International Journal of Control, Automation and Systems 15(3) (2017) 1405-1414
http://dx.doi.org/10.1007/s12555-016-0364-4
ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Pinning Control of Complex Network Synchronization: A Recurrent Neural Network Approach

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Abstract: Using recurrent high order neural networks for identification, a new scheme for pinning control of complex networks with changing unknown coupling strengths is proposed for achieving synchronization. The robust behavior of the control system is investigated via simulations.

Keywords: Chaos control, complex network, neural network, pinning control.

1. INTRODUCTION

Complex dynamical networks have received a great deal of attention since the publication of the seminal articles ([1], [2] and [3]). Complex systems and networks are used to model and analyze processes and phenomena consisting of interacting elements named nodes, and to control their global and/or individual behaviors ([4], [5] and [6]). Their possible applications are in diverse fields, from biological and chemical systems to electronic circuits and social networks [5]. The models used to describe complex networks in the continuous-time settings are derived from graph theory and other frameworks such as the Kuramoto model of linear coupling oscillators [7]. Models have been developed with different structures and coupling characteristics like the small-world model [1], the E-R random graph model [8] and the scale-free model [9].

Synchronization is a process wherein many identical or different systems adjust a given property of their motions throughout to a suitable coupling strength configuration, or forced by an external input [10, 11]. The emergence of collective and synchronized dynamics in a large network of coupled units has been investigated since the beginning of the 1990 in different contexts and in various fields, ranging from biology and ecology to semiconductor lasers to electronic circuits [5]. There are many events where synchronization is a desirable feature; examples include identical oscillators in cardiac peacemaker cells or waves propagation in the brain [2]. Results have demonstrated that synchronization takes place only if some structural and coupling conditions are fulfilled. One example is the master stability function [12]; another is the Wu-Chua conjecture, which correlates the coupling strength with the structural Laplacian matrix [13]. To guarantee synchronization, efficient control techniques may be applied [6].

The basic idea of pinning control is to utilize the network structure to contribute to its regulation; to this end a local control action is applied to a small number of nodes [14, 15]. How many and which nodes to select is still the key problem. Comparations between random and specific pinning have been investigated, for different topologies ([16], [12] and [13]). Measures like degree distribution, clustering coefficient, average shortest path length, efficiency, betweness, coreness and asorativity have been used to characterize the importance of nodes and their neighborhoods. In order to find the best selection of pinned nodes to guarantee a desired behavior for the whole network [5], in this work we focus on the degrees of the nodes.

Most studies focus on stabilization control, where weights or coupling strengths between nodes are considered as an equal and fixed value for all links; other studies consider the coupling strengths as adaptive variables [13, 17]. On the other hand, the coupling strengths for a real network could be unknown, and might change over time. The change of the coupling strengths has been rarely studied. Consequently, the problem presented in this paper is the design of a robust control law which guarantees stability for nonlinear systems coupled by a complex network in the presence of non-modeled dynamics of the nodes with changes in coupling strengths.

Adaptive neural control schemes could offer a solution for the problem described above. Artificial neural networks have become an useful tool for control engineering thanks to their applicability on modeling, state estimation

Manuscript received June 21, 2016; accepted July 9, 2016. Recommended by Editor-in-Chief Young Hoo Joo.

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and control of nonlinear systems ([18] and [19]). Using neural networks, control algorithms can be developed to be robust against uncertainties, modeling errors and parameter changes. Neural networks consist of a number of interconnected processing elements (neurons). The way in which the neurons are interconnected determines its structure [19].

Since the publication of [20], there has been continuously increasing interest in applying neural networks to identification and control of nonlinear systems. Lately, the use of recurrent neural networks is being developed, which allows more efficient modeling [18, 21]. Three representative books ([22], [19] and [23]) have reviewed the applications of recurrent neural networks to nonlinear system identification and control. In particular, while [22] uses off-line learning, [19] analyzes adaptive identification and control by means of on-line learning, where stability of the closed-loop system is established based on the Lyapunov methodology. In [19], trajectory tracking is reduced to a linear model-following problem, with application to DC electric motors. In [23], analysis of recurrent neural networks for identification, estimation and control is developed, with applications to chaos, robotics and chemical processes control.

Chaotic attractors have been used to demonstrate the effectiveness of pinning control schemes in simulations and implementations due to their special characteristics [**?**]. Different techniques have been proposed to achieve chaos control [24]; including for instance, linear state space feedback [25], Lyapunov methods [26], adaptive control [27], linear matrix inequalities [28] and bang-bang control [29], among others. Most of the chaos control methods have the disadvantage of requiring the system parameters to be known; artificial neural networks provide as a solution to this problem. In this paper, we propose an identification and control scheme based on recurrent high order neural networks (RHONN) for pinning control of weighted complex networks with unknown node dynamics. The paper is organized as follows: in Section 2, preliminaries are given; Section 3 presents a neural network identification scheme for pinned nodes in a complex network and a control scheme for stabilizing control of the complex network, followed by a simulation study in Section 4. Finally, conclusions are drawn in Section 5. A preliminary version of this paper was presented earlier in a conference [30].

2. FUNDAMENTALS

2.1. Preliminaries

Throughout this paper, $\mathbb{R}$, $\mathbb{R}^n$, $\mathbb{R}^{N\times N}$ stand for spaces of real numbers, n-dimensional vectors and $N\times N$-dimensional matrices; $\|\cdot\|$ denotes the Euclidean norm; I_n stands for the $n\times n$ identity matrix.

Definition 1 [12]**:** The Kronecker product of two matrices A and B is

$$A\otimes B=\begin{pmatrix} a_{11}B & \cdots & a_{1m}B \\ \vdots & \ddots & \vdots \\ a_{n1}B & \cdots & a_{nm}B \end{pmatrix},$$

where if A is an $n\times m$ matrix and B is a $p\times q$ matrix, then $A\otimes B$ is an $np\times mq$ matrix.

Definition 2 [12]**:** The product $A\otimes f(x_i,t)$ is defined by

$$A\otimes f(x_i,t)= \begin{pmatrix} a_{11}f(x_1,t)+a_{12}f(x_2,t)+\cdots+a_{1m}f(x_m,t) \\ \vdots \\ a_{n1}f(x_1,t)+a_{n2}f(x_2,t)+\cdots+a_{nm}f(x_m,t) \end{pmatrix},$$

where if A is an $n\times m$ matrix and f is a $p\times 1$ function, then $A\otimes f(x_i,t)$ is a $np\times 1$ vector.

Definition 3 [12]**:** Matrix A is reducible if there exists a permutation matrix P such that PAP^T is of the form $\left(\begin{smallmatrix} B & C \\ 0 & D\end{smallmatrix}\right)$, where B and D are square matrices. Matrix A is irreducible if it is not reducible.

Lemma 1 [12]**:** If Q is a real symmetric matrix the set Υ consisting of all matrices with zero row sums, which have only nonpositive off-diagonal elements, then Q is positive semi-definite and has a zero eigenvalue associated with the eigenvector $(1,1,\ldots,1)$. Furthermore, Q can be decomposed as $Q=M^TM$, where M is a matrix in a class of matrices such that its row i consists of all zeros except one entry β_i and one entry $-\beta_i$ for some nonzero β_i. Furthermore, if Q is irreducible, then the zero eigenvalue has multiplicity 1.

Definition 4 [12]**:** A function $\xi\colon \mathbb{R}^n\times\mathbb{R}\to\mathbb{R}^n$ is uniformly increasing if there exists $\theta>0$ such that for all x, y, t,

$$(x-y)^TP(\xi(x,t)-\xi(y,t))\geq\theta\|x-y\|^2. \tag{1}$$

Definition 5 [12]**:** Given a square matrix V, a function $\xi\colon \mathbb{R}^n\times\mathbb{R}\to\mathbb{R}^n$ is V-uniformly increasing if $V\xi$ is uniformly increasing.

Definition 6 [12]**:** A function $\xi\colon \mathbb{R}^n\times\mathbb{R}\to\mathbb{R}^n$ is (V-uniformly) decreasing if $-\xi$ is (V-uniformly) increasing.

Corollary 1 [31]**:** Let $x=0$ be an equilibrium point for a nonlinear system of the form $\dot{x}=f(x,t)$. Let $V:\mathbb{R}^n\to\mathbb{R}$ be a continuously differentiable, radially unbounded, positive definite function, such that $\dot{V}(x)\leq 0$ for all $x\in\mathbb{R}^n$. Let $S=\{x\in\mathbb{R}^n \mid \dot{V}(x)=0\}$ and suppose that no solution can stay permanently in S, except the trivial solution. Then, the origin is globally asymptotically stable.

2.2. Complex networks

This subsection is taken from ([12] and [13]).

In general, a complex network with N identical linearly and diffusively coupled nodes, with each node being an n-dimensional dynamical system can be described as follows:

$$\dot{x}_i = f(x_i) + \sum_{j=1, j\neq i}^{N} c_{ij} a_{ij} \Gamma (x_j - x_i) \quad i = 1, 2, \ldots, N, \quad (2)$$

where $x_i = (x_1, x_2, ..., x_n)^T \in \mathbb{R}^n$ is the state vector of node i, the constant $c_{ij} > 0$ represents the coupling strength between node i and node j, $\Gamma = (\gamma_{pq}) \in \mathbb{R}^{n\times n}$ is a matrix linking coupled variables, and if some pairs (p,q), $1 \leq p, q \leq n$, has $\gamma_{pq} \neq 0$, it means two coupled nodes are linked through their pth and qth state variables, respectively.

In network (2), the coupling matrix $A = (a_{ij}) \in \mathbb{R}^{N\times N}$ represents the structural configuration of the network, which is assumed in this paper to be a scale-free network described by the BA model [12]. If there is a connection between node i and node j ($i \neq j$), then $a_{ij} = a_{ji} = 1$; otherwise, $a_{ij} = a_{ji} = 0$ ($i \neq j$). The degree k_i of node i is defined to be the number of its outreaching connections, and $\sum_{j=1, j\neq i}^{N} a_{ij} = \sum_{j=1, j\neq i}^{N} a_{ji} = k_i$ for $i = 1, 2, ..., N$. Let the diagonal elements of A be $a_{ii} = -k_i$, $i = 1, 2, \ldots, N$. Then, the coupling matrix A is symmetric and the matrix $-A$ is in Υ. Let Υ_i be the subset consisting of all irreducible matrices in Υ.

Assume the network is connected in the sense of having no isolated clusters. Then, the symmetric coupling matrix A is irreducible. From Lemma 1, zero is an eigenvalue of $-A$ with multiplicity 1, and other eigenvalues of $-A$ are strictly positive.

Let $x_s(t)$ be a solution of an isolated node of the network, which is assumed to exist and to be unique, satisfying

$$\dot{x}_s = f(x_s), \quad (3)$$

where x_s is an homogeneous equilibrium point.

The objective is to obtain a pinning control scheme which synchronize the entire network (2) to x_s on the manifold

$$x_1 = x_2 = \ldots = x_N = x_s \qquad f(x_s) = 0. \quad (4)$$

To achieve (4), the pinning control strategy is applied on a small fraction $\delta (0 < \delta \ll 1)$ of the nodes in network (2). Suppose that nodes $i_1, i_2, \ldots, i_l$ are selected, where $l = [\delta N]$ stands for the smaller but nearest integer to the real number δN. This controlled network is described as

$$\dot{x}_i = f(x_i, t) - \sum_{j=1}^{N} g_{ij} \Gamma x_j + u_i \quad i = 1, 2, \ldots l \quad (5)$$

$$\dot{x}_i = f(x_i, t) - \sum_{j=1}^{N} g_{ij} \Gamma x_j \quad i = l+1, \ldots, N, \quad (6)$$

where $g_{ij} = -c_{ij} a_{ij}$, and the coupling strength c_{ii} satisfies

$$c_{ii} a_{ii} + \sum_{j=1, j\neq i}^{N} c_{ij} a_{ij} = 0. \quad (7)$$

Without loss of generality, we rearrange the order of nodes in the network such that the pinned nodes $i = 1, 2, ...l$, are the first l nodes in the rearranged network.

The following local linear negative feedback control law is used:

$$u_i = -c_{ii} d_i \Gamma (x_i - x_s), \quad (8)$$

where the feedback gain $d_i > 0$, $i = 1, 2, \ldots, l$.

Define the following matrices:

$$D' = diag(c_{11} d_1, c_{22} d_2, ... c_{ll} d_l, 0, ..., 0) \in \mathbb{R}^{N\times N}, \quad (9)$$

$$D = diag(d_1, d_2, ... d_l, 0, ..., 0) \in \mathbb{R}^{N\times N}. \quad (10)$$

Substituting (8) into ((5) and (6)), one can re-arrange the controlled network and write it by using the Kronecker product as

$$\dot{X} = I_N \otimes [f(x_i, t)] - [(G + D') \otimes \Gamma] X + (D' \otimes \Gamma) \bar{X}, \quad (11)$$

where $\bar{X} = (x_s^T, x_s^T, \ldots, x_s^T)^T$, and the elements g_{ij} of the symmetric irreducible matrix $G = (g_{ij}) \in \mathbb{R}^{N\times N}$ are defined as $g_{ij} = -c_{ij} a_{ij}$.

It is easy to see that G is positive semi-definite, and $G + D'$ is positive definite with the minimal eigenvalue $\sigma_{min}(G + D') > 0$.

Theorem 1 [12]**:** Assume that $f(x_i)$ is Lipschitz continuous in x with a Lipschitz constant $L_c^f > 0$. If Γ is symmetric and positive definite, then the controlled network (5 and 6) is globally stable about the homogenous state x_s, provided that $\frac{(L_c^f)}{\sigma_{min}(\Gamma)} > 0$ such that

$$\sigma_{min}(G + D') > \frac{(L_c^f)}{\sigma_{min}(\Gamma)}, \quad (12)$$

where $\sigma_{min}(\Gamma)$ and $\sigma_{min}(G + D')$ are the minimal eigenvalues of matrices Γ and $G + D'$, respectively.

Theorem 2 [12]**:** Assume that the node $\dot{x}_i = f(x_i)$ is chaotic for all $i = 1, 2, \ldots, N$, with the maximum positive Lyapunov exponent $h_{max} > 0$. If $c_{ij} = c$, $d_i = cd$ and $\Gamma = I_m$, then the controlled network (11) is locally asymptotically stable on the homogenous state x_s, provided that

$$c > \frac{h_{max}}{\sigma_{min}(-A + diag(d, \ldots, d, 0, \ldots, 0))}, \quad (13)$$

where σ_{min} stands for the minimal eigenvalue of the matrix.

2.3. Recurrent higher-order neural networks

In a recurrent neural network, the outputs of a neuron are feedback to the same neuron or some neurons in the preceding layers. Signals flow in forward and backward directions [32]. Artificial recurrent neural networks are mostly based on the Hopfield model [33].

In [34], Recurrent Higher-Order Neural Networks (RHONN) are defined as

$$\dot{\chi}_i = -\lambda_i \chi_i + \sum_{j=1}^{L} w_{ij} \prod_{j \in I_k} y_j^{\delta_j(\kappa)}, \qquad i = 1,2,\dots,n, \quad (14)$$

where χ_i is the ith neuron state, L is the number of higher-order connections, $\{I_1, I_2, \dots, I_L\}$ is a collection of non-ordered subsets of $\{1,2,\dots,m+n\}$, $\lambda_i > 0$, w_{ij} are the adjustable weights of the neural network, $\delta_j(\kappa)$ are nonnegative integers, and y is a vector defined by

$$\begin{aligned} y =&[y_1,\dots,y_n,y_{n+1},\dots,y_{n+m}]^T \\ =&[S(\chi_{i1}),\dots,S(\chi_{in}),S(u_{i1}),\dots,S(u_{im})] \end{aligned} \quad (15)$$

with $u_i = [u_{i1}, u_{i2}, \dots, u_{im}]$ being the input to the neural network and with a smooth sigmoid function $S(\chi_i) = \frac{1}{1+e^{-\beta\chi}} + \varepsilon$, in which β is a positive constant and ε is a small positive real number, so, $S(\chi_i) \in [\varepsilon, \varepsilon+1]$. As can be seen, (15) includes higher-order terms.

By defining a vector

$$\begin{aligned} z(\chi_i,u_i) =&[z_1(\chi_i,u_i),\dots,z_L(\chi_i,u_i)]^T \\ =&[\Pi_{j\in I_1} y_j^{\delta_j(1)}, \Pi_{j\in I_2} y_j^{\delta_j(2)}, \dots, \Pi_{j\in I_L} y_j^{\delta_j(L)}]^T. \end{aligned} \quad (16)$$

Equation (14) ROHNN can be written as

$$\dot{\chi}_i = -\lambda_i \chi_i + w_i^T z(\chi_i,u_i), \qquad i = 1,\dots,n, \quad (17)$$

where $w_i = [w_{i,1}, w_{i,2} \dots w_{i,L}]^T$. In this paper, consider

$$y = [y_1,\dots,y_n]^T = [S(\chi_{i1}),\dots,S(\chi_{in})]. \quad (18)$$

If the RHONN is affine in the control, then reformulating (17) in a matrix form yields

$$\dot{\chi}_i = \Lambda\chi_i + W_i z(\chi_i) + W_{ig} u_i \quad (19)$$

where $\chi_i \in \mathbb{R}^n$, $W_i \in \mathbb{R}^{n\times L}$, $W_{ig} \in \mathbb{R}^{n\times n}$, $z(\chi_i) \in \mathbb{R}^L$, $u_i \in \mathbb{R}^n$, and $\Lambda = -\lambda I_n$ with $\lambda > 0$.

3. THE IDENTIFICATION AND CONTROL SCHEME

In this section, an adaptive control scheme (Fig. 1) is proposed. It is composed by a recurrent neural identifier and a controller for the pinned nodes in the complex network, where the former is used to build an on-line model for the unknown plant and the later to force the unknown node dynamics to converge to an equilibrium point.

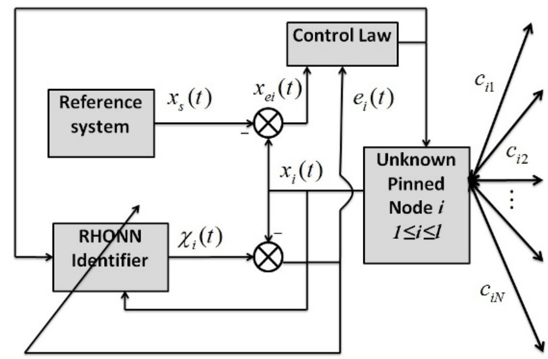

Fig. 1. Control diagram.

3.1. Neural Identifier

In this subsection, a neural network identifier for unknown pinned nodes is designed. Without losts of generality, proceed with only one pinned node according to [35]. The weight adaptation law is taken from ([19] and [34]). Under the assumption that all the states are available for measurement and use, a recurrent neural network is designed for on-line identification of the unknown ith node system ($i = 1,2,\dots,l$).

Consider the unknown nonlinear plant for the ith pinned node as

$$\begin{aligned} \dot{x}_i =&F_i(x_i,u_i) = f(x_i,t) - \sum_{j=1}^{N} g_{ij}\Gamma x_j(t) + u_i \\ &\triangleq \hat{f}(x_i,t) + \hat{g}_1(x_i,t)d + \hat{g}_2(x_i,t)u_i \end{aligned} \quad (20)$$

in accordance with [32].

Taking into account that $f(x_i)$ is unknown, with x_i available for measurement, one can model (20) by a recurrent neural network as in (19).

Assumption 1 [12]**:** For the given nonlinear $f(x)$, there is a matrix T such that $f(x)+Tx$ is V-uniformly decreasing for some symmetric and positive definite matrix V.

Now, we propose the following recurrent neural network in a Series-Parallel structure:

$$\dot{\chi}_i = \Lambda\chi_i + W_i z(x_i) + \omega_{ier} + u_i, \quad (21)$$

where W_i are the values of the on-line estimated network weights, which minimize the modeling error ω_{ier}.

Assumption 2 [32]**:** For every bounded state x_i and for every bounded $w_{ij} \in W_i$, the system (21) is bounded.

Assumption 3 [32]**:** The given node dynamics can be completely described, without any modelling error, by the neural network of the form

$$\dot{x}_i = \Lambda x_i + W_i^* z(x_i) + u_i, \quad (22)$$

where W_i^* are the constant weights to be determined and all other elements are as defined above.

Then, we define the identification error as $e_i = \chi_i - x_i$, whose dynamics satisfy

$$\begin{aligned} \dot{e}_i &= \dot{\chi}_i - \dot{x}_i, \\ \dot{e}_i &= \Lambda e_i + \tilde{W}_i z(x_i) + \omega_{ier}, \\ \tilde{W}_i &= W_i - W_i^*. \end{aligned} \tag{23}$$

Select the weight adaptation law as in [19], namely,

$$tr\left\{\dot{\tilde{W}}_i^T \tilde{W}_i\right\} = -\gamma e_i^T \tilde{W}_i z(x_i), \tag{24}$$

which has elements as

$$\dot{w}_{i,j} = -\gamma e_{i_i}^T \tilde{W}_i z(x_i), \;\; i = 1,2,...,n, \; j = 1,2,...,L.$$

With this adaptation law, the modeling error $\dot{\omega}_{er} = -\rho\,\omega_{er}$ with $\rho > 0$ will converge to zero. For the respective stability analysis on (23), we refer the reader to [32].

3.2. Stabilization

In this subsection, an adaptive neural control law is designed for pinned nodes to stabilize its trajectory onto the homogeneous state x_s as defined in (4). The problem of regulation by pinning control for a complex network can be solved even by pinning only one node [35], which is also applied here, by pinning just the node with the greatest degree. The structure of the control law is derived from the one presented in [12], so that a local robust feedback controller is obtained.

Dynamics of the pinned nodes so selected is identificated by a RHONN. A robust controller is used on such identifier implementation, which guarantees stabilization of the error between the plant and the desired equilibrium point x_s.

Theorem 3: The unknown pinned network ((5) and (6)), whose changing coupling strengths c remain above the limit given by (13), can become locally asymptotically stable at the homogeneous state x_s under the control law

$$u_i = -cd\Gamma(\chi_i - x_s) \quad i = 1,2,\ldots,l, \tag{25}$$

where χ_i are the identificated states of the pinned node by a RHONN in the form of (21), and $d > 0$ are the feedback gains.

Proof: It is required to stabilize the errors between the ROHNN states and the desired equilibrium point (3). To apply the Lyapunov methodology [31] define the stabilization error for pinned nodes as $x_{ei} = x_i - x_s$, $i = 1,2,...,l$, and obtain its derivative from (21) and (5) as follows:

$$\begin{aligned} \dot{x}_{ei} =& \dot{x}_i - \dot{x}_s \triangleq \dot{\chi}_i + \omega_{ier} - \dot{x}_s \\ =& \Lambda\chi_i + W_i z(x_i) + \omega_{ier} + u_i - f(x_s). \end{aligned} \tag{26}$$

This can be rewritten as

$$\begin{aligned} \dot{x}_{ei}(t) =& \Lambda x_{ei} + W_i z(x_i) + \alpha(x_s) + \omega_{ier} + u_i \\ =& \hat{f}(x_{ei}, e_i, W_i) + \omega_{ier} + \hat{g}_2(x_{ei}, e_i, W_i) u_i, \end{aligned} \tag{27}$$

where $\hat{f}(x_{ei}, e_i, W_i) = \Lambda x_{ei} + W_i z(x_i) + \alpha(x_s)$, $\hat{g}_2(x_{ei}, e_i, W_i) = I_n$ and $\alpha(x_s) = \Lambda x_s - f(x_s)$.

Note that $(x_{ei}, \tilde{W}_i, e_i) = (0,0,0)$ is an equilibrium point for (27) without disturbances. Now, consider the next Lyapunov function candidate

$$V = \frac{1}{2} \parallel e_i \parallel^2 + \frac{1}{2} \parallel x_{ei} \parallel^2 + \frac{1}{2\gamma} tr\left\{\tilde{W}_i^T \tilde{W}_i\right\}, \qquad \gamma > 0, \tag{28}$$

where e_i and $\tilde{W}_i$ are defined in (23). Its time derivative along the trajectories of (27), with the control law (25), is

$$\begin{aligned} \dot{V} =& -\lambda \parallel e_i \parallel^2 + e_i^T \tilde{W}_i z(x_i) + \frac{1}{\gamma} tr\left\{\dot{\tilde{W}}_i^T \tilde{W}_i\right\} \\ & -\lambda \parallel x_{ei} \parallel^2 + x_{ei}^T W_i z(x_i) + x_{ei}^T(\alpha(x_s) + \omega_{ier}) \\ & -cd x_{ei}^T \Gamma(\chi_i - x_s). \end{aligned} \tag{29}$$

Replacing the weight adaptation law (24) in (29), and taking into account the property of $-x^T\Gamma x \leq -\sigma_{min}(\Gamma)\|x\|^2$ where $\sigma_{min}(\Gamma)$ is the minimum eigenvalue of matrix Γ, and then reordering terms, one obtains

$$\begin{aligned} \dot{V} \leq& -\lambda \parallel e_i \parallel^2 + e_i^T \tilde{W}_i z(x_i) - e_i^T \tilde{W}_i z(x_i) \\ & -(\lambda + cd\sigma_{min}(\Gamma)) \parallel x_{ei} \parallel^2 + x_{ei}^T W_i z(x_i) \\ & + x_{ei}^T(\alpha(x_s, e_i) + \omega_{ier}). \end{aligned} \tag{30}$$

After eliminating the term $e_i^T \tilde{W}_i z(x_i)$, one has

$$\begin{aligned} \dot{V} \leq& -\lambda \parallel e_i \parallel^2 - (\lambda + cd\sigma_{min}(\Gamma)) \parallel x_{ei} \parallel^2 \\ & + x_{ei}^T W_i z(x_i) + x_{ei}^T(\alpha_s(x_s) + \omega_{ier}). \end{aligned} \tag{31}$$

In the fourth term of (31), x_s is a constant; consequently, $\alpha_s(x_s)$ is bounded. It follows that the part of this term, which includes the uncertain term ω_{ier}, is also bounded from above and is vanishing because $\dot{\omega}_{er} = -\rho\,\omega_{er}$. Therefore, the last two terms in (31) are bounded. Finally, by selecting d adequately in the second term, $\dot{V}$ is negative definite, even when c change but remain above the threshold define in (13). It follows from the Barbalat's Lemma [31] and Corollary 1 that the pinned nodes are asymptotically stables at x_s.

Next, the stability of non-pinned nodes dynamics (6) is analyzed.

First, write ((5) and (6)) as in (11). Since $c_{ij} = c$ and $D' = diag[cd, cd, \ldots, cd, 0, \ldots, 0]$, one has $\sigma_{min}(G + D') = c\sigma_{min}[(-A + D)] > 0$ by definition (recall that $-A$ is a positive semi-definite matrix in W_i). Then, determine a $d > 0$ such that (12) and (13) are fulfilled. Finally, by Theorem 2, the entire controlled dynamical network ((5) and (6)) is locally stable at the homogeneous state x_s.

The neural network absorbs variations of the coupling strengths, so that a proper adjustment can be accomplished on the control law.

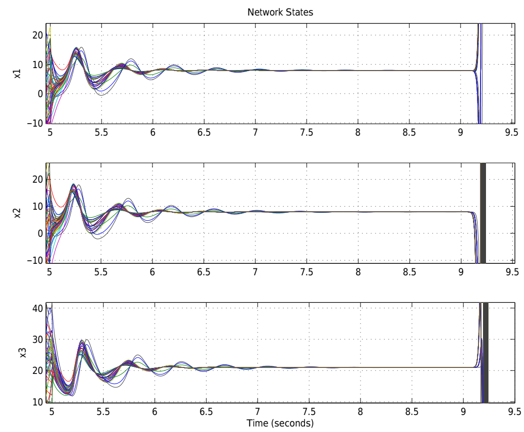

Fig. 2. State time evolution under the proportional control scheme.

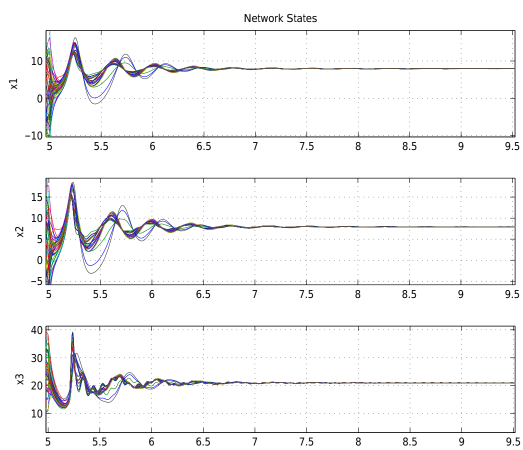

Fig. 3. State time evolution under the proposed control scheme.

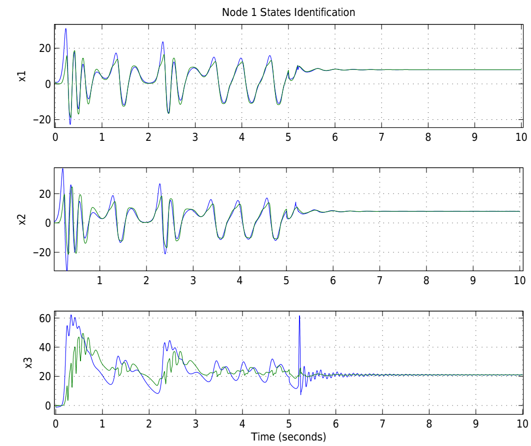

Fig. 4. Node state vs. identified state for Node 1.

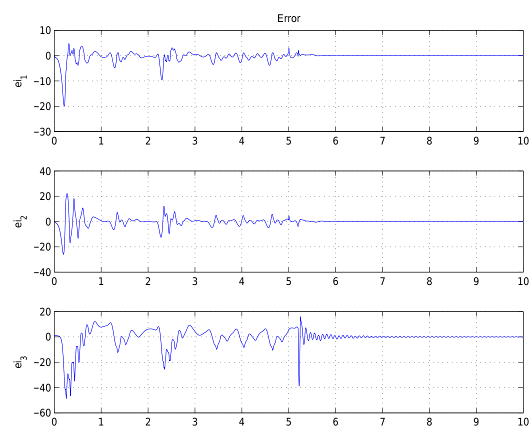

Fig. 5. Identification error for Node 1.

4. SIMULATION EXAMPLES

Consider a 50-node scale free network with degree distribution $\Delta(k_i) \approx k_i^{-2}$. Each node is selected as a chaotic Chen system [12] defined by

$$
\begin{aligned}
\dot{x}_1 &= \hat{a}(x_2 - x_1),\\
\dot{x}_2 &= (\hat{c} - \hat{a})x_1 - x_1 x_3 + \hat{c}x_2,\\
\dot{x}_3 &= x_1 x_2 - \hat{b}x_3.
\end{aligned} \tag{32}
$$

The parameters in (32) are selected as $\hat{a} = 35$, $\hat{b} = 3$ and $\hat{c} = 28$, so that an unstable equilibrium point exists at $x_s = [7.9373, 7.9373, 21]$. This equilibrium point is selected as the homogeneous stationary state, at which the complex network is going to be synchronized. The maximum positive Lyapunov exponent is $h_{ie} \approx 2.01745$ [12]. The Γ matrix is taken as I_3. In the implementation of the RHONN, set $z(x_i) \in \mathbb{R}^{10}$ in (17).

Two control algorithms are compared: the proportional control scheme presented in [12] and the neural network scheme proposed in this paper. Just one node is pinned, which selected as the one with the highest degree. For both control schemes, coupling strengths c at node connections are set initially higher than the minimum value required by (13). Then, the control law is incepted. Once the complex network is stabilized, the coupling strengths are changed to lower values, but still above their minimum values required by (13).

For both control algorithms, the simulation is carried out as follows:

Initially, from $t = 0$ to $t = 5$, the systems at the nodes run without any connection, i.e. $c = 0$. At $t = 5$ the coupling strengths are set to $c = 30$, so that the complex network is connected according to a predefined scale-free distribution. Subsequently, at $t = 5.2$ the control law is incepted. After stabilization is achieved, starting at $t = 9$, the coupling strengths change from $c = 30$ to $c = 23$. For both controllers $d = 1000$ and $c_{min} \approx 21.55$.

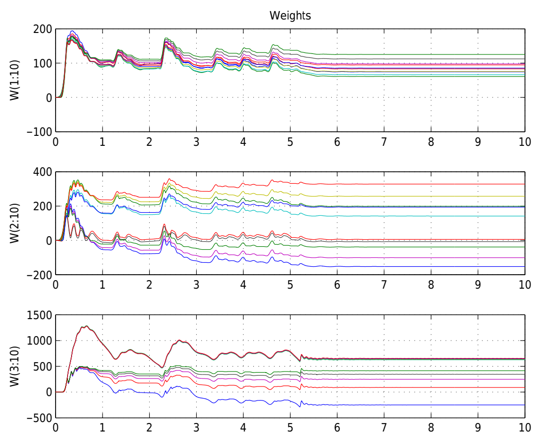

Fig. 6. Weights evolution for Node 1 with identification.

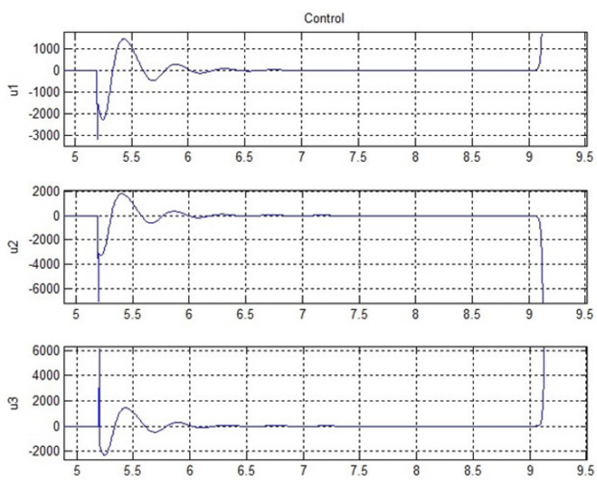

Fig. 7. Control signals for the proportional control scheme.

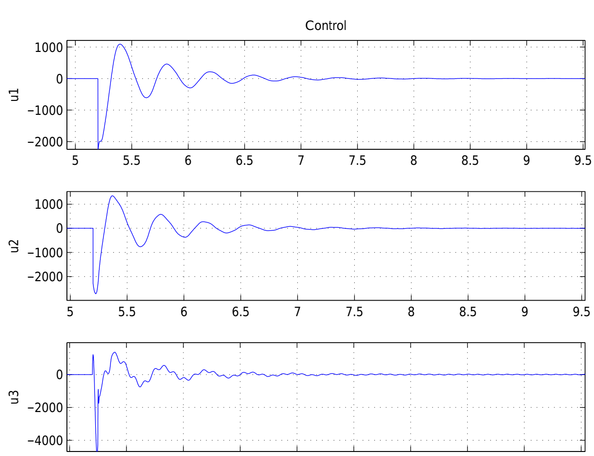

Fig. 8. Control signals for the proposed control scheme.

Fig. 2 and Fig. 3 show that the states of the entire network have been regulated to x_s. In Fig. 2, the network loses its regulation when the coupling strengths are changed at $t = 9$; in Fig. 3, with the robust control law (25), the network evolution stays at the stabilization state.

In Fig. 4, real state vs identified state of Node 1 are presented, followed by the identification error for Node 1 in Fig. 5. Fig. 6 shows the evolution of the neural-network weights in the identification of Node 1. Fig. 7 and Fig. 8 display the control signals for both controllers. The network maintains synchronization with the proposed control scheme.

5. CONCLUSIONS

This paper develops a new pinning control scheme for complex networks, from a recurrent higher-order neural network approach. It is based on a neural identifier and a proportional controller. By means of this novel scheme, it is possible to stabilize a complex network even in the presence of varying coupling strengths, with a robust property. Simulation results illustrate the applicability and effectiveness of the proposed scheme.

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