International Journal of Control, Automation and Systems 15(3) (2017) 1227-1235
http://dx.doi.org/10.1007/s12555-016-0078-7
ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Robust Output Tracking Control for a Class of Uncertain Nonlinear Systems Using Extended State Observer

Yan Zhao, Jiang-Bo Yu*, and Jie Tian

Abstract: This paper investigates the global robust output tracking control problem via output feedback for a class of nonlinear uncertain systems with the integral input-to-state stable (iISS) dynamic uncertainties. By performing the coordinates transformation and employing an extended state observer (ESO), it can be seen that the robust set-point tracking control is well solved. The proposed control strategy simplifies the control design procedure. This control scheme finds its application in the mass-spring mechanical system. It is shown that the asymptotic tracking control for any desired displacement can be achieved in the mass-spring mechanical system in the case of unknown parameters. The simulation results demonstrate that the proposed control scheme has a better tracking performance. This verifies the effectiveness of the developed method.

Keywords: Dynamic uncertainty, extended state observer, nonlinear systems, output feedback, tracking control

1. INTRODUCTION

The nonlinear control theory is an active research direction because of its widespread applications in the real world [1]. During the past two decades, the input-to-state stability (ISS) invented by Sontag ED in [2] has now become one of the central properties in the study of stability of perturbed nonlinear systems. As an integral variant of ISS, iISS is another meaningful but much weaker notion. As stated in [3], iISS is a most natural concept and has been widely used as well as ISS in nonlinear feedback control design and analysis. In recent years, the research on iISS has received much attention in the control community. The work [4] presents a unifying framework for global output feedback regulation control from ISS to iISS. More recently, the technique of changing supply rates for iISS systems has been discussed in [5, 6]. More results along this direction can be found in [7–11], etc.

The set-point tracking control is one of the active topics in nonlinear control community. The global set-point tracking control is to design a feedback controller in order to achieve the tracking control of constant reference signals starting from any initial conditions. It is extensively studied by many researchers in recent decades. Freeman and Kokotović in [12] proposed a global set-point tracking controller for a class of nonlinear systems with full-state information and no zero dynamics. Characterizing the dynamic uncertainties via ISS and ISS-Lyapunov functions, Jiang and Mareels in [13] further extended the previous results. However, the decay rate in the ISS dissipation inequality is monotone or non-oscillatory in [13]. This problem is further investigated in case of unknown control coefficients in [14] by state feedback, and output feedback in [15], whereas a filter and an observer are needed to compensate the unknown parameter vector, and this makes the control scheme much complicated.

As a further development of the existing work in [13–15], the purpose of this paper is to continue studying this issue using only the output information under the weaker iISS conditions, which accommodates the oscillatory dissipation. A dynamic output feedback control scheme will be proposed with the help of a $(n+1)$-order observer due to the unknown equilibrium. Moreover, the mass-spring system as a nonlinear benchmark example provides a simple yet practical method for modeling a wide variety of vibrating systems, see [16, 17]. It is interesting to note that the proposed control scheme could be applied to the robust set-point tracking control for the mass-spring system. With only the angle measured for arbitrary initial amplitude, the external force is quantitatively worked out, which allows the mass displacement to any desired set position in the presence of uncertain viscous friction.

Manuscript received February 10, 2016; revised June 30, 2016 and August 11, 2016; accepted August 28, 2016. Recommended by Associate Editor Do Wan Kim under the direction of Editor Jessie (Ju H.) Park. This work was supported by the Outstanding Middle-age and Young Scientist Award Foundation of Shandong Province under grant BS2015DX008, the National Natural Science Foundation of China under grant 61304008, 61471409, 61403237, and the Natural Science Foundation of Shandong Province of China under grant ZR2013FQ033. The authors would like to thank the anonymous reviewers for their valuable and constructive comments for improving the quality of this work. The authors also express their gratitude to associate Prof. Lei Yu for helpful discussions on PID control in Soochow University, China.

Yan Zhao, Jiang-Bo Yu, and Jie Tian are with the School of Science, Shandong Jianzhu University, Jinan 250101, P. R. China (e-mails: zhaoyan@sdjzu.edu.cn, jbyu2002@163.com, tjie9801@163.com).
* Corresponding author.

Our main contributions are composed of three parts.

(i). This paper investigates the global set-point tracking control for a class of nonlinear uncertain systems with much weaker iISS inverse dynamics than ISS in [13, 14].

(ii). By performing a coordinate transformation and constructing an extended $(n+1)$-order state observer, we present a novel set-point tracking control scheme which is robust and easier implemented because of less variables than earlier works [15].

(iii). The developed control scheme is applied to the mass-spring mechanical system. It is shown that any desired displacement can be regulated using only the displacement information.

2. PROBLEM STATEMENT

In this paper, we focus on the following class of nonlinear uncertain systems described by

$$\begin{aligned} \dot{\eta} &= q(\eta, y) \\ \dot{\xi}_1 &= \xi_2 + \theta_1(\eta, y) \\ &\vdots \\ \dot{\xi}_n &= u + \theta_n(\eta, y) \\ y &= \xi_1, \end{aligned} \tag{1}$$

where $u, y \in R$ is the control input, output and $\eta \in R^{\iota}$, $\xi = (\xi_1, \xi_2, \cdots, \xi_n)^T \in R^n$ are the states. It is further assumed that the states $(\xi_2, \cdots, \xi_n)$ as well as η which is also referred to as dynamic uncertainties are not assumed to be measurable. For each $1 \le i \le n$, nonlinear vector field $\theta_i : R^{\iota} \times R \to R$ is a uncertain function. The uncertain functions $q(\cdot), \theta_i(\cdot)$ are assumed to be locally Lipschitz.

Given a desired reference signal $y_r \in R$, the control objective here is to find, if possible, a smooth, dynamic, output feedback law of the form

$$\dot{\kappa} = \nu(\kappa, y),\ u = \mu(\kappa, y) \tag{2}$$

such that the tracking error $y(t) - y_r \to 0$ as $t \to \infty$ and the other signals $(\eta(t), \xi(t), \kappa(t))$ are bounded.

The following assumptions are needed for system (1):

Assumption 1: For any desired reference signal $y_r \in R$, there exists a unique $\eta_r \in R^{\iota}$ such that $q(\eta_r, y_r) = 0$.

Assumption 2: For any pair of (η_r, y_r) , there are two unknown constants $p_{i1}, p_{i2} > 0$, such that for all (η, y),

$$\begin{aligned} &\left|\theta_i(\eta, y) - \theta_i(\eta_r, y_r)\right| \\ &\le p_{i1} k_{i1}\big(|\eta - \eta_r|\big) + p_{i2} k_{i2}\big(|y - y_r|\big), \end{aligned}$$

where $k_{i1}(\cdot)$ and $k_{i2}(\cdot)$ are smooth functions vanishing at the origin and independent of (η_r, y_r).

Assumption 3: Letting $x_1 = y - y_r$ and $\zeta = \eta - \eta_r$, the derived system

$$\dot{\zeta} = q(\zeta + \eta_r, x_1 + y_r) = q_0(\zeta, x_1) \tag{3}$$

has an iISS-Lyapunov function $U_0(\zeta)$ satisfying

$$\frac{\partial U_0}{\partial \zeta}(\zeta) q_0(\zeta, x_1) \le -\alpha_0(|\zeta|) + \gamma_0(|x_1|), \tag{4}$$

where $\alpha_0(\cdot)$ is a positive-definite continuous function and γ_0 is a class-$\mathcal{K}_\infty$ function.

Remark 1: The system (1) can cover a large class of nonlinear systems with dynamic uncertainties. For example, the classical systems in output feedback form could be transformed into such class of nonlinear systems (1) via suitable coordinates change, see [4, 18], etc.

Remark 2: The similar assumptions on system (1) can be found in [13, 14]. However, different from the state feedback control therein, the control task here is to achieve the set-point tracking by output feedback. Due to the unmeasured η in the nonlinear uncertainties $\theta_i(\cdot)$, how to achieve this control task is a more challenging control problem than the work in [13, 14].

Remark 3: According to [3], one knows that the η-subsystem is iISS, and the function pair (α_0, γ_0) is the supply rates. Unlike the global exponentially stable condition imposed in [19], iISS can characterize a broader class of dynamic uncertainties, such as [7, 8], etc.

3. OUTPUT FEEDBACK CONTROL DESIGN

3.1. State transformation

To begin with, we perform the coordinates changes

$$x_i = \xi_i - \xi_{ir} (i = 1, \cdots, n),\ x_{n+1} = -u_r \tag{5}$$

with $\xi_{1r} = y_r$, $\xi_{ir} = -\theta_{(i-1)r}(\eta_r, y_r) (i = 2, \cdots, n)$, $u_r = -\theta_{nr}(\eta_r, y_r)$, and we obtain

$$\begin{aligned} \dot{\zeta} &= q(\eta, y) - q(\eta_r, y_r) = q_0(\zeta, x_1) \\ \dot{x}_1 &= x_2 + \theta_1(\eta, y) - \theta_1(\eta_r, y_r) \\ &\vdots \\ \dot{x}_{n-1} &= x_n + \theta_{n-1}(\eta, y) - \theta_{n-1}(\eta_r, y_r) \\ \dot{x}_n &= u + x_{n+1} + \theta_n(\eta, y) - \theta_n(\eta_r, y_r). \end{aligned} \tag{6}$$

Next, we will design a dynamic output feedback controller using only x_1 for the transformed system (6).

3.2. Observer design

We design the linear extended state observer (LESO):

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + l_1(x_1 - \hat{x}_1) \\ &\vdots \\ \dot{\hat{x}}_n &= u + \hat{x}_{n+1} + l_n(x_1 - \hat{x}_1) \\ \dot{\hat{x}}_{n+1} &= l_{n+1}(x_1 - \hat{x}_1), \end{aligned} \tag{7}$$

where $l_i (i = 1, \cdots, n+1)$ are constant design parameters to be determined later.

Remark 4: From the coordinates change (5), it can be seen that the unknown term of $-u_r$ as an external disturbance because of unknown $\theta_{nr}(\eta_r,y_r)$, is formulated as an augmented state. And then, a LESO is designed to rebuild the unmeasured states $(x_2,\cdots,x_n)$ and x_{n+1}. The LESO has strong robustness and could account for the nonlinearities and the modeling uncertainties, see [20–23].

Let the error $e_i = x_i - \hat{x}_i (i=1,\cdots,n+1)$, and we get

$$\dot{e} = Ae + \Theta(\eta,y,\eta_r,y_r), \tag{8}$$

where

$$e = \begin{bmatrix} e_1 \\ \vdots \\ e_{n+1} \end{bmatrix}, A = \begin{bmatrix} -l_1 & & & \\ \vdots & & I_n & \\ -l_{n+1} & 0 & \cdots & 0 \end{bmatrix},$$

$$\Theta(\eta,y,\eta_r,y_r) = \begin{bmatrix} \theta_1(\eta,y)-\theta_1(\eta_r,y_r) \\ \vdots \\ \theta_n(\eta,y)-\theta_n(\eta_r,y_r) \\ 0 \end{bmatrix}.$$

Choose $l_i(i=1,\cdots,n+1)$ such that A is Hurwitz, and hence there exists a positive definite matrix P satisfying

$$A^TP + PA = -2I. \tag{9}$$

In order to obtain a computable gain function, one has the following scaled error system

$$\dot{\bar{e}} = A\bar{e} + \frac{1}{\rho}\Theta(\eta,y,\eta_r,y_r), \tag{10}$$

with $\bar{e} = \frac{1}{\rho}e$, $\rho = \max\{1, \|P\|p_{i1}, \|P\|p_{i2} | i=1,\cdots,n\}$.

Proposition 1: Let $V_{\bar{e}} = \bar{e}^TP\bar{e}$, and we get

$$\dot{V}_{\bar{e}} \le -\bar{e}^T\bar{e} + k_1^2(|\zeta|) + k_2^2(|x_1|) \tag{11}$$

with smooth functions k_1, k_2 taking values zero at zero.

Proof: According to (9)(10) and the definition of ρ, by completing the squares, a straight computation shows that

$$\dot{V}_{\bar{e}} \le -\bar{e}^T\bar{e} + \sum_{i=1}^{n}\left(k_{i1}^2(|\zeta|) + k_{i2}^2(|x_1|)\right). \tag{12}$$

Take the following smooth functions

$$k_1(|\zeta|) = \Big(\sum_{i=1}^{n} k_{i1}^2(|\zeta|)\Big)^{\frac{1}{2}},\ k_2(|x_1|) = \Big(\sum_{i=1}^{n} k_{i2}^2(|x_1|)\Big)^{\frac{1}{2}},$$

and we arrive at

$$\dot{V}_{\bar{e}} \le -\bar{e}^T\bar{e} + k_1^2(|\zeta|) + k_2^2(|x_1|). \tag{13}$$

This completes the proof.

Remark 5: From Assumption 2, $k_{i2}(\cdot)(i=1,2,\cdots,n)$ vanishing at the origin, in view of $k_2(|x_1|) = \left(\sum_{i=1}^{n} k_{i2}^2(|x_1|)\right)^{\frac{1}{2}}$, one get $k_2(0)=0$. As a result,

$$k_2(|x_1|) = k_2(0) + |x_1|\int_0^1 \dot{k}_2(s|x_1|)ds \tag{14}$$

$$= |x_1|\int_0^1 \dot{k}_2(s|x_1|)ds. \tag{15}$$

3.3. Controller design

In this subsection, for the following augmented system

$$\begin{aligned} &\dot{\bar{e}} = A\bar{e} + \frac{1}{\rho}\Theta(\eta,y,\eta_r,y_r) \\ &\dot{\zeta} = q_0(\zeta,x_1) \\ &\dot{x}_1 = \hat{x}_2 + \rho\bar{e}_2 + \theta_1(\eta,y) - \theta_1(\eta_r,y_r) \\ &\quad\vdots \\ &\dot{\hat{x}}_n = u + \hat{x}_{n+1} + l_n(x_1 - \hat{x}_1) \\ &\dot{\hat{x}}_{n+1} = l_{n+1}(x_1 - \hat{x}_1) \\ &y = x_1, \end{aligned} \tag{16}$$

we develop a global set-point tracking control procedure.

Step 1: In view of the subsystem $\dot{x}_1 = x_2 + \theta_1(\eta,y) - \theta_1(\eta_r,y_r)$, we define $z_1 = x_1$, $z_2 = \hat{x}_2 - \alpha_1$ where α_1 is the virtual control input. Choose the Lyapunov function

$$V_1 = V_{\bar{e}} + \frac{1}{2}z_1^2 \tag{17}$$

and from the x_1 subsystem, its time derivative satisfies

$$\begin{aligned} \dot{V}_1 \le & z_1\big(\alpha_1 + e_2 + \theta_1(\eta,y) - \theta_1(\eta_r,y_r)\big) + z_1z_2 \\ & - \bar{e}^T\bar{e} + k_1^2(|\zeta|) + k_2^2(|z_1|). \end{aligned} \tag{18}$$

According to (15) and $z_1 = x_1$, the following holds

$$k_2^2(|z_1|) = z_1^2\Big(\int_0^1 \dot{k}_2(s|z_1|)ds\Big)^2. \tag{19}$$

By completing the squares, we have

$$z_1e_2 = z_1\rho\bar{e}_2 \le \frac{1}{2n}\bar{e}^T\bar{e} + \frac{n}{2}\rho^2z_1^2. \tag{20}$$

Similar to the calculations in (14) and (15), one gets

$$\begin{aligned} & z_1\big(\theta_1(\eta,y) - \theta_1(\eta_r,y_r)\big) \\ & \le |z_1|\big(p_{11}k_{11}(|\zeta|) + p_{12}k_{12}(|x_1|)\big) \\ & \le k_{11}^2(|\zeta|) + \frac{p_{11}^2}{4}z_1^2 + p_{12}|z_1|k_{12}(|x_1|) \\ & \le k_1^2(|\zeta|) + \frac{p_{11}^2}{4}z_1^2 + p_{12}z_1^2\int_0^1 \dot{k}_{12}(s|x_1|)ds. \end{aligned} \tag{21}$$

Based on the above calculation, we get

$$\begin{aligned} & z_1e_2 + z_1\big(\theta_1(\eta,y) - \theta_1(\eta_r,y_r)\big) \\ & \le \frac{1}{2n}\bar{e}^T\bar{e} + k_1^2(|\zeta|) + z_1^2\varphi_1(x_1)p^* \end{aligned} \tag{22}$$

with $\varphi_1(x_1) = 1 + \int_0^1 \dot{k}_{12}(\lambda|x_1|)d\lambda + (\int_0^1 \dot{k}_2(\lambda|z_1|)d\lambda)^2$ and $p^* = \max\{\frac{p_{11}^2}{4} + \frac{n}{2}\rho^2, p_{12}, \frac{p_{12}^2}{4}\}$.

Due to the unknown p^*, we augment V_1 as follows

$$\overline{V}_1 = V_1 + \frac{1}{2\lambda}(\hat{p} - p^*)^2, \tag{23}$$

where $\lambda > 0$ is a design parameter. According to (18) (22), the following holds

$$\dot{\overline{V}}_1 \le -(1 - \frac{1}{2n})\bar{e}^T\bar{e} + z_1\big(\alpha_1 + z_1\varphi_1(x_1)\hat{p}\big)$$

$$+\frac{1}{\lambda}\tilde{p}(\lambda z_1^2\varphi_1(x_1)-\dot{\hat{p}})+2k_1^2(|\zeta|)+z_1z_2. \quad (24)$$

Choose the virtual control law and updating function as follows

$$\alpha_1(x_1,\hat{p})=-\nu_1(z_1)z_1-z_1\varphi_1(x_1)\hat{p},$$
$$\tau_1=\lambda z_1^2\varphi_1(x_1), \quad (25)$$

where $\nu_1(\cdot)$ is a smooth function, and we get

$$\dot{\bar{V}}_1\leq-(1-\frac{1}{2n})\bar{e}^T\bar{e}-\nu_1(z_1)z_1^2+2k_1^2(|\zeta|)$$
$$+\frac{1}{\lambda}\tilde{p}(\tau_1-\dot{\hat{p}})+z_1z_2. \quad (26)$$

Step i **($2\leq i\leq n$):** Suppose we have designed the virtual control α_j and updating function τ_j $(1\leq j\leq i-1)$, such that, with $z_j=\hat{x}_j-\alpha_{j-1}(x_1,\hat{x}_1,\cdots,\hat{x}_{j-1},\hat{p})(1\leq j\leq i-1)$, the time derivative of the following function along solutions of equation (16)

$$V_{i-1}=V_{i-2}+\frac{1}{2}z_{i-1}^2 \quad (27)$$

satisfies

$$\dot{V}_{i-1}\leq-(1-\frac{i-1}{2n})\bar{e}^T\bar{e}-z_1^2\big(\nu_1(z_1)-i+2\big)$$
$$-\sum_{j=2}^{i-1}\nu_jz_j^2+ik_1^2(|\zeta|)+z_{i-1}z_i$$
$$+\big(\frac{1}{\lambda}\tilde{p}+\sum_{j=1}^{i-2}z_{j+1}\frac{\partial\alpha_j}{\partial\hat{p}}\big)(\tau_{i-1}-\dot{\hat{p}}), \quad (28)$$

where $\nu_j(2\leq j\leq i-1)$ are positive design parameters.

We prove in the sequel that a similar property holds for the $\hat{x}_i$ subsystem of equation (16).

Set $z_{i+1}=\hat{x}_{i+1}-\alpha_i$, and we consider the following Lyapunov function

$$V_i=V_{i-1}+\frac{1}{2}z_i^2. \quad (29)$$

Noticing that the variable z_i satisfies

$$\dot{z}_i=\hat{x}_{i+1}+l_i(x_1-\hat{x}_1)-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{x}_j}\dot{\hat{x}}_j-\frac{\partial\alpha_{i-1}}{\partial x_1}\hat{x}_2$$
$$-\frac{\partial\alpha_{i-1}}{\partial x_1}\big(\theta_1(\eta,y)-\theta_1(\eta_r,y_r)\big)-\frac{\partial\alpha_{i-1}}{\partial x_1}e_2$$
$$-\frac{\partial\alpha_{i-1}}{\partial\hat{p}}\dot{\hat{p}}. \quad (30)$$

Similar to (20) and (21), there exists smooth function $\varphi_i(x_1,\hat{x}_1,\cdots,\hat{x}_i,\hat{p})=\big((\int_0^1\dot{k}_{12}(\lambda|x_1|)d\lambda)^2+1\big)\ (\frac{\partial\alpha_{i-1}}{\partial x_1})^2$, such that

$$-z_i\frac{\partial\alpha_{i-1}}{\partial x_1}e_2-z_i\frac{\partial\alpha_{i-1}}{\partial x_1}\big(\theta_1(\eta,y)-\theta_1(\eta_r,y_r)\big)$$
$$\leq\frac{1}{2n}\bar{e}^T\bar{e}+k_1^2(\zeta)+z_1^2+z_i^2\varphi_i(\cdot)p^*. \quad (31)$$

Then, it can be verified that

$$\dot{V}_i\leq-(1-\frac{i}{2n})\bar{e}^T\bar{e}-z_1^2\big(\nu_1(z_1)-i+1\big)$$
$$-\sum_{j=2}^{i-1}\nu_jz_j^2+\big(\frac{1}{\lambda}\tilde{p}+\sum_{j=1}^{i-2}z_{j+1}\frac{\partial\alpha_j}{\partial\hat{p}}\big)(\tau_{i-1}-\dot{\hat{p}})$$
$$+z_i\Big(\alpha_i+z_{i-1}+l_i(x_1-\hat{x}_1)-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{x}_j}\dot{\hat{x}}_j$$
$$-\frac{\partial\alpha_{i-1}}{\partial x_1}\hat{x}_2-\frac{\partial\alpha_{i-1}}{\partial\hat{p}}\dot{\hat{p}}+z_i\varphi_i(\cdot)\hat{p}\Big)$$
$$+z_i^2\varphi_i\tilde{p}+(i+1)k_1^2(|\zeta|)+z_iz_{i+1}. \quad (32)$$

Choose the virtual control law and updating function

$$\alpha_i=-\nu_iz_i-z_{i-1}-l_i(x_1-\hat{x}_1)+\frac{\partial\alpha_{i-1}}{\partial x_1}\hat{x}_2$$
$$+\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{x}_j}\dot{\hat{x}}_j-z_i\varphi_i\hat{p}+\frac{\partial\alpha_{i-1}}{\partial\hat{p}}\tau_i$$
$$-z_i\varphi_i(\cdot)\lambda\sum_{j=1}^{i-2}z_{j+1}\frac{\partial\alpha_j}{\partial\hat{p}}$$
$$\tau_i=\tau_{i-1}+\lambda z_i^2\varphi_i(x_1,\hat{x}_1,\cdots,\hat{x}_i,\hat{p}), \quad (33)$$

then, one can get

$$\dot{V}_i\leq-(1-\frac{i}{2n})\bar{e}^T\bar{e}-z_1^2\big(\nu_1(z_1)-i+1\big)$$
$$-\sum_{j=2}^{i}\nu_jz_j^2+(i+1)k_1^2(|\zeta|)+z_iz_{i+1}$$
$$+\big(\frac{1}{\lambda}\tilde{p}+\sum_{j=1}^{i-1}z_{j+1}\frac{\partial\alpha_j}{\partial\hat{p}}\big)(\tau_i-\dot{\hat{p}}). \quad (34)$$

So far, if $i=n$, the actual adaptive controller and the updating law are given by

$$u=\alpha_n(x_1,\hat{x}_1,\cdots,\hat{x}_n,\hat{p})-\hat{x}_{n+1}, \quad (35)$$
$$\dot{\hat{p}}=\tau_n\ =\ \tau_{n-1}+\lambda z_n^2\varphi_n(x_1,\hat{x}_1,\cdots,\hat{x}_n,\hat{p}), \quad (36)$$

and the function

$$V_n=\bar{e}^TP\bar{e}+\sum_{j=1}^{n}\frac{1}{2}z_j^2+\frac{1}{2\lambda}(\hat{p}-p^*)^2 \quad (37)$$

satisfies

$$\dot{V}_n\leq-\frac{1}{2}\bar{e}^T\bar{e}-z_1^2(\nu_1(z_1)-n+1)-\sum_{j=2}^{n}\nu_jz_j^2$$
$$+(n+1)k_1^2(|\zeta|). \quad (38)$$

At this stage, we have completed the design procedure of the output feedback control by the recursive design. In the next section it will be shown that the dynamic output feedback controller (35) could achieve the control objective proposed in Section 2.

4. MAIN RESULTS

Now, we are ready to state the main results in this paper.

Theorem 1: Under Assumptions 1-3, suppose the following local conditions hold:

$$k_{i1}^2(s) = O\big(\alpha_0(s)\big)(i=1,\cdots,n), \gamma_0(s) = O(s^2). \quad (39)$$

Moreover in case α_0 is bounded, we have the following additional condition

$$\limsup_{s\to\infty} \frac{k_{i1}^2(s)}{\alpha_0(s)} < \infty, \ i=1,\cdots,n. \quad (40)$$

Then, the global asymptotic regulation control is achieved by the output feedback controller (35). More precisely, the following convergence property holds:

$$a) \lim_{t\to\infty} \big(|y(t)-y_r| + |\eta(t)-\eta_r|\big) = 0;$$
$$b) \lim_{t\to\infty} \big(|\xi_1(t)-\xi_{1r}| + \cdots + |\xi_n(t)-\xi_{nr}|\big) = 0.$$

Proof: To begin with, it is easy to see that the right-hand sides of the closed-loop system are locally Lipschitz. Assume that the solution is defined on a right-maximal interval $[0,T_f)$ with $0 < T_f \le \infty$. We will establish that $T_f = \infty$. Next, we construct another iISS-Lyapunov function to handle the unmeasured state ζ

$$V_0(\zeta) = \int_0^{U_0(\zeta)} \sigma(s)ds, \quad (41)$$

where $\sigma(\cdot)$ is a positive continuous function. In view of (39), (40) and **Proposition 1**, using the changing supply function technique for iISS systems, there exists a positive continuous function $\widetilde{\sigma}(\cdot)$, such that the time derivative of $V_0(\zeta)$ along the solutions of ζ system satisfies

$$\dot{V}_0 \le -k_1^2(|\zeta|) + \widetilde{\sigma}(x_1)\gamma_0(|x_1|). \quad (42)$$

Consequently, for the whole closed-loop system, consider the following Lyapunov function

$$V_c(\zeta,z,\bar{e},\hat{p}) = V_n + hV_0(\zeta) \quad (43)$$

where $h > 0$ is a designed constant. With the help of (38) and (42), the derivative of V_c with respect to time satisfies

$$\dot{V}_c \le -\frac{1}{2}\bar{e}^T\bar{e} - \sum_{j=2}^{n} \nu_j z_j^2 - (h-n-1)k_1^2(|\zeta|) - \Big(z_1^2\big(\nu_1(z_1)-n+1\big) - h\widetilde{\sigma}(z_1)\gamma_0(|z_1|)\Big). \quad (44)$$

Considering $\gamma_0(s) = O(s^2)$ and $\tilde{\sigma}(\cdot)$ is a positive continuous function, there exists a function $\nu_1(\cdot)$ such that,

$$h > n+1+\bar{\nu}_k, \ \bar{\nu}_k > 0,$$
$$z_1^2\big(\nu_1(z_1)-n+1\big) - h\widetilde{\sigma}(z_1)\gamma_0(|z_1|) \ge z_1^2. \quad (45)$$

Substituting (45) into (44) yields that

$$\dot{V}_c \le -\frac{1}{2}\bar{e}^T\bar{e} - z_1^2 - \sum_{j=2}^{n} \nu_j z_j^2 - \bar{\nu}_k k_1^2(|\zeta|). \quad (46)$$

Thus, we conclude that $V_c(\zeta,z,\bar{e},\hat{p})$ is bounded and in turn, the solution $(\zeta(t),z(t),\bar{e}(t),\hat{p}(t))$ is bounded. Therefore, there is no finite escape on $[0,T_f)$, and further, we derive that for the closed-loop system there is a unique solution that is defined for all $t > 0$, thus $T_f = \infty$.

In the sequel, we will prove the convergence properties $a)$ and $b)$. With the help of (46), we know from LaSalle's invariance principle(see [18]) that

$$\lim_{t\to\infty} \big(|\bar{e}(t)| + |z(t)| + k_1(|\zeta(t)|)\big) = 0. \quad (47)$$

Since $k_1(\cdot)$ is positive definite, then $\zeta(t) \to 0$ as $t \to \infty$. Considering $z_1(t) \to 0$ and $z_1 = x_1$, we can get $x_1(t) \to 0$ as $t \to \infty$, and hence, convergence property $a)$ is proved. From $z_1(t) \to 0$, we can get $\alpha_1(x_1,\hat{p}) \to 0$ according to (30). In view of $z_2 = \hat{x}_2 - \alpha_1$, together with $z_2(t) \to 0$ and $\alpha_1(t) \to 0$, we can get $\hat{x}_2 \to 0$. From $x_2 = \hat{x}_2 + e_2$ and $e_2 \to 0$, we get $x_2 \to 0$. Using the similar recursive method, it can be concluded that $x(t)$ converges to the origin. Therefore, convergence property $b)$ is proved. As a result, the proof can be completed.

5. ILLUSTRATIVE EXAMPLE

5.1. Application to the mass-spring mechanical systems

In this section, we apply our output feedback tracking control strategy into the mass-spring control system. The mass-spring system is a typical and popular model for analysis of vibrational phenomena in engineering and technology, and hence it is also known as the mass-spring-damping system [16, 17]. The dynamics of a mass attached to a spring is described by

$$\ddot{y} + c\dot{y} + k(1+a^2y^2)y = F, \quad (48)$$

where y is the displacement from a reference position which is considered as the measured output, $\dot{y}$ viewed as the velocity is not measured for the feedback, F is an external force which is considered as the control input, c,k,a are unknown constant parameters. The control objective is to give the external force F such that the mass displacement y can approach any desired position y_r using only the information of the system output y.

Towards this end, we choose the following new state variables

$$\xi_1 = y, \ \xi_2 = \dot{\xi}_1 + c\xi_1, \ u = F, \quad (49)$$

and then, (48) is turned into

$$\begin{aligned} \dot{\xi}_1 &= \xi_2 - c\xi_1 \\ \dot{\xi}_2 &= u - k\xi_1 - ka^2\xi_1^3 \\ y &= \xi_1, \end{aligned} \quad (50)$$

which falls into the class of investigated system (1). Let

$$\xi_{1r} = y_r, \ \xi_{2r} = c\xi_{1r}, \ u_r = k\xi_{1r} + ka^2\xi_{1r}^3,$$

$$x_1 = \xi_1 - \xi_{1r},\ x_2 = \xi_2 - \xi_{2r},\ x_3 = u_r, \tag{51}$$

then (50) can be further changed into

$$\begin{aligned} \dot{x}_1 &= x_2 - cx_1 \\ \dot{x}_2 &= u + x_3 - kx_1 - ka^2(\xi_1^3 - \xi_{1r}^3). \end{aligned} \tag{52}$$

Design the following extended state observer

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + l_1(x_1 - \hat{x}_1) \\ \dot{\hat{x}}_2 &= u + \hat{x}_3 + l_2(x_1 - \hat{x}_1) \\ \dot{\hat{x}}_3 &= l_3(x_1 - \hat{x}_1) \end{aligned} \tag{53}$$

and the observer error $e = [e_1, e_2, e_3]^T$, then we have

$$\dot{e} = Ae + \Theta(y, y_r) \tag{54}$$

with

$$\Theta(y, y_r) = \begin{bmatrix} -cx_1 \\ -kx_1 - ka^2(\xi_1^3 - \xi_{1r}^3) \\ 0 \end{bmatrix}.$$

Choose $l_i (i = 1, 2, 3)$ such that A is Hurwitz, and hence choose a $P > 0$ satisfying $A^T P + PA = -2I$.

By simple calculations, the scaled-error dynamics is

$$\dot{\bar{e}} = A\bar{e} + \frac{1}{\rho}\Theta(y, y_r) \tag{55}$$

with $\bar{e} = \frac{1}{\rho}e$ and $\rho = \max\{1, \|P\|(c^2 + k^2)^{\frac{1}{2}}, \|P\||k|a^2\}$.

For the augmented system composed of (52), (53), (55), using the proposed method in Section 3, we can obtain the following controller and the updating law

$$\begin{aligned} u =& - \nu_1 z_1 - \nu_2 z_2 - \hat{x}_3 - l_2(x_1 - \hat{x}_1) + \frac{\partial \alpha_1}{\partial x_1}\hat{x}_2 \\ &+ \frac{\partial \alpha_1}{\partial \hat{p}}\dot{\hat{p}} - \hat{p} z_2 (\frac{\partial \alpha_1}{\partial x_1})^2, \end{aligned} \tag{56}$$

$$\dot{\hat{p}} = \tau_2 = \tau_1 + \lambda z_2^2 (\frac{\partial \alpha_1}{\partial x_1})^2. \tag{57}$$

The simulation results are shown in Fig.1. It can be seen that the system output $y = \xi_1$ can be regulated to any desired reference signal y_r. The control input $u = F$ has a good performance. In addition, the observer works well and the other signals in closed-loop system are bounded. The simulation is carried out with the reference signal $y_r = 1$ and the parameters $k = 1,\ a = 1,\ c = 1$. The initial conditions are chosen as $\hat{x}_1(0) = 0.5,\ \hat{x}_2(0) = 0,\ \hat{x}_3(0) = 1,\ \hat{p}(0) = 0.5$, and the design parameters $l_1 = 6,\ l_2 = 11,\ l_3 = 6,\ \nu_1 = 1,\ \nu_2 = 1,\ \lambda = 1$.

5.2. Comparison

For illustrating the effectiveness and superiority of the control scheme derived herein, the results of this paper will be compared with those by other control methods, such as the output regulation control with reduced-order observer in [15], the proportional-integral-derivative

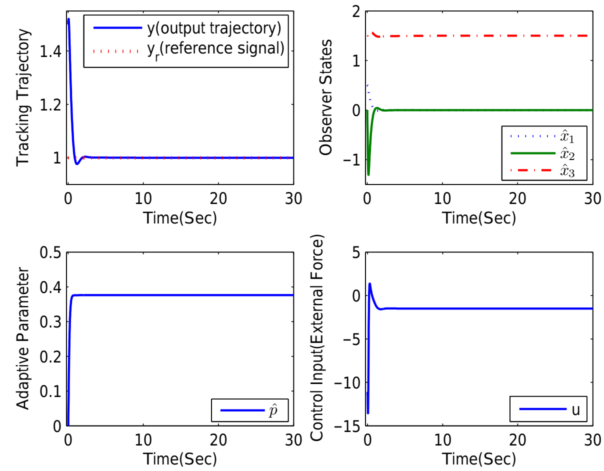


Fig. 1. The output in closed-loop system (48)-(57).

(PID) control, and adaptive control design with K-filters [18]. For comparison purposes, the simulations are carried out with the same reference signal $y_r = 1$, parameters $k = 1,\ a = 1,\ c = 1$, as well as the same initial conditions $y(0) = 1.5, \dot{y}(0) = 0$.

Case 1 (Output regulation control based reduced-order observer [15]): Take the notations

$$\begin{aligned} \phi_1^T(y) &= [y, 0, 0],\ \phi_2^T(y) = [0, y, y^3], \\ \theta^T &= [-c, -k, -ka^2]. \end{aligned}$$

Like in [15], we design the reduced-order observer

$$\dot{\hat{x}}_1 = u - L_1(\hat{x}_1 + L_1 y),\ L_1 > 0, \tag{58}$$

and the dynamic auxiliary signals

$$\dot{\Xi} = -L_1\Xi + \phi_2(y) - L_1\phi_1(y) \tag{59}$$

with $\Xi^T = [\Xi_1,\ \Xi_2,\ \Xi_3]$. Using the adaptive backstepping technique, one can design the following output feedback controller and the adaptive law

$$\begin{aligned} u =& - \nu_2 e_2 - e_1 - e_2(\frac{\partial \alpha_1}{\partial e_1})^2 + L_1(\hat{x}_1 + L_1 y) \\ &+ \frac{\partial \alpha_1}{\partial e_1}(\hat{x}_1 + L_1 y + (y + \Xi_1)\hat{\theta}_1 + \Xi_2\hat{\theta}_2 + \Xi_3\hat{\theta}_3) \\ &+ \frac{\partial \alpha_1}{\partial \hat{\theta}^T}\dot{\hat{\theta}} + \frac{\partial \alpha_1}{\partial \Xi^T}\dot{\Xi}, \end{aligned} \tag{60}$$

$$\dot{\hat{\theta}} = (e_1 - e_2\frac{\partial \alpha_1}{\partial e_1})[y + \Xi_1, \Xi_2, \Xi_3]^T \tag{61}$$

with $e_1 = x_1 - y_r,\ e_2 = \hat{x}_1 - \alpha_1,\ \alpha_1 = -\nu_1 e_1 - e_1 - L_1 y - (y + \Xi_1)\hat{\theta}_1 - \Xi_2\hat{\theta}_2 - \Xi_3\hat{\theta}_3$. The initial conditions are chosen as $\hat{x}_1(0) = 0.5,\ \Xi_1(0) = 0,\ \Xi_2(0) = 0.5,\ \Xi_3(0) = 1, \hat{\theta}_1(0) = 0,\ \hat{\theta}_2(0) = 0,\ \hat{\theta}_3(0) = 1$ and the design parameters $L_1 = 1,\ \nu_1 = 1,\ \nu_2 = 1$.

Case 2 (PID controller): The PID controller is designed as

$$u = k_I e_0 + k_P e_1 + k_D e_2, \tag{62}$$

where the PID control gains tuned via error-and-try method as in [20–22] are $k_I = 10, k_P = 40, k_D = 1$, and the initial values are $e_0(0) = 0, e_1(0) = 0.5, e_2(0) = 1$.

Case 3 (Adaptive controller with K-filters [18]): For the mass-spring mechanical system (48), we choose the following new state variables

$$x_1 = y,\ x_2 = \dot{x}_1 + cx_1,\ u = F, \tag{63}$$

and adopt the following notations:

$$\theta_1 = c,\ \theta_2 = k,\ \theta_3 = ka^2, \varphi_1(y) = [-y,\ 0]^T,$$
$$\varphi_2(y) = [0,\ -y]^T, \varphi_3(y) = [0,\ -y^3]^T,$$
$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

then the system (48) is turned into

$$\dot{x} = Ax + \sum_{j=1}^{3} \theta_j \varphi_j(y) + bu. \tag{64}$$

Design the following K-filters

$$\begin{aligned} &\dot{\varsigma} = A_0\varsigma + ly \\ &\dot{\chi}_j = A_0\chi_j + \varphi_j(y), 1 \le j \le 3 \\ &\dot{\upsilon}_0 = A_0\upsilon_0 + bu \end{aligned} \tag{65}$$

with

$$A_0 = \begin{bmatrix} -l_1 & 1 \\ -l_2 & 0 \end{bmatrix}, l = \begin{bmatrix} l_1 \\ l_2 \end{bmatrix}, \varsigma = \begin{bmatrix} \varsigma_1 \\ \varsigma_2 \end{bmatrix},$$
$$\upsilon_0 = \begin{bmatrix} \upsilon_{01} \\ \upsilon_{02} \end{bmatrix}, \chi_j = \begin{bmatrix} \chi_{j1} \\ \chi_{j2} \end{bmatrix}, j = 0, 1, 2, 3.$$

Using adaptive backstepping design approach, we can construct the following dynamic output feedback controller and adaptive law

$$\begin{aligned} u =& -c_2z_2 - z_1 + \frac{\partial \alpha_1}{\partial y}\left(\varsigma_2 + \upsilon_{02} + \hat{\theta}^T\omega_1(y,\chi)\right) \\ &+ l_2\varsigma_1 - dz_2\left(\frac{\partial \alpha_1}{\partial y}\right)^2 + \frac{\partial \alpha_1}{\partial \varsigma_2}(-l_2\varsigma_1 + l_2y) \\ &+ \sum_{i=1}^{3} \frac{\partial \alpha_1}{\partial \chi_{i2}}\left(-l_2\chi_{i1} + \varphi_{i2}(y)\right) + \frac{\partial \alpha_1}{\partial \hat{\theta}^T}\tau_2, \end{aligned} \tag{66}$$

$$\dot{\hat{\theta}} = (z_1 - \frac{\partial \alpha_1}{\partial y}z_2)\omega_1(y,\chi) \tag{67}$$

with $y = x_1$, $z_1 = x_1 - y_r$, $z_2 = \upsilon_{02} - \alpha_1$, $\alpha_1 = -c_1z_1 - dz_1 - \varsigma_2 - \omega_1^T\hat{\theta}$, and $\omega_1^T = [-y + \chi_{12}, \chi_{22}, \chi_{32}]$. The initial conditions are chosen as $x_1(0) = 1.5, x_2(0) = 1.5$, $\varsigma_1(0) = 1$, $\varsigma_2(0) = 0.5$, $\chi_{11}(0) = 1$, $\chi_{12}(0) = 0$, $\chi_{21}(0) = 1$, $\chi_{22}(0) = 1$, $\chi_{31}(0) = 0.5$, $\chi_{32}(0) = 1.5$, $\upsilon_{01}(0) = 0.5$, $\upsilon_{02}(0) = 1.5$, $\hat{\theta}_1(0) = 1$, $\hat{\theta}_2(0) = 0$, $\hat{\theta}_3(0) = 0.5$ and the design parameters $c_1 = 1$, $c_2 = 10$, $d = 1$, $l_1 = 2$, $l_2 = 1$.

Remark 6: The simulation results are shown in Fig. 2. From the comparison and analysis of the four control schemes, the primary observations are listed as follows:

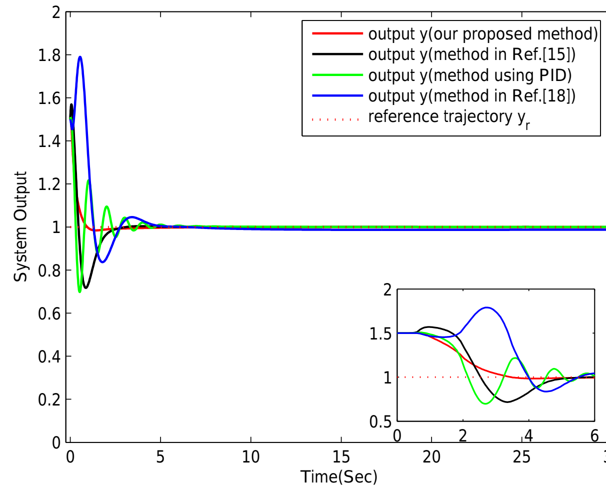


Fig. 2. The system outputs using different methods.

(i) The tracking objectives with the four methods are all well obtained. However, compared with the other three control schemes, our result in this paper yields faster convergence with less transitional fluctuation, less control efforts and faster adjustment time in terms of performance. This shows that the proposed output feedback set-point tracking control scheme based ESO can quickly handle unmodeled factors better than the other controllers; **(ii)** Different from Case 1 and Case 3, since the number of the online adaptive parameter is only one, the computation burden is significantly reduced accordingly using the current control method proposed in this paper, which is more feasible in engineering.

Remark 7: As an anonymous reviewer pointed out, it is found that the proposed output-feedback control scheme is more robust than the PID controller. Comparing with the PID controller, our method is more insensitive to the changes of the system parameters, since the control scheme employs the ESO and adaptive control technique to estimate modeling uncertainties and compensate them in controller. To illustrate this point, the following simulation plotted in Fig.3 is carried out in such a way that the control is fixed while the simulated system assumes the parameter $k = 1$ in the first 20s and assumes $k = 10$ in the remaining seconds. From the simulation results, it can be seen that the proposed robust tracking control scheme works well no matter how the parameters of the mass-spring control system vary.

6. CONCLUSION

The global robust output tracking control problem is investigated for a class of nonlinear systems with dynamic uncertainties and uncertain nonlinearities. The studied system covers a larger variety of nonlinear uncertain systems than existing results, which allows the oscillatory

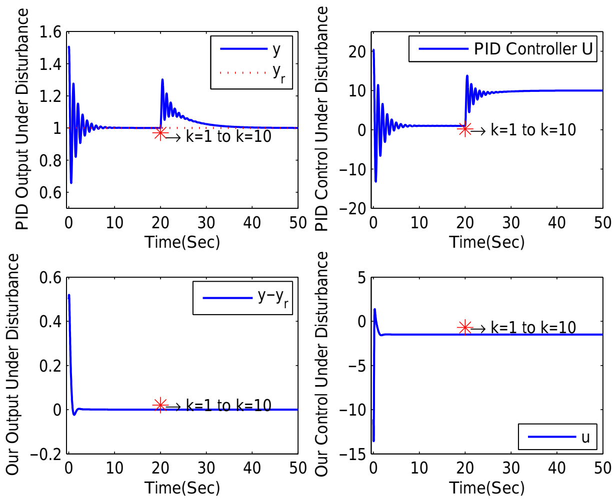


Fig. 3. The response using PID and current method under disturbance.

decay rate for the iISS dynamic uncertainties. Different from the existing related results, we employ a novel ESO to present a systematic output feedback set-point tracking control scheme. It requires only fewer dynamic variables and no redesign when the tracked signals change, and then the computation burden can be reduced accordingly; therefore, it is more convenient to implement this algorithm in practice. The mass-spring mechanical system is used to verify the proposed algorithm, and the simulation results give a good control performance. One interesting subject of future study is how to achieve the robust set-point tracking control with time delay [24].

REFERENCES

[1] I. Karafyllis and Z. P. Jiang, *Stability and Stabilisation of Nonlinear Systems*, Springer-Verlag, London, 2011.

[2] E. D. Sontag, "Smooth stabilization implies comprime factorization," *IEEE Trans. Autom. Control*, vol. 34, no. 4, pp. 435-443, 1989.

[3] D. Angeli, E. D. Sontag, and Y. Wang, "A characterization of integral input to state stability," *IEEE Trans. Autom. Control*, vol. 45, no. 6, pp. 1082-1087, 2000.

[4] Z. P. Jiang, I. Mareels, D. J. Hill, and J. Huang, "A unifying framework for global regulation via nonlinear output feedback from ISS to iISS," *IEEE Trans. Autom. Control*, vol. 49, no. 4, pp. 49-562, 2004.

[5] X. Yu, Y. Q. Wu, and X. J. Xie, "Reduced-order observer-based output feedback regulation for a class of nonlinear systems with iISS inverse dynamics," *Int. J. Control*, vol. 85, no. 12, pp. 1942-1951, 2012. [click]

[6] D. B. Xu, J. Huang, and Z. P. Jiang, "Global adaptive output regulation for a class of nonlinear systems with iISS inverse dynamics using output feedback," *Automatica*, vol. 49, no. 7, pp. 2184-2191, 2013. [click]

[7] Y. Q. Wu, J. B. Yu, and Y. Zhao, "Further results on global asymptotic regulation control for a class of nonlinear systems with iISS inverse dynamics," *IEEE Trans. Autom. Control*, vol. 56, no. 4, pp. 941-946, 2011. [click]

[8] X. M. Sun and W. Wang, "Integral input-to-state stability for hybrid delayed systems with unstable continuous dynamics," *Automatica*, vol. 48, no. 9, pp. 2359-2364, 2012. [click]

[9] S. H. Ding, J. D. Wang, and W. X. Zheng, "Second-order sliding mode control for nonlinear uncertain systems bounded by positive functions," *IEEE Trans. Ind. Electron.*, vol. 62, no. 9, pp. 5899-5909, 2015.

[10] S. H. Ding, S. H. Li, and W. X. Zheng, "Nonsmooth stabilization of a class of nonlinear cascaded systems," *Automatica*, vol. 48, no. 10, pp. 2597-2606, 2012. [click]

[11] J. B. Yu, A.L. Shi and J. L. Yao, "Global asymptotic regulation control by output feedback for a class of nonlinear systems," *Proc. of the 34th Chinese Control Conf.*, pp. 895-900, 2015.

[12] R. Freeman and P. V. Kokotović, *Robust Nonlinear Control Design*, Birkhäuser, Boston, 1996.

[13] Z. P. Jiang and I. Mareels, "Robust nonlinear integral control," *IEEE Trans. Autom. Control*, vol. 46, no. 8, pp. 1336-1342, 2001. [click]

[14] J. B. Yu and Y. Q. Wu, "Global set-point tracking control for a class of nonlinear systems and its application in continuously stirred tank reactor systems," *IET Control Theory Appl.*, vol. 6, no. 12, pp. 1965-1971, 2012. [click]

[15] J. B. Yu, J. Z. Wang, C. X. Zhang, and Y. Q. Wu, "Output feedback regulation control for a class of uncertain nonlinear systems," *ASME J. Dyn. Syst. Meas. Control*, vol. 137, no. 4, pp. 041019-1-5, 2015. [click]

[16] K. Dupree, C. H. Liang, G. Hu, and W. E. Dixon, "Adaptive Lyapunov-based control of a robot and mass-spring system undergoing an impact collision," *IEEE Trans. on Systems, Man, and Cybernetics, Part B: Cybernetics*, vol. 38, no. 4, pp. 1050-1061, 2008. [click]

[17] H. Y. Li, Y. N. Peng, and P. Shi, "Switched fuzzy output feedback control and its application to Mass-spring-damping system," *IEEE Trans. Fuzzy Systems*, vol. 24, no. 6, pp. 1259-1269, 2016.

[18] M. Krstić, I. Kanellakopoulos, and P. V. Kokotović, *Nonlinear and Adaptive Control Design*, Wiley, New York, 1995.

[19] Z. P. Jiang and D. J. Hill, "A robust adaptive backstepping scheme for nonlinear systems with unmodeled dynamics," *IEEE Trans. Autom. Control*, vol. 44, no. 9, pp. 1705-1711, 1999. [click]

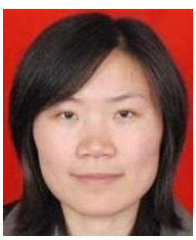
[20] J. Y. Yao, Z. X. Jiao, and D. W. Ma, "Extended-state-observer-based output feedback nonlinear robust control of hydraulic systems with Backstepping," *IEEE Trans. Ind. Electron.*, vol. 61, no. 11, pp. 6285-6293, 2014. [click]

[21] J. Y. Yao, Z. X. Jiao, and D. W. Ma, "Adaptive robust control of DC motors with extended state observer," *IEEE Trans. Ind. Electron.*, vol. 61, no. 7, pp. 3630-3637, 2014. [click]

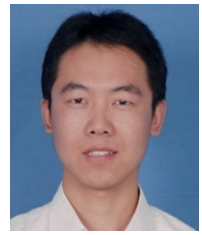
[22] J. Y. Yao, Z. X. Jiao, and D. W. Ma, "Output feedback robust control of direct current motors with nonlinear friction compensation and disturbance rejection," *ASME J. Dyn. Syst. Meas. Control*, vol. 137, no. 4, pp. 041004, 2015. [click]

[23] H. L. Xing, D. H. Li, J. Li, and C. H. Zhang, "Linear extended state observer based sliding mode disturbance decoupling control for nonlinear multivariable systems with uncertainty," *Int J Contr Autom Syst*, vol. 14, no. 4, pp. 1-10, 2016.

[24] W. Y. Yu, S. T. Liu, and F. F. Zhang, "Global output feedback regulation of uncertain nonlinear systems with unknown time delay," *Int J Contr Autom Syst*, vol. 13, no. 2, pp. 1-9, 2015.



Yan Zhao received the B.S. and M.S. degrees in Applied Mathematics and Control Theory from Qufu Normal University, and the Ph.D. degree in Control Science and Engineering from Southeast University, in 2007, 2010 and 2014, respectively. She is currently a Lecturer in School of Science, Shandong Jianzhu University. Her research interests include stochastic nonholonomic systems control and nonlinear systems control.



Jiang-Bo Yu received the B.S. and M.S. degrees in Applied Mathematics and Control Theory from Qufu Normal University, and the Ph.D. degree in Control Science and Engineering from Southeast University, in 2006, 2009 and 2012, respectively. He is currently an Associate Professor in School of Science, Shandong Jianzhu University. His research interests include robust adaptive control and nonlinear systems control.

Jie Tian received her B.S., M.S. and Ph.D. degrees in Applied Mathematics from Qufu Normal University, in 2002, 2005 and 2008, respectively. She is currently an Associate Professor in School of Science, Shandong Jianzhu University. Her research interests include stochastic nonlinear control.