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Decentralized Iterative Learning Control for Large-scale Interconnected Linear Systems with Fixed Initial Shifts

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Abstract: This paper deals with the problem of iterative learning control for large-scale interconnected linear systems in the presence of fixed initial shifts. According to the characteristics of the systems, iterative learning control laws are proposed for such large-scale interconnected linear systems based on the PD-type learning schemes. The proposed controller of each subsystem only relies on local output variables without any information exchanges with other subsystems. Using the contraction mapping method, we show that the schemes can guarantee the output of the system converges uniformly to the corresponding output limiting trajectory over the whole time interval along the iteration axis. Simulation examples illustrate the effectiveness of the proposed method.

Keywords: Decentralized control, fixed initial shifts, iterative learning control, large-scale interconnected linear systems, PD-type learning schemes.

1. INTRODUCTION

Iterative learning control (ILC) has a well-established research history as shown in [1–7]. By generating a correct control signal from the previous control execution, it can achieve perfect tracking performance on the finite time interval. Owing to its simplicity and effectiveness, ILC has been found to be a good alternative in many areas and applications, e.g., see [8] for detailed results. Nowadays, ILC is playing a more and more important role in controlling repeatable processes.

In the process of ILC design, an interesting question is how to properly set the initial value of the iterative system at each iteration, such that the output trajectory of the iterative system can converge to the desired reference trajectory. In the early work, a common assumption about this question is that the initial condition at each iteration should be equal to the initial condition of the desired reference trajectory [1–4], or within its neighborhood [5–7]. In the case of perturbed initial conditions, boundedness of the tracking error is established and the error bound is shown to be proportional to the bound on initial condition errors [5–7]. Recently, more attention has been paid to the performance of ILC in the presence of fixed initial shifts, and initial rectifying action has been introduced in learning algorithm [9–12]. For a class of partially irregular multivariable plants, paper [9] utilized initial impulse rectifying to eliminate the effect of the fixed initial shifts so that a complete reference trajectory tracking over the whole time interval was achieved. In the case of fixed initial shifts, paper [10] first gave the output limiting trajectory under the action of D-type, and furthermore, the P-type errors were added in the D-type learning schemes, and the obtained PD-type learning schemes were used to get the modified output limiting trajectory, which can converge asymptotically to the desired reference trajectory as time increases. In [11], the I-type errors were added in the PD-type learning schemes in [10], and the obtained PID-type learning schemes can guarantee the output of the system converges uniformly to the corresponding output limiting trajectory, which also can converge asymptotically to the desired reference trajectory as time increases. And the two parameter matrices (corresponding to "P" and "I") can be used to adjust the output limiting trajectory and the convergence speed when time approaches to infinity. Paper [12] addressed the fixed initial shift problem in ILC for affine nonlinear systems with system relative degree, and the uniform convergence of the output trajectory to a desired one jointed smoothly with a specified transient trajectory from the starting position was ensured in the presence of fixed initial shifts. Paper [13] proposed a feedback-aided PD-type learning algorithm to solve the fixed initial shift problem for linear time-invariant systems. Up to now, because of its simplicity (corresponding

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to “PID”) and effectiveness, the PD-type learning schemes have been the most commonly used tools to solve the problem of fixed initial shifts [10, 12, 13].

Large-scale interconnected system is a compound system which is composed of interconnected subsystems. In many practical control problems, the systems have large-scale system model, such as power system, chemical engineering, large space structure, computer communication networks and so on. Because of the reliability of implementation, real-time and economics, decentralized control has become an active branch of the large-scale system theory [14–18], and some results for large-scale interconnected linear systems have been obtained based on decentralized ILC [19–21] in recent years. More recently, the decentralized ILC design for large-scale interconnected nonlinear systems has been also proposed in [22–24]. It is noticed that a common assumption in [19–24] is that the initial condition of each subsystem should be equal to the initial condition of the corresponding desired reference trajectory [19–22], or within its neighborhood [23, 24]. This observation motivates our present study.

Based on the work in [10, 13], this paper studies the problem of iterative learning control algorithm of large-scale interconnected linear systems in the presence of fixed initial shifts. The decentralized PD-type learning algorithms are proposed, and the corresponding output limiting trajectory over the whole time interval under the action of the PD-type learning schemes is given. In the process of the decentralized ILC design, two kinds of convergence conditions are proposed, one is discussed in [10, 13] and the other is not. Correspondingly, two different methods of proof are given.

In this paper, the following notational conventions are adopted: for n-dimensional Euclidean space R^n, $\|x\|$ denotes Euclidean norm of a vector $x = [\, x_1 \;\; x_2 \;\; \cdots \;\; x_n \,]^{\mathrm{T}}$. For a matrix A, $\|A\|$ denotes its induced norm. For a function f: $[0,T] \to R^n$ and a real number $\lambda > 0$, $\|f(\cdot)\|_s$ denotes the supreme norm defined by $\|f(\cdot)\|_s = \sup\limits_{t\in[0,T]} \|f(t)\|$; $\|f(\cdot)\|_\lambda$ denotes the λ-norm defined by $\|f(\cdot)\|_\lambda = \sup\limits_{t\in[0,T]} \mathrm{e}^{-\lambda t}\|f(t)\|$. From [2] we know, $\|f(\cdot)\|_s$ and $\|f(\cdot)\|_\lambda$ are equivalent for a finite constant λ. Thus, convergence results can be proved using either of them. We use I to represent the identity matrix.

2. PROBLEM DESCRIPTION

Consider the following large-scale interconnected linear system [19, 20]:

$$\begin{cases} \dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + \sum\limits_{j=1, j\neq i}^{N} D_{ij} x_j(t), \\ y_i(t) = C_i x_i(t), \end{cases} \tag{1}$$

where $i = 1, 2, \cdots, N$ represent each subsystem, $t \in [0,T]$. $x_i(t) \in R^{n_i}$, $u_i(t) \in R^{r_i}$, $y_i(t) \in R^{m_i}$ represent the state, control input and output of the ith subsystem respectively, A_i,B_i,C_i, and D_{ij} are matrices with appropriate dimensions.

It is assumed that the system (1) is repeatable over $t \in [0,T]$. Rewrite the system (1) at each iteration as:

$$\begin{cases} \dot{x}_{ik}(t) = A_i x_{ik}(t) + B_i u_{ik}(t) + \sum\limits_{j=1, j\neq i}^{N} D_{ij} x_{jk}(t), \\ y_{ik}(t) = C_i x_{ik}(t), \end{cases} \tag{2}$$

where the subscript k is employed to mark the iteration index. The task of the ILC is to find the appropriate learning schemes, so that the output sequence $y_{ik}(t)$ of the ith subsystem has the good tracking performance. $i = 1, 2, \cdots, N$.

Before giving our decentralized iterative learning control laws, we introduce for the large-scale system (2) the following assumptions:

Assumption 1: For each iteration index k, the initial value of the ith subsystem is always set to the fixed value x_{i0}, i.e., $x_{ik}(0) = x_{i0}$, $i = 1, 2, \cdots, N$.

Assumption 2: The matrix $C_i B_i$ is full rank, $i = 1, 2, \cdots, N$.

Lemma 1 [25]**:** If $\{a_k\}$, $k \in \{0, 1, \cdots, \infty\}$ is a sequence of real numbers such that

$$|a_{k+1}| \le \hat{\rho}\, |a_k| + \beta, \quad 0 \le \hat{\rho} < 1, \quad \beta > 0,$$

then

$$\limsup_{k\to\infty} |a_k| \le \frac{\beta}{1-\hat{\rho}}.$$

3. MAIN RESULTS

Construct the decentralized PD-type learning schemes for the system (2) as follows:

$$u_{i(k+1)}(t) = u_{ik}(t) + \Gamma_i(\dot{e}_{ik}(t) + L_i e_{ik}(t)), \tag{3}$$

where $\Gamma_i \in R^{r_i \times m_i}$ and $L_i \in R^{m_i \times m_i}$ are the learning gain matrices, and all the eigenvalues of the matrix L_i have positive real part. While $e_{ik}(t) = y_{id}(t) - y_{ik}(t)$ is the tracking error of each subsystem at kth iteration, and $y_{id}(t)$ is the corresponding desired reference trajectory. $i = 1, 2, \cdots, N$.

It is assumed that there exists a $u^*_{id}(t)$, such that

$$\begin{cases} \dot{x}^*_{id}(t) = A_i x^*_{id}(t) + B_i u^*_{id}(t) + \sum\limits_{j=1, j\neq i}^{N} D_{ij} x^*_{id}(t), \\ y^*_{id}(t) = C_i x^*_{id}(t), \end{cases} \tag{4}$$

where $y^*_{id}(t) = y_{id}(t) - e^*_i(t)$, $e^*_i(t) = \mathrm{e}^{-L_i t}(y_{id}(0) - C_i x_{i0})$. $i = 1, 2, \cdots, N$.

For the system (2) that the dimension of the input is less than or equal to the dimension of the output, i.e., $r_i \le m_i$, $i = 1, 2, \cdots, N$, we have:

Theorem 1: Consider the system (1) satisfying (4) and Assumptions 1-2 hold true. If there exist the gain matrices $\Gamma_i \in R^{r_i \times m_i} (i = 1, 2, \cdots, N)$ such that

$$\rho = \max_{1 \le i \le N} \|I - \Gamma_i C_i B_i\| < 1, \tag{5}$$

then the output $y_{ik}(t)$ of each subsystem converges uniformly to the corresponding output limiting trajectory $y^*_{id}(t)$ over the whole time interval $[0, T]$, under the action of the learning scheme (3) , i.e., $\lim_{k\to\infty} \|e^*_{ik}\|_s = 0$, where $e^*_{ik}(t) = y^*_{id}(t) - y_{ik}(t)$.

Proof: From $y^*_{id}(t) = y_{id}(t) - \mathrm{e}^{-L_i t}(y_{id}(0) - C_i x_{i0})$, we have

$$y^*_{id}(0) = C_i x_{i0}.$$

It follows from the system (4) that $y^*_{id}(0) = C_i x^*_{id}(0)$. Thus, it is reasonable to set the initial value as:

$$x^*_{id}(0) = x_{i0}. \tag{6}$$

By definition,

$$\begin{aligned} e_{ik}(t) &= e^*_{ik}(t) + e^*_i(t) \\ &= e^*_{ik}(t) + \mathrm{e}^{-L_i t}(y_{id}(0) - C_i x_{i0}). \end{aligned} \tag{7}$$

We differentiate both sides of the above expression to obtain

$$\dot{e}_{ik}(t) = \dot{e}^*_{ik}(t) - L_i \mathrm{e}^{-L_i t}(y_{id}(0) - C_i x_{i0}). \tag{8}$$

Form (7) and (8), we have

$$\begin{aligned} &\dot{e}_{ik}(t) + L_i e_{ik}(t) \\ &= \dot{e}^*_{ik}(t) - L_i \mathrm{e}^{-L_i t}(y_{id}(0) - C_i x_{i0}) + L_i e_{ik}(t) \\ &= \dot{e}^*_{ik}(t) - L_i(e_{ik}(t) - e^*_{ik}(t)) + L_i e_{ik}(t) \\ &= \dot{e}^*_{ik}(t) + L_i e^*_{ik}(t). \end{aligned} \tag{9}$$

Denote $\Delta u^*_{ik}(t) = u^*_{id}(t) - u_{ik}(t)$, $\Delta x^*_{ik}(t) = x^*_{id}(t) - x_{ik}(t)$, $i = 1, 2, \cdots, N$. From (2)-(4) and (9), we get

$$\begin{aligned} &\Delta u^*_{i(k+1)}(t) \\ &= u^*_{id}(t) - u_{i(k+1)}(t) \\ &= u^*_{id}(t) - u_{ik}(t) - (u_{i(k+1)}(t) - u_{ik}(t)) \\ &= \Delta u^*_{ik}(t) - \Gamma_i(\dot{e}_{ik}(t) + L_i e_{ik}(t)) \\ &= \Delta u^*_{ik}(t) - \Gamma_i(\dot{e}^*_{ik}(t) + L_i e^*_{ik}(t)) \\ &= \Delta u^*_{ik}(t) - \Gamma_i(C_i \dot{x}^*_{id}(t) - C_i \dot{x}_{ik}(t) \\ &\quad + L_i(C_i x^*_{id}(t) - C_i x_{ik}(t))) \\ &= \Delta u^*_{ik}(t) - \Gamma_i L_i C_i \Delta x^*_{ik}(t) \\ &\quad - \Gamma_i C_i \left(A_i \Delta x^*_{ik}(t) + B_i \Delta u^*_{ik}(t) + \sum_{j=1, j\ne i}^{N} D_{ij} \Delta x^*_{jk}(t) \right) \\ &= (I - \Gamma_i C_i B_i)\Delta u^*_{ik}(t) - (\Gamma_i L_i C_i + \Gamma_i C_i A_i)\Delta x^*_{ik}(t) \\ &\quad - \Gamma_i C_i \sum_{j=1, j\ne i}^{N} D_{ij} \Delta x^*_{jk}(t). \end{aligned}$$

Taking the λ- norm on both sides of the above expression yields

$$\begin{aligned} \|\Delta u^*_{i(k+1)}\|_\lambda \le{}& \|I - \Gamma_i C_i B_i\| \, \|\Delta u^*_{ik}\|_\lambda \\ &+ c_{i1} \|\Delta x^*_{ik}\|_\lambda + \sum_{j=1, j\ne i}^{N} d_{ij} \left\|\Delta x^*_{jk}\right\|_\lambda, \end{aligned}$$

where $c_{i1} = \|\Gamma_i L_i C_i + \Gamma_i C_i A_i\|$ and $d_{ij} = \|\Gamma_i C_i D_{ij}\|$. $i = 1, 2, \cdots, N$. Taking sum of the above expression for i from 1 to N and combining with (5), we have

$$\begin{aligned} \sum_{i=1}^{N} \|\Delta u^*_{i(k+1)}\|_\lambda &\le \rho \sum_{i=1}^{N} \|\Delta u^*_{ik}\|_\lambda + c_1 \sum_{i=1}^{N} \|\Delta x^*_{ik}\|_\lambda \\ &\quad + (N-1)c_2 \sum_{i=1}^{N} \|\Delta x^*_{ik}\|_\lambda \\ &= \rho \sum_{i=1}^{N} \|\Delta u^*_{ik}\|_\lambda + c_3 \sum_{i=1}^{N} \|\Delta x^*_{ik}\|_\lambda, \end{aligned} \tag{10}$$

where $c_1 = \max_{1\le i\le N} c_{i1}$, $c_2 = \max_{1\le i,j\le N, i\ne j} d_{ij}$, and $c_3 = c_1 + (N-1)c_2$.

It follows from (2) and (4) that

$$\begin{aligned} \frac{\mathrm{d}(\Delta x^*_{ik}(t))}{\mathrm{d}t} ={}& A_i \Delta x^*_{ik}(t) + B_i \Delta u^*_{ik}(t) \\ &+ \sum_{j=1, j\ne i}^{N} D_{ij} \Delta x^*_{jk}(t). \end{aligned}$$

Integrating both sides of the above expression over $[0, t]$ and combining with (6), we can obtain

$$\begin{aligned} \Delta x^*_{ik}(t) ={}& \int_0^t A_i \Delta x^*_{ik}(\xi)\mathrm{d}\xi + \int_0^t B_i \Delta u^*_{ik}(\xi)\mathrm{d}\xi \\ &+ \sum_{j=1, j\ne i}^{N} \int_0^t D_{ij} \Delta x^*_{jk}(\xi)\mathrm{d}\xi + x_{i0} - x_{ik}(0). \end{aligned}$$

Taking Euclidean norm on both sides of the above expression yields

$$\begin{aligned} &\|\Delta x^*_{ik}(t)\| \\ &\le \|A_i\| \int_0^t \|\Delta x^*_{ik}(\xi)\| \mathrm{d}\xi + \|B_i\| \int_0^t \|\Delta u^*_{ik}(\xi)\| \mathrm{d}\xi \\ &\quad + \sum_{j=1, j\ne i}^{N} \|D_{ij}\| \int_0^t \left\|\Delta x^*_{jk}(\xi)\right\| \mathrm{d}\xi + \|x_{i0} - x_{ik}(0)\| \\ &= \|A_i\| \int_0^t \mathrm{e}^{\lambda\xi}\mathrm{e}^{-\lambda\xi} \|\Delta x^*_{ik}(\xi)\| \mathrm{d}\xi \\ &\quad + \|B_i\| \int_0^t \mathrm{e}^{\lambda\xi}\mathrm{e}^{-\lambda\xi} \|\Delta u^*_{ik}(\xi)\| \mathrm{d}\xi \\ &\quad + \sum_{j=1, j\ne i}^{N} \|D_{ij}\| \int_0^t \mathrm{e}^{\lambda\xi}\mathrm{e}^{-\lambda\xi} \left\|\Delta x^*_{jk}(\xi)\right\| \mathrm{d}\xi \end{aligned}$$

$$
\begin{aligned}
&+\|x_{i0}-x_{ik}(0)\| \\
&\leq \frac{e^{\lambda t}-1}{\lambda}\,(\|A_i\|\,\|\Delta x_{ik}^*\|_\lambda+\|B_i\|\,\|\Delta u_{ik}^*\|_\lambda \\
&+\sum_{j=1,j\neq i}^{N}\|D_{ij}\|\,\left\|\Delta x_{jk}^*\right\|_\lambda)+\|x_{i0}-x_{ik}(0)\|.
\end{aligned}
$$

Therefore

$$
\begin{aligned}
&\|\Delta x_{ik}^*\|_\lambda=\sup_{t\in[0,T]} e^{-\lambda t}\,\|\Delta x_{ik}^*(t)\| \\
&\leq (\|A_i\|\,\|\Delta x_{ik}^*\|_\lambda+\|B_i\|\,\|\Delta u_{ik}^*\|_\lambda \\
&+\sum_{j=1,j\neq i}^{N}\|D_{ij}\|\,\left\|\Delta x_{jk}^*\right\|_\lambda)\sup_{t\in[0,T]}\frac{1-e^{-\lambda t}}{\lambda} \\
&+\|x_{i0}-x_{ik}(0)\|\sup_{t\in[0,T]} e^{-\lambda t} \\
&=\frac{1-e^{-\lambda T}}{\lambda}(\|A_i\|\,\|\Delta x_{ik}^*\|_\lambda+\|B_i\|\,\|\Delta u_{ik}^*\|_\lambda \\
&+\sum_{j=1,j\neq i}^{N}\|D_{ij}\|\,\left\|\Delta x_{jk}^*\right\|_\lambda)+\|x_{i0}-x_{ik}(0)\|.
\end{aligned}
$$

Taking sum of the above expression for i from 1 to N, we have

$$
\begin{aligned}
&\sum_{i=1}^{N}\|\Delta x_{ik}^*\|_\lambda\leq\frac{1-e^{-\lambda T}}{\lambda}(c_4\sum_{i=1}^{N}\|\Delta x_{ik}^*\|_\lambda \\
&+c_5\sum_{i=1}^{N}\|\Delta u_{ik}^*\|_\lambda+(N-1)c_6\sum_{i=1}^{N}\|\Delta x_{ik}^*\|_\lambda) \\
&+\sum_{i=1}^{N}\|x_{i0}-x_{ik}(0)\|, \\
&=\frac{1-e^{-\lambda T}}{\lambda}\left(c_7\sum_{i=1}^{N}\|\Delta x_{ik}^*\|_\lambda+c_5\sum_{i=1}^{N}\|\Delta u_{ik}^*\|_\lambda\right) \\
&+\sum_{i=1}^{N}\|x_{i0}-x_{ik}(0)\|,
\end{aligned}
$$

where $c_4=\max_{1\leq i\leq N}\|A_i\|$, $c_5=\max_{1\leq i\leq N}\|B_i\|$, $c_6=\max_{1\leq i,j\leq N,i\neq j}\|D_{ij}\|$, and $c_7=c_4+(N-1)c_6$. Taking λ so that

$$
\frac{1-e^{-\lambda T}}{\lambda}c_7<1,
$$

we get

$$
\begin{aligned}
\sum_{i=1}^{N}\|\Delta x_{ik}^*\|_\lambda\leq &c_8\frac{1-e^{-\lambda T}}{\lambda}\sum_{i=1}^{N}\|\Delta u_{ik}^*\|_\lambda \\
&+\frac{c_8}{c_5}\sum_{i=1}^{N}\|x_{i0}-x_{ik}(0)\|, \qquad (11)
\end{aligned}
$$

while

$$
c_8=\frac{c_5}{1-\dfrac{1-e^{-\lambda T}}{\lambda}c_7}.
$$

Substituting (11) into (10) results

$$
\begin{aligned}
\sum_{i=1}^{N}\|\Delta u_{i(k+1)}^*\|_\lambda\leq&\hat{\rho}\sum_{i=1}^{N}\|\Delta u_{ik}^*\|_\lambda \\
&+c_9\sum_{i=1}^{N}\|x_{i0}-x_{ik}(0)\|, \qquad (12)
\end{aligned}
$$

while

$$
\hat{\rho}=\rho+c_3c_8\frac{1-e^{-\lambda T}}{\lambda},\quad c_9=\frac{c_3c_8}{c_5}.
$$

By Assumption 1 and (12), we have

$$
\sum_{i=1}^{N}\|\Delta u_{i(k+1)}^*\|_\lambda\leq\hat{\rho}\sum_{i=1}^{N}\|\Delta u_{ik}^*\|_\lambda. \qquad (13)
$$

Since $0\leq\rho<1$ by (5), it is possible to choose λ sufficiently large so that $\hat{\rho}<1$. Then, (13) is a contraction in $\sum_{i=1}^{N}\|\Delta u_{ik}^*\|_\lambda$. It follows from (11), (13) and Assumption 1 that

$$
\lim_{k\to\infty}\sum_{i=1}^{N}\|\Delta x_{ik}^*\|_\lambda=0.
$$

Therefore, we have

$$
\lim_{k\to\infty}\|e_{ik}^*\|_s=0,\quad i=1,2,\cdots,N.
$$

This completes the proof. □

Remark 1: Theorem 1 extends the corresponding result of Theorem 2 in [10] to large-scale interconnected systems.

Remark 2: If Assumption 1 is replaced with the following assumption: for all k, the repeatability of the initial setting is satisfied within an admissible deviation level, i.e., $\|x_{i0}-x_{ik}(0)\|\leq\varepsilon_i$, $i=1,2,\cdots,N$, $k=0,1,2,\cdots$, where $\varepsilon_1,\varepsilon_2,\cdots,\varepsilon_N$ are positive constants, then the output tacking error $\sum_{i=1}^{N}\|e_{ik}^*\|_s$ is bounded. A brief explanation is given as follows:

From (12), we have

$$
\sum_{i=1}^{N}\|\Delta u_{i(k+1)}^*\|_\lambda\leq\hat{\rho}\sum_{i=1}^{N}\|\Delta u_{ik}^*\|_\lambda+c_9\sum_{i=1}^{N}\varepsilon_i.
$$

So it can be derived by Lemma 1 that

$$
\limsup_{k\to\infty}\sum_{i=1}^{N}\|\Delta u_{ik}^*\|_\lambda\leq\frac{c_9}{1-\hat{\rho}}\sum_{i=1}^{N}\varepsilon_i,
$$

which together with (11) implies

$$
\limsup_{k\to\infty}\sum_{i=1}^{N}\|\Delta x_{ik}^*\|_s=\limsup_{k\to\infty}\left\{\sum_{i=1}^{N}\sup_{t\in[0,T]}\|\Delta x_{ik}^*(t)\|\right\}
$$

$$\leq \limsup_{k\to\infty}\left\{\sum_{i=1}^{N}\sup_{t\in[0,T]}\left\{e^{\lambda T}e^{-\lambda t}\|\Delta x_{ik}^*(t)\|\right\}\right\}$$
$$= e^{\lambda T}\limsup_{k\to\infty}\sum_{i=1}^{N}\|\Delta x_{ik}^*\|_\lambda$$
$$\leq e^{\lambda T}c_8\frac{1-e^{-\lambda T}}{\lambda}\limsup_{k\to\infty}\sum_{i=1}^{N}\|\Delta u_{ik}^*\|_\lambda + e^{\lambda T}\frac{c_8}{c_5}\sum_{i=1}^{N}\varepsilon_i$$
$$\leq e^{\lambda T}c_8\frac{1-e^{-\lambda T}}{\lambda}\frac{c_9}{1-\hat{\rho}}\sum_{i=1}^{N}\varepsilon_i + e^{\lambda T}\frac{c_8}{c_5}\sum_{i=1}^{N}\varepsilon_i.$$

Therefore, $\sum_{i=1}^{N}\|e_{ik}^*\|_s$ is bounded.

For the system (2) that the dimension of the output is less than or equal to the dimension of the input, i.e., $m_i \leq r_i$, $i=1,2,\cdots,N$, we have:

Theorem 2: Consider the system (1) satisfying (4) and Assumptions 1-2 hold true. If there exist the gain matrices $\Gamma_i \in R^{r_i\times m_i}(i=1,2,\cdots,N)$ such that

$$\rho = \max_{1\leq i\leq N}\|I - C_iB_i\Gamma_i\| < 1, \tag{14}$$

then the output $y_{ik}(t)$ of each subsystem converges uniformly to the corresponding output limiting trajectory $y_{id}^*(t)$ of over the whole time interval $[0,T]$, under the action of the learning scheme (3) , i.e., $\lim_{k\to\infty}\|e_{ik}^*\|_s = 0$, where $e_{ik}^*(t) = y_{id}^*(t) - y_{ik}(t)$.

Proof: Denote $\delta x_{ik}(t) = x_{i(k+1)}(t) - x_{ik}(t)$, $\delta u_{ik}(t) = u_{i(k+1)}(t) - u_{ik}(t)$. From (2), (3) and (9), we have

$$\begin{aligned}
&\frac{d(\delta x_{ik}(t))}{dt}\\
&= A_i\delta x_{ik}(t) + B_i\delta u_{ik}(t) + \sum_{j=1,j\neq i}^{N}D_{ij}\delta x_{jk}(t)\\
&= A_i\delta x_{ik}(t) + B_i\Gamma_i(\dot{e}_{ik}(t) + L_ie_{ik}(t))\\
&\quad + \sum_{j=1,j\neq i}^{N}D_{ij}\delta x_{jk}(t)\\
&= A_i\delta x_{ik}(t) + B_i\Gamma_i(\dot{e}_{ik}^*(t) + L_ie_{ik}^*(t))\\
&\quad + \sum_{j=1,j\neq i}^{N}D_{ij}\delta x_{jk}(t).
\end{aligned}\tag{15}$$

It is easy to yield that

$$e_{i(k+1)}^*(t) = e_{ik}^*(t) - (y_{i(k+1)}(t) - y_{ik}(t)).$$

From (15), we can obtain

$$\begin{aligned}
&\dot{e}_{i(k+1)}^*(t)\\
&= \dot{e}_{ik}^*(t) - (\dot{y}_{i(k+1)}(t) - \dot{y}_{ik}(t))\\
&= \dot{e}_{ik}^*(t) - C_i\frac{d(\delta x_{ik}(t))}{dt}\\
&= \dot{e}_{ik}^*(t) - C_i(A_i\delta x_{ik}(t) + B_i\Gamma_i(\dot{e}_{ik}^*(t) + L_ie_{ik}^*(t))\\
&\quad + \sum_{j=1,j\neq i}^{N}D_{ij}\delta x_{jk}(t))\\
&= (I - C_iB_i\Gamma_i)\dot{e}_{ik}^*(t) - C_i(A_i\delta x_{ik}(t) + B_i\Gamma_iL_ie_{ik}^*(t)\\
&\quad + \sum_{j=1,j\neq i}^{N}D_{ij}\delta x_{jk}(t)).
\end{aligned}$$

Taking the λ- norm on both sides of the above expression yields

$$\begin{aligned}
\|\dot{e}_{i(k+1)}^*\|_\lambda &\leq \|I - C_iB_i\Gamma_i\|\,\|\dot{e}_{ik}^*\|_\lambda + c_{10}\|\delta x_{ik}\|_\lambda\\
&\quad + c_{11}\|e_{ik}^*\|_\lambda + c_{12}\sum_{j=1,j\neq i}^{N}\|\delta x_{jk}\|_\lambda,
\end{aligned}$$

where $c_{10} = \max_{1\leq i\leq N}\|C_iA_i\|$, $c_{11} = \max_{1\leq i\leq N}\|C_iB_i\Gamma_iL_i\|$, and $c_{12} = \max_{1\leq i,j\leq N, i\neq j}\|C_iD_{ij}\|$. Taking sum of the above expression for i from 1 to N and combining with (14) , we have

$$\begin{aligned}
\sum_{i=1}^{N}\|\dot{e}_{i(k+1)}^*\|_\lambda &\leq \rho\sum_{i=1}^{N}\|\dot{e}_{ik}^*\|_\lambda + c_{10}\sum_{i=1}^{N}\|\delta x_{ik}\|_\lambda\\
&\quad + c_{11}\sum_{i=1}^{N}\|e_{ik}^*\|_\lambda + c_{12}(N-1)\sum_{i=1}^{N}\|\delta x_{ik}\|_\lambda\\
&= \rho\sum_{i=1}^{N}\|\dot{e}_{ik}^*\|_\lambda + c_{13}\sum_{i=1}^{N}\|\delta x_{ik}\|_\lambda + c_{11}\sum_{i=1}^{N}\|e_{ik}^*\|_\lambda,
\end{aligned}\tag{16}$$

where $c_{13} = c_{10} + c_{12}(N-1)$. It follows from (2),(4) and (6) that

$$e_{ik}^*(t) = \int_0^t \frac{d(e_{ik}^*(\xi))}{d\xi}d\xi + C_i(x_{i0} - x_{ik}(0)).$$

Then we have

$$\|e_{ik}^*\|_\lambda \leq \frac{1-e^{-\lambda T}}{\lambda}\|\dot{e}_{ik}^*\|_\lambda + \|C_i\|\,\|x_{i0} - x_{ik}(0)\|. \tag{17}$$

Substituting (17) into (16) results

$$\begin{aligned}
\sum_{i=1}^{N}\|\dot{e}_{i(k+1)}^*\|_\lambda &\leq \left(\rho + c_{11}\frac{1-e^{-\lambda T}}{\lambda}\right)\sum_{i=1}^{N}\|\dot{e}_{ik}^*\|_\lambda\\
&\quad + c_{13}\sum_{i=1}^{N}\|\delta x_{ik}\|_\lambda + c_{11}\sum_{i=1}^{N}\|C_i\|\,\|x_{i0} - x_{ik}(0)\|.
\end{aligned}\tag{18}$$

Integrating both sides of (15) over $[0,t]$, we can obtain

$$\begin{aligned}
\delta x_{ik}(t) &= \int_0^t A_i\delta x_{ik}(\xi)d\xi\\
&\quad + \int_0^t B_i\Gamma_i\dot{e}_{ik}^*(\xi)d\xi + \int_0^t B_i\Gamma_iL_ie_{ik}^*(\xi)d\xi\\
&\quad + \sum_{j=1,j\neq i}^{N}\int_0^t D_{ij}\delta x_{jk}(\xi)d\xi + x_{i(k+1)}(0) - x_{ik}(0)\\
&= \int_0^t A_i\delta x_{ik}(\xi)d\xi + \int_0^t B_i\Gamma_i\dot{e}_{ik}^*(\xi)d\xi\\
&\quad + \int_0^t B_i\Gamma_iL_ie_{ik}^*(\xi)d\xi
\end{aligned}$$

$$+\sum_{j=1,j\neq i}^{N}\int_0^t D_{ij}\delta x_{jk}(\xi)\mathrm{d}\xi$$
$$+x_{i(k+1)}(0)-x_{i0}+x_{i0}-x_{ik}(0).$$

Denote $c_{14}=\max\limits_{1\le i\le N}\|B_i\Gamma_i\|$, $c_{15}=\max\limits_{1\le i\le N}\|B_i\Gamma_iL_i\|$. As in the proof of Theorem 1, we can derive

$$\begin{aligned}&\sum_{i=1}^{N}\|\delta x_{ik}\|_\lambda\\ &\le c_{16}\frac{1-\mathrm{e}^{-\lambda T}}{\lambda}\sum_{i=1}^{N}\|\dot e_{ik}^*\|_\lambda+c_{17}\frac{1-\mathrm{e}^{-\lambda T}}{\lambda}\sum_{i=1}^{N}\|e_{ik}^*\|_\lambda\\ &\quad+c_{18}\sum_{i=1}^{N}\left(\left\|x_{i(k+1)}(0)-x_{i0}\right\|+\|x_{i0}-x_{ik}(0)\|\right),\end{aligned}\qquad(19)$$

where

$$c_{16}=\frac{c_{14}}{1-\dfrac{1-\mathrm{e}^{-\lambda T}}{\lambda}c_7},\quad c_{17}=\frac{c_{15}}{1-\dfrac{1-\mathrm{e}^{-\lambda T}}{\lambda}c_7},$$
$$c_{18}=\frac{1}{1-\dfrac{1-\mathrm{e}^{-\lambda T}}{\lambda}c_7}.$$

From (17) and (19), we have

$$\begin{aligned}\sum_{i=1}^{N}\|\delta x_{ik}\|_\lambda&\le c_{16}\frac{1-\mathrm{e}^{-\lambda T}}{\lambda}\sum_{i=1}^{N}\|\dot e_{ik}^*\|_\lambda\\ &\quad+c_{17}\frac{1-\mathrm{e}^{-\lambda T}}{\lambda}\frac{1-\mathrm{e}^{-\lambda T}}{\lambda}\sum_{i=1}^{N}\|\dot e_{ik}^*\|_\lambda\\ &\quad+c_{17}\frac{1-\mathrm{e}^{-\lambda T}}{\lambda}\sum_{i=1}^{N}\|C_i\|\,\|x_{i0}-x_{ik}(0)\|\\ &\quad+c_{18}\sum_{i=1}^{N}\left(\left\|x_{i(k+1)}(0)-x_{i0}\right\|+\|x_{i0}-x_{ik}(0)\|\right)\\ &=\left(c_{16}+c_{17}\frac{1-\mathrm{e}^{-\lambda T}}{\lambda}\right)\frac{1-\mathrm{e}^{-\lambda T}}{\lambda}\sum_{i=1}^{N}\|\dot e_{ik}^*\|_\lambda\\ &\quad+c_{17}\frac{1-\mathrm{e}^{-\lambda T}}{\lambda}\sum_{i=1}^{N}\|C_i\|\,\|x_{i0}-x_{ik}(0)\|\\ &\quad+c_{18}\sum_{i=1}^{N}\left(\left\|x_{i(k+1)}(0)-x_{i0}\right\|+\|x_{i0}-x_{ik}(0)\|\right).\end{aligned}$$

Substituting the above expression into (18) results

$$\begin{aligned}\sum_{i=1}^{N}\|\dot e_{i(k+1)}^*\|_\lambda&\le\bar\rho\sum_{i=1}^{N}\|\dot e_{ik}^*\|_\lambda\\ &\quad+c_{13}c_{17}\frac{1-\mathrm{e}^{-\lambda T}}{\lambda}\sum_{i=1}^{N}\|C_i\|\,\|x_{i0}-x_{ik}(0)\|\\ &\quad+c_{13}c_{18}\sum_{i=1}^{N}\left(\left\|x_{i(k+1)}(0)-x_{i0}\right\|+\|x_{i0}-x_{ik}(0)\|\right)\\ &\quad+c_{11}\sum_{i=1}^{N}\|C_i\|\,\|x_{i0}-x_{ik}(0)\|,\end{aligned}\qquad(20)$$

where

$$\begin{aligned}0<\bar\rho&=\rho+c_{11}\frac{1-\mathrm{e}^{-\lambda T}}{\lambda}\\ &\quad+c_{13}\left(c_{16}+c_{17}\frac{1-\mathrm{e}^{-\lambda T}}{\lambda}\right)\frac{1-\mathrm{e}^{-\lambda T}}{\lambda}<1,\end{aligned}$$

when we choose λ sufficiently large. By Assumption 1 and (20), we have

$$\sum_{i=1}^{N}\|\dot e_{i(k+1)}^*\|_\lambda\le\bar\rho\sum_{i=1}^{N}\|\dot e_{ik}^*\|_\lambda.\qquad(21)$$

It follows from (21) that

$$\lim_{k\to\infty}\sum_{i=1}^{N}\|\dot e_{ik}^*\|_\lambda=0.$$

Therefore, we have

$$\lim_{k\to\infty}\|\dot e_{ik}^*\|_s=0,\quad i=1,2,\cdots,N.$$

Since $e_{ik}^*(0)=0$ by (6) and Assumption 1, we can derive

$$\lim_{k\to\infty}\|e_{ik}^*\|_s=0,\quad i=1,2,\cdots,N.$$

This completes the proof. □

Remark 3: It is worth pointing out that this kind of convergence condition such as (14) is not discussed in [10, 13].

Remark 4: Similar to Remark 2, the boundedness of $\sum\limits_{i=1}^{N}\|\dot e_{ik}^*\|_s$ can be obtained by (20) and Lemma 1. And furthermore, $\sum\limits_{i=1}^{N}\|e_{ik}^*\|_s$ is bounded.

Remark 5: Assumption 2 can guarantee the gain matrices Γ_i in Theorem 1,2 are existing.

Remark 6: Generally speaking, PD-type learning algorithms can guarantee the output limiting trajectory of the system asymptotically converges to the desired reference trajectory as time increases [10, 11, 13]. It follows from the expression of $y_{id}^*(t)$ that the decentralized PD-type learning algorithms in this paper also have the same properties.

4. SIMULATION EXAMPLES

1) Systems that the dimension of the input is less than the dimension of the output. Consider the following inter-

connected linear system:

$$\begin{cases} \dot{x}_{1k}(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x_{1k}(t) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u_{1k} \\ \quad + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x_{2k}(t), \\ y_{1k}(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x_{1k}(t), \\ \dot{x}_{2k}(t) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} x_{2k}(t) + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u_{2k} \\ \quad + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x_{1k}(t), \\ y_{2k}(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x_{2k}(t), \end{cases}$$

where the subscript k is employed to mark the iteration index. It is easy to yield that the above system satisfies Assumption 2. Set the initial values at each iteration to the fixed values $x_{10} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $x_{20} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, and take $T = 1$,

$$L_1 = L_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \Gamma_1 = \Gamma_2 = \begin{bmatrix} 0.2 & 0.2 \end{bmatrix},$$

then

$$|1 - \Gamma_1 C_1 B_1| = |1 - \Gamma_2 C_2 B_2| = 0.4 < 1.$$

For the given desired reference trajectories:

$$y_{1d}(t) = \begin{bmatrix} 2e^t - 2e^{-t} \\ e^t - e^{-t} \end{bmatrix}, \quad y_{2d}(t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

we have

$$\begin{aligned} y^*_{1d}(t) =& y_{1d}(t) - e^{-L_1 t}(y_{1d}(0) - C_1 x_{10}) \\ =& \begin{bmatrix} 2e^t - 2e^{-t} \\ e^t - e^{-t} \end{bmatrix} \\ & - e^{\begin{bmatrix} -t & 0 \\ 0 & -t \end{bmatrix}} \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right) \\ =& \begin{bmatrix} 2e^t \\ e^t \end{bmatrix}, \\ y^*_{2d}(t) =& y_{2d}(t) - e^{-L_2 t}(y_{2d}(0) - C_2 x_{20}) \\ =& \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ & - e^{\begin{bmatrix} -t & 0 \\ 0 & -t \end{bmatrix}} \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right) \\ =& \begin{bmatrix} e^{-t} \\ 2e^{-t} \end{bmatrix}. \end{aligned}$$

Take the initial controls $u_{10}(t) = u_{20}(t) = 0$. Under the action of the learning scheme (3), and by using the mathematical software Matlab, it is easy to see that $\|e^*_{ik}\|_s$ $(i = 1,2)$ tend to zero as $k \to \infty$ (shown in Figs. 1 and 2).

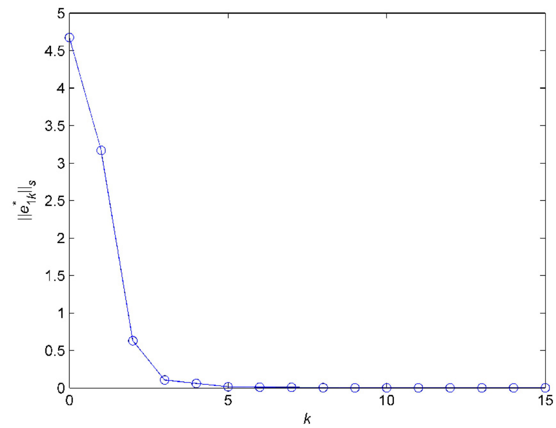

Fig. 1. Tracking errors of y_{1k}.

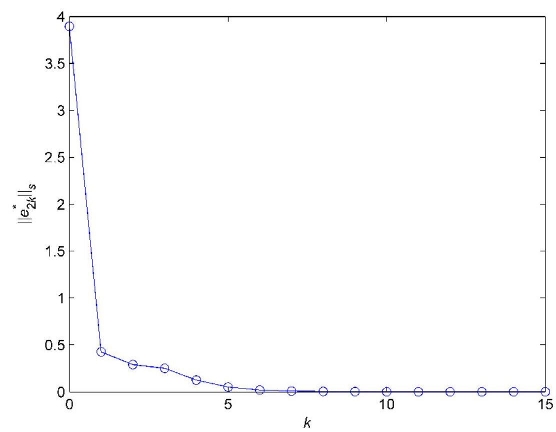

Fig. 2. Tracking errors of y_{2k}.

2) Systems that the dimension of the output is less than the dimension of the input. Consider the following interconnected linear system:

$$\begin{cases} \dot{x}_{1k}(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x_{1k}(t) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} u_{1k} \\ \quad + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x_{2k}(t), \\ y_{1k}(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} x_{1k}(t), \\ \dot{x}_{2k}(t) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} x_{2k}(t) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} u_{2k} \\ \quad + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x_{1k}(t), \\ y_{2k}(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} x_{2k}(t), \end{cases}$$

where the subscript k is employed to mark the iteration index. It is easy to yield that the above system satisfies Assumption 2. Set the initial values at each iteration to the fixed values $x_{10} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$,$x_{20} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, and take $T = 1$,

$$L_1 = L_2 = 1, \quad \Gamma_1 = \Gamma_2 = \begin{bmatrix} 0.25 \\ 0.25 \end{bmatrix},$$

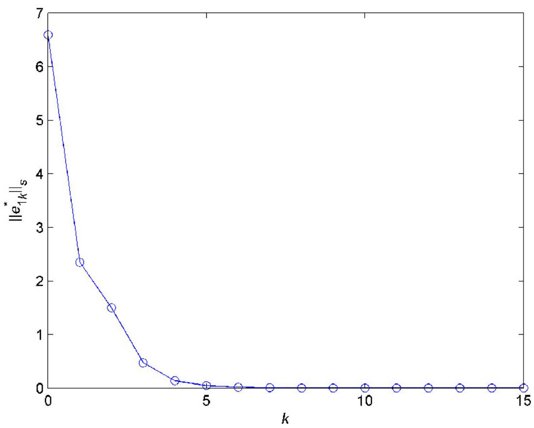

Fig. 3. Tracking errors of y_{1k}.

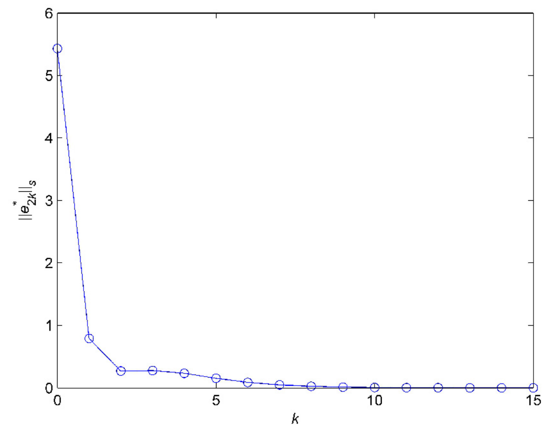

Fig. 4. Tracking errors of y_{2k}.

then

$$|1-C_1B_1\Gamma_1| = |1-C_2B_2\Gamma_2| = 0.5 < 1.$$

For the given desired reference trajectories:

$$y_{1d}(t) = 3\mathrm{e}^{t} - 3\mathrm{e}^{-t}, \;\; y_{2d}(t) = 0,$$

we have

$$\begin{aligned} y^*_{1d}(t) =& y_{1d}(t) - \mathrm{e}^{-L_1 t}(y_{1d}(0) - C_1 x_{10}) \\ =& 3\mathrm{e}^{t} - 3\mathrm{e}^{-t} - \mathrm{e}^{-t}\left(0 - \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}\right) \\ =& 3\mathrm{e}^{t}, \\ y^*_{2d}(t) =& y_{2d}(t) - \mathrm{e}^{-L_2 t}(y_{2d}(0) - C_2 x_{20}) \\ =& 0 - \mathrm{e}^{-t}\left(0 - \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) \\ =& 3\mathrm{e}^{-t}. \end{aligned}$$

Take the initial controls $u_{10}(t) = u_{20}(t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Under the action of the learning scheme (3), and by using the mathematical software Matlab, it is easy to see that $\|e^*_{ik}\|_s\,(i=1,2)$ tend to zero as $k \to \infty$ (shown in Figs. 3 and 4).

3) Systems that the dimension of the output is equal to the dimension of the input. In order to illustrate the application of the results, we consider the two identical pendulums which are coupled by a spring and subject to two distinct inputs [20] as shown in Fig. 5.

We choose the state vectors as

$$\begin{aligned} x_1(t) &= \begin{bmatrix} \theta_1(t) & \dot{\theta}_1(t) \end{bmatrix}^{\mathrm{T}}, \\ x_2(t) &= \begin{bmatrix} \theta_2(t) & \dot{\theta}_2(t) \end{bmatrix}^{\mathrm{T}}. \end{aligned}$$

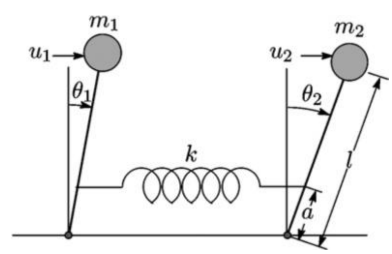

Fig. 5. The coupled inverted pendulums.

Then, the interconnected system can be described by

$$\begin{cases} \dot{x}_{1k}(t) = \begin{bmatrix} 0 & 1 \\ \frac{g}{l} & 0 \end{bmatrix} x_{1k}(t) + \begin{bmatrix} 0 \\ \frac{1}{ml^2} \end{bmatrix} u_{1k} \\ \qquad + \begin{bmatrix} 0 & 0 \\ -\frac{\bar{k}a^2}{ml^2} & 0 \end{bmatrix} x_{1k}(t) + \begin{bmatrix} 0 & 0 \\ \frac{\bar{k}a^2}{ml^2} & 0 \end{bmatrix} x_{2\bar{k}}(t), \\ y_{1k}(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} x_{1k}(t), \\ \dot{x}_{2k}(t) = \begin{bmatrix} 0 & 1 \\ \frac{g}{l} & 0 \end{bmatrix} x_{2k}(t) + \begin{bmatrix} 0 \\ \frac{1}{ml^2} \end{bmatrix} u_{2k} \\ \qquad + \begin{bmatrix} 0 & 0 \\ \frac{\bar{k}a^2}{ml^2} & 0 \end{bmatrix} x_{1k}(t) + \begin{bmatrix} 0 & 0 \\ -\frac{\bar{k}a^2}{ml^2} & 0 \end{bmatrix} x_{2k}(t), \\ y_{2k}(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} x_{2k}(t), \end{cases}$$

where the subscript k is employed to mark the iteration index, $\bar{k}$ and g are spring and gravity constants. For simulation, we give the following parameters as in [20]:

$$\frac{g}{l} = 1.0, \quad \frac{1}{ml^2} = 1.0, \quad \frac{\bar{k}}{m} = 2.0, \quad \frac{a}{l} = 0.5.$$

Set the initial values at each iteration to the fixed values $x_{10} = \begin{bmatrix} 1.5 \\ 0.5 \end{bmatrix}$, $x_{20} = \begin{bmatrix} 0.5 \\ 1.5 \end{bmatrix}$, and take $T = 1$,

$$L_1 = 1, \quad L_2 = 2, \quad \Gamma_1 = \Gamma_2 = 0.5,$$

then

$$|1-C_1B_1\Gamma_1| = |1-C_2B_2\Gamma_2| = 0.5 < 1.$$

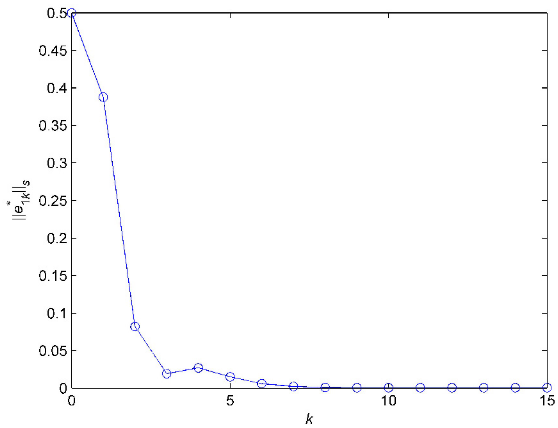

Fig. 6. Tracking errors of y_{1k}.

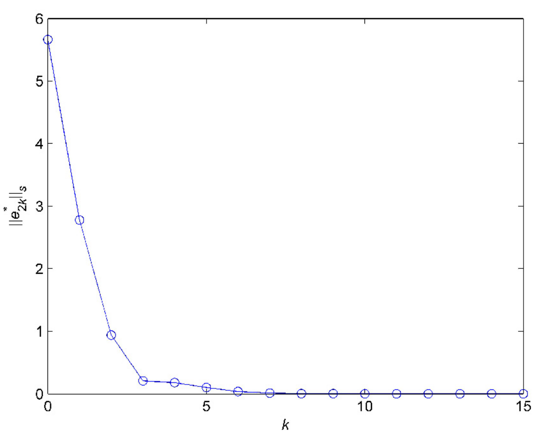

Fig. 7. Tracking errors of y_{2k}.

For the given desired reference trajectories:

$$y_{1d}(t) = 2e^t - 2e^{-t}, \quad y_{2d}(t) = 0,$$

we have

$$y^*_{1d}(t) = y_{1d}(t) - e^{-L_1 t}(y_{1d}(0) - C_1 x_{10})$$
$$= 2e^t - 2e^{-t} - e^{-t}\left(0 - \begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} 1.5 \\ 0.5 \end{bmatrix}\right) = 2e^t,$$
$$y^*_{2d}(t) = y_{2d}(t) - e^{-L_2 t}(y_{2d}(0) - C_2 x_{20})$$
$$= 0 - e^{-2t}\left(0 - \begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} 0.5 \\ 1.5 \end{bmatrix}\right) = 2e^{-2t}.$$

Take the initial controls $u_{10}(t) = u_{20}(t) = 0$. Under the action of the learning scheme (3), and by using the mathematical software Matlab, it is easy to see that $\|e^*_{ik}\|_s$ $(i = 1,2)$ tend to zero as $k \to \infty$ (shown in Figs. 6 and 7).

5. CONCLUSIONS

This paper studies the decentralized ILC problem for large-scale interconnected linear systems in the presence of fixed initial shifts. Two kinds of the system structure are considered in this paper, one is that the dimension of the input is less than or equal to the dimension of the output, and the other is that the dimension of the output is less than or equal to the dimension of the input. By using the PD-type learning schemes, the corresponding output limiting trajectory over the whole time interval under the action of the PD-type learning schemes is given, and the convergence theorems of the output tracking errors are established based on the contraction mapping method. The simulation results are consistent with theoretical analysis. Since the PD-type ILC for the system with fixed initial shifts has mainly focused on single system until now, the result of this paper extends the range of the application of PD-type ILC to some extent.

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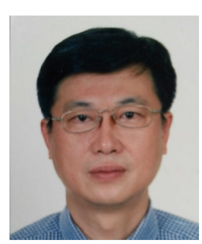

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