International Journal of Control, Automation and Systems 15(4) (2017) 1592-1599
http://dx.doi.org/10.1007/s12555-016-0297-y
ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Exponential H_∞ Control for Singular Systems with Time-varying Delay

Jiemei Zhao* and Zhonghui Hu

Abstract: This paper studies the exponential admissibility and H_∞ control problems for a class of singular systems with time-varying delay in state. Firstly, an exponential admissibility criterion is obtained based on linear matrix inequalities (LMIs). It is worth mentioning that the derivative of the time-varying delay does not need to be smaller than one. Based on the proposed condition, a new delay-dependent H_∞ controller is also given, which guarantees the admissibility and the H_∞ performance γ. Numerical examples are given to illustrate the effectiveness of the proposed method.

Keywords: Exponential admissibility, H_∞ control, singular systems, time-varying delay.

1. INTRODUCTION

Time-delay is frequently encountered in various systems, such as economical, telecommunications, biology systems, and other areas. Generally, time-delay is regarded as the main source of instability and poor performance in a system [1–6]. Recently, much attention has been paid to the problem of H_∞ control for uncertain time-delay systems [7, 8].

Singular systems are often referred to as implicit systems, which have a wide range of applications in many practical systems, such as electric circuit systems, chemical process, economy systems, and so on [9, 10]. However, the study of such systems is much more complicated than that for standard state-space systems for two reasons. First, the existence and uniqueness of a solution for singular system are not always guaranteed. Second, the system can also have undesired impulsive behavior. Therefore, singular systems have received a lot of attention in the last few decades, especially in the field of stability and stabilization [11–18], filtering [19–21], passivity [22–24] and H_∞ control [25–29]. Among them, exponential stability is a valuable research topic because it has faster convergence rate than asymptotic stability generally. For example, in [11], the global exponential stability problem of singular systems with multiple time-varying delays was addressed. The exponential admissibility of switched singular systems with time-varying delays was investigated in [12, 14]. In [15], exponential stability for singular systems with interval time-varying delay was studied. In [17], the exponential stabilization problem for singular systems with time-varying delays and nonlinear perturbations were discussed by using sliding mode control method. Very recently, in [30, 31], some new delay-dependent criteria for the exponential stability of singular systems with mixed interval time-varying delays were proposed. However, these results in the above literatures [11, 12, 14, 15, 17, 20, 21, 24, 30, 31] all assumed the derivatives of time-varying delays to be smaller than one, which may restrict the applications of those results. It brings some conservative in the process of system stability analysis. Therefore, the problem of exponential stability analysis for singular systems with time-varying delays without restrictions on derivative of delays still remains open, which motivates the present study.

In this paper, exponentially admissible and H_∞ controller designed problems are considered for time-varying delay singular system. The main contributions of this paper as follows:

1) A exponential admissibility criterion is proposed.

2) Based on the result proposed in 1), an H_∞ controller is proposed to ensure the time-varying delay singular system is exponentially admissible with a disturbance attenuation level γ.

3) The H_∞ controller will reduce the effect of the noise or disturbance with bounded energy.

4) The proposed algorithm is generally since the derivative of the time-varying delay does not need to be smaller than one.

Notations: Throughout this paper, $\mathcal{R}^n$ denotes the n-dimensional Euclidean space, $\mathcal{R}^{m\times n}$ is the set of all $m\times n$ real matrices; The notation $P > 0$ ($P \geq 0$) indicates that

Manuscript received May 15, 2016; revised September 5, 2016 and October 10, 2016; accepted November 21, 2016. Recommended by Associate Editor Sing Kiong Nguang under the direction of Editor Myo Taeg Lim. This work was supported by the Natural Science Foundation of Hubei Province of China(2016CFB273), the Research and Innovation Initiatives of WHPU (2015d9).

Jiemei Zhao is with the School of Mathematics and Computer Science, Wuhan Polytechnic University, Wuhan 430023, China (e-mail: jiemeizhao@163.com). Zhonghui Hu is with the 709th Research Institute of China Shipbuilding Industry Corporation, Wuhan 430074, China (e-mail: huzhonghui@gmail.com).
* Corresponding author.

P is a real symmetric and positive (semi-) definite matrix. $\|x\|$ refers to the Euclidean norm of the vector x; The notation $\|x\|_t = \sup_{t-h\le s\le t} x(s)$, where $x(t)$ be a continuous function. I is the identity matrix with appropriate dimension. Matrices, if not explicitly stated, are assumed to have compatible dimensions. The symmetric terms in a symmetric matrix are denoted by $*$. $\lambda_M(\cdot)$ and $\lambda_m(\cdot)$ denote the maximum and minimum eigenvalue of the responding matrix, respectively.

2. PROBLEM FORMULATION

Consider the following uncertain time-varying delay singular system:

$$\begin{cases} E\dot{x}(t) = Ax(t) + A_d x(t-d(t)) + Bu(t) \\ \qquad + B_\omega \omega(t), \\ z(t) = Cx(t), \\ x(t) = \varphi(t), t \in [-\tau, 0], \end{cases} \tag{1}$$

where $x(t)$ is the state vector, $u(t)$ is the input vector, $z(t)$ is the controlled output vector, $\omega(t)$ is the disturbance input which belongs to $L_2[0,\infty)$, $d(t)$ represents time delay in the state satisfying

$$0 \le d(t) \le \tau, \ \dot{d}(t) \le \mu, \tag{2}$$

$\varphi(t)$ is a compatible vector valued initial function, E, A, A_d, B, B_ω and C are known real constant matrices of appropriate dimensions, $\mathrm{rank}(E) = r < n$.

The nominal unforced singular time-varying delay system of (1) can be written as

$$E\dot{x}(t) = Ax(t) + A_d x(t-d(t)). \tag{3}$$

According to system (3), the following definition is introduced.

Definition 1 [33]**:** 1) the system (3) is said to be regular if $\det(sE-A)$ is not identically zero.

2) the system (3) is said to be impulse free if it is regular and $\deg(\det(sE-A)) = \mathrm{rank}(E)$.

3) the system (3) is said to be exponentially stable if there exist $\lambda(\alpha) \ge 1$ and $\alpha > 0$ such that $\|x(t)\| \le \lambda(\alpha)\|x(t_0)\|e^{-\alpha t}$, for all $t \in [t_0, +\infty)$.

4) the system (3) is said to be exponentially admissible if it is regular, impulse-free and exponentially stable.

For the system (1) and a given positive scalar γ, the H_∞ performance measure is

$$J = \int_0^\infty (z^T(t)z(t) - \gamma\omega^T(t)\omega(t))\mathrm{dt}. \tag{4}$$

Definition 2 [5]**:** The system (1) is said to be exponentially admissible with the H_∞ performance $\gamma > 0$ if it is exponentially admissible when $\omega(t) \equiv 0$, and satisfies $J < 0$ for all $t \ge 0$ under the zero initial condition.

Lemma 1 [17]**:** Let $f(t) : [t_0 - h, \infty) \to [0, \infty)$ be a non-negative continuous function and satisfies

$$f(t) \le k_1 e^{-\alpha(t-t_0)} + k_2\|f\|_t,$$

where $k_1 > 0, 0 < k_2 < 1$, then there exists a scalar $k > 0 (k \le \alpha)$ such that

$$f(t) \le \left(\ \|f\|_{t_0} + \tfrac{k_1}{1-k_2 e^{kh}} \ \right) e^{-\alpha(t-t_0)}.$$

3. EXPONENTIAL ADMISSIBILITY CRITERION

In the following, the exponential admissibility criterion for the system (3) will be given.

Since $\mathrm{rank}(E) = r < n$, there exist two nonsingular matrices F and H such that

$$\begin{aligned} \bar{E} &= FEH = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \\ \bar{A} &= FAH = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \\ \bar{A}_d &= FA_dH = \begin{bmatrix} A_{d,11} & A_{d,12} \\ A_{d,21} & A_{d,22} \end{bmatrix}. \end{aligned} \tag{5}$$

Now, let $y(t) = H^{-1}x(t)$, then the system

$$\bar{E}\dot{y}(t) = \bar{A}y(t) + \bar{A}_d y(t-d(t)). \tag{6}$$

is equivalent to the system (3).

Theorem 1: For given scalars $\tau > 0$ and $\alpha > 0$, the time-varying delay singular system (3) is exponentially admissible, if there exist matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $Z_1 > 0$, $Z_2 > 0$, matrices G_1, G_2, G_3, U_1, U_2, V and S such that

$$\|A_{22}^{-1}A_{d,22}\| < 1, \tag{7}$$

and

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & A^T U_2 \\ * & \Omega_{22} & 0 & \Omega_{24} & \Omega_{25} \\ * & * & \Omega_{33} & 0 & 0 \\ * & * & * & \Omega_{44} & 0 \\ * & * & * & * & \Omega_{55} \end{bmatrix} < 0, \tag{8}$$

where

$$\begin{aligned} \Omega_{11} &= 2\alpha E^T PE + (E^T P + SR^T)A \\ &\quad + A^T(PE + RS^T) + E^T G_1 + G_1^T E \\ &\quad + e^{2\alpha\tau}(Q_1 + Q_2) - e^{-2\alpha\tau}E^T Z_1 E, \\ \Omega_{12} &= (E^T P + SR^T)A_d + E^T G_2 - G_1^T E + A^T U_1, \\ \Omega_{13} &= e^{-2\alpha\tau}E^T Z_1 E, \\ \Omega_{14} &= E^T G_3 - G_1^T, \\ \Omega_{22} &= -(1-\mu)Q_2 - E^T G_2 - G_2^T E + A_d^T U_1 + U_1^T A_d, \end{aligned}$$

$$\Omega_{24} = -E^T G_3 - G_2^T,$$
$$\Omega_{25} = -U_1^T + A_d^T U_2,$$
$$\Omega_{33} = -Q_1 - e^{-2\alpha\tau} E^T Z_1 E,$$
$$\Omega_{44} = -G_3 - G_3^T - \tau Z_2,$$
$$\Omega_{55} = \tau^2(Z_1 + Z_2) + RV + V^T R^T - U_2 - U_2^T.$$

and R is any matrix with full column and satisfies $E^T R = 0$.

Proof: From (5), then R parameterized as $R = F^T \begin{bmatrix} 0 \\ \Phi \end{bmatrix}$, $\Phi \in \mathcal{R}^{(n-r)\times(n-r)}$ is any nonsingular matrix. Let

$$\bar{P} = F^{-T} P F^{-1} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix},$$
$$\bar{Z}_1 = F^{-T} Z_1 F^{-1} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix},$$
$$\bar{G}_1 = F^{-T} G_1 H = \begin{bmatrix} G_{1,11} & G_{1,12} \\ G_{1,21} & G_{1,22} \end{bmatrix},$$
$$\bar{S} = H^T S = \begin{bmatrix} S_{11} \\ S_{21} \end{bmatrix}.$$

It follows from (8) that $\Omega_{11} < 0$ and $Q_i > 0$, $i = 1, 2$, we can get

$$\begin{aligned}\Theta =&(E^T P + SR^T)A + A^T(PE + RS^T) + 2\alpha E^T PE \\ &+ E^T G_1 + G_1^T E - e^{-2\alpha\tau} E^T Z_1 E < 0.\end{aligned}$$

Pre- and post-multiplying Θ by H^T and H, respectively, yields

$$\begin{bmatrix} \star & \star \\ \star & A_{22}^T \Phi S_{21}^T + S_{21} \Phi^T A_{22} \end{bmatrix} < 0,$$

where $\star$ represent irrelevant to the result of the following discussion.

Obviously,

$$A_{22}^T \Phi S_{21}^T + S_{21} \Phi^T A_{22} < 0, \tag{9}$$

and thus A_{22} is nonsingular. Otherwise, supposing A_{22} is singular, there must exist a non-zero vector $\zeta \in \mathcal{R}^{n-r}$ which ensures $A_{22}\zeta = 0$. And we can conclude that $\zeta^T(A_{22}^T \Phi S_{21}^T + S_{21}\Phi^T A_{22})\zeta = 0$, and this contradicts (9). So A_{22} is nonsingular. That is, the singular system (3) is regular and impulse free.

Consider the following Lyapunov function

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t), \tag{10}$$

where

$$V_1(t) = e^{2\alpha t} x^T(t) E^T P E x(t),$$
$$V_2(t) = e^{2\alpha\tau} \int_{t-\tau}^{t} e^{2\alpha s} x^T(s) Q_1 x(s) \mathrm{d}s,$$
$$V_3(t) = e^{2\alpha\tau} \int_{t-d(t)}^{t} e^{2\alpha s} x^T(s) Q_2 x(s) \mathrm{d}s,$$
$$V_4(t) = \tau \int_{-\tau}^{0} \int_{t+\theta}^{t} e^{2\alpha s} \dot{x}^T(s) E^T Z_1 E \dot{x}(s) \mathrm{d}s\mathrm{d}\theta.$$

We calculate the derivation of $V(t)$ along the solution of system (3), then

$$\begin{aligned}\dot{V}_1(t) =&2e^{2\alpha t} x^T(t) E^T P[\alpha E x(t) + E\dot{x}(t)] \\ =&2e^{2\alpha t} x^T(t) E^T P \\ &\times [\alpha E x(t) + A x(t) + A_d x(t - d(t))],\end{aligned} \tag{11}$$

$$\begin{aligned}\dot{V}_2(t) =&e^{2\alpha t} x^T(t) e^{2\alpha\tau} Q_1 x(t) \\ &- e^{2\alpha t} x^T(t-\tau) Q_1 x(t-\tau),\end{aligned} \tag{12}$$

$$\begin{aligned}\dot{V}_3(t) \le&e^{2\alpha t} x^T(t)(e^{2\alpha\tau} Q_2) x(t) \\ &- (1-\mu) e^{2\alpha t} x^T(t-d(t)) Q_2 x(t-d(t)),\end{aligned} \tag{13}$$

$$\begin{aligned}\dot{V}_4(t) =&e^{2\alpha t} \dot{x}^T(t) E^T(\tau^2 Z_1) E\dot{x}(t) \\ &- \tau \int_{t-\tau}^{t} e^{2\alpha s} \dot{x}^T(s) E^T Z_1 E \dot{x}(s) \\ \le&e^{2\alpha t} \dot{x}^T(t) E^T(\tau^2 Z_1) E\dot{x}(t) \\ &- \tau e^{2\alpha t} \int_{t-\tau}^{t} e^{-2\alpha\tau} \dot{x}^T(s) E^T Z_1 E \dot{x}(s) \mathrm{d}s.\end{aligned} \tag{14}$$

In fact

$$\begin{aligned}&- \tau \int_{t-\tau}^{t} \dot{x}^T(s) E^T Z_1 E \dot{x}(s) \mathrm{d}s \\ &\le -[\int_{t-\tau}^{t} \dot{x}^T(s) \mathrm{d}s] E^T Z_1 E [\int_{t-\tau}^{t} \dot{x}(s) \mathrm{d}s] \\ &\le \begin{bmatrix} x(t) \\ x(t-\tau) \end{bmatrix}^T \begin{bmatrix} -E^T Z_1 E & E^T Z_1 E \\ E^T Z_1 E & -E^T Z_1 E \end{bmatrix} \\ &\quad \times \begin{bmatrix} x(t) \\ x(t-\tau) \end{bmatrix}.\end{aligned} \tag{15}$$

From the Newton-Leibniz formulation $x(t) - x(t - d(t)) - \int_{t-d(t)}^{t} \dot{x}(s)\mathrm{d}s = 0$, the following equation is true for the matrices G_1, G_2 and G_3 of appropriate dimensions

$$\begin{aligned}&2e^{2\alpha t}[Ex(t) - Ex(t-d(t)) - \int_{t-d(t)}^{t} E\dot{x}(s)\mathrm{d}s]^T \\ &\quad \times [G_1 x(t) + G_2 x(t-d(t)) + G_3 \int_{t-d(t)}^{t} E\dot{x}(s)\mathrm{d}s] \\ &= 0.\end{aligned} \tag{16}$$

In addition, for matrices $Z_2 > 0$ and U_1, U_2, we have

$$\begin{aligned}0 \le&e^{2\alpha t} \tau^2 [\dot{x}^T(t) E^T Z_2 E \dot{x}(t) \\ &- \int_{t-d(t)}^{t} \dot{x}(s) E^T Z_2 E \dot{x}(s) \mathrm{d}s] \\ \le&\tau^2 e^{2\alpha t} \dot{x}^T(t) E^T Z_2 E \dot{x}(t)\end{aligned}$$

$$-\tau e^{2\alpha t}[\int_{t-d(t)}^{t}\dot{x}^T(s)E^T\mathrm{d}s]Z_2[\int_{t-d(t)}^{t}E\dot{x}(s)\mathrm{d}s], \tag{17}$$

and

$$2e^{2\alpha t}[-E\dot{x}(t)+Ax(t)+A_dx(t-d(t))]^T \times[U_1x(t-d(t))+U_2E\dot{x}(t)]=0. \tag{18}$$

Noting $E^TR=0$, we can deduce

$$2e^{2\alpha t}\dot{x}^T(t)E^TR[S^Tx(t)+VE\dot{x}^T(t)]=0, \tag{19}$$

where V and S are any matrices of appropriate dimensions.

It follows from (11)-(19) that

$$\dot{V}(t)\le\xi^T(t)\Omega\xi(t)$$

where

$$\xi(t)=[x^T(t)\ x^T(t-d(t))\ x^T(t-\tau)\ \int_{t-d(t)}^{t}\dot{x}^T(s)E^T\mathrm{d}s\ \ \dot{x}^T(t)E^T]^T.$$

It follows from $\dot{V}(t)\le 0$ that $V(t)\le V(t_0)$.

From (10) and

$$\begin{aligned}&\dot{x}^T(s)E^TE\dot{x}(s)\\ &\le 3[x^T(t)A^TAx(t)+x^T(t-d(t))A_d^TA_dx(t-d(t))]\\ &\le 3[\lambda_M(A^TA)+\lambda_M(A_d^TA_d)]\|x\|_{t_0}^2,\end{aligned}$$

we can easily get

$$V(t_0)\le\eta e^{2\alpha t_0}\|x\|_{t_0}^2,$$

where $\eta=\lambda_M(E^TPE)+\tau e^{2k\tau}(\lambda_M(Q_1)+\lambda_M(Q_2))+3\tau^3\lambda_M(Z_1)[\lambda_M(A^TA)+\lambda_M(A_d^TA_d)]$.

The system (6) equivalent to the following one:

$$\begin{cases}\dot{y}_1(t)=A_{11}y_1(t)+A_{12}y_2(t)\\ \qquad +A_{d,11}y_1(t-d(t))+A_{d,12}y_2(t-d(t)),\\ 0=A_{21}y_1(t)+A_{22}y_2(t)\\ \qquad +A_{d,21}y_1(t-d(t))+A_{d,22}y_2(t-d(t)).\end{cases} \tag{20}$$

Notice that $x^T(t)E^TPEx(t)=y_1^T(t)P_{11}y_1(t)$, hence, $e^{2\alpha t}\lambda_m(P_{11})\|y_1(t)\|^2\le V(t)\le V(t_0)\le\eta e^{2\alpha t_0}\|x(t_0)\|^2$, that is,

$$\|y_1(t)\|\le\sqrt{\frac{\eta}{\lambda_m(P_{11})}}\|x\|_{t_0}e^{-\alpha(t-t_0)}. \tag{21}$$

Using (21), we have

$$\begin{aligned}&\|A_{21}y_1(t)+A_{d,21}y_1(t-d(t))\|\\ &\le\sqrt{\frac{\eta}{\lambda_m(P_{11})}}(\|A_{21}\|+\|A_{d,21}\|e^{\alpha\tau})\|x\|_{t_0}e^{-\alpha(t-t_0)}\\ &:=\bar{\eta}e^{-\alpha(t-t_0)}.\end{aligned}$$

Using the second equation of (20), we have

$$\begin{aligned}y_2(t)=&-A_{22}^{-1}A_{d,22}y_2(t-d(t))\\ &-A_{22}^{-1}[A_{21}y_1(t)+A_{d,21}y_1(t)].\end{aligned} \tag{22}$$

So

$$\begin{aligned}\|y_2(t)\|=&\|A_{22}^{-1}A_{d,22}\|\|y_2(t-d(t))\|\\ &+\|A_{22}^{-1}\|\|A_{21}y_1(t)+A_{d,21}y_1(t)\|\\ \le&\|A_{22}^{-1}A_{d,22}\|\|y_2\|_t+\|A_{22}^{-1}\|\bar{\eta}e^{-\alpha(t-t_0)},\end{aligned}$$

where $\|y_2\|_t=\sup_{-\tau\le\theta\le 0}\|y_2(t+\theta)\|$.

Applying Lemma 1 and $\|A_{22}^{-1}A_{d,22}\|<1$, then there exists a scalar $k>0$ such that

$$\|y_2(t)\|=\left[\ \|y_2\|_{t_0}+\frac{\|A_{22}^{-1}\|\bar{\eta}}{1-\|A_{22}^{-1}A_{d,22}\|e^{k\tau}}\ \right]e^{-k(t-t_0)}. \tag{23}$$

From (21) and (23), we get

$$\|x(t)\|=\|Hy(t)\|\le\|H\|(\|y_1(t)\|+\|y_2(t)\|)\le\rho e^{-kt}\|\varphi\|_\tau,$$

where

$$\rho=\frac{\|H\|\left[\eta\|x\|_{t_0}+\|y_2\|_{t_0}+\frac{\|A_{22}^{-1}\|\bar{\eta}}{1-\|A_{22}^{-1}A_{d,22}\|e^{k\tau}}\right]e^{\alpha t_0}}{\|\varphi\|_\tau},$$

$$\|\varphi\|_\tau=\sup_{-\tau\le s\le 0}\varphi(s),$$

which shows that the system (3) is exponentially stable and has the exponential decay rate k from Definition 1 (3). From Definition 1 (4), the system (3) is exponentially admissible. □

Remark 1: In [11, 12, 14, 15, 17, 30, 31], the derivatives of time delays required to be smaller than one. In this paper, we remove this limiting condition. Therefore, Theorem 1 has less conservative.

Remark 2: The key techniques in Theorem 1 as follows: 1) In order to relax the limit of μ, some important terms are added in Ω_{22} by introducing free-weighting matrix method. 2) The singular system (3) is restricted system equivalent to system (20) by the regularity and non-impulsiveness characteristics of system (3). Then Lemma 3 is employed to get the bound of $\|y_2(t)\|$. This is the key to prove the exponential admissibility.

4. H_∞ CONTROL

Substituting the control law $u(t)=Kx(t)$ to the system (1), then

$$\begin{cases}E\dot{x}(t)=(A+BK)x(t)+A_dx(t-d(t))\\ \qquad +B_\omega\omega(t),\\ z(t)=Cx(t),\\ x(t)=\varphi(t),\ \ t\in[-\tau,0].\end{cases} \tag{24}$$

Next, we will design a controller $u(t) = Kx(t)$ to guarantee the closed-loop system is exponentially admissible with a H_∞ performance γ.

Theorem 2: For given scalars $\tau > 0$, $\alpha > 0$ and $\gamma > 0$, the time-varying delay singular system (1) controlled by $u(t) = WP^{-1}x(t)$ is exponentially admissible with a disturbance attenuation level γ, if there exist matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $Z_1 > 0$, $Z_2 > 0$, matrices G_1, G_2, G_3, U_1, U_2, V, W and S such that the following LMIs hold:

$$\bar{\Omega} = \begin{bmatrix} \bar{\Omega}_{11} & \bar{\Omega}_{12} & e^{-2\alpha\tau}EZ_1E^T & EG_3 - G_1^T & AU_2 & (EP+SR^T)C^T & B_\omega \\ * & \bar{\Omega}_{22} & 0 & -EG_3 - G_2^T & \bar{\Omega}_{25} & U_1^TC^T & 0 \\ * & * & \bar{\Omega}_{33} & 0 & 0 & 0 & 0 \\ * & * & * & \Omega_{44} & 0 & U_2^TC^T & 0 \\ * & * & * & * & \Omega_{55} & 0 & 0 \\ * & * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0, \quad (25)$$

$$\|\bar{A}_{22}^{-1}A_{d,22}\| < 1, \quad (26)$$

where $\bar{A}_{22}$ is a block of $F(A+BK)H = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix}$, F, H and $A_{d,22}$ are defined as in (5), R is any matrix with full column and satisfies $ER = 0$, Ω_{44} and Ω_{55} are defined as in (8),

$$\begin{aligned} \bar{\Omega}_{11} &= 2\alpha EPE^T + (EP+SR^T)A^T + A(PE^T+RS^T) \\ &\quad + EW^TB^T + BWE^T + EG_1 + G_1^TE^T \\ &\quad + e^{2\alpha\tau}(Q_1+Q_2) - e^{-2\alpha\tau}EZ_1E^T, \\ \bar{\Omega}_{12} &= (EP+SR^T)A_d^T + EG_2 - G_1^TE^T + AU_1 + A_dU_1, \\ \bar{\Omega}_{22} &= -(1-\mu)Q_2 - EG_2 - G_2^TE^T, \\ \bar{\Omega}_{25} &= -U_1^T + A_dU_2, \\ \bar{\Omega}_{33} &= -Q_1 - e^{-2\alpha\tau}EZ_1E^T. \end{aligned}$$

Proof: For the system (1) with $u(t) = 0$, choose the Lyapunov function (10), we can get the system (1) is regular, impulse free and exponentially stable by Theorem 1.

Under zero initial condition (i.e., $V(0) = 0$) and $V(\infty) \geq 0$ of system (1), Then

$$\begin{aligned} J &= \int_0^\infty \left(z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) \right) \mathrm{d}t \\ &\quad - e^{-2\alpha t}\dot{V}(\infty) \\ &\leq \int_0^\infty \bar{\xi}^T(t)\Upsilon\bar{\xi}(t)\mathrm{d}t, \end{aligned}$$

where

$$\bar{\Omega} = \begin{bmatrix} \Omega_{11}+C^TC & \Omega_{12} & e^{-2\alpha\tau}E^TZ_1E & E^TG_3 - G_1^T & A^TU_2 & (E^TP+SR^T)B_\omega \\ * & \Omega_{22} & 0 & -E^TG_3 - G_2^T & \Omega_{25} & U_1^TB_\omega \\ * & * & \Omega_{33} & 0 & 0 & 0 \\ * & * & * & \Omega_{44} & 0 & 0 \\ * & * & * & * & \Omega_{55} & U_2^TB_\omega \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0, \quad (27)$$

and

$$\bar{\xi}(t) = [x^T(t) \quad x^T(t-\tau) \quad x^T(t-d(t)) \quad \int_{t-d(t)}^t \dot{x}^T(s)E^T\mathrm{d}s \quad \dot{x}^T(t)E^T \quad \omega^T(t)]^T.$$

Since $\det(sE-(A+BK)) = \det(sE^T-(A+BK)^T)$, the pair $(E, A+BK)$ is regular and impulse free if and only if the pair $(E^T, (A+BK)^T)$ is regular and impulse free. Moreover, since the solution of $\det(sE-(A+BK)-e^{-ds}A_d) = 0$ is the same as that of $\det(sE^T-(A+BK)^T-e^{-ds}A_d^T) = 0$ and

$$\begin{aligned} &\|G(s)\|_\infty \\ &= \sup_{\omega\in[0,+\infty)} \lambda_M\{C(j\omega E-(A+BK)-d^{-dj\omega}A_d)^{-1}B_\omega)\} \end{aligned}$$

is equal to

$$\begin{aligned} &\|H(s)\|_\infty \\ &= \sup_{\omega\in[0,+\infty)} \lambda_M\{B_\omega^T(j\omega E^T-(A+BK)^T-d^{-dj\omega}A_d^T)^{-1}C^T\} \end{aligned}$$

as long as the regularity, absence of impulses, and stability with H_∞ performance are our only concern, the system (24) is equivalent to the system

$$\begin{cases} E^T\dot{x}(t) = (A+BK)^Tx(t) \\ \qquad\qquad + A_d^Tx(t-d(t)) + C^T\omega(t), \\ z(t) = B_\omega^Tx(t). \end{cases} \quad (28)$$

Hence, replacing $E, A+BK, A_d, C$ and B_ω in (27) by $E^T, (A+BK)^T, A_d^T, B_\omega^T$ and C^T, respectively, and setting $W = KP$ yields (25). □

Remark 3: Theorem 2 proposes a new version of the bound real lemma (BRL) for the singular time-delay systems (1) with $u(t) = 0$.

5. NUMERICAL EXAMPLES

In this section, three numerical examples will be presented to show the validity of the main results derived above.

Example 1: Consider singular time-delay system (3) with [20]

$$E = \begin{bmatrix} 9 & 3 \\ 6 & 2 \end{bmatrix}, A = \begin{bmatrix} -13.1 & -13.7 \\ -15.4 & -23.8 \end{bmatrix},$$
$$A_d = \begin{bmatrix} -18.6 & -10.4 \\ -25.2 & -16.8 \end{bmatrix}.$$

In this example, we choose $\mu = 0.5$. Table 1 lists the allowable upper bound of the time-delay. It is seen from Table 1 that the stability criterion proposed here gives less conservative results than those in [11,20,21,24,32]. Table 2 gives the allowable upper bound of the time-delay with different α. Table 2 shows that when α increased, the upper bound of τ decreased.

Example 2: Consider the time-varying delay singular system (3) with the following parameters

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A = \begin{bmatrix} -0.6 & 0.5 \\ 0.5 & -1 \end{bmatrix}, A_d = \begin{bmatrix} 0.7 & 0 \\ -3 & 0.2 \end{bmatrix},$$

$d(t) = 1.3 + 1.2\cos(t), \mu = 1.2$. In this case, the methods in papers [11, 12, 14, 15, 17, 30, 31] are invalid, i.e., the methods in these literatures are not feasible to this example. Assume the initial state is $x(t) = [0.2 \; -0.75]^T$. The trajectories of the state responses of system are given in Fig. 1. From Fig. 1, we find that the corresponding state responses converge to zeros. Table 3 gives the allowable upper bounds for various α. It can be seen that when α increase, the maximum allowable $d(t)$ descend. Let $\tau = 2, \alpha = 0.3$. Since $|-1 \times 0.2| = 0.2 < 1$, the condition (7) is satisfied. By using the MATLAB LMI Control Toolbox, we can find a solution to the LMI (8) in Theorem 1 as follows:

$$P = \begin{bmatrix} 27.9873 & 0 \\ 0 & 85.6665 \end{bmatrix},$$
$$Q_1 = \begin{bmatrix} 0.6208 & -0.2068 \\ -0.2068 & 0.3713 \end{bmatrix},$$
$$Q_2 = \begin{bmatrix} 0.3211 & -0.0409 \\ -0.0409 & 0.0774 \end{bmatrix},$$
$$Z_1 = \begin{bmatrix} 0.7475 & 0 \\ 0 & 85.6665 \end{bmatrix},$$
$$Z_2 = 10^3 \times \begin{bmatrix} 0.0109 & -0.4868 \\ -0.4868 & 3.6510 \end{bmatrix}.$$

Example 3: Consider the time-varying delay singular system (1) with the following parameters

$$E = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \; A = \begin{bmatrix} -0.3 & 0.1 \\ 0.5 & -0.8 \end{bmatrix},$$

Table 1. Delay bounds of different cases.

Methods	Upper bound τ	Upper bound τ
[11]($\alpha \to 0$), [20]	2.1121 ($d_1 = 1.4$)	2.5852 ($d_1 = 2.2$)
[21]	2.2314 ($d_1 = 1.4$)	2.6777 ($d_1 = 2.2$)
[24]	2.3372 ($d_1 = 1.4$)	2.7494 ($d_1 = 2.2$)
[32] ($m = 9$)	2.3360 ($d_1 = 1.4$)	2.7007 ($d_1 = 2.2$)
Theorem 1	4.2550 ($\alpha = 0.15$)	2.7494 ($\alpha = 0.231$)

Table 2. Delay bounds of different cases.

Methods	$\alpha = 0.15$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.6$
Theorem 1	4.2550	3.1890	2.1258	1.0630

Table 3. Allowable upper bounds for various α.

Methods	$\mu = 1.2$
[11, 12, 14, 15, 17, 30, 31]	-
Theorem 1 ($\alpha = 0.3$)	3.2856
Theorem 1 ($\alpha = 0.6$)	2.9750
Theorem 1 ($\alpha = 0.9$)	2.9732

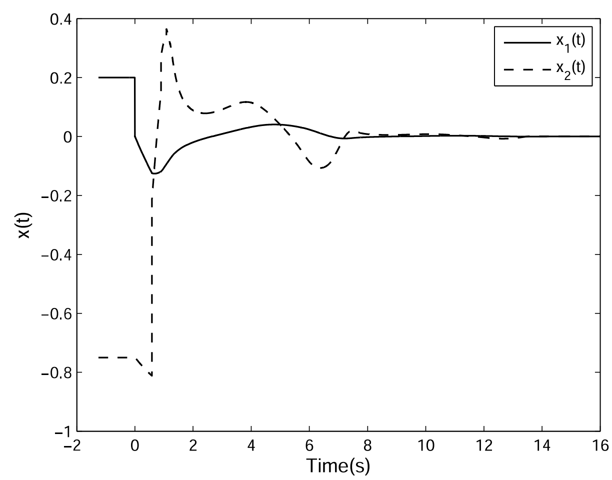

Fig. 1. State responses of the considered singular time-delay in Example 2.

Table 4. Maximum allowed time-delay.

Methods	$\gamma = 1$	$\gamma = 1.5$	$\gamma = 2$
Theorem 2 ($\mu = 1.3$)	1.1480	1.5672	1.6008
Theorem 2 ($\mu = 1.5$)	1.1479	1.4101	1.5010

$$B_\omega = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}, \; A_d = \begin{bmatrix} 0.6 & -0.1 \\ 1 & 0.3 \end{bmatrix},$$
$$B = \begin{bmatrix} 0 \\ 1.5 \end{bmatrix}, \; C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T.$$

Let $\alpha = 0.8$ and $d(t) = 0.5 + 1.5\cos(t)$. Table 4 gives the allowable upper bounds for various μ.

According to Theorem 2, when $\tau = 2, \gamma = 2$, the corresponding state feedback controller gain

$$K = \begin{bmatrix} -92.7806 & -9.2106 \end{bmatrix}.$$

6. CONCLUSION

In this paper, the problems of exponential admissibility criterion and H_∞ controller designed for time-varying delay singular systems are investigated. Firstly, a criterion is established, which covers more delay rate changes and guarantees the time-varying delay singular system is to be exponentially admissible. Based on this criterion, an H_∞ control algorithm is obtained to ensure singular time-delay system to be exponential admissibility with a disturbance attenuation level γ. All the obtained results are formulated in terms of strict LMIs, which are checked easily and free of the decomposition of the given system. Numerical examples demonstrate the usefulness of the main results of the proposed methods.

Recently, the new approaches to deal with the time delay in state are provided (see [34–36]). How to improved research technique to deal with time-delay is our next work. The proposed method can also be enriched by recent results [37–39], so extension of these results will be investigated to hand the switched singular or time-delay systems in future work.

REFERENCES

[1] W. Yu, S. Liu, and F. Zhang, "Global output feedback regulation of uncertain nonlinear systems with unknown time delay," *International Journal of Control, Automation and Systems*, vol. 13, no. 2, pp. 327-335, 2015. [click]

[2] M. Kazerooni, A. Khayatian, and A. A. Safavi. "Robust delay dependent fault estimation for a class of interconnected nonlinear time delay systems," *International Journal of Control, Automation and Systems*, vol. 14, no. 2, pp. 569-578, 2016. [click]

[3] Y. Sheng, Y. Shen, and M. Zhu, "Delay-dependent global exponential stability for delayed recurrent neural networks," *IEEE Transactions on Neural Networks and Learning Systems*, 2016.

[4] X. Zhang, Y. Han, L. Wu, and J. Zou, "M-matrix-based globally asymptotic stability criteria for genetic regulatory networks with time-varying discrete and unbounded distributed delays," *Neurocomputing*, vol. 174, pp. 1060-1069, 2016. [click]

[5] S. Xu and J. Lam, *Robust Control and Filtering of Singular Systems*, Springer, Berlin, 2006.

[6] J. Cheng, H. Wang, S. Chen, and J. Yang, "Robust delay-derivative-dependent state-feedback control for a class of continuous-time system with time-varying delays," *Neurocomputing*, vol. 173, pp. 827-834, 2016. [click]

[7] J. Cheng, H. Zhu, S. Zhong, Y. Zeng, and X. Dong, "Finite-time H_∞ control for a class of Markovian jump systems with mode-dependent time-varying delays via new Lyapunov functionals," *ISA transactions*, vol. 52, no. 6, pp. 768-774, 2013.

[8] X. Gao, L. Lian, and W. Qi, "H_∞ control for sochastic time-delayed Markovian switching systems with partly known transition rates and input saturation," *International Journal of Control, Automation and Systems*, vol. 14, no. 3, pp. 637-646, 2016. [click]

[9] L. Dai, *Singular Control Systems*, Springer-Verlag, Berlin, Germany, 1989.

[10] F. Lewis, "A survey of linear singular systems," *Circuits Syst. Signal Process*, vol. 5, no. 1, pp. 3-36, 1986. [click]

[11] A. Haidar and E. K. Boukas, "Exponential stability of singular systems with multiple time-varying delays," *Automatica*, vol. 45, no. 2, pp: 539-545, 2009. [click]

[12] X. Ding, Xiu Liu, and S. Zhong, "Delay-independent criteria for exponential admissibility of switched descriptor delayed systems," *Applied Mathematics and Computation*, vol. 228, pp. 432-445, 2014. [click]

[13] J. Zhao, Z. Hu, and L. Zhang, "Stability and stabilization for discrete-time singular systems with infinite distributed delays and actuator failures," *International Journal of Control, Automation and Systems*, vol. 10, no. 4, pp. 721-726, 2012. [click]

[14] J. Zhao, L. Zhang, and X. Qi, "A necessary and sufficient condition for stabilization of switched descriptor time-delay systems under arbitrary switching," *Asian Journal of Control*, vol. 18, no. 1, pp. 266-272, 2016. [click]

[15] J. X. Lin, X. L. Zhao, and S. M. Fei, "New delay-range-dependent exponential estimates for singular systems with time-varying delay," *International Journal of Control, Automation, and Systems*, vol. 9, no. 2, 218-227, 2011. [click]

[16] M. Kchaou, F. Tadeo, M. Chaabane, and A. Toumi, "Delay-dependent robust observer-based control for discrete-time uncertain singular systems with interval time-varying state delay," *International Journal of Control, Automation and Systems*, vol. 12, no. 1, pp. 12-22, 2014. [click]

[17] Y. Ding, H. Zhu, and S. Zhong, "Exponential stabilization using sliding mode control for singular systems with time-varying delays and nonlinear perturbations", *Communications in Nonlinear Science and Numerical Simulation*, vol. 16, no. 10, pp. 4099-4107, 2011. [click]

[18] Y. Cui, J. Lam, Z. Feng, and J. Shen, "Robust admissibility and admissibilisation of uncertain discrete singular time-delay systems," *International Journal of Systems Science*, vol. 47, no. 15, 1-10, 2015.

[19] Z. Hu, H. Zhu, and J. Zhao, "Further results on H_∞ filtering for a class of discrete-time singular systems with interval time-varying delay," *Circuits, Systems, and Signal Processing*, vol. 32, no. 3, pp. 1081-1095, 2013.

[20] Z. G. Wu, H. Y. Su, and J. Chu, "H_∞ filtering for singular systems with time-varying delay," *International Journal of Robust and Nonlinear Control*, vol. 20, no. 11, pp. 1269-1284, 2010. [click]

[21] X. Zhu, Y. Wang, and Y. Gan, "H_∞ filtering for continuous-time singular systems with time-varying delay," *International Journal of Adaptive Control and Signal Processing*, vol. 25, no. 2, pp. 137-154, 2011. [click]

[22] L. G. Wu and W. X. Zheng, "Passivity-based sliding mode control of uncertain singular time-delay systems," *Automatica*, vol. 45, no. 9, pp. 2120-2127, 2009. [click]

[23] Z. G. Wu, J. H. Park, H. Y. Su, and J. Chu, "Delay-dependent passivity for singular Markov jump systems with time-delays," *Communications in Nonlinear Science and Numerical Simulation*, vol. 18, no. 3, pp. 669-681, 2013. [click]

[24] Z. G. Wu, J. H. Parka, H. Y. Su, and J. Chu, "Reliable passive control for singular systems with time-varying delays," *Journal of Process Control*, vol. 23, no. 8, pp. 1217-1228, 2013. [click]

[25] X. Dong and M. Xiao, "H_∞ control of singular systems via delta operator method," *International Journal of Control, Automation and Systems*, vol. 13, no. 3, pp. 643-651, 2015. [click]

[26] M. Chadli and M. Darouach, "Novel bounded real lemma for discrete-time descriptor systems: application to H_∞ control design," *Automatica*, vol. 48. no. 2, pp. 449-453, 2012. [click]

[27] Y. Zhang, P. Shi, S. K. Nguang, and Y. Song, "Robust finite-time H_∞ control for uncertain discrete-time singular systems with Markovian jumps," *IET Control Theory & Applications*, vol. 8, no. 12, pp. 1105-1111, 2014. [click]

[28] L. H. Xie and C. E. de Souza. "Robust H_∞ control for linear systems with norm-bounded time-varying uncertainty," *IEEE Trans. Autom. Control*, vol. 37, no. 8, pp. 1188-1191, 1992. [click]

[29] S. Kririm, A. Hmamed, and F. Tadeo. "Analysis and design of H_∞ controllers for 2D singular systems with delays," *Circuits, Systems, and Signal Processing*, vol. 35, no. 5, pp. 1579-1592, 2016.

[30] H. Chen and P. Hu, "New result on exponential stability for singular systems with two interval time-varying delays," *IET Control Theory & Applications*, vol. 7, no. 15, pp. 1941-1949, 2013.

[31] L. V. Hien, L. H. Vu, and V. N. Phat, "Improved delay-dependent exponential stability of singular systems with mixed interval time-varying delays," *IET Control Theory & Applications*, vol. 9, no. 9, pp. 1364-1372, 2015. [click]

[32] Z. G. Wu, H. Y. Su, J. H. Park, and J. Chu, "Dissipativity analysis for singular systems with time-varying delays," *Applied Mathematics and Computation*, vol. 218, pp. 4605-4613, 2011. [click]

[33] E. K. Boukas, *Control of Singular Systems with Random Abrupt Changes*, Springer, New York, 2008.

[34] H. Li, Y. Gao, P. Shi, and X. Zhao, "Output-feedback control for T-S fuzzy delta operator systems with time-varying delays via an input-output approach," *IEEE Transactions on Fuzzy Systems*, vol. 23, no. 4, pp. 1100-1112, 2015. [click]

[35] H. Li, Y. Gao, L. Wu, and H. K. Lam, "Fault detection for T-S fuzzy time-delay systems: delta operator and input-output methods," *IEEE transactions on cybernetics*, vol. 45, no. 2, pp. 229-241, 2015.

[36] X. Su, P. Shi, L. Wu, and Y. Song, "A novel control design on discrete-time Takagi-Sugeno fuzzy systems with time-varying delays," *IEEE Transactions on Fuzzy Systems*, vol. 21, no. 4, pp. 655-671, 2013.

[37] M. Chadli, H. R. Karimi, and P. Shi, "On stability and stabilization of singular uncertain Takagi-Sugeno fuzzy systems," *Journal of the Franklin Institute*, vol. 351, no. 3, pp. 1453-1463, 2014. [click]

[38] Y. Liu, Y. Kao, S. Gu, and H. R. Karimi, "Soft variable structure controller design for singular systems," *Journal of the Franklin Institute*, vol. 352, no. 4, pp. 1613-1626, 2015. [click]

[39] Y. Liu, Y. Kao, H. R. Karimi, and Z. Gao, "Input-to-state stability for discrete-time nonlinear switched singular systems," *Information Sciences*, vol. 358, pp. 18-28, 2016. [click]

Jiemei Zhao received her M.S. degree in Applied Mathematics and the Ph.D. degree in Systems Engineering from the Harbin Engineering University, Harbin, China, in 2008 and 2013, respectively. Since 2013, she is a lecturer with the School of Mathematics and Computer Science, Wuhan Polytechnic University, Wuhan, China. Her research interests include nonlinear control systems, singular systems, neural networks and memristors.

Zhonghui Hu received his Ph.D. degree in Systems Engineering from the Harbin Engineering University, Harbin, China, in 2013. Since 2013, he works at the 709th Research Institute of China Shipbuilding Industry Corporation, China. His research interests include nonlinear control systems.