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Stability and Asynchronous Stabilization for a Class of Discrete-time Switched Nonlinear Systems with Stable and Unstable Subsystems

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Abstract: The stability analysis and asynchronous stabilization problems for a class of discrete-time switched nonlinear systems with stable and unstable subsystems are investigated in this paper. The Takagi-Sugeno (T-S) fuzzy model is used to represent each nonlinear subsystem. Through using the T-S fuzzy model, the studied systems are modeled into the switched T-S fuzzy systems. By using the switching fuzzy-basis-dependent Lyapunov functions (FLFs) approach and mode-dependent average dwell time (MDADT) technique, the stability conditions for the open-loop switched T-S fuzzy systems with unstable subsystems and asynchronous stabilization conditions for the closed-loop switched T-S fuzzy systems with unstable subsystems are obtained. Both the stability results and asynchronous stabilization results are derived in terms of linear matrix inequalities (LMIs). Finally two numerical examples are provided to illustrate the effectiveness of the results obtained.

Keywords: Discrete-time switched nonlinear systems, mode-dependent average dwell time, switching fuzzy-basis-dependent Lyapunov functions, Takagi-Sugeno fuzzy model, unstable subsystems.

1. INTRODUCTION

During the past decades, the switched systems attracted considerable attention because of many physical or man-made systems possessing switching features [1]. A typical switched system is composed of a finite number of continuous-time or discrete-time subsystems and a switching signal. The stability analysis problem is the main concern in the study of switched systems [1–9]. So far, two stability issues have been addressed, i.e., the stability under arbitrary switching and stability under constrained switching. As one typical example of constrained switching, the mode-dependent average dwell time (MDADT) method is proposed in [10], which can relax the conservativeness of the results obtained by the average dwell time logic [11]. As we know, switched systems with unstable subsystems are inevitably encountered in many real plants. Recently, some efforts have been made to study the switched linear systems with unstable subsystems [12–15].

In recent years, the switched nonlinear systems have attracted more and more attention. Due to the nonlinearity, it is difficult to analyze the switched nonlinear systems directly. The T-S fuzzy model is proven to be an effective tool in approximating most complex nonlinear systems [16], which utilizes local linear system description for each rule. The last several decades have witnessed more and more applications of T-S fuzzy model [17–29]. Furthermore, the T-S fuzzy model has been extended to study switched nonlinear systems [30–32].

It is noted that the results based on a common quadratic Lyapunov function (QLF) might be conservative since a common Lyapunov matrix should be found for all subsystems. To relax the conservativeness, the basis-dependent Lyapunov functions have received a great deal of attention in the stability analysis and controller synthesis for discrete-time systems [33–36]. In this paper, the switching fuzzy-basis-dependent Lyapunov functions (FLFs) are used by using the switching information and structural information of membership function in the rule base. The candidate Lyapunov function is switching according to the system switching among serval FLFs.

In practice, when the switched system is switching among the subsystems, the matched controller or filter of each subsystem can not be operating immediately. It inevitably takes some time to identify the system model and apply the matched controller or filter. Therefore asynchronous behaviors generally exist. The asynchronous be-

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haviors usually bring unsatisfactory performance or even make the system out of control. Recently, several work have explored the effect of asynchronous behaviors on the control of switched systems [31,32].

The novelty of our work can be summarized as follows: The stability analysis and asynchronous stabilization problems for a class of switched nonlinear systems with unstable subsystems are investigated. Especially, our paper is the first work to study the asynchronous stabilization problem for switched systems. Furthermore, the results obtained in our work can also be reduced to study the stability analysis and asynchronous stabilization problems for switched linear systems.

The rest of this paper is organized as follows. Section 2 gives preliminaries and problem formulation. The main results are given in Section 3. Two numerical examples are provided in Section 4. Finally, some conclusions are given in Section 5.

Notations: The notations $\tilde{h}_{pl}$ and h_{pm} represent $h_{pl}(k+1)$ and $h_{pm}(k)$, respectively. The notation $\|\cdot\|$ refers to the Euclidean vector norm. The superscript "T" stands for matrix transpose. The symbol "$*$" in a matrix stands for the transposed elements in the symmetric positions. We use $P>0(\geq 0,<0,\leq 0)$ to denote a positive definite (semi-positive definite, negative definite, semi-negative definite) matrix P. $\mathbb{R}^n$ denotes the $n-$dimensional Euclidean space. $\mathcal{C}^1$ denotes the space of continuously differentiable functions. A continuous function $\alpha:[0,\infty)\to[0,\infty)$ is said to be of class $\mathcal{K}$ if it is strictly increasing and $\alpha(0)=0$. If α is also unbounded, then it is said to be of class $\mathcal{K}_\infty$. A function $\beta:[0,\infty)\times[0,\infty)\to[0,\infty)$ is said to be of class $\mathcal{KL}$ if $\beta(\cdot,t)$ is of class $\mathcal{K}$ for each fixed $t\geq 0$ and $\beta(s,t)$ decreases to 0 as $t\to\infty$ for each fixed $s\geq 0$. If not explicitly stated, matrices are assumed to have compatible dimensions.

2. PRELIMINARIES AND PROBLEM FORMULATION

Consider the following discrete-time switched nonlinear system

$$x(k+1)=f_{\sigma(k)}(x(k),u(k)),\quad x(k_0)=x_0, \tag{1}$$

where $x(k)\in\mathbb{R}^n$ is the state vector, $u(k)\in\mathbb{R}^m$ is the control input vector. $\sigma(k)$ is a piecewise constant function of time, which is called the switching signal and takes its value in a finite set $\mathcal{I}=\{1,\ldots,N\}$ where N is the number of subsystems. For a switching sequence $0<k_1<k_2\cdots<k_i<k_{i+1}<\cdots,\sigma(k)$ is continuous from right everywhere. Let k_i^- represent the previous instant of discrete instant k_i. When $k\in[k_i,k_{i+1})$, we say that the $\sigma(k_i)$ subsystem is activated. $f_{\sigma(k)}(\cdot)$ are nonlinear functions. Here, we consider $f_{\sigma(k)}(\cdot)$ can be either stable or unstable. Without loss of generality, we suppose that there are $g(1\leq g\leq N)$ stable subsystems and $N-g$ unstable subsystems. For brevity, we denote $\mathcal{S}\triangleq\{1,2,\cdots,g\},\mathcal{U}\triangleq\{g+1,g+2,\cdots,N\}$. $\mathcal{S}$ and $\mathcal{U}$ are the sets of stable subsystems and unstable subsystems, respectively.

In this paper, each nonlinear subsystem is represented by the following T-S fuzzy model.

Rule m for subsystem p: **IF** $z^p_{m1}(k)$ is M^p_{m1} and $\cdots$ and $z^p_{mj}(k)$ is M^p_{mj}, **THEN**

$$x(k+1)=A_{pm}x(k)+B_{pm}u(k), \tag{2}$$

where z^p_{mh} are the premise variables, and M^p_{mh} are the fuzzy sets $(h=1,2,\cdots,j)$. A_{pm} and B_{pm} are constant matrix.

By using "fuzzy blending", the final output of the p subsystem is inferred as follows:

$$x(k+1)=\sum_{m=1}^{r}h_{pm}\left[A_{pm}x(k)+B_{pm}u(k)\right] \tag{3}$$

with $w_{pm}(k)=\prod_{h=1}^{j}M^p_{mh}(z^p_{mh}(k))$, $h_{pm}=\frac{w_{pm}(k)}{\sum_{m=1}^{r}w_{pm}(k)}$, and $M^p_{mh}(z^p_{mh}(k))$ is the grade of membership of $z^p_{mh}(k)$ in M^p_{mh}, $m\in\mathbb{R}_p$, $\mathbb{R}_p=\{1,2,\cdots,r\}$ and r is the number of IF-THEN rules. It is assumed that $w_{pm}(k)\geq 0$ for all $k\geq 0$. Then, the normalized membership function h_{pm} satisfies

$$h_{pm}\geq 0,\quad \sum_{m=1}^{r}h_{pm}=1. \tag{4}$$

In view of asynchronous behaviors, the controller $u(k)$ is divided into two parts $\bar{u}(k)$ and $\hat{u}(k)$, where $\bar{u}(k)$ denotes the unmatched controller, and $\hat{u}(k)$ represents the matched controller. For the fuzzy model (3), the following fuzzy controller can be constructed:

$$\begin{cases}\bar{u}(k)=\sum\limits_{n=1}^{r}h_{qn}K_{qn}x(k), & k\in[k_i,\bar{k}_i)\\ \hat{u}(k)=\sum\limits_{n=1}^{r}h_{pn}K_{pn}x(k), & k\in[\bar{k}_i,k_{i+1}],\end{cases} \tag{5}$$

where the notation $\bar{k}_i$ $(k_i\leq\bar{k}_i<k_{i+1})$ denotes the starting-operating instant of the matched controller, K_{qn} and K_{pn} are constant matrices. Substituting (5) into (3), we obtain the following closed-loop switched T-S fuzzy system:

$$\begin{cases}x(k+1)=\bar{A}_p(k)x(k), & \forall k\in[k_i,\bar{k}_i)\\ x(k+1)=\hat{A}_p(k)x(k), & \forall k\in[\bar{k}_i,k_{i+1}),\end{cases} \tag{6}$$

where the $\bar{A}_p(k)$ and $\hat{A}_p(k)$ are defined as follows:

$$\begin{aligned}\bar{A}_p(k)&=\sum_{m=1}^{r}\sum_{n=1}^{r}h_{pm}h_{qn}\bar{A}_{pmn}\\&=\sum_{m=1}^{r}\sum_{n=1}^{r}h_{pm}h_{qn}(A_{pm}+B_{pm}K_{qn}),\\ \hat{A}_p(k)&=\sum_{m=1}^{r}\sum_{n=1}^{r}h_{pm}h_{pn}\hat{A}_{pmn}\end{aligned}$$

$$= \sum_{m=1}^{r}\sum_{n=1}^{r} h_{pm}h_{pn}(A_{pm}+B_{pm}K_{pn}).$$

In the end, some definitions and lemmas are presented, which are helpful to obtain the main results of our work.

Definition 1 [1]**:** The switched system (1) with $u(k)\equiv 0$ is globally uniformly asymptotically stable (GUAS) if there exists a class $\mathcal{KL}$ function β such that for all switching signals $\sigma(k)$ and all initial conditions $x(k_0)$, the solutions of (1) satisfy the following inequality

$$\| x(k) \| \le \beta(\| x(k_0) \|, k), \forall k \ge k_0. \tag{7}$$

Definition 2 [10]**:** For any $k_2 > k_1 \ge 0$, $p \in \mathcal{I}$, let $N_{\sigma_p}(k_2,k_1)$ denote the switching numbers of the p^{th} subsystem activated over the interval $[k_1,k_2]$, $T_p(k_2,k_1)$ denote the total running time of the p^{th} subsystem over the interval $[k_1,k_2]$, and N_{0p} denote the mode-dependent chatter bounds. We say that $\sigma(k)$ has a MDADT τ_{ap} if there exist two positive numbers N_{0p} and $\tau_{ap} > 0$ such that the following inequality holds

$$N_{\sigma_p}(k_2,k_1) \le N_{0p} + T_p(k_2,k_1)/\tau_{ap}. \tag{8}$$

Lemma 1 [35]**:** Given two matrices $A \in \mathbb{R}^{m\times n}, B \in \mathbb{R}^{m\times n}$, and matrix $P > 0 \in \mathbb{R}^{m\times m}$, then

$$A^T PB + B^T PA \le A^T PA + B^T PB. \tag{9}$$

3. MAIN RESULTS

3.1. Stability analysis

Lemma 2 [14]**:** Consider the switched nonlinear system (1) with $u(k)\equiv 0$, and let $\varsigma_p > -1$, $\mu_p > 1$ be given constants. Suppose that there exist $\mathcal{C}^1$ functions $V_p(x(k)) : \mathbb{R}^n \to \mathbb{R}$, and class $\mathcal{K}_\infty$ functions κ_1, κ_2, $\forall p \in \mathcal{I}$, such that

$$\kappa_1(\| x(k) \|) \le V_p(x(k)) \le \kappa_2(\| x(k) \|), \tag{10}$$

$$\Delta V_p(x(k)) \le \varsigma_p V_p(x(k)), \tag{11}$$

and $\forall \left(\sigma(k_i) = p, \sigma(k_i^-) = q\right) \in \mathcal{I}\times\mathcal{I}, p \neq q$, such that

$$V_p(x(k_i)) \le \mu_p V_q(x(k_i^-)), \tag{12}$$

then the system (1) with $u(k)\equiv 0$ is GUAS with marginal γ^* for any switching signal satisfying

$$\begin{cases} \tau_{ap} \ge -\dfrac{\ln \mu_p}{\ln(1+\varsigma_p)}, & (-1 < \varsigma_p < 0, p \in \mathcal{S}), \\ \tau_{ap} \ge \tau_{ap}^*, & (\varsigma_p > 0, \forall \tau_{ap}^* > 0, p \in \mathcal{U}), \\ \dfrac{T^-}{T^+} > \dfrac{\ln\gamma_+ - \ln\gamma^*}{\ln\gamma^* - \ln\gamma_-}, & (0 < \gamma^- < \gamma^* < 1), \end{cases} \tag{13}$$

where $T^- = \sum_{p=1}^{g} T_p(k,k_0)$ and $T^+ = \sum_{p=g+1}^{N} T_p(k,k_0)$ correspond to the total running time of stable and unstable subsystems, respectively, $\gamma_- = \max_{p\in\mathcal{S}}(\mu_p^{\frac{1}{\tau_{ap}}}\overline{\varsigma}_p)$, $\gamma_+ = \max_{p\in\mathcal{U}}(\mu_p^{\frac{1}{\tau_{ap}}}\overline{\varsigma}_p)$ and $\overline{\varsigma}_{\sigma(k_i)} = 1 + \varsigma_{\sigma(k_i)}$.

Remark 1: Lemma 2 shows if the ratio of running time of stable subsystems and unstable subsystems is no less than a certain lower-bound, the discrete-time switched system with unstable subsystems can still be stable.

Without control input, the open-loop switched T-S fuzzy system for (6) is listed as follow:

$$x(k+1) = \sum_{m=1}^{r} h_{pm}(k)A_{pm}x(k), \quad \forall k \in [k_i,k_{i+1}). \tag{14}$$

The stability conditions for the system (14) with unstable subsystems can be summarized in the following theorem.

Theorem 1: Consider the switched T-S fuzzy system (14), and let $\varsigma_p > -1$, $\mu_p > 1$ be given constants. If there exist matrices $P_{pm} > 0, \forall(p,q) \in \mathcal{I}\times\mathcal{I}, p \neq q$, such that

$$\begin{bmatrix} (1+\varsigma_p)P_{pm} & A_{pm}^T P_{pl} \\ * & P_{pl} \end{bmatrix} > 0, \quad m,l \in \mathbb{R}_p, \tag{15}$$

$$P_{pl} \le \mu_p P_{qm}, \tag{16}$$

then the system (14) is GUAS with marginal γ^* for any switching signal satisfying (13).

Proof: Choose switching FLFs as follows:

$$V_p(k) = x^T(k)P_p(k)x(k), \tag{17}$$

where $P_p(k) = \sum_{m=1}^{r} h_{pm}P_{pm}$.

$\forall k \in [k_i, k_{i+1})$, along the system (14), we can obtain

$$\begin{aligned} &\Delta V_p(x(k)) - \varsigma_p V_p(x(k)) \\ &= x^T(k)\Big\{\sum_{l=1}^{r}\tilde{h}_{pl}\Big\{\sum_{m=1}^{r}\sum_{n=1}^{r}h_{pm}h_{pn} \\ &\quad [A_{pm}^T P_{pl}A_{pn} - (1+\varsigma_p)P_{pm}]\Big\}\Big\}x(k) \qquad (18)\\ &= x^T(k)\Big\{\sum_{l=1}^{r}\tilde{h}_{pl}\Big\{\sum_{m=1}^{r}h_{pm}^2[A_{pm}^T P_{pl}A_{pm} - (1+\varsigma_p)P_{pm}] \\ &\quad + \sum_{m=1}^{r}\sum_{m<n}^{r}h_{pm}h_{pn}\times[A_{pm}^T P_{pl}A_{pn} + A_{pn}^T P_{pl}A_{pm} \\ &\quad - (1+\varsigma_p)P_{pm} \; -(1+\varsigma_p)P_{pn}]\Big\}\Big\}x(k). \end{aligned}$$

By Lemma 1, from (18), we can further obtain

$$\begin{aligned} &\Delta V_p(x(k)) - \varsigma_p V_p(x(k)) \\ &\le x^T(k)\Big\{\sum_{l=1}^{r}\tilde{h}_{pl}\Big\{\sum_{m=1}^{r}h_{pm}^2[A_{pm}^T P_{pl}A_{pm} - (1+\varsigma_p)P_{pm}] \\ &\quad + \sum_{m=1}^{r}\sum_{m<n}^{r}h_{pm}h_{pn}[A_{pm}^T P_{pl}A_{pm} + A_{pn}^T P_{pl}A_{pn} \qquad (19)\\ &\quad - (1+\varsigma_p)P_{pm} \; -(1+\varsigma_p)P_{pn}]\Big\}\Big\}x(k). \end{aligned}$$

By (15) and Schur complement theorem, (19) implies

$$\Delta V_p(x(k)) \le \varsigma_p V_p(x(k)). \tag{20}$$

From (16), it is not difficult to get

$$V_p(k_i) \leq \mu_p V_q(k_i^-). \tag{21}$$

By Lemma 2, we can conclude that the system (14) is GUAS with marginal γ^* for any switching signal satisfying (13). The proof of Theorem 1 is ended. □

Remark 2: By setting $P_{pm} = P_p$, the switching FLFs will be reduced to the QLFs. So the results in Theorem 1 might have less conservativeness than those results based on the QLFs method.

Remark 3: If the nonlinearity is removed from our work, and setting $P_{pm} = P_p$, the results shown in Theorem 1 of our work will be reduced to Theorem 1 of [14]. Therefore we can conclude that our work can include Theorem 1 of [14] as a special case.

3.2. Asynchronous controller design

In this subsection, let $T(k_i, k_{i+1})$ represent the length of the running time interval of each subsystem. It can be seen from (6) that the $T(k_i, k_{i+1})$ can be divided into two parts: $T_\uparrow(k_i, k_{i+1})$ and $T_\downarrow(k_i, k_{i+1})$. $T_\uparrow(k_i, k_{i+1})$ represents the running time of the subsystem subject to the unmatched controller. $T_\downarrow(k_i, k_{i+1})$ denotes the running time of the subsystem subject to the matched controller. In view of asynchronous switching, the generic scheme of control for switched systems includes two parts: I) the mode activation sensor estimating the active subsystems; II) the subsystem controller for each activated subsystem, which is shown in Fig. 1.

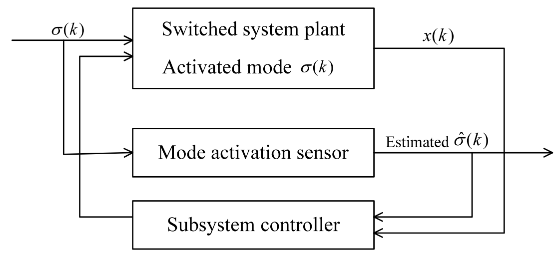

Fig. 1. A generic scheme of controller for switched systems.

Lemma 3: Consider the close-loop switched nonlinear system (1), and let $\mu_p > 1$, $\varsigma_p > -1$, $\varsigma_c > 0$ be given constants. Suppose that there exist $\mathcal{C}^1$ functions $V_p(x(k)) : \mathbb{R}^n \to \mathbb{R}$, and class $\mathcal{K}_\infty$ functions κ_1, κ_2, $\forall \left(\sigma(k_i) = p, \sigma(k_i^-) = q\right) \in \mathcal{I} \times \mathcal{I}, p \neq q$ satisfying

$$\kappa_1(\|x(k)\|) \leq V_p(x(k)) \leq \kappa_2(\|x(k)\|), \tag{22}$$

$$\Delta V_p(x(k)) \leq \begin{cases} \varsigma_c V_p(x(k)), k \in T_\uparrow(k_i, k_{i+1}) \\ \varsigma_p V_p(x(k)), k \in T_\downarrow(k_i, k_{i+1}), \end{cases} \tag{23}$$

and

$$V_p(x(k_i)) \leq \mu_p V_q(x(k_i^-)), \tag{24}$$

then the closed-loop system (1) is GUAS with marginal γ_* for any switching signal satisfying

$$\begin{cases} \tau_{ap} \geq -\frac{\ln \mu_p}{\ln(1+\varsigma_p)}, (-1 < \varsigma_p < 0, p \in \mathcal{S}), \\ \tau_{ap} \geq \tau_{ap}^*, (\varsigma_p > 0, \forall \tau_{ap}^* > 0, p \in \mathcal{U}), \\ T_- > T_+\alpha + T_c\beta, (0 < \gamma_- < \gamma_* < 1), \end{cases} \tag{25}$$

where $T_- = \sum_{p=1}^{g} T_p(k, k_0)$ and $T_+ = \sum_{p=g+1}^{N} T_p(k, k_0)$ denote the total running time of stable and unstable subsystems in the synchronous state, respectively, T_c represents the total time of the system in the asynchronous state, $\gamma_- = \max\limits_{p \in \mathcal{S}}(\mu_p^{\frac{1}{\tau_{ap}}} \overline{\varsigma}_p)$, $\gamma_+ = \max\limits_{p \in \mathcal{U}}(\mu_p^{\frac{1}{\tau_{ap}}} \overline{\varsigma}_p)$, $\alpha = \frac{\ln \gamma_+ - \ln \gamma_*}{\ln \gamma_* - \ln \gamma_-}$, $\beta = \frac{\ln \overline{\varsigma}_c - \ln \gamma_*}{\ln \gamma_* - \ln \gamma_-}$, $\overline{\varsigma}_c = (1+\varsigma_c)$ and $\overline{\varsigma}_p = (1+\varsigma_p)$.

Proof: $\forall k \in [k_i, k_{i+1})$, supposing $k_0 = 0$, by (23) and (24), we can obtain

$$\begin{aligned} &V_{\sigma(k)}(x(k)) \\ &\leq \overline{\varsigma}_{\sigma(k_i)}^{(k-k_i-T_i)} \overline{\varsigma}_c^{T_i} V_{\sigma(k_i)}(x(k_i)) \\ &\leq \mu_{\sigma(k_i)} \overline{\varsigma}_{\sigma(k_i)}^{(k-k_i-T_i)} \overline{\varsigma}_c^{T_i} V_{\sigma(k_{i-1})}(x(k_i)) \\ &\leq \mu_{\sigma(k_i)} \overline{\varsigma}_{\sigma(k_i)}^{(k-k_i-T_i)} \overline{\varsigma}_c^{T_i} \overline{\varsigma}_{\sigma(k_{i-1})}^{(k_i-k_{i-1}-T_{i-1})} \\ &\quad \overline{\varsigma}_c^{T_{i-1}} V_{\sigma(k_{i-1})}(x(k_{i-1})) \\ &\leq \cdots \\ &\leq \prod_{s=1}^{i} \mu_{\sigma(k_s)} \overline{\varsigma}_{\sigma(k_i)}^{(k-k_i-T_i)} \cdots \overline{\varsigma}_{\sigma(k_0)}^{(k_1-k_0-T_0)} \\ &\quad \overline{\varsigma}_c^{(T_i+T_{i-1}+\cdots+T_0)} V_{\sigma(0)}(x(0)). \end{aligned} \tag{26}$$

Since there are g stable subsystems and $N-g$ unstable subsystems. By Definition 2 and from (26), we can obtain

$$\begin{aligned} &V_{\sigma(k)}(x(k)) \\ &\leq \prod_{p=1}^{g} \mu_p^{N_{\sigma_p}(k,0)} \prod_{p=g+1}^{N} \mu_p^{N_{\sigma_p}(k,0)} \prod_{p=1}^{g} \overline{\varsigma}_p^{T_p(k,0)} \\ &\quad \prod_{p=g+1}^{N} \overline{\varsigma}_p^{T_p(k,0)} \overline{\varsigma}_c^{T_c(k,0)} V_{\sigma(0)}(x(0)) \\ &\leq \prod_{p=1}^{g} \mu_p^{\left(N_{0p}+\frac{T_p(k,0)}{\tau_{ap}}\right)} \prod_{p=g+1}^{N} \mu_p^{\left(N_{0p}+\frac{T_p(k,0)}{\tau_{ap}}\right)} \\ &\quad \prod_{p=1}^{g} \overline{\varsigma}_p^{T_p(k,0)} \prod_{p=g+1}^{N} \overline{\varsigma}_p^{T_p(k,0)} \overline{\varsigma}_c^{T_c(k,0)} V_{\sigma(0)}(x(0)) \\ &= K \prod_{p=1}^{g} \left(\mu_p^{\frac{1}{\tau_{ap}}} \overline{\varsigma}_p\right)^{T_p(k,0)} \\ &\quad \prod_{p=g+1}^{N} \left(\mu_p^{\frac{1}{\tau_{ap}}} \overline{\varsigma}_p\right)^{T_p(k,0)} \overline{\varsigma}_c^{T_c(k,0)} V_{\sigma(0)}(x(0)), \end{aligned} \tag{27}$$

where $K = \exp\left\{\sum_{p=1}^{N} N_{0p} \ln \mu_p\right\}$. From (27), for $p \in \mathcal{S}$, if supposing

$$\mu_p^{\frac{1}{\tau_{ap}}} \overline{\varsigma}_p < 1, \tag{28}$$

we can conclude $V_{\sigma(k)}(x(k)) \to 0$ as $k \to \infty$. The inequality (28) is equivalent to

$$\tau_{ap} \geq -\frac{\ln \mu_p}{\ln(1+\varsigma_p)}. \tag{29}$$

By letting $\gamma_- = \max_{p\in\mathcal{S}}(\mu_p^{\frac{1}{\tau_{ap}}}\overline{\varsigma}_p)$, $\gamma_+ = \max_{p\in\mathcal{U}}(\mu_p^{\frac{1}{\tau_{ap}}}\overline{\varsigma}_p)$, $T_- = \sum_{p=1}^{g} T_p(k,k_0)$, $T_+ = \sum_{p=g+1}^{N} T_p(k,k_0)$, (27) implies

$$\begin{aligned} V_{\sigma(k)}(k) &\leq K\gamma_-^{T_-}\gamma_+^{T_+}\overline{\varsigma_c}^{T_c}V_{\sigma(0)}(x(0)) \\ &\leq K\gamma^{*(k-k_0)}V_{\sigma(0)}(x(0)) \\ &= K\gamma^{*(k-k_0)}V_{\sigma(0)}(x(0)). \end{aligned} \tag{30}$$

By Definition 1 and (22), we conclude that $V_{\sigma(k)}(x(k)) \to 0$ as $k \to \infty$ with marginal γ_* as long as the MDADT satisfies (25). The proof of Lemma 3 is completed. □

Remark 4: In Lemma 3, we consider the close-loop switched system with stable and unstable subsystems. The reason for this is that not all subsystems can be stabilized by a controller. For example, in some cases, the controller gain can not be too big, namely, some strong unstable subsystems can not be stabilized by constrained controllers. Therefore, it is reasonable for us to consider the closed-loop switched systems containing unstable subsystems.

Theorem 2: Consider the switched T-S fuzzy system (6), and let $\mu_p > 1$, $\varsigma_p > -1$, $\varsigma_c > 0$ be given constants. If there exist symmetric matrices $P_{pm} > 0$, Q_{pm}, W_p and X_{pm}, $\forall (p,q) \in \mathcal{I}\times\mathcal{I}, p \neq q$, $m,n,l \in \mathbb{R}_p$, such that

$$P_{pm} \leq \mu_p P_{ql}, \tag{31}$$

$$\begin{bmatrix} (1+\varsigma_c)(W_q + W_q^T - Q_{pm}) & \bar{\Omega}_{pmn} \\ * & Q_{pl} \end{bmatrix} > 0, \tag{32}$$

$m \leq n, l \in \mathbb{R}_p$,

$$\begin{bmatrix} (1+\varsigma_p)(W_p + W_p^T - Q_{pm}) & \hat{\Omega}_{pmn} \\ * & Q_{pl} \end{bmatrix} > 0, \tag{33}$$

$m \leq n, l \in \mathbb{R}_p$

where

$$\bar{\Omega}_{pmn} = \frac{W_q^T A_{pm}^T + W_q^T A_{pn}^T + X_{qn}^T B_{pm}^T + X_{qm}^T B_{pn}^T}{2},$$
$$\hat{\Omega}_{pmn} = \frac{W_p^T A_{pm}^T + W_p^T A_{pn}^T + X_{pn}^T B_{pm}^T + X_{pm}^T B_{pn}^T}{2},$$

then the system (6) is GUAS with marginal γ_* for any switching signal satisfying (25), and the controller gains are given by

$$K_{pn} = X_{pn}W_p^{-1}. \tag{34}$$

Proof: I) By the system (6) and switching FLFs (17), considering $k \in T_\uparrow(k_i, k_{i+1})$, we can obtain

$$\begin{aligned} &\Delta V_p(x(k)) - \varsigma_c V_p(x(k)) \\ &= x^T(k)\sum_{l=1}^{r}\tilde{h}_{pl}\Big\{\sum_{m=1}^{r}\sum_{n=1}^{r}\sum_{u=1}^{r}\sum_{v=1}^{r} h_{pm}h_{pn} \\ &\quad h_{pu}h_{pv}\left[\bar{A}_{pmn}^T P_{pl}\bar{A}_{puv} - (1+\varsigma_c)P_{pm}\right]\Big\}x(k) \\ &= \frac{1}{4}x^T(k)\Big\{\sum_{l=1}^{r}\tilde{h}_{pl}\Big\{\sum_{m=1}^{r}\sum_{n=1}^{r}\sum_{u=1}^{r}\sum_{v=1}^{r} h_{pm}h_{pn} \\ &\quad h_{pu}h_{pv}\left[(\bar{A}_{pmn}+\bar{A}_{pnm})^T P_{pl}(\bar{A}_{puv}+\bar{A}_{pvu})\right. \\ &\quad \left.-4(1+\varsigma_c)P_{pm}\right]\Big\}\Big\}x(k) \\ &= \frac{1}{8}x^T(k)\Big\{\sum_{l=1}^{r}\tilde{h}_{pl}\Big\{\sum_{m=1}^{r}\sum_{n=1}^{r}\sum_{u=1}^{r}\sum_{v=1}^{r} h_{pm}h_{pn}h_{pu} \\ &\quad h_{pv}\left[(\bar{A}_{pmn}+\bar{A}_{pnm})^T P_{pl}(\bar{A}_{puv}+\bar{A}_{pvu})\right. \\ &\quad +(\bar{A}_{puv}+\bar{A}_{pvu})^T P_{pl}(\bar{A}_{pmn}+\bar{A}_{pnm}) \\ &\quad \left.-8(1+\varsigma_c)P_{pm}\right]\Big\}\Big\}x(k). \end{aligned} \tag{35}$$

According to Lemma 1, we can further obtain

$$\begin{aligned} &\Delta V_p(x(k)) - \varsigma_c V_p(x(k)) \\ &\leq \frac{1}{8}x^T(k)\Big\{\sum_{l=1}^{r}\tilde{h}_{pl}\Big\{\sum_{m=1}^{r}\sum_{n=1}^{r}\sum_{u=1}^{r}\sum_{v=1}^{r} h_{pm}h_{pn}h_{pu} \\ &\quad h_{pv}\left[(\bar{A}_{pmn}+\bar{A}_{pnm})^T P_{pl}(\bar{A}_{pmn}+\bar{A}_{pnm})\right. \\ &\quad +(\bar{A}_{puv}+\bar{A}_{pvu})^T P_{pl}(\bar{A}_{puv}+\bar{A}_{pvu}) \\ &\quad \left.-8(1+\varsigma_c)P_{pm}\right]\Big\}\Big\}x(k) \\ &= \frac{1}{4}x^T(k)\Big\{\sum_{l=1}^{r}\tilde{h}_{pl}\Big\{\sum_{m=1}^{r}\sum_{n=1}^{r} h_{pm}h_{pn} \\ &\quad \left[(\bar{A}_{pmn}+\bar{A}_{pnm})^T P_{pl}(\bar{A}_{pmn}+\bar{A}_{pnm})\right. \\ &\quad \left.-4(1+\varsigma_c)P_{pm}\right]\Big\}\Big\}x \\ &= x^T(k)\Big\{\sum_{l=1}^{r}\tilde{h}_{pl}\Big\{\sum_{m=1}^{r} h_{pm}^2\left[\bar{A}_{pmm}^T P_{pl}\bar{A}_{pmm}\right. \\ &\quad \left.-(1+\varsigma_c)P_{pm}\right] + 2\sum_{m=1}^{r}\sum_{m<n}^{r} h_{pm}h_{pn} \\ &\quad \left[\left(\frac{\bar{A}_{pmn}+\bar{A}_{pnm}}{2}\right)^T P_{pl}\left(\frac{\bar{A}_{pmn}+\bar{A}_{pnm}}{2}\right)\right. \\ &\quad \left.-(1+\varsigma_c)P_{pm}\right]\Big\}\Big\}x(k). \end{aligned} \tag{36}$$

By Schur complement, from (32), we can obtain

$$\bar{\Omega}_{pmn}Q_{pl}^{-1}\bar{\Omega}_{pmn}^T < (1+\varsigma_c)(W_q + W_q^T - Q_{pm}). \tag{37}$$

The inequality $(Q_{pm} - W_q)^T Q_{pm}^{-1}(Q_{pm} - W_q) \geq 0$ implies that $-W_q^T Q_{pm}^{-1} W_q \leq Q_{pm} - (W_q + W_q^T)$. Then it follows from (37) that

$$\bar{\Omega}_{pmn}Q_{pl}^{-1}\bar{\Omega}_{pmn}^T < (1+\varsigma_c)W_q^T Q_{pm}^{-1} W_q. \tag{38}$$

By pre-multiplying W_q^{-T} and post-multiplying W_q^{-1}, (38) can be transformed into

$$\left(\frac{\bar{A}_{pmn}+\bar{A}_{pnm}}{2}\right)^T Q_{pl}^{-1}\left(\frac{\bar{A}_{pmn}+\bar{A}_{pnm}}{2}\right)$$

$$< (1+\varsigma_c)Q_{pm}^{-1}. \tag{39}$$

Letting $Q_{pm}^{-1} = P_{pm}$, from (36) and (39), we can get

$$\Delta V_p(x(k)) \leq \varsigma_c V_p(x(k)). \tag{40}$$

II) Similarly, for $k \in T_{\downarrow}(k_i, k_{i+1})$, by (33) and Lemma 1, we can obtain

$$\Delta V_p(x(k)) \leq \varsigma_p V_p(x(k)). \tag{41}$$

From (31), it is not difficult to get

$$V_p(k_i) \leq \mu_p V_q(k_i^-). \tag{42}$$

By Lemma 3, we can conclude that the system (6) is GUAS with marginal γ_* for any switching signal satisfying (25). The proof is completed. □

Remark 5: All parameters (μ_p, ς_p and ς_c) of Theorem 2 have their physical meaning. Specifically speaking, the parameter $-1 < \varsigma_p < 0$ denotes the decline rate of the Lyapunov function, which corresponds to the convergence rate of the stable subsystem in the synchronous state. The parameter $\varsigma_p > 0$ represents the increasing rate of the Lyapunov function, which corresponds to the divergence rate of the unstable subsystem in the synchronous state. The parameter $\varsigma_c > 0$ denotes the increasing rate of the Lyapunov function, which corresponds to the divergence rate of the switched system in the asynchronous state. The parameter $\mu_p \geq 1$ denotes the increasing rate bound from the q subsystem to the p subsystem. In practice, these parameters can be designed according to the system performance requirements. By designing the value of these parameters, we can make the systems reach the desired performance.

Remark 6: In our results, in order to reduce the fuzzy approximation error, much more fuzzy rules should be used, which will cause to greater computational complexity.

Remark 7: By setting $P_{pm} = P_p$, the corresponding asynchronous stabilization conditions based on the QLFs method can be obtained. So the results in Theorem 2 might have less conservativeness than those results based on the QLFs method.

4. NUMERICAL EXAMPLE

Example 1 (Stability analysis)**:** Consider the open-loop switched T-S fuzzy system (14) consisting of two subsystems, and each subsystem has two fuzzy rules, where

$$A_{11} = \begin{bmatrix} 0.5 & 0.05 \\ -0.02 & 0.6 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 0.65 & -0.02 \\ -0.03 & 0.5 \end{bmatrix},$$
$$A_{21} = \begin{bmatrix} 1.3 & 0.15 \\ 0.2 & 1.2 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} 1.2 & 0.18 \\ 0.03 & 1.15 \end{bmatrix}.$$

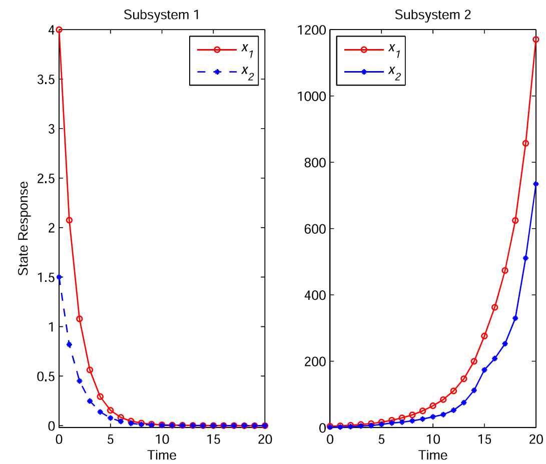

Fig. 2. State response of subsystem 1 and subsystem 2.

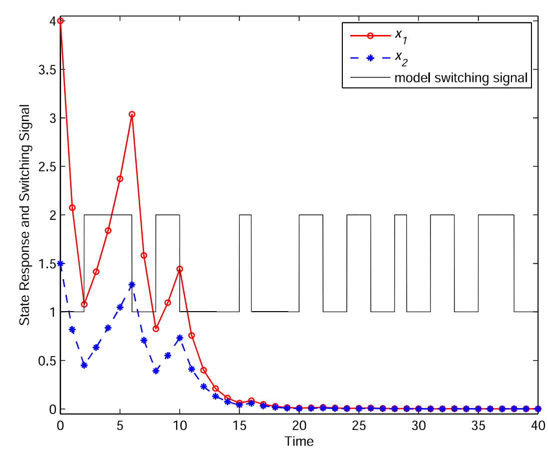

Fig. 3. State response of the open-loop switched T-S fuzzy system with $\tau_{a1} = 2.556$, $\tau_{a2} = 2.125$ and $\gamma^* = 0.9$.

The fuzzy membership functions are taken as

$$h_{11} = 1 - \frac{1}{1+\exp(-2x_1/(0.5-0.2\pi))},$$
$$h_{12} = 1 - h_{11},$$
$$h_{21} = \sin^2(x_2+2), \quad h_{22} = 1 - h_{21}.$$

The premise variables for the subsystem 1 and 2 are x_1 and x_2, respectively. The state response of the subsystem 1 and subsystem 2 is shown in Fig. 2. As shown in Fig. 2 that the subsystem 1 is stable while the subsystem 2 is unstable.

The initial condition is assumed to be $x(0) = [4, 1.5]^T$, and the parameters are given as $\varsigma_1 = -0.57, \varsigma_2 = 1.05, \mu_1 = 1.1, \mu_2 = 1.2$. By (13), we can obtain $\tau_{a1} \geq 0.1129$. The MDADT of the unstable subsystem 2 can be an arbitrary positive integer. Moreover, the ratio of running time of the stable subsystem 1 and the unstable subsystem 2 should be more than a lower bound. Here we choose $\tau_{a1} = 2.556$, $\tau_{a2} = 2.125$ and $\gamma^* = 0.9$. From (13), we can calculate $\gamma^- = 0.4463$, $\gamma^+ = 2.2337$

and $T^-/T^+ > 1.296$. The state response of the system (14) is shown in Fig. 3, where the switching signal with MDADT $\tau_{a1} = 2.556$, $\tau_{a2} = 2.125$ and $T^-/T^+ = 1.3529$ which satisfy the conditions specified by (13).

Example 2: (Asynchronous stabilization) Consider the following discrete-time closed-loop switched T-S fuzzy system with two subsystems, and each subsystem also has two fuzzy rules, where

$$A_{11} = \begin{bmatrix} 2.5 & 0.05 \\ 0.05 & 1.1 \end{bmatrix},\ A_{12} = \begin{bmatrix} 2 & 0 \\ 0 & 1.1 \end{bmatrix},$$
$$A_{21} = \begin{bmatrix} 0.7 & -0.1 \\ 0.21 & 1.2 \end{bmatrix},\ A_{22} = \begin{bmatrix} 0.8 & 0.11 \\ 0.41 & 1.3 \end{bmatrix},$$
$$B_{11} = \begin{bmatrix} 0.9 \\ 0 \end{bmatrix},\ B_{12} = \begin{bmatrix} 0.8 \\ 0 \end{bmatrix},$$
$$B_{21} = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix},\ B_{22} = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix}.$$

The fuzzy membership functions are same as in Example 1. The premise variables for the subsystem 1 and 2 are x_1 and x_2, respectively. The state response of the the subsystem 1 and subsystem 2 is shown in Fig. 4. It is shown in Fig. 4 that both of the two subsystems are unstable.

The parameters are given as $\varsigma_1 = 0.4$, $\varsigma_2 = -0.3$, $\varsigma_c = 0.85$, $\mu_1 = 1.1$, $\mu_2 = 1.2$, $T_c = 2$. By (25), we can calculate $\tau_{a2} \geq 0.5112$. The MDADT of the unstable subsystem 1 can be an arbitrary positive integer. Moreover, the total running time of the stable subsystem 2 and the unstable subsystem 1 in the matched state and the total running time in the asynchronous state should be satisfied the constraint described by (25). In this example, we set $\tau_{a1} = 2.75$, $\tau_{a2} = 18.75$ and $\gamma^* = 0.9$. From (25), we can get $\gamma_- = 0.7068$, $\gamma_+ = 1.4494$, $\alpha = 1.9719$, $\beta = 2.9818$.

Considering the feedback matrices, a set of feasible controller gains can be given as follows:

$$K_{11} = \begin{bmatrix} -1.6651 & -1.4052 \end{bmatrix},$$
$$K_{12} = \begin{bmatrix} -1.6675 & -1.3621 \end{bmatrix},$$
$$K_{21} = \begin{bmatrix} -1.2857 & -2.7501 \end{bmatrix},$$
$$K_{22} = \begin{bmatrix} -2.0125 & -3.2426 \end{bmatrix}.$$

By assuming the initial condition to be $x(0) = [3, 4]^T$, and setting $\tau_{a1} = 2.75$, $\tau_{a2} = 18.75$, $\gamma^* = 0.9$, the state response of the closed-loop switched T-S fuzzy system with stable subsystem and unstable subsystems is shown in Fig. 5, where $T_- = 75$, $T_+ = 11$ and $T_c = 14$, which satisfies $T_- > T_+\alpha + T_c\beta$. As shown in Fig. 5 although there exist unstable subsystems and asynchronous switching which usually makes the system unstable, the closed-loop switched T-S fuzzy system is still stable.

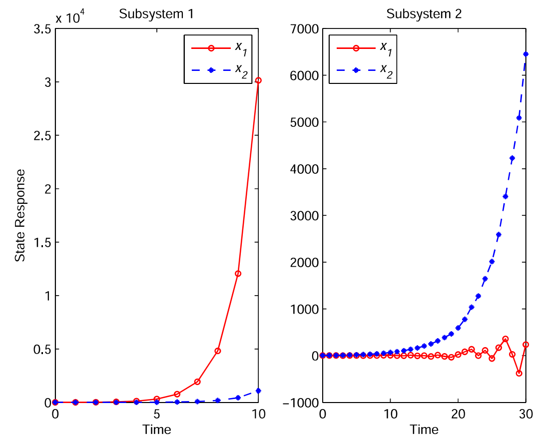

Fig. 4. State response of subsystem 1 and subsystem 2.

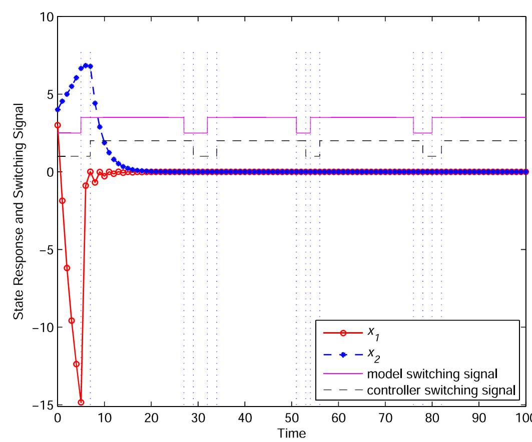

Fig. 5. State response of the closed-loop switched T-S fuzzy system with $\tau_{a1} = 2.75$, $\tau_{a2} = 18.75$, and $\gamma^* = 0.9$.

5. CONCLUSIONS

In this paper, the stability analysis and asynchronous stabilization problems for a class of discrete-time switched nonlinear systems with stable and unstable subsystems are investigated. By using the T-S fuzzy model, the studied switched nonlinear system is modelled into the switched T-S fuzzy system. By using the switching FLFs approach and MDADT technique, stability conditions and asynchronous stabilization conditions are obtained for the studied system in terms of LMIs. A set of state-feedback fuzzy controllers are designed, which can ensure the stability of the closed-loop switched T-S fuzzy system with asynchronous behaviors. Finally, two numerical examples are given to demonstrate the effectiveness of our results. The potential extension of our current work to the stability analysis and controller synthesis for switched systems with time-delays [37] will be a future work. Furthermore, extending our current work to other underlying systems under the network-based environment with time-delays, packet dropouts will be another interesting future work [38, 39]. It has been shown that the piecewise Lyapunov functions method is another way to relax the conservativeness inherent in common Lyapunov function method [18]. The T-S fuzzy affine dynamic model has much improved function approximation capabilities [19, 23]. On the other hand, the dissipativity and l_2-l_∞ are also two important approaches for the control synthesis of dynamical systems [40–43]. Therefore, it would be an interesting and challenge work to investigate the dissipativity or l_2-l_∞ control problem for the T-S fuzzy affine systems with unstable subsystems via the piecewise Lyapunov functions method.

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