $\Omega_{i,j}$, in other words, $x^TQ_ix=x^TQ_jx\geq 0$. The hyper-surface $\Omega_{i,j}$ could be described by a set of formulas, such that the following inequalities hold:

$$
\begin{aligned}
&x_1^T(\widetilde{A}_{1i}^T(Q_{1i}-Q_{1j})+(Q_{1i}-Q_{1j})\widetilde{A}_{1i})x_1\\
&+x_2^TA_{2i}^T(Q_{1i}-Q_{1j})x_1+x_1^T(Q_{1i}-Q_{1j})A_{2i}x_2\\
&+\omega^TG_i^T(Q_{1i}-Q_{1j})x_1+x_1^T(Q_{1i}-Q_{1j})G_i\omega<0, \qquad (34)
\end{aligned}
$$

and

$$
\begin{aligned}
&x_1^T(\widetilde{A}_{1j}^T(Q_{1i}-Q_{1j})+(Q_{1i}-Q_{1j})\widetilde{A}_{1j})x_1\\
&+x_2^TA_{2j}^T(Q_{1i}-Q_{1j})x_1+x_1^T(Q_{1i}-Q_{1j})A_{2j}x_2\\
&+\omega^TG_j^T(Q_{1i}-Q_{1j})x_1+x_1^T(Q_{1i}-Q_{1j})G_j\omega>0. \qquad (35)
\end{aligned}
$$

The derivative of the state on the hyper-surface satisfying

$$
\begin{aligned}
\dot{x}_1(t) =& \rho(\tilde{A}_{1i}x_1(t)+A_{2i}x_2(t)+G_i\omega(t))\\
&+(1-\rho)(\tilde{A}_{1j}x_1(t)+A_{2j}x_2(t)+G_j\omega(t))\\
=& \tilde{A}_{1ij}x_1(t)+\tilde{A}_{2ij}x_2(t)+\tilde{G}_{ij}\omega(t), \qquad (36)
\end{aligned}
$$

where $\rho\in[0,1]$.

In view of (13) and (19), we have

$$
\begin{aligned}
&x_1^T(\tilde{A}_{1i}^TP_{1j}+P_{1j}\tilde{A}_{1i})x_1+x_2^TA_{2i}^TP_{1j}x_1\\
&+x_1^TP_{1j}A_{2i}x_2+\omega^TG_i^TP_{1j}x_1+x_1^TP_{1j}G_i\omega\\
&< x_1^T(\tilde{A}_{1i}^TP_{1i}+P_{1i}\tilde{A}_{1i})x_1+x_2^TA_{2i}^TP_{1i}x_1\\
&+x_1^TP_{1i}A_{2i}x_2+\omega^TG_i^TP_{1i}x_1+x_1^TP_{1i}G_i\omega\\
&< \alpha\min_{i\in\underline{P}}(\lambda_{min}(P_{1i}))x_1^Tx_1+\gamma^2\omega^T\omega, \qquad (37)
\end{aligned}
$$

and

$$
\begin{aligned}
&x_1^T(\tilde{A}_{1j}^TP_{1i}+P_{1i}\tilde{A}_{1j})x_1+x_2^TA_{2j}^TP_{1i}x_1\\
&+x_1^TP_{1i}A_{2j}x_2+\omega^TG_j^TP_{1i}x_1+x_1^TP_{1i}G_j\omega\\
&< x_1^T(\tilde{A}_{1j}^TP_{1j}+P_{1j}\tilde{A}_{1j})x_1+x_2^TA_{2j}^TP_{1j}x_1\\
&+x_1^TP_{1j}A_{2j}x_2+\omega^TG_j^TP_{1j}x_1+x_1^TP_{1j}G_j\omega\\
&< \alpha\min_{j\in\underline{P}}(\lambda_{min}(P_{1j}))x_1^Tx_1+\gamma^2\omega^T\omega, \qquad (38)
\end{aligned}
$$

which implies that

$$
\begin{aligned}
&x_1^T[\rho(\tilde{A}_{1i}^TP_{1i}+P_{1i}\tilde{A}_{1i})+(1-\rho)(\tilde{A}_{1j}^TP_{1i}+P_{1i}\tilde{A}_{1j})]x_1\\
&+x_2^T[\rho A_{2i}^TP_{1i}+(1-\rho)A_{2j}^TP_{1i}]x_1\\
&+x_1^T[\rho P_{1i}A_{2i}+(1-\rho)P_{1i}A_{2j}]x_2
\end{aligned}
$$

$$
\begin{aligned}
&+\omega^T[\rho G_i^T P_{1i}+(1-\rho)G_j^T P_{1i}]x_1\\
&+x_1^T[\rho P_{1i}G_i+(1-\rho)P_{1i}G_j]\omega\\
&=x_1^T(\tilde{A}_{1ij}^T P_{1i}+P_{1i}\tilde{A}_{1ij})x_1+x_2^T\tilde{A}_{2ij}^T P_{1i}x_1+x_1^T P_{1i}\tilde{A}_{2ij}x_2\\
&+\omega^T\tilde{G}_{ij}^T P_{1i}x_1+x_1^T P_{1i}\tilde{G}_{ij}\omega\\
&<\alpha\lambda_3 x_1^T x_1+\alpha\lambda_3 x_2^T x_2+\gamma^2\omega^T\omega\\
&<\alpha x_1^T P_{1i}x_1+\alpha x_2^T P_{2i}x_2+\gamma^2\omega^T\omega\\
&<\alpha x^T P_i x+\gamma^2\omega^T\omega, \qquad (39)
\end{aligned}
$$

and

$$
\begin{aligned}
&x_1^T[\rho(\tilde{A}_{1j}^T P_{1j}+P_{1j}\tilde{A}_{1j})+(1-\rho)(\tilde{A}_{1i}^T P_{1j}+P_{1j}\tilde{A}_{1i})]x_1\\
&+x_2^T[\rho A_{2j}^T P_{1j}+(1-\rho)A_{2i}^T P_{1j}]x_1\\
&+x_1^T[\rho P_{1j}A_{2j}+(1-\rho)P_{1j}A_{2i}]x_2\\
&+\omega^T[\rho G_j^T P_{1j}+(1-\rho)G_i^T P_{1j}]x_1\\
&+x_1^T[\rho P_{1j}G_j+(1-\rho)P_{1j}G_i]\omega\\
&=x_1^T(\tilde{A}_{1ji}^T P_{1j}+P_{1j}\tilde{A}_{1ji})x_1+x_2^T\tilde{A}_{2ji}^T P_{1j}x_1+x_1^T P_{1j}\tilde{A}_{2ji}x_2\\
&+\omega^T\tilde{G}_{ji}^T P_{1j}x_1+x_1^T P_{1j}\tilde{G}_{ji}\omega\\
&<\alpha\lambda_3 x_1^T x_1+\alpha\lambda_3 x_2^T x_2+\gamma^2\omega^T\omega\\
&<\alpha x_1^T P_{1i}x_1+\alpha x_2^T P_{2i}x_2+\gamma^2\omega^T\omega\\
&<\alpha x^T P_i x+\gamma^2\omega^T\omega. \qquad (40)
\end{aligned}
$$

Thus, we can obtain that

$$\dot{V}(x)<\alpha V_i(x(t))+\gamma^2\omega^T\omega. \qquad (41)$$

It is clear that Eqs. (20) and (24) can also be attained in case 2. The rest of proof is the same as in Case 1. This completes the proof.

Furthermore, the sufficient criteria of finite-time boundedness and has a prescribed H_∞ performance level γ for the cascade nonlinear switched system (1) with $\omega\equiv 0$ are presented as follows.

Corollary 1: Consider the cascade nonlinear switched system (1) with $u(t)\equiv 0$. For any $i,j\in\Theta$, $i\neq j$, let $\overline{P}_i=R^{-\frac{1}{2}}P_iR^{-\frac{1}{2}}, P_i=diag\{P_{1i},P_{2i}\}$, and suppose that there exist positive constants $\alpha>0$, $\gamma>0$, $\theta_i>0$, $\eta_{ij}>0$, and matrices $P_i>0$, $Q_i=Q_i^T$, $W>0$ with appropriate dimensions, such that

$$
\begin{bmatrix}
\Xi_i & A_{2i}\overline{P}_{2i} & G_i & \overline{P}_{1i}C_i^T\\
* & -\alpha\lambda_3 I & 0 & 0\\
* & * & -\gamma^2 I & H_i^T\\
* & * & * & -I
\end{bmatrix}<0, \qquad (42)
$$

$$\overline{P}_i^{-1}-\overline{P}_j^{-1}+\eta_{ij}(Q_i-Q_j)=0, \qquad (43)$$

$$\theta_1 Q_1+\cdots+\theta_m Q_m\geq 0, \qquad (44)$$

$$\frac{c_1}{\lambda_2}+\gamma^2 d<\frac{c_2}{\lambda_1}e^{-\alpha T_f}, \qquad (45)$$

where

$$\Xi_i=A_{1i}\overline{P}_{1i}+\overline{P}_{1i}A_{1i}^T-\alpha\lambda_3 I.$$

If the switching signals satisfying

$$\sigma(x)=arg\Big(\max_{i\in\Theta}x^T(t)Q_ix(t)\Big), \qquad (46)$$

then switched system (1) with control input $u_i(t)=K_ix_1(t)=X_i\overline{P}_{1i}^{-1}x_1(t)$ is finite-time finite-time bounded with H_∞ performance γ for any state-dependent switching signal (46) with respect to $(0,c_2,T_f,d,\gamma,R,\sigma(x))$, where $\lambda_1=\max_{\forall i\in\Theta}(\lambda_{max}(P_i))$, $\lambda_2=\min_{\forall i\in\Theta}(\lambda_{min}(P_i))$, $\lambda_3=\min_{\forall i\in\Theta}(\lambda_{min}(\overline{P}_i^{-1}))$.

Remark 5: It is easy to solve the conditions in Theorem 1 when Assumption 2 has been establish. In view of Assumption 1, it can be seen that $x(t)=x(t^-),\forall t\in[0,T_f]$. in this paper, thus we can conclude that $x^T(t_k)P_ix(t_k)=x^T(t_k^-)P_ix(t_k^-), i=\sigma(t_k)$. In this case, based on multiple Lyapunov-like functions, some sufficient conditions can be given.

Remark 6: From the perspective of mathematics, note that the criteria proposed in Theorem 1 are not the conventional sense linear matrix inequalities (LMIs). In fact, these are the bilinear matrix inequalities (BMIs) according to the consequence of uncertain parameters. Whereas once a few values are fixed for α, η_{ij}, θ_i, γ, the criteria can be converted to LMIs conditions. In addition, conditions (15) can be insured by the following criteria, $\forall i\in\Theta$:

$$k_1 I<\overline{P}_i<k_2 I, \qquad (47)$$

$$\overline{k_2}<\overline{k_1}\frac{c_2}{c_1}e^{-\alpha T_f}, \qquad (48)$$

and

$$\frac{c_1}{\overline{k_1}}+\gamma^2 d<\frac{c_2}{\overline{k_2}}e^{-\alpha T_f}, \qquad (49)$$

where $\overline{k_1}=k_1\lambda_{min}(R)$, $\overline{k_2}=k_2\lambda_{max}(R)$, and R is a deterministic positive definite matrix with appropriate dimensions.

Remark 7: As discussed in [27], in many cases, it is difficult to find a common Lyapunov-like function, thus, we choose the multiple Lyapunov-like functions method in this paper.

Remark 8: Finite-time stabilization is different from the concept of Lyapunov Asymptotic stabilization. The proof of finite-time stabilization does not require the derivative negative definite of lyapunov function, which is necessary in the proof of Lyapunov Asymptotic stabilization. Where the control objective of finite-time stabilization is to require that the solution of the system does not exceed a certain bound during a fixed finite-time interval. This can explain the Eqs. (20) and (41) are non-negative definite.

□

4. NUMERICAL EXAMPLE

In this section, a numerical example is presented to demonstrate the potential and validity of our developed theoretical results.

Example 1: Consider the cascade nonlinear switched system with two subsystems as follows:

$$A_{11}=\begin{bmatrix}0 & -0.9\\ 1.9 & 0\end{bmatrix},\ A_{12}=\begin{bmatrix}0 & -1.9\\ 1.1 & 0\end{bmatrix},$$
$$A_{21}=\begin{bmatrix}-0.4\\ 0.3\end{bmatrix},\ A_{22}=\begin{bmatrix}0.3\\ -0.5\end{bmatrix},$$
$$B_1=\begin{bmatrix}0.8\\ 0\end{bmatrix},\ B_2=\begin{bmatrix}0\\ 0.6\end{bmatrix},$$
$$G_1=\begin{bmatrix}0.3\\ -0.1\end{bmatrix},\ G_2=\begin{bmatrix}-0.4\\ 0.2\end{bmatrix},$$
$$C_1=[-0.2\ 0.3],\ C_2=[0.3\ -0.1]$$
$$D_1=[0.8],\ D_2=[0.6],\ H_1=[-0.4],\ H_2=[0.3],$$
$$f_{21}=-0.9x_3-0.7x_3\sin^2 x_3,$$
$$f_{22}=-0.7x_3-0.7x_3\cos^2 x_3,$$

And the exogenous disturbance input $\omega(t)$ is chosen to be of the forms

$$\omega(t)=0.2\sin(10t).$$

The corresponding parameters are assigned as follows:

$$c_1=0,\ c_2=10,\ T_f=10,\ R=I,\ \alpha=0.4,\ \eta=1,$$
$$d=0.3,\ \gamma=0.9,$$

where $\theta_1=\theta_2=1$ are some properly chosen constants. x^TRx is a measurement of the state x, and R is a real symmetric and positive definite matrix. From (15), we can obtain an appropriate value for γ. According to the method designed in the paper, we can calculate that

$$X_1=[1.5266\ -5.2688],\ X_2=[-6.8211\ -0.1593],$$
$$P_1=\begin{bmatrix}9.9301 & 0.9047 & 0\\ 0.9047 & 14.8048 & 0\\ 0 & 0 & 2.7313\end{bmatrix},$$
$$P_2=\begin{bmatrix}14.1824 & -0.6141 & 0\\ -0.6141 & 5.5389 & 0\\ 0 & 0 & 2.3913\end{bmatrix},$$
$$Q_1=\begin{bmatrix}-0.0051 & 0.0016 & 0\\ 0.0016 & 0.0851 & 0\\ 0 & 0 & 0.1073\end{bmatrix},$$
$$Q_2=\begin{bmatrix}0.0253 & -0.0124 & 0\\ -0.0124 & -0.0284 & 0\\ 0 & 0 & 0.0553\end{bmatrix}.$$

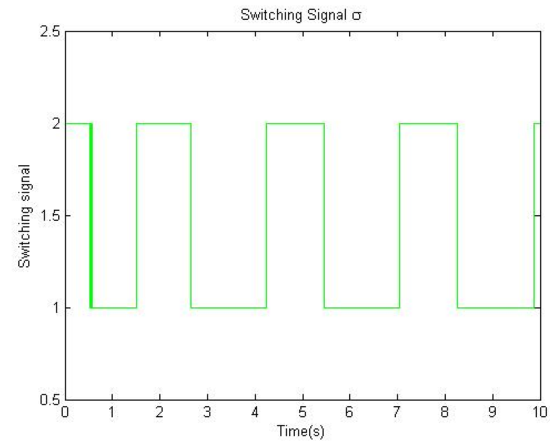

Fig. 1. Switching Signal σ.

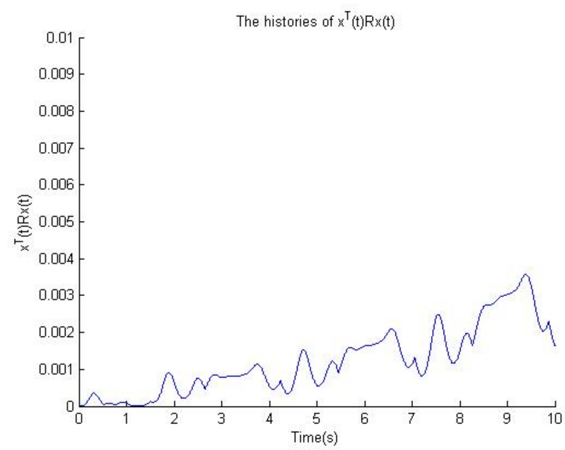

Fig. 2. The histories of $x^T(t)Rx(t)$.

By solving the matrix equalities in Theorem 1, we have the following controller gain

$$u_1=[0.1872\ -0.3673]x_1,$$
$$u_2=[-0.4845\ -0.0825]x_1.$$

It can be well seen from the curves that the cascade nonlinear switched system (1) is finite-time bounded with H_∞ performance $\gamma=0.9$ with respect to $(0,10,10,0.3,0.9,I,\sigma(x))$.

Here, we choose the switching signal satisfying (16) as shown in Fig. 1. The trajectory of $x^T(t)Rx(t)$ is shown in Fig. 2 for the given initial condition $x(0)=[0\ 0\ 0]^T$. It can be clearly observed that, the trajectory may arise in some interval, but the upper bound holds below c_2 for the whole $[0,T_f]$, hence we can conclude that the cascade nonlinear switched system (1) is finite-time bounded with respect to $(0,10,10,0.3,0.9,I,\sigma(x))$. The controlled output response of the cascade nonlinear switched system (1) under the state-dependent switching signal (16) is given in Fig. 3, therefore, the resulting closed-loop system is

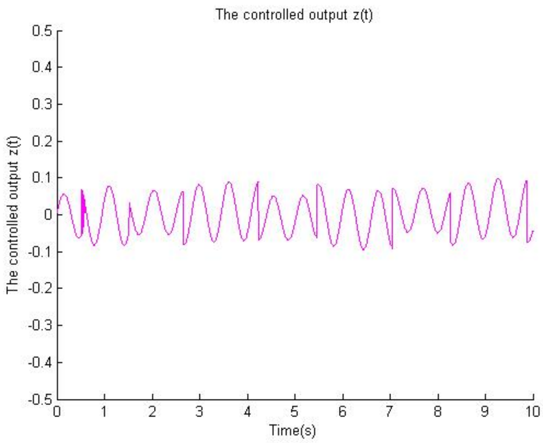

Fig. 3. The controlled output z(t).

finite-time bounded with H_∞ performance $\gamma = 0.9$, which illustrates that the method we proposed is feasible.

5. CONCLUSION

The finite-time H_∞ control problem of the class of cascade nonlinear switched systems has been investigated in this paper. The system under consideration is composed of two subsystems: a linear switched part and a nonlinear part, which are also switched systems. Using the state-dependent switching method, the weighted H_∞ performance criterion for the class of cascade nonlinear switched systems is derived and the corresponding switched state feedback controllers are designed. Subsequently, sufficient conditions have been given to guarantee the solvability of the H_∞ control problem. The obtained results are expected to be extended to issues such as output feedback and estimation for the underlying systems.

REFERENCES

[1] Z. Sun and S. S. Ge, *Stability Theory of Switched Dynamical Systems*, Springer-Verlag, London, 2011.

[2] D. Liberzon, *Switching in Systems and Control*, Birkhäuser, Boston, 2003.

[3] L. J. Long and J. Zhao, "Practical output tracking of switched nonlinear systems in p-normal form with unstable subsystems," *International Journal of Systems Science*, vol. 47, no. 11, pp. 2709-2721, 2016. [click]

[4] B. Niu, H. R. Karimi, H. Q. Wang, and Y. L. Liu, "Adaptive output-feedback controller design for switched nonlinear stochastic systems with a modified average Dwell-time method," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 47, no. 7, pp. 1371-1382, 2017.

[5] R. Wang and J. Zhao, "Guaranteed cost control for a class of uncertain delay systems with actuator failures based on switching method," *International Journal of Control, Automation, and Systems*, vol. 5, no. 5, pp. 492-500, 2007.

[6] L. H. Zhao and G. O. Shi, "A model following control system for nonlinear descriptor systems," *Journal of Northeast Dianli University*, vol. 32, no. 4, pp. 87-90, 2012.

[7] Y. Kang, D. H. Zhai, G. P. Liu, Y. B. Zhao, and P. Zhao, "Stability analysis of a class of hybrid stochastic retarded systems under asynchronous switching," *IEEE Transactions on Automatic Control*, vol. 59, no. 6, pp. 1511-1523, 2014. [click]

[8] B. Niu, Y. J. Liu, G. D. Zong, Z. Y. Han, and J. Fu, "Command filter-based adaptive neural tracking controller design for uncertain switched nonlinear output-constrained systems," *IEEE Transactions on Cybernetics*, vol. 47, no. 10, pp. 3160-3171, 2017.

[9] J. Li and G. H. Yang, "Simultaneous fault detection and control for switched systems with actuator faults," *International Journal of Systems Science*, vol. 47, no. 10, pp. 2411-2427, 2016. [click]

[10] Q. Y. Su, L. J. Long, and J. Zhao, "Stabilization of a class of state constrained switched nonlinear systems in p-normal form," *International Journal of Robust and Nonlinear Control*, vol. 24, no. 10, pp. 1550-1562, 2014. [click]

[11] J. Li and G. H. Yang, "Asynchronous fault detection filter design approach for discrete-time switched linear systems," *International Journal of Robust and Nonlinear Control*, vol. 24, no. 1, pp. 70-96, 2014. [click]

[12] W. W. Che, G. H. Yang, and X. Z. Jin, "Sparse structured non-fragile H_∞ controller design for linear systems," *International Journal of Control, Automation, and Systems*, vol. 11, no. 4, pp. 704-710, 2013. [click]

[13] H. Y. Li, Y. B. Gao, P. Shi, and X. D. Zhao, "Output-feedback control for T-S fuzzy delta operator systems with time-varying delays via an input-output approach," *IEEE Transactions on Fuzzy Systems*, vol. 23, no. 4, pp. 1100-1112, 2015. [click]

[14] H. Y. Li, J. H. Wang, H. K. Lam, Q. Zhou, and H. P. Du, "Adaptive sliding mode control for interval type-2 fuzzy systems," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 46, no. 12, pp. 1654-1663, 2016. [click]

[15] Y. B. Gao, H. Y. Li, L. G. Wu, H. R. Karimi, and H. K. Lam, "Optimal control of discrete-time interval type-2 fuzzy-model-based systems with $\mathcal{D}$-stability constraint and control saturation," *Signal Processing*, vol. 120, pp. 409-421, 2016. [click]

[16] F. Amato, G. Carannante, and G. D. Tommasi, "Input-output finite-time stability of linear systems: Necessary and sufficient conditions," *IEEE Trans. Autom. Control*, vol. 57, no. 12, pp. 3051-3063, 2012.

[17] F. Amato, G. De Tommasib, and A. Pironti, "Necessary and sufficient conditions for finite-time stability of impulsive dynamical linear systems," *Automatica*, vol. 49, no. 8, pp. 2546-2550, 2013. [click]

[18] L. X. Zhang, S. Wang, H. R. Karimi, and A. Jasra, "Robust finite-time control of switched linear systems and application to a class of servomechanism systems," *IEEE/ASME Trans. Mechatronics*, vol. 20, no. 5, pp. 2476-2485, 2015. [click]

[19] S. P. Huang and Z. R. Xiang, "Finite-time stabilization of a class of switched stochastic nonlinear systems under arbitrary switching," *International Journal of Robust and Nonlinear Control*, vol. 26, no. 10, pp. 2136-2152, 2016. [click]

[20] T. T. Liu, B. W. Wu, L. L. Liu, and Y. E. Wang, "Asynchronously finite-time control of discrete impulsive switched positive time-delay systems," *Journal of the Franklin Institute-Engineering and Applied Mathematics*, vol. 352, no. 10, pp. 4503-4514, 2015.

[21] J. Fu, R. Ma and T. Chai, "Global finite-time stabilization of a class of switched nonlinear systems with the powers of positive odd rational numbers," *Automatica*, vol. 54, no. 4, pp. 360-373, 2015. [click]

[22] A. Isidori, *Nonlinear Control Systems*, 3rd edition, Springer, Berlin, 1995.

[23] G. Zhai, B. Hu, K. Yasuda, and A. N. Michel, "Disturbance attenuation properties of time-controlled switched systems," *J. Franklin Inst.*, pp. 765-779, 2001. [click]

[24] G. Zhai, Y. Sun, X. Chen, and A. N. Michel, "Stability and L_2-gain analysis of switched symmetric systems with time delay," *Proc. Amer. Control Conf.*, Denver, CO, pp. 2682-2687, 2003.

[25] S. Zhao and J. Zhao, "H_∞ control for cascade minimum-phase switched nonlinear systems," *J. Control Theory Appl.*, pp. 163-167, 2005. [click]

[26] M. Wang, J. Zhao, and G. M. Dimirovski, "H_∞ control for a class of cascade switched nonlinear systems," *Asian Journal of Control*, vol. 10, no. 6, pp. 724-729, 2008. [click]

[27] H. B. Du, X. Z. Lin and S. H. Li, "Finite-time boundedness and stabilization of switched linear systems," *Kybernetika*, vol. 46, no. 5, pp. 870-889, 2010.

[28] H. Liu and X. D. Zhao, "Finite-time H_∞ control of switched systems with mode-dependent average dwell time," *Journal of the Franklin Institute*, vol. 351, pp. 1301-1315, 2014. [click]

[29] S. Pettersson, "Synthesis of switched linear systems," *Proceedings of the 42nd IEEE Conference on Decision Control*, pp. 5283-5288, 2003.

[30] H. Liu, Y. Shen, and X. D. Zhao, "Finite-time stabilization and boundedness of switched linear system under state-dependent switching," *Journal of the Franklin Institute*, vol. 350, pp. 541-555, 2013. [click]

Qingyu Su received his B.Sc. degree in electrical automation in 2005, and his M.Sc. degree in control theory and application in 2008, both from Liaoning Technical University, China. He completed his Ph.D. degree in control theory and application in 2013 at Northeastern University, China. Now he is an Associate Professor at the School of Automation Engineering, Northeast Dianli University, China. From October 2015 to October 2016, he was a visiting scholar at the Intelligent Systems and Biomedical Robotics Group (ISR), University of Portsmouth, UK. He has published 12 articles on SCI Journals and 8 papers on international conferences. His research interests include switched systems, nonlinear control systems and power systems.

Xiaolong Jia received the B.S. degree in school of control and computer engineering from North China Electric Power University, Beijing, China, in 2012. He is currently working toward an M.S. degree in the school of automation engineering, Northeast Dianli University, Jilin, China. He has published 2 articles on SCI Journals and 1 paper on international conferences. His research interests include switched systems, nonlinear control systems and finite-time control problems.