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Adaptive Iterative Learning Controller with Input Learning Technique for a Class of Uncertain MIMO Nonlinear Systems

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Abstract: In this paper, an adaptive iterative learning controller (AILC) with input learning technique is presented for uncertain multi-input multi-output (MIMO) nonlinear systems in the normal form. The proposed AILC learns the internal parameter of the state equation as well as the input gain parameter, and also estimates the desired input using an input learning rule to track the whole history of command trajectory. The features of the proposed control scheme can be briefly summarized as follows: 1) To the best of authors' knowledge, the AILC with input learning is first developed for uncertain MIMO nonlinear systems in the normal form; 2) The convergence of learning input error is ensured; 3) The input learning rule is simple; therefore, it can be easily implemented in industrial applications. With the proposed AILC scheme, the tracking error and desired input error converge to zero as the repetition of the learning operation increases. Single-link and two-link manipulators are presented as simulation examples to confirm the feasibility and performance of the proposed AILC.

Keywords: Adaptive control, iterative learning control, multi-input multi-output systems, nonlinear systems, robot manipulators, uncertain systems.

1. INTRODUCTION

Iterative learning control of nonlinear systems has been attracted a lot of attention from the control community [1–5]. The basic idea behind iterative learning control is that the knowledge obtained from the previous iteration is used to improve the control input for the current iteration. Hence, the control input in each iteration is adjusted using the tracking error signals obtained from the previous trial. In its initial stage, the iterative learning controller was developed for nonlinear systems that have constant input and output gains [6]. In this scheme, the time derivative of the output error was used to modify the control input for the next iteration. A learning scheme that does not use the time derivative of the output/state variable was proposed for robotic systems in [7], and it was extended to a class of nonlinear systems whose input gain satisfies the global Lipschitz condition [8]. ILC laws based on the relative degree of nonlinear system were proposed in [9]. Further, a P-type learning controller was proposed for nonlinear time-varying system in [10]. Since then, the iterative learning scheme has been extended to a general nonlinear system in [11]. However, these ILC techniques have problems such as the requirement of the global Lipschitz condition, initial resetting condition, and low convergence rate.

To overcome these difficulties, an adaptive iterative learning controller (AILC) was introduced in the early-90's. The AILC accommodates the adaptive control technique that adjusts the control parameters over successive repetitions. The AILC scheme was first proposed for uncertain robotic system in which the uncertainty is in linear parameters [12–14]. Afterward, the results were extended to a class of nonlinear systems. The state-tracking problems for the parametric uncertain system in the normal form [15] and with time-varying parameters [16] were solved, where parameter adjustment was performed in the time domain [15] and iteration domain [16], respectively. A large class of uncertain nonlinear system was considered in [17, 18] using the hybrid system parameter adjustment technique. Recently, multi-input multi-output (MIMO) nonlinear systems with iteration-varying initial error and reference trajectory were considered in [19].

A new AILC approach, an AILC with the input learning

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technique was introduced in [20], in which the unknown periodic desired input is learned. Using this technique, the state feedback problem was solved for the single-input single-output (SISO) feedback linearizable system with matched uncertainty in [21] and the output feedback problem was solved for a class of minimum-phase nonlinear systems with output dependent nonlinearities in [22, 23]. Since then, the AILCs with the input learning have been proposed for nonlinear systems in the normal form and the cascaded form in [24], a class of partially feedback linearizable system in [25], a class of nonlinear systems in a lower triangular form with general unknown periodic uncertainties in [26], and for relative degree one systems with output dependent uncertainties and for nonlinear systems with matching uncertainties in [27]; these works are only limited to SISO systems. Comparing to SISO systems, MIMO systems cover larger industrial applications such as excavators, chemical reactors, robot manipulators, and unmanned vehicles. Thus, the AILC with input learning must be further investigated in uncertain MIMO nonlinear systems.

Motivated by the aforementioned discussion, in this paper, the AILC with input learning technique is developed for uncertain MIMO nonlinear systems in the normal form. The proposed controller consists of three learning rules that estimate the unknown parameters in the system and input gain, and the unknown desired input. After the sufficient learning of the control input occurs, the feedback control gain can be decreased, and so the proposed AILC becomes more robust to practical problems such as actuator saturation, unmodelled dynamics, and noise vulnerability than the AILC without input learning. The main advantages of the proposed control scheme are that 1) To the best of our knowledge, the AILC with input learning is first developed for uncertain MIMO nonlinear systems in the normal form; 2) The convergence of learning input error is ensured; 3) The input learning rule is simple; therefore, it can be easily implemented in industrial applications. The proposed controller achieves asymptotic convergence of both the tracking error and desired input error as it performs a repetitive task. Simulation results on robotic applications are presented to validate the effectiveness of proposed control scheme. It is also worth highlighting that two-link manipulator is formulated as uncertain MIMO nonlinear system in the normal form for the first time.

The remainder of this paper is organized as follows. Uncertain SISO/MIMO nonlinear systems in the normal form are described in Section 2. The AILC with input learning is proposed and shows its asymptotic convergence for both the SISO/MIMO systems in Section 3. The simulation results are presented in Section 4 and the conclusions are discussed in Section 5.

In the subsequent discussion, the following notation and definitions will be used. R^n is the n-dimensional Euclidean space over R endowed with L^2-norm $||x|| = (\Sigma_{i=1}^{n}|x_i|^2)^{\frac{1}{2}}$. For any $m\times n$ matrix B, $\lambda_{min}(B)$ is the smallest eigenvalue. A hat over a variable (i.e. $(\hat{\cdot})$) denotes the estimated value of $(\cdot)$.

The signum function $\mathrm{sgn}(\cdot)$ is defined as

$$\mathrm{sgn}(x) \equiv \begin{cases} -1 & \text{if} \quad x<0 \\ 0 & \text{if} \quad x=0 \\ 1 & \text{if} \quad x>0, \end{cases} \tag{1}$$

where x is scalar variable. The projection operation $\mathrm{Proj}(\cdot)$ is described as

$$\mathrm{Proj}(\hat{x}) \equiv \begin{cases} -x^b & \text{if} \quad \hat{x}\le -x^b \\ \hat{x} & \text{if} \quad -x^b<\hat{x}<x^b \\ x^b & \text{if} \quad \hat{x}\ge x^b, \end{cases} \tag{2}$$

where x^b is the upper bound of $|x|$. For notational brevity, the time variable t is omitted from the time-dependent variables except for the variables in the adaptive learning law.

2. PROBLEM FORMULATION AND PRELIMINARIES

2.1. Uncertain SISO nonlinear system in the normal form

Consider uncertain SISO nonlinear system in the normal form as:

$$\begin{aligned} \dot{x}_1 &= x_2, \\ &\vdots \\ \dot{x}_{n-1} &= x_n, \\ \dot{x}_n &= f(\mathbf{x})+g(\mathbf{x})u, \\ y &= x_1, \end{aligned} \tag{3}$$

where $\mathbf{x}=[x_1,\dots,x_n]\in R^n$, $y\in R$, and $u\in R$ are the state, output, and control input, respectively. The functions $f(\mathbf{x})$ and $g(\mathbf{x})$ are bounded Lipschitz continuous functions of $\mathbf{x}$ and can be linearly parameterized as

$$\begin{aligned} f(\mathbf{x}) &= \Sigma_{i=1}^{n_1} f_i(\mathbf{x})\theta_{1i} = \mathbf{w_1^T}\boldsymbol{\theta_1}, \\ g(\mathbf{x}) &= \Sigma_{j=1}^{n_2} g_j(\mathbf{x})\theta_{2j} = \mathbf{w_2^T}\boldsymbol{\theta_2}, \end{aligned} \tag{4}$$

where $\mathbf{w_1^T}=[f_1(\mathbf{x}),\dots,f_{n_1}(\mathbf{x})]\in R^{n_1}$, $\mathbf{w_2^T}=[g_1(\mathbf{x}),\dots,g_{n_2}(\mathbf{x})] \in R^{n_2}$ with $f_i(\mathbf{x})\in R$, $g_j(\mathbf{x})\in R$ being known bounded uniformly continuous functions, $\boldsymbol{\theta_1}=[\theta_{11},\dots,\theta_{1n_1}]^T\in R^{n_1}$, $\boldsymbol{\theta_2}=[\theta_{21},\dots,\theta_{2n_2}]^T\in R^{n_2}$ with $\theta_{1i}\in R$, $\theta_{2j}\in R$ being unknown parameters.

Further, consider the desired system:

$$\begin{aligned} \dot{x}_{1d} &= x_{2d}, \\ &\vdots \end{aligned}$$

$$\dot{x}_{n-1d} = x_{nd},$$
$$\dot{x}_{nd} = f(\mathbf{x_d}) + g(\mathbf{x_d})u_d,$$
$$y_d = x_{1d}, \tag{5}$$

where $\mathbf{x_d} = [x_{1d},...,x_{nd}] \in R^n$, $y_d \in R$, and $u_d \in R$ are the desired state, output, and input, respectively. Again, $f(\mathbf{x_d})$ and $g(\mathbf{x_d})$ can be linearly parameterized as $f(\mathbf{x_d}) = \mathbf{w_{1d}^T}\boldsymbol{\theta_1}$ and $g(\mathbf{x_d}) = \mathbf{w_{2d}^T}\boldsymbol{\theta_2}$.

For the proposed controller design, the following assumptions, corollary, and lemma are introduced.

Assumption 1: The desired system is invertible; that is, for each y_d, this system has a unique solution u_d that can be obtained using inverse dynamics.

Assumption 2: Let t_i, $i = 0,1,2,...$, be a sequence satisfying $t_0 = 0$ and $t_{i+1} - t_i = T > 0$. The desired input $u_d(t)$ is uniformly continuous on each interval $[t_i,t_{i+1})$.

Assumption 3: The system parameter θ_1, input gain parameter θ_2, desired input u_d, and desired output y_d are periodic with T; that is, $\boldsymbol{\theta_1}(t) = \boldsymbol{\theta_1}(t-T)$, $\boldsymbol{\theta_2}(t) = \boldsymbol{\theta_2}(t-T)$, $u_d(t) = u_d(t-T)$, and $y_d(t) = y_d(t-T)$ $\forall t \in [T,\infty)$.

Assumption 4: The system parameter θ_1, input gain parameter θ_2, desired input u_d, desired output y_d, and its derivatives $y_d^{(1)},...,y_d^{(n)}$ are all bounded.

Assumption 5: There exists a positive constant $\bar{g}$ such that $0 < \bar{g} < |g(\mathbf{x})|$ for all x, and the sign of the input gain $g(\mathbf{x})$ is known.

Corollary 1:(Barbalat's Corollary). If $h(t)$, $\dot{h}(t) \in L_\infty$ and $h(t) \in L_2$, then $\lim_{t\to\infty} h(t) = 0$.

Lemma 1:(Generalized Barbalet's Lemma). Let t_i, $i = 0,1,2,...$, be a sequence satisfying $t_0 = 0$, $t_{i+1} - t_i \geq \tau > 0$. If the differentiable function $h(t)$ has a finite limit as $t \to \infty$ and if $\dot{h}(t)$ is uniformly continuous on each interval $[t_i,t_{i+1})$, then $\dot{h}(t) \to 0$ as $t \to \infty$.

Proof: The proof is quite similar to that of lemma 1 in [28]. Suppose that $\dot{h}(t)$ does not approach zero as $t \to \infty$. Then there exists an infinite sequence $\{\tau_{s_1},\tau_{s_2},...\}$, $\tau_{s_k} \in [t_{s_k},t_{s_{k+1}})$, $\lim_{k\to\infty} \tau_{s_k} = +\infty$ such that $|\dot{h}(\tau_{s_k})| > \varepsilon_0$ for some positive number ε_0.

Since $\lim_{t\to\infty} h(t)$ exists, by Cauchy's convergence criterion, for any $\varepsilon > 0$, there exists a positive number T, such that for any $T_2 > T_1 > T$, $|h(T_2) - h(T_1)| < \varepsilon$, that is,

$$\left| \int_{T_1}^{T_2} \dot{h}(t)dt \right| < \varepsilon. \tag{6}$$

Let $\delta = \min\{\delta_{min},\tau\}$. Then, $t_{s_{k+1}} - t_{s_k} > \tau \geq \delta$. We have either

$$(\tau_{s_k} - \frac{\delta}{2}, \tau_{s_k}] \subset [t_{s_k}, t_{s_k+1}), \tag{7}$$

or

$$[\tau_{s_k}, \tau_{s_k} + \frac{\delta}{2}) \subset [t_{s_k}, t_{s_k+1}). \tag{8}$$

When (7) holds, by uniform continuity, for any $t \in (\tau_{s_k} - \delta/2, \tau_{s_k}]$, we have

$$|\dot{h}(\tau_{s_k})| - |\dot{h}(t)| \leq |\dot{h}(\tau_{s_k}) - \dot{h}(t)| \leq \frac{\varepsilon_0}{2}. \tag{9}$$

From (9) and $|\dot{h}(\tau_{s_k})| > \varepsilon_0$, we have

$$|\dot{h}(t)| \geq |\dot{h}(\tau_{s_k})| - \frac{\varepsilon_0}{2} > \frac{\varepsilon_0}{2}. \tag{10}$$

By continuity of $\dot{h}(t)$ over $(\tau_{s_k} - \delta/2, \tau_{s_k}]$ and (10), $\dot{h}(t)$ does not change sign for any $t \in (\tau_{s_k} - \delta/2, \tau_{s_k}]$. Then

$$\left| \int_{\tau_{s_k}-\delta/2}^{\tau_{s_k}} \dot{h}(s)ds \right| = \int_{\tau_{s_k}-\delta/2}^{\tau_{s_k}} |\dot{h}(s)|ds > \int_{\tau_{s_k}-\delta/2}^{\tau_{s_k}} \frac{\varepsilon_0}{2} ds > \frac{\delta\varepsilon_0}{4} > 0. \tag{11}$$

Similarly, when (8) holds, we have

$$\left| \int_{\tau_{s_k}}^{\tau_{s_k}+\delta/2} \dot{h}(s)ds \right| > \frac{\delta\varepsilon_0}{4} > 0. \tag{12}$$

Both (11) and (12) contradict (6) for sufficiently large τ_{s_k}. Thus, $\lim_{t\to\infty} \dot{h}(t) = 0$.

2.2. Uncertain MIMO nonlinear system in the normal form

Consider the following uncertain MIMO nonlinear system in the normal form:

$$\begin{aligned}
\dot{x}_{11} &= x_{12},\\
&\vdots\\
\dot{x}_{1n_1-1} &= x_{1n_1},\\
\dot{x}_{1n_1} &= f_1(\mathbf{x}) + \Sigma_{i=1}^{p} g_{1i}(\mathbf{x})u_i,\\
\dot{x}_{21} &= x_{22},\\
&\vdots\\
\dot{x}_{pn_p-1} &= x_{pn_p},\\
\dot{x}_{pn_p} &= f_p(\mathbf{x}) + \Sigma_{i=1}^{p} g_{pi}(\mathbf{x})u_i,\\
y_1 &= x_{11},\\
y_2 &= x_{21},\\
&\vdots\\
y_p &= x_{p1},
\end{aligned} \tag{13}$$

where $\mathbf{x} = [x_{11},...,x_{pn_p}] \in R^n$ with $n = n_1 + \cdots + n_p$, $y_i \in R$, and $u_i \in R$ for all $i = 1,...,p$. The functions $f_i(\mathbf{x})$ and $g_{ij}(\mathbf{x})$ are bounded Lipschitz continuous functions of $\mathbf{x}$ for all $i = 1,...,p$ and $j = 1,...,p$.

Combining the u_i-related equations in (13), we have

$$\begin{bmatrix} y_1^{(n_1)} \\ \vdots \\ y_p^{(n_p)} \end{bmatrix} = \begin{bmatrix} f_1(\mathbf{x}) \\ \vdots \\ f_p(\mathbf{x}) \end{bmatrix} + \mathbf{A}(\mathbf{x}) \begin{bmatrix} u_1 \\ \vdots \\ u_p \end{bmatrix}, \tag{14}$$

where the decoupling matrix is

$$\mathbf{A}(\mathbf{x}) = \begin{bmatrix} g_{11}(\mathbf{x}) & \cdots & g_{1p}(\mathbf{x}) \\ \vdots & \ddots & \vdots \\ g_{p1}(\mathbf{x}) & \cdots & g_{pp}(\mathbf{x}) \end{bmatrix}. \tag{15}$$

In a MIMO system, we assume that $[f_1(\mathbf{x}),\ldots,f_p(\mathbf{x})]^T$ and $\mathbf{A}(\mathbf{x})$ can be linearly parameterized as:

$$[f_1(\mathbf{x}),\ldots,f_p(\mathbf{x})]^T = \Sigma_{i=1}^{n_1}\mathbf{w_{1i}^T}\theta_{\mathbf{1i}} = \mathbf{W_1^T}\theta_{\mathbf{1}}, \tag{16}$$

$$\mathbf{A}(\mathbf{x}) = \Sigma_{i=1}^{n_2}\mathbf{A_i}(\mathbf{x})\theta_{2i}, \tag{17}$$

where $\mathbf{W_1^T} = [\mathbf{w_{11}^T},\ldots,\mathbf{w_{1n_1}^T}] \in R^{p\times n_1}$ with $\mathbf{w_{1i}^T}$ being a known column vector, $\mathbf{A_i}(\mathbf{x}) \in R^{p\times n_2}$ is a known matrix, $\theta_{\mathbf{1}} = [\theta_{11},\ldots,\theta_{1n_1}] \in R^{n_1}$ with $\theta_{1i} \in R$ being an unknown parameter, $\theta_{2i} \in R$ is an unknown parameter.

Further, consider the desired system:

$$\begin{aligned}
\dot{x}_{1d1} &= x_{1d2},\\
&\vdots\\
\dot{x}_{1dn_1-1} &= x_{1dn_1},\\
\dot{x}_{1dn_1} &= f_1(\mathbf{x_d}) + \Sigma_{i=1}^{p} g_{1i}(\mathbf{x_d})u_{id},\\
\dot{x}_{2d1} &= x_{2d2},\\
&\vdots\\
\dot{x}_{pdn_p-1} &= x_{pdn_p},\\
\dot{x}_{pdn_p} &= f_p(\mathbf{x_d}) + \Sigma_{i=1}^{p} g_{pi}(\mathbf{x_d})u_{id},\\
y_{1d} &= x_{1d1},\\
y_{2d} &= x_{2d1},\\
&\vdots\\
y_{pd} &= x_{pd1},
\end{aligned} \tag{18}$$

where $\mathbf{x_d} = [x_{1d1},\ldots,x_{pdn_p}] \in R^n$, $y_{id} \in R$, and $u_{id} \in R$ for all $i = 1,\ldots,p$.

Combining the u_{id}-related equations in (18), we have

$$\begin{bmatrix} y_{1d}^{(n_1)} \\ \vdots \\ y_{pd}^{(n_p)} \end{bmatrix} = \begin{bmatrix} f_1(\mathbf{x_d}) \\ \vdots \\ f_p(\mathbf{x_d}) \end{bmatrix} + \mathbf{A}(\mathbf{x_d}) \begin{bmatrix} u_{1d} \\ \vdots \\ u_{pd} \end{bmatrix}, \tag{19}$$

where the decoupling matrix is

$$\mathbf{A}(\mathbf{x_d}) = \begin{bmatrix} g_{11}(\mathbf{x_d}) & \cdots & g_{1p}(\mathbf{x_d}) \\ \vdots & \ddots & \vdots \\ g_{p1}(\mathbf{x_d}) & \cdots & g_{pp}(\mathbf{x_d}) \end{bmatrix}. \tag{20}$$

Both $[f_1(\mathbf{x_d}),\ldots,f_p(\mathbf{x_d})]^T$ and $\mathbf{A}(\mathbf{x_d})$ can be linearly parameterized as $[f_1(\mathbf{x_d}),\ldots,f_p(\mathbf{x_d})]^T = \mathbf{W_{1d}^T}\theta_{\mathbf{1}}$, $\mathbf{A}(\mathbf{x_d}) = \Sigma_{i=1}^{n_2}\mathbf{A_i}(\mathbf{x_d})\theta_{\mathbf{2i}}$. Denoting $\mathbf{u_d} = [u_{1d},\ldots,u_{pd}]^T$ and $\mathbf{y_d} = [y_{1d},\ldots,y_{pd}]^T$, we have the same assumptions 1-4 as in the SISO system, and another assumption 5: $\mathbf{A}(\mathbf{x})$ must be a positive definite matrix for all $\mathbf{x}$.

3. ADAPTIVE ITERATIVE LEARNING CONTROLLER DESIGN

3.1. Controller design for the SISO system

Consider the u-related equation in (3) for the uncertain SISO nonlinear system, and the u_d-related equation in (5)

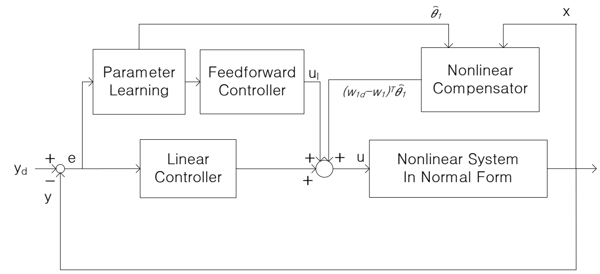

Fig. 1. Schematic diagram of the adaptive iterative learning control system.

for its desired system

$$y^{(n)} = f(\mathbf{x}) + g(\mathbf{x})u, \tag{21}$$

$$y_d^{(n)} = f(\mathbf{x_d}) + g(\mathbf{x_d})u_d. \tag{22}$$

Subtracting (21) from (22) and linearly parameterizing the uncertain terms, we have

$$\begin{aligned} y_d^{(n)} - y^{(n)} &= f(\mathbf{x_d}) - f(\mathbf{x}) + g(\mathbf{x_d})u_d - g(\mathbf{x})u \\ &= \mathbf{w_{1d}^T}\theta_1 - \mathbf{w_1^T}\theta_1 + \mathbf{w_{2d}^T}\theta_2\mathbf{u_d} - \mathbf{w_2^T}\theta_2\mathbf{u}. \end{aligned} \tag{23}$$

The control law is

$$u = \frac{1}{\mathbf{w_2^T}\hat{\theta}_2}(v + \mathbf{w_{2d}^T}\hat{\theta}_2 u_l), \tag{24}$$

where $v = a_1 e^{(n-1)} + \cdots + a_n e + (\mathbf{w_{1d}} - \mathbf{w_1})^{\mathbf{T}}\hat{\theta}_{\mathbf{1}} \in R$, $\mathbf{e} = [e,\ldots,e^{(n-1)}]^T \in R^n$, and $e = y_d - y$. The positive constants $a_1,\ldots,a_n$ are chosen such that $s^n + a_1 s^{n-1} + \cdots + a_n$ becomes a Hurwitz polynomial. By using adaptive learning laws, $\hat{\theta}_{\mathbf{1}} \in R^{n_1}$, $\hat{\theta}_{\mathbf{2}} \in R^{n_2}$, and $u_l \in R$ respectively learn the system parameter $\theta_{\mathbf{1}}$, input gain parameter $\theta_{\mathbf{2}}$, and desired input u_d. This control scheme (Fig. 1) consists of three parts. The feedback term $a_1 e^{(n-1)} + \cdots + a_n e$ in v makes the closed loop system stable within a uniform error bound. The term $(\mathbf{w_{1d}} - \mathbf{w_1})^{\mathbf{T}}\hat{\theta}_{\mathbf{1}}$ in v compensates for the nonlinear part of the system. The learning input u_l estimates the unknown desired input u_d and also compensates for the nonlinear part of the system. Substituting (24) into (23) yields

$$\begin{aligned} e^{(n)} &= \mathbf{w_{1d}^T}\theta_1 - \mathbf{w_1^T}\theta_1 + \mathbf{w_{2d}^T}\theta_2 u_d - a_1 e^{(n-1)} - \cdots \\ &\quad - a_n e - (\mathbf{w_{1d}} - \mathbf{w_1})^{\mathbf{T}}\hat{\theta}_{\mathbf{1}} - \mathbf{w_{2d}^T}\hat{\theta}_2 u_l \\ &\quad - \frac{\mathbf{w_2^T}(\theta_2 - \hat{\theta}_2)}{\mathbf{w_2^T}\hat{\theta}_2}(v + \mathbf{w_{2d}^T}\hat{\theta}_2 u_l). \end{aligned} \tag{25}$$

Moving the error terms to the left-hand side of (25), we obtain the error dynamics as:

$$\begin{aligned} & e^{(n)} + a_1 e^{(n-1)} + \cdots + a_n e \\ &= (\mathbf{w_{1d}} - \mathbf{w_1})^{\mathbf{T}}\tilde{\theta}_{\mathbf{1}} + (\mathbf{w_{2d}^T}u_l - \mathbf{w_2^T}u)\tilde{\theta}_2 + \mathbf{w_{2d}^T}\theta_2\tilde{u}_l \\ &= \mathbf{w_{1e}^T}\tilde{\theta}_1 + \mathbf{w_{2e}^T}\tilde{\theta}_2 + \mathbf{w_{2d}^T}\theta_2\tilde{u}_l \end{aligned}$$

$$= \mathbf{w_e^T}\tilde{\theta} + \mathbf{w_{2d}^T}\theta_2\tilde{u}_l, \tag{26}$$

where $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1 \in R^{n_1}$, $\tilde{\theta}_2 = \theta_2 - \hat{\theta}_2 \in R^{n_2}$, $\tilde{u}_l = u_d - u_l \in R$, $\mathbf{w_{1e}^T} = (\mathbf{w_{1d}} - \mathbf{w_1})^\mathbf{T} \in R^{n_1}$, $\mathbf{w_{2e}^T} = \mathbf{w_{2d}^T}u_l - \mathbf{w_2^T}u \in R^{n_2}$, $\mathbf{w_e^T} = [\mathbf{w_{1e}^T}, \mathbf{w_{2e}^T}] \in R^{n_1+n_2}$, and $\tilde{\theta} = [\tilde{\theta}_1^T, \tilde{\theta}_2^T]^T \in R^{n_1+n_2}$.

If the bias term $\mathbf{w_e^T}\tilde{\theta} + \mathbf{w_{2d}^T}\theta_2\tilde{u}_l$ on the right hand side of (26) is zero, then the closed-loop system would be asymptotically stable. However, the bias term prevents the tracking error $\mathbf{e}$ from being zero. Hence an adaptive learning mechanism must be used to estimate and cancel the bias term. In the following theorem, we propose adaptive learning laws for the system parameter θ_1, input gain parameter θ_2, and desired input u_d.

Theorem 1: Consider the uncertain SISO nonlinear system in (3) with Assumptions 1-5. Consider the control law as in (24) and choose $\hat{\theta}_1(t)$, $\hat{\theta}_2(t)$, $u_l(t)$ as

$$\hat{\theta}_1(t) = \hat{\theta}_{1pr}(t-T) + \beta_1\mathbf{w_{1e}}\mathbf{b^T}\mathbf{Pe}, \tag{27}$$

$$\hat{\theta}_2(t) = \hat{\theta}_{2pr}(t-T) + \beta_2\mathbf{w_{2e}}\mathbf{b^T}\mathbf{Pe}, \tag{28}$$

$$u_l(t) = u_{lpr}(t-T) + \beta_3\mathbf{sgn}(\mathbf{w_{2d}^T}\theta_2)\mathbf{b^T}\mathbf{Pe}, \tag{29}$$

where

$$\hat{\theta}_{1pr}(t-T) = \mathrm{Proj}(\hat{\theta}_1(t-T)) = [\mathrm{Proj}(\hat{\theta}_{11}(t-T)), ..., \mathrm{Proj}(\hat{\theta}_{1n_1}(t-T))]^T, \tag{30}$$

$$\hat{\theta}_{2pr}(t-T) = \mathrm{Proj}(\hat{\theta}_2(t-T)) = [\mathrm{Proj}(\hat{\theta}_{21}(t-T)), ..., \mathrm{Proj}(\hat{\theta}_{2n_2}(t-T))]^T, \tag{31}$$

$$u_{lpr}(t-T) = \mathrm{Proj}(u_l(t-T)), \tag{32}$$

and β_1, β_2, β_3 are positive learning gains. If the control scheme satisfies the condition that $\mathbf{w_2^T}\hat{\theta}_2$ is bounded away from zero, then the tracking error $\mathbf{e}$, parameter errors $\tilde{\theta}_1$ and $\tilde{\theta}_2$, and desired input error $\tilde{u}_l$ are bounded, and $\mathbf{e}$ asymptotically converges to the origin. Moreover, $\tilde{u}_l$ asymptotically converges to zero.

Proof: Applying the control input in (24) to the dynamics in the normal form in (3), we have (26) and error dynamics in the state-space form as:

$$\dot{\mathbf{e}} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & \cdots & -a_1 \end{bmatrix}\mathbf{e} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}(\mathbf{w_e^T}\tilde{\theta} + \mathbf{w_{2d}^T}\theta_2\tilde{u}_l) = \mathbf{Ae} + \mathbf{b}(\mathbf{w_e^T}\tilde{\theta} + \mathbf{w_{2d}^T}\theta_2\tilde{u}_l), \tag{33}$$

and obtain $\mathbf{P}$ by solving the Lyapunov equation $\mathbf{PA} + \mathbf{A^TP} = -\mathbf{Q}$. Then, using this $\mathbf{P}$, we first choose the Lyapunov function candidate as:

$$V = \mathbf{e^TPe} + \int_{t-T}^{t}\left[\frac{1}{\beta_1}\tilde{\theta}_1^\mathbf{T}(\tau)\tilde{\theta}_1(\tau) + \frac{1}{\beta_2}\tilde{\theta}_2^\mathbf{T}(\tau)\tilde{\theta}_2(\tau) + \frac{|\mathbf{w_{2d}^T}\theta_2|}{\beta_3}\tilde{u}_l^2(\tau)\right]d\tau. \tag{34}$$

After taking the time derivative on each side of (34), we get

$$\begin{aligned}\dot{V} &= \dot{\mathbf{e}}^\mathbf{T}\mathbf{Pe} + \mathbf{e^TP}\dot{\mathbf{e}} \\ &\quad + \frac{1}{\beta_1}(\tilde{\theta}_1^\mathbf{T}(t)\tilde{\theta}_1(t) - \tilde{\theta}_1^\mathbf{T}(t-T)\tilde{\theta}_1(t-T)) \\ &\quad + \frac{1}{\beta_2}(\tilde{\theta}_2^\mathbf{T}(t)\tilde{\theta}_2(t) - \tilde{\theta}_2^\mathbf{T}(t-T)\tilde{\theta}_2(t-T)) \\ &\quad + \frac{|\mathbf{w_{2d}^T}\theta_2|}{\beta_3}(\tilde{u}_l^2(t) - \tilde{u}_l^2(t-T)) \\ &\leq \dot{\mathbf{e}}^\mathbf{T}\mathbf{Pe} + \mathbf{e^TP}\dot{\mathbf{e}} \\ &\quad + \frac{1}{\beta_1}(\tilde{\theta}_1^\mathbf{T}(t)\tilde{\theta}_1(t) - \tilde{\theta}_{1pr}^\mathbf{T}(t-T)\tilde{\theta}_{1pr}(t-T)) \\ &\quad + \frac{1}{\beta_2}(\tilde{\theta}_2^\mathbf{T}(t)\tilde{\theta}_2(t) - \tilde{\theta}_{2pr}^\mathbf{T}(t-T)\tilde{\theta}_{2pr}(t-T)) \\ &\quad + \frac{|\mathbf{w_{2d}^T}\theta_2|}{\beta_3}(\tilde{u}_l^2(t) - \tilde{u}_{lpr}^2(t-T)).\end{aligned} \tag{35}$$

Subtracting $\theta_1(t) = \theta_1(t-T)$, $\theta_2(t) = \theta_2(t-T)$, $u_d(t) = u_d(t-T)$ from (27)–(29), we have

$$\tilde{\theta}_1(t) = \tilde{\theta}_{1pr}(t-T) - \beta_1\mathbf{w_{1e}}\mathbf{b^T}\mathbf{Pe}, \tag{36}$$

$$\tilde{\theta}_2(t) = \tilde{\theta}_{2pr}(t-T) - \beta_2\mathbf{w_{2e}}\mathbf{b^T}\mathbf{Pe}, \tag{37}$$

$$\tilde{u}_l(t) = \tilde{u}_{lpr}(t-T) - \beta_3\mathbf{sgn}(\mathbf{w_{2d}^T}\theta_2)\mathbf{b^T}\mathbf{Pe}, \tag{38}$$

where $\tilde{\theta}_{1pr}(t-T) = \theta_1(t-T) - \hat{\theta}_{1pr}(t-T)$, $\tilde{\theta}_{2pr}(t-T) = \theta_2(t-T) - \hat{\theta}_{2pr}(t-T)$, $\tilde{u}_{lpr}(t-T) = u_d(t-T) - u_{lpr}(t-T)$.

Applying (36)–(38) to (35) gives

$$\begin{aligned}\dot{V} &\leq \mathbf{e^T}(\mathbf{PA} + \mathbf{A^TP})\mathbf{e} + 2\mathbf{e^TPbw_{1e}^T}\tilde{\theta}_1(t) \\ &\quad + 2\mathbf{e^TPbw_{2e}^T}\tilde{\theta}_2(t) + 2\mathbf{e^TPbw_{2d}^T}\theta_2\tilde{u}_l(t) \\ &\quad - 2\mathbf{e^TPbw_{1e}^T}\tilde{\theta}_{1pr}(t-T) - 2\mathbf{e^TPbw_{2e}^T}\tilde{\theta}_{2pr}(t-T) \\ &\quad - 2\mathbf{e^TPbw_{2d}^T}\theta_2\tilde{u}_{lpr}(t-T) + \beta_1\mathbf{e^TPbw_{1e}^Tw_{1e}b^TPe} \\ &\quad + \beta_2\mathbf{e^TPbw_{2e}^Tw_{2e}b^TPe} + \beta_3\mathbf{e^TPb}|\mathbf{w_{2d}^T}\theta_2|\mathbf{b^TPe} \\ &= \mathbf{e^T}(\mathbf{PA} + \mathbf{A^TP})\mathbf{e} + 2\mathbf{e^TPbw_{1e}^T}(\tilde{\theta}_1(t) - \tilde{\theta}_{1pr}(t-T)) \\ &\quad + 2\mathbf{e^TPbw_{2e}^T}(\tilde{\theta}_2(t) - \tilde{\theta}_{2pr}(t-T)) \\ &\quad + 2\mathbf{e^TPbw_{2d}^T}\theta_2(\tilde{u}_l(t) - \tilde{u}_{lpr}(t-T)) \\ &\quad + \beta_1\mathbf{e^TPbw_{1e}^Tw_{1e}b^TPe} + \beta_2\mathbf{e^TPbw_{2e}^Tw_{2e}b^TPe} \\ &\quad + \beta_3\mathbf{e^TPb}|\mathbf{w_{2d}^T}\theta_2|\mathbf{b^TPe} \\ &= -\mathbf{e^TQe} - \beta_1\mathbf{e^TPbw_{1e}^Tw_{1e}b^TPe} \\ &\quad - \beta_2\mathbf{e^TPbw_{2e}^Tw_{2e}b^TPe} - \beta_3\mathbf{e^TPb}|\mathbf{w_{2d}^T}\theta_2|\mathbf{b^TPe} \\ &= -\mathbf{e^TMe} \\ &\leq 0,\end{aligned} \tag{39}$$

where $\mathbf{M} = \mathbf{Q} + \beta_1\mathbf{Pbw_{1e}^Tw_{1e}b^TP} + \beta_2\mathbf{Pbw_{2e}^Tw_{2e}b^TP} + \beta_3\mathbf{Pb}|\mathbf{w_{2d}^T}\theta_2|\mathbf{b^TP}$ is a positive definite matrix.

Because $V(t)$ is decreasing, we have $V(t) \leq V(0) < \infty$. Thus, $\mathbf{e} \in L_\infty$. $\tilde{\theta}_1 \in L_\infty$, $\tilde{\theta}_2 \in L_\infty$, $\tilde{u}_l \in L_\infty$, $\mathbf{w_1^T} \in L_\infty$, and $\mathbf{w_2^T} \in L_\infty$. Therefore, it can be easily derived that $\dot{\mathbf{e}} = \mathbf{Ae} + \mathbf{b}(\mathbf{w_e^T}\tilde{\theta} + \mathbf{w_{2d}^T}\theta_2\tilde{u}_l) \in L_\infty$. From $\dot{V} \leq -\mathbf{e^T Q e}$, we have

$$\int_0^\infty \dot{V}dt \leq -\lambda_{min}(\mathbf{Q})\int_0^\infty ||\mathbf{e}||_2^2 dt, \tag{40}$$

and

$$\int_0^\infty ||\mathbf{e}||_2^2 dt \leq \frac{V(\infty)-V(0)}{-\lambda_{min}(\mathbf{Q})} = \frac{V(0)}{\lambda_{min}(\mathbf{Q})} - \frac{V(\infty)}{\lambda_{min}(\mathbf{Q})} < \frac{V(0)}{\lambda_{min}(\mathbf{Q})} < \infty. \tag{41}$$

Therefore, $\mathbf{e} \in L_2$. By corollary 1, $\mathbf{e} \to 0$ as $t \to \infty$. As a result, this closed loop system becomes asymptotically stable.

Moreover, $\mathbf{e} \to 0$ implies that $e^{(1)}, \ldots, e^{(n-1)} \to 0$, and $\mathbf{w_e^T} \to 0$. Let $e^{(n)} \to 0$ as $t \to \infty$. Then, all terms except $\mathbf{w_{2d}^T}\theta_2\tilde{u}_l$ in (26) become zero as $t \to \infty$. Therefore, $\mathbf{w_{2d}^T}\theta_2\tilde{u}_l$ needs to become zero as $t \to \infty$. Because $0 < \bar{g} < |\mathbf{w_{2d}^T}\theta_2|$ from Assumption 5, the desired input error $\tilde{u}_l \to 0$.

The remaining part of the proof is to show that $e^{(n)} \to 0$ as $t \to \infty$. Let t_i, $i = 0, 1, 2, \ldots$, be a sequence satisfying $t_0 = 0$ and $t_{i+1} - t_i = T > 0$. To establish $e^{(n)} \to 0$ as $t \to \infty$, we need to verify that $e^{(n)} = \mathbf{w_{1d}^T}\theta_1 + \mathbf{w_{2d}^T}\theta_2 u_d - \mathbf{w_1^T}\theta_1 - \mathbf{w_2^T}\theta_2 u$ is uniformly continuous on each interval $[t_i, t_{i+1})$. This in turn requires u to be a bounded uniformly continuous function on each interval $[t_i, t_{i+1})$. First, it can be shown that the parameters $\hat{\theta}_1$, $\hat{\theta}_2$, and u_l are bounded functions from the parameter learning rules in (27)–(29), and $\mathbf{e} \to 0$. Further we assume that $\hat{\theta}_1$, $\hat{\theta}_2$, and u_l are uniformly continuous functions on each interval $[t_i, t_{i+1})$. Because $\mathbf{w_2^T}\hat{\theta}_2$ is a uniformly continuous function on each interval $[t_i, t_{i+1})$ and is bounded away from zero, $1/(\mathbf{w_2^T}\hat{\theta}_2)$ then becomes a bounded uniformly continuous function on each interval $[t_i, t_{i+1})$. $\mathbf{e}$ is also a bounded uniformly continuous function because $\mathbf{e} \in L_\infty$ and $\dot{\mathbf{e}} \in L_\infty$. Hence, it follows that the control input $u = 1/(\mathbf{w_2^T}\hat{\theta}_2) \cdot (a_1 e^{(n-1)} + \cdots + a_n e + (\mathbf{w_{1d}} - \mathbf{w_1})^{\mathbf{T}}\hat{\theta}_1 + \mathbf{w_{2d}^T}\hat{\theta}_2 u_l)$ is a bounded uniformly continuous function on each interval $[t_i, t_{i+1})$. Consequently, according to Lemma 1, $e^{(n)} \to 0$ as $t \to \infty$. □

Remark 1: The estimated parameters $\hat{\theta}_1$, $\hat{\theta}_2$, and u_l are the sums of the tracking error $\mathbf{e}$ on each interval $[t_i, t_{i+1})$ as in the learning rules in (27)–(29); therefore, these parameters are differentiable functions. Further, we use the forgetting factor technique for the learning rules in (27)–(29) as:

$$\hat{\theta}_1(t) = \lambda_1\hat{\theta}_{1pr}(t-T) + \beta_1\mathbf{w_{1e}b^T Pe}, \tag{42}$$

$$\hat{\theta}_2(t) = \lambda_2\hat{\theta}_{2pr}(t-T) + \beta_2\mathbf{w_{2e}b^T Pe}, \tag{43}$$

$$u_l(t) = \lambda_3 u_{lpr}(t-T) + \beta_3\mathbf{sgn}(\mathbf{w_{2d}^T}\theta_2)\mathbf{b^T Pe}, \tag{44}$$

where $0 < \lambda_1, \lambda_2, \lambda_3 < 1$ are the forgetting factors, and the projection operator is redefined as $\mathrm{Proj}(x) \equiv \tanh(x/x^b)$ where $x^b \in R$ is the upper bound of x. Then, assuming that the derivatives of $\mathbf{w_1}$ and $\mathbf{w_2}$ are bounded, the derivatives of $\hat{\theta}_1$, $\hat{\theta}_2$, and u_l become bounded on each interval $[t_i, t_{i+1})$. Because every function that is differentiable and has a bounded derivative is uniformly continuous, the assumption that $\hat{\theta}_1$, $\hat{\theta}_2$, and u_l are uniformly continuous functions on each interval $[t_i, t_{i+1})$ in theorem 1 is satisfied.

Remark 2: The boundedness of $1/\mathbf{w_2^T}\hat{\theta}_2$ in the control input u in (24) cannot be ensured by the learning rule in (28), because this learning rule can generate the estimate $\mathbf{w_2^T}\hat{\theta}_2$ arbitrarily close or even equal to zero. To guarantee that $\mathbf{w_2^T}\hat{\theta}_2$ is bounded away from zero, we first assume that $\mathrm{sgn}(\theta_{2i})$ and a lower bound $\bar{\theta}_{2i} > 0$ for $|\theta_{2i}|$ are known and the sign of w_{2i} is the same as the sign of θ_{2i} for all $i = 1, \ldots, n_2$. Then, we modify the adaptive learning rule in (28) using the projection technique in [29] as follows:

$$\hat{\theta}_{2i}(t) = \begin{cases} & \text{if } |\hat{\theta}_{2i}(t)| > \bar{\theta}_{2i}(t) \text{ or} \\ \hat{\theta}_{2ipr}(t-T) & \text{if } |\hat{\theta}_{2i}(t)| = \bar{\theta}_{2i}(t) \text{ and} \\ +\beta_2(\mathbf{w_{2e}b^T P_1\hat{e}})(i) & (\mathbf{w_{2e}b^T P_1\hat{e}})(i) \\ & \cdot\mathrm{sgn}(\hat{\theta}_{2i}(t)) \geq 0, \\ \hat{\theta}_{2ipr}(t-T) & \text{otherwise}, \end{cases} \tag{45}$$

where the initial value $\hat{\theta}_{2i}(t)$ is chosen such that $\hat{\theta}_{2i}(t)\mathrm{sgn}(\theta_{2i}(t)) \geq \bar{\theta}_{2i}$ for all $t \in [0, T)$.

Remark 3: The control input and parameter learning rules require knowledge of the state variables. Although we used the output variable y throughout this paper, we assume that the state variables are available.

Remark 4: The fundamental operating principle of the ILC is to observe the periodic output signals for one period, and then to generate the control output for the next period. The ILC has been successfully used for hard disk drive, process control, and power converter. These systems have periodic reference signals and suffer from the periodic disturbances. An adaptive learning controller has also been dealt with these systems, and so periodicity of the signals can be ensured.

However, in practice, the error between ideal periodic signals and actual almost periodic signals can be accumulated as the iteration increases, and it affects to the performance of the control system. To solve this problem, several solutions have been proposed. A supervisory adaptive scheme has been proposed to estimate the period from on-line measurements [30, 31]. In order to improve the capability of repetitive controllers for the case where the periodicity cannot be ensured, a robust repetitive controller has been proposed in [32]. Robustness of repetitive controller can be achieved for small variations in the period by using multiple memory-loops and correct design of the coefficients.

Remark 5: Neural network/adaptive fuzzy based adaptive control techniques have been applied to a class of nonlinear systems in [33–38]. These controllers can also estimate and compensate the unknown parameters in nonlinear systems so that they can precisely control these systems. However, in practical applications, it requires to set much more learning parameters compared to the proposed adaptive controller.

3.2. Controller design for the MIMO system

We review below the dynamic equation (14) for the uncertain MIMO nonlinear system and the dynamic equation (19) for its desired system:

$$\begin{bmatrix} y_1^{(n_1)} \\ \vdots \\ y_p^{(n_p)} \end{bmatrix} = \begin{bmatrix} f_1(\mathbf{x}) \\ \vdots \\ f_p(\mathbf{x}) \end{bmatrix} + \mathbf{A}(\mathbf{x}) \begin{bmatrix} u_1 \\ \vdots \\ u_p \end{bmatrix}, \tag{46}$$

$$\begin{bmatrix} y_{1d}^{(n_1)} \\ \vdots \\ y_{pd}^{(n_p)} \end{bmatrix} = \begin{bmatrix} f_1(\mathbf{x_d}) \\ \vdots \\ f_p(\mathbf{x_d}) \end{bmatrix} + \mathbf{A}(\mathbf{x_d}) \begin{bmatrix} u_{1d} \\ \vdots \\ u_{pd} \end{bmatrix}. \tag{47}$$

Subtracting (46) from (47) and linearly parameterizing the uncertain terms, we have

$$\begin{aligned} \begin{bmatrix} y_{1d}^{(n_1)} \\ \vdots \\ y_{pd}^{(n_p)} \end{bmatrix} - \begin{bmatrix} y_1^{(n_1)} \\ \vdots \\ y_p^{(n_p)} \end{bmatrix} &= \begin{bmatrix} f_1(\mathbf{x_d}) \\ \vdots \\ f_p(\mathbf{x_d}) \end{bmatrix} - \begin{bmatrix} f_1(\mathbf{x}) \\ \vdots \\ f_p(\mathbf{x}) \end{bmatrix} \\ &\quad + \mathbf{A}(\mathbf{x_d}) \begin{bmatrix} u_{1d} \\ \vdots \\ u_{pd} \end{bmatrix} - \mathbf{A}(\mathbf{x}) \begin{bmatrix} u_1 \\ \vdots \\ u_p \end{bmatrix} \\ &= \mathbf{W_{1d}^T}\theta_1 - \mathbf{W_1^T}\theta_1 \\ &\quad + \mathbf{A}(\mathbf{x_d})\mathbf{u_d} - \mathbf{A}(\mathbf{x})\mathbf{u}. \end{aligned} \tag{48}$$

The control law is

$$\mathbf{u} = \hat{\mathbf{A}}(\mathbf{x})^{-1}(\mathbf{v} + \hat{\mathbf{A}}(\mathbf{x_d})\mathbf{u_l}), \tag{49}$$

where $\mathbf{v} = [v_1, \dots, v_p]^T \in R^p$, $v_i = a_{i1}e_i^{(n_i-1)} + \cdots + a_{in_i}e_i + ((\mathbf{W_{1d}} - \mathbf{W_1})^\mathbf{T}\hat{\theta}_1)(i) \in R$ for all $i = 1, \dots, p$, $\mathbf{e} = [\mathbf{e_1}, \dots, \mathbf{e_p}]^T \in R^{n_1+\cdots+n_p}$, $\mathbf{e_i} = [e_i, \dots, e_i^{(n_i-1)}] \in R^{n_i}$, and $e_i = y_{id} - y_i \in R$ for all $i = 1, \dots, p$. The positive constants $a_{i1}, \dots, a_{in_i}$ are chosen such that $s^{n_i} + a_{i1}s^{n_i-1} + \cdots + a_{in_i}$ becomes a Hurwitz polynomial for each $i = 1, \dots, p$. The vectors $\hat{\theta}_1 \in R^{n_1}$, $\hat{\theta}_2 \in R^{n_2}$, and $\mathbf{u_l} \in R^p$ are to be learned by the adaptive learning laws.

Substituting (49) into (48) yields

$$\begin{aligned} \begin{bmatrix} e_1^{(n_1)} \\ \vdots \\ e_p^{(n_p)} \end{bmatrix} &= \mathbf{W_{1d}^T}\theta_1 - \mathbf{W_1^T}\theta_1 + \mathbf{A}(\mathbf{x_d})\mathbf{u_d} - \mathbf{v} \\ &\quad - \hat{\mathbf{A}}(\mathbf{x_d})\mathbf{u_l} + \hat{\mathbf{A}}(\mathbf{x})\hat{\mathbf{A}}(\mathbf{x})^{-1}(\mathbf{v} + \hat{\mathbf{A}}(\mathbf{x_d})\mathbf{u_l}) \\ &\quad - \mathbf{A}(\mathbf{x})\hat{\mathbf{A}}(\mathbf{x})^{-1}(\mathbf{v} + \hat{\mathbf{A}}(\mathbf{x_d})\mathbf{u_l}). \end{aligned} \tag{50}$$

Assigning $\mathbf{v}$ and moving the error terms to the left-hand side of (50), we have

$$\begin{aligned} &\begin{bmatrix} e_1^{(n_1)} + a_{11}e_1^{(n_1-1)} + \cdots + a_{1n_1}e_1 \\ \vdots \\ e_p^{(n_p)} + a_{p1}e_p^{(n_p-1)} + \cdots + a_{pn_p}e_p \end{bmatrix} \\ &= \mathbf{W_{1d}^T}\theta_1 - \mathbf{W_1^T}\theta_1 + \mathbf{A}(\mathbf{x_d})\mathbf{u_d} - (\mathbf{W_{1d}} - \mathbf{W_1})^\mathbf{T}\hat{\theta}_1 \\ &\quad - \hat{\mathbf{A}}(\mathbf{x_d})\mathbf{u_l} - (\mathbf{A}(\mathbf{x}) - \hat{\mathbf{A}}(\mathbf{x}))\hat{\mathbf{A}}(\mathbf{x})^{-1}(\mathbf{v} + \hat{\mathbf{A}}(\mathbf{x_d})\mathbf{u_l}) \\ &= (\mathbf{W_{1d}} - \mathbf{W_1})^\mathbf{T}\tilde{\theta}_1 + \mathbf{A}(\mathbf{x_d})\mathbf{u_d} - \hat{\mathbf{A}}(\mathbf{x_d})\mathbf{u_l} - \mathbf{A}(\mathbf{x_d})\mathbf{u_l} \\ &\quad + \mathbf{A}(\mathbf{x_d})\mathbf{u_l} - (\mathbf{A}(\mathbf{x}) - \hat{\mathbf{A}}(\mathbf{x}))\mathbf{u} \\ &= (\mathbf{W_{1d}} - \mathbf{W_1})^\mathbf{T}\tilde{\theta}_1 + \mathbf{A}(\mathbf{x_d})\tilde{\mathbf{u}}_\mathbf{l} + (\mathbf{A}(\mathbf{x_d}) - \hat{\mathbf{A}}(\mathbf{x_d}))\mathbf{u_l} \\ &\quad - (\mathbf{A}(\mathbf{x}) - \hat{\mathbf{A}}(\mathbf{x}))\mathbf{u} \\ &= (\mathbf{W_{1d}} - \mathbf{W_1})^\mathbf{T}\tilde{\theta}_1 + \Sigma_{i=1}^{n_2}\mathbf{A_i}(\mathbf{x_d})(\theta_{\mathbf{2i}} - \hat{\theta}_{\mathbf{2i}})\mathbf{u_l} \\ &\quad - \Sigma_{i=1}^{n_2}\mathbf{A_i}(\mathbf{x})(\theta_{\mathbf{2i}} - \hat{\theta}_{\mathbf{2i}})\mathbf{u} + \mathbf{A}(\mathbf{x_d})\tilde{\mathbf{u}}_\mathbf{l} \\ &= (\mathbf{W_{1d}} - \mathbf{W_1})^\mathbf{T}\tilde{\theta}_1 + \Sigma_{i=1}^{n_2}(\mathbf{A_i}(\mathbf{x_d})\mathbf{u_l} - \mathbf{A_i}(\mathbf{x})\mathbf{u})\tilde{\theta}_{2i} \\ &\quad + \mathbf{A}(\mathbf{x_d})\tilde{\mathbf{u}}_\mathbf{l} \\ &= \mathbf{W_{1e}^T}\tilde{\theta}_1 + \mathbf{W_{2e}^T}\tilde{\theta}_2 + \mathbf{A}(\mathbf{x_d})\tilde{\mathbf{u}}_\mathbf{l} \\ &= \mathbf{W_e^T}\tilde{\theta} + \mathbf{A}(\mathbf{x_d})\tilde{\mathbf{u}}_\mathbf{l}, \end{aligned} \tag{51}$$

where $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1 \in R^{n_1}$, $\tilde{\theta}_2 = \theta_2 - \hat{\theta}_2 \in R^{n_2}$, $\tilde{\mathbf{u}}_\mathbf{l} = \mathbf{u_d} - \mathbf{u_l} \in R^p$, $\mathbf{W_{1e}^T} = (\mathbf{W_{1d}} - \mathbf{W_1})^\mathbf{T}$, $\mathbf{W_{2e}^T} = [\mathbf{A_1}(\mathbf{x_d})\mathbf{u_l} - \mathbf{A_1}(\mathbf{x})\mathbf{u}, \dots, \mathbf{A_{n_2}}(\mathbf{x_d})\mathbf{u_l} - \mathbf{A_{n_2}}(\mathbf{x})\mathbf{u}] \in R^{p\times n_2}$, $\mathbf{W_e^T} = [\mathbf{W_{1e}^T}, \mathbf{W_{2e}^T}] \in R^{\,p\times(n_1+n_2)}$, and $\tilde{\theta} = [\tilde{\theta}_1^T, \tilde{\theta}_2^T]^T \in R^{n_1+n_2}$.

As we did for the SISO system, we propose the adaptive learning laws with input $\mathbf{u}$ in (49) in the following theorem.

Theorem 2: Consider the uncertain MIMO nonlinear system in (13) with the same assumptions. We define the control law as in (49) and choose $\hat{\theta}_1(t)$, $\hat{\theta}_2(t)$, $\mathbf{u_l}(t)$ as

$$\hat{\theta}_1(t) = \hat{\theta}_{\mathbf{1pr}}(t-T) + \beta_1\mathbf{W_{1e}}\mathbf{B^T}\mathbf{Pe}, \tag{52}$$

$$\hat{\theta}_2(t) = \hat{\theta}_{\mathbf{2pr}}(t-T) + \beta_2\mathbf{W_{2e}}\mathbf{B^T}\mathbf{Pe}, \tag{53}$$

$$\mathbf{u_l}(t) = \mathbf{u_{lpr}}(t-T) + \beta_3\mathbf{B^T}\mathbf{Pe}, \tag{54}$$

where

$$\begin{aligned} \hat{\theta}_{\mathbf{1pr}}(t-T) &= \mathrm{Proj}(\hat{\theta}_1(t-T)) \\ &= [\mathrm{Proj}(\hat{\theta}_{11}(t-T)), \dots, \mathrm{Proj}(\hat{\theta}_{1n_1}(t-T))]^T, \end{aligned} \tag{55}$$

$$\begin{aligned} \hat{\theta}_{\mathbf{2pr}}(t-T) &= \mathrm{Proj}(\hat{\theta}_2(t-T)) \\ &= [\mathrm{Proj}(\hat{\theta}_{21}(t-T)), \dots, \mathrm{Proj}(\hat{\theta}_{2n_2}(t-T))]^T, \end{aligned} \tag{56}$$

$$\begin{aligned} \mathbf{u_{lpr}}(t-T) &= \mathrm{Proj}(\mathbf{u_l}(t-T)) \\ &= [\mathrm{Proj}(u_{l1}(t-T)), \dots, \mathrm{Proj}(u_{lp}(t-T))]^T, \end{aligned} \tag{57}$$

and β_1, β_2, β_3 are the positive adaptation gains. If the control scheme satisfies the condition that $\hat{\mathbf{A}}(\mathbf{x})$ is invertible, then the tracking error $\mathbf{e}$, parameter errors $\tilde{\theta}_1$ and $\tilde{\theta}_2$, and desired input error $\tilde{\mathbf{u}}_\mathbf{l}$ are bounded, and $\mathbf{e}$ asymptotically converges to the origin. Moreover, $\tilde{\mathbf{u}}_\mathbf{l}$ asymptotically converges to zero.

Proof: Applying the control input from (49) to the dynamics in the normal form in (14), we rewrite (51) and the error dynamics in the state-space form as:

$$\dot{\mathbf{e}}_\mathbf{i} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -a_{in_i} & -a_{i(n_i-1)} & \cdots & -a_{i1} \end{bmatrix} \mathbf{e}_\mathbf{i} + \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix} (\mathbf{W}_\mathbf{e}^\mathbf{T}\tilde{\theta} + \mathbf{A}(\mathbf{x_d})\tilde{\mathbf{u}}_\mathbf{l})(i) = \mathbf{A_i}\mathbf{e_i} + \mathbf{b}(\mathbf{W}_\mathbf{e}^\mathbf{T}\tilde{\theta} + \mathbf{A}(\mathbf{x_d})\tilde{\mathbf{u}}_\mathbf{l})(i). \quad (58)$$

We obtain $\mathbf{P_i}$ by solving $\mathbf{P_i A_i} + \mathbf{A_i^T P_i} = -\mathbf{Q_i}$ and construct the matrices $\mathbf{P}$, $\mathbf{Q}$, $\mathbf{A}$, and $\mathbf{B}$ by diagonalizing $\mathbf{P_i}$, $\mathbf{Q_i}$, $\mathbf{A_i}$, and $\mathbf{b}$ for each $i = 1,...,p$. Then, we obtain $\mathbf{P}$ by using $\mathbf{PA} + \mathbf{A^T P} = -\mathbf{Q}$. Using this $\mathbf{P}$, we choose the Lyapunov function candidate as:

$$V = \mathbf{e^T P e} + \int_{t-T}^{t} \left[\frac{1}{\beta_1}\tilde{\theta}_\mathbf{1}^\mathbf{T}(\tau)\tilde{\theta}_\mathbf{1}(\tau) + \frac{1}{\beta_2}\tilde{\theta}_\mathbf{2}^\mathbf{T}(\tau)\tilde{\theta}_\mathbf{2}(\tau) + \frac{1}{\beta_3}\tilde{\mathbf{u}}_\mathbf{l}^\mathbf{T}(\tau)\mathbf{A}(\mathbf{x_d})\tilde{\mathbf{u}}_\mathbf{l}(\tau) \right] d\tau. \quad (59)$$

By taking the time derivative on both sides of (59), we have

$$\begin{aligned} \dot{V} &= \dot{\mathbf{e}}^\mathbf{T}\mathbf{Pe} + \mathbf{e^T P}\dot{\mathbf{e}} \\ &\quad + \frac{1}{\beta_1}(\tilde{\theta}_\mathbf{1}^\mathbf{T}(t)\tilde{\theta}_\mathbf{1}(t) - \tilde{\theta}_\mathbf{1}^\mathbf{T}(t-T)\tilde{\theta}_\mathbf{1}(t-T)) \\ &\quad + \frac{1}{\beta_2}(\tilde{\theta}_\mathbf{2}^\mathbf{T}(t)\tilde{\theta}_\mathbf{2}(t) - \tilde{\theta}_\mathbf{2}^\mathbf{T}(t-T)\tilde{\theta}_\mathbf{2}(t-T)) \\ &\quad + \frac{1}{\beta_3}(\tilde{\mathbf{u}}_\mathbf{l}^\mathbf{T}(t)\mathbf{A}(\mathbf{x_d})\tilde{\mathbf{u}}_\mathbf{l}(t) - \tilde{\mathbf{u}}_\mathbf{l}^\mathbf{T}(t-T)\mathbf{A}(\mathbf{x_d})\tilde{\mathbf{u}}_\mathbf{l}(t-T)) \\ &\le \dot{\mathbf{e}}^\mathbf{T}\mathbf{Pe} + \mathbf{e^T P}\dot{\mathbf{e}} \\ &\quad + \frac{1}{\beta_1}(\tilde{\theta}_\mathbf{1}^\mathbf{T}(t)\tilde{\theta}_\mathbf{1}(t) - \tilde{\theta}_\mathbf{1pr}^\mathbf{T}(t-T)\tilde{\theta}_\mathbf{1pr}(t-T)) \\ &\quad + \frac{1}{\beta_2}(\tilde{\theta}_\mathbf{2}^\mathbf{T}(t)\tilde{\theta}_\mathbf{2}(t) - \tilde{\theta}_\mathbf{2pr}^\mathbf{T}(t-T)\tilde{\theta}_\mathbf{2pr}(t-T)) \\ &\quad + \frac{1}{\beta_3}(\tilde{\mathbf{u}}_\mathbf{l}^\mathbf{T}(t)\mathbf{A}(\mathbf{x_d})\tilde{\mathbf{u}}_\mathbf{l}(t) - \tilde{\mathbf{u}}_\mathbf{lpr}^\mathbf{T}(t-T)\mathbf{A}(\mathbf{x_d}) \\ &\quad \times \tilde{\mathbf{u}}_\mathbf{lpr}(t-T)). \end{aligned} \quad (60)$$

Subtracting $\theta_\mathbf{1}(t) = \theta_\mathbf{1}(t-T)$, $\theta_\mathbf{2}(t) = \theta_\mathbf{2}(t-T)$, and $\mathbf{u_d}(t) = \mathbf{u_d}(t-T)$ from (52)–(54), we have

$$\tilde{\theta}_\mathbf{1}(t) = \tilde{\theta}_\mathbf{1pr}(t-T) - \beta_1\mathbf{W_{1e}B^T Pe}, \quad (61)$$

$$\tilde{\theta}_\mathbf{2}(t) = \tilde{\theta}_\mathbf{2pr}(t-T) - \beta_2\mathbf{W_{2e}B^T Pe}, \quad (62)$$

$$\tilde{\mathbf{u}}_\mathbf{l}(t) = \tilde{\mathbf{u}}_\mathbf{lpr}(t-T) - \beta_3\mathbf{B^T Pe}, \quad (63)$$

where $\tilde{\theta}_\mathbf{1pr}(t-T) = \theta_\mathbf{1}(t-T) - \hat{\theta}_\mathbf{1pr}(t-T)$, $\tilde{\theta}_\mathbf{2pr}(t-T) = \theta_\mathbf{2}(t-T) - \hat{\theta}_\mathbf{2pr}(t-T)$, $\tilde{\mathbf{u}}_\mathbf{lpr}(t-T) = \mathbf{u_d}(t-T) - \mathbf{u_{lpr}}(t-T)$.

Applying (61)–(63) to (60), we obtain

$$\begin{aligned} \dot{V} &\le \mathbf{e^T(PA + A^T P)e} + 2\mathbf{e^T PBW_{1e}^T}\tilde{\theta}_\mathbf{1}(t) \\ &\quad + 2\mathbf{e^T PBW_{2e}^T}\tilde{\theta}_\mathbf{2}(t) + 2\mathbf{e^T PBA}(\mathbf{x_d})\tilde{\mathbf{u}}_\mathbf{l}(t) \\ &\quad - 2\mathbf{e^T PBW_{1e}^T}\tilde{\theta}_\mathbf{1pr}(t-T) - 2\mathbf{e^T PBW_{2e}^T}\tilde{\theta}_\mathbf{2pr}(t-T) \\ &\quad - 2\mathbf{e^T PBA}(\mathbf{x_d})\tilde{\mathbf{u}}_\mathbf{lpr}(t-T) \\ &\quad + \beta_1\mathbf{e^T PBW_{1e}^T W_{1e} B^T Pe} \\ &\quad + \beta_2\mathbf{e^T PBW_{2e}^T W_{2e} B^T Pe} + \beta_3\mathbf{e^T PBA}(\mathbf{x_d})\mathbf{B^T Pe} \\ &= \mathbf{e^T(PA + A^T P)e} + 2\mathbf{e^T PBW_{1e}^T}(\tilde{\theta}_\mathbf{1}(t) - \tilde{\theta}_\mathbf{1pr}(t-T)) \\ &\quad + 2\mathbf{e^T PBW_{2e}^T}(\tilde{\theta}_\mathbf{2}(t) - \tilde{\theta}_\mathbf{2pr}(t-T)) \\ &\quad + 2\mathbf{e^T PBA}(\mathbf{x_d})(\tilde{\mathbf{u}}_\mathbf{l}(t) - \tilde{\mathbf{u}}_\mathbf{lpr}(t-T)) \\ &\quad + \beta_1\mathbf{e^T PBW_{1e}^T W_{1e} B^T Pe} \\ &\quad + \beta_2\mathbf{e^T PBW_{2e}^T W_{2e} B^T Pe} + \beta_3\mathbf{e^T PBA}(\mathbf{x_d})\mathbf{B^T Pe} \\ &= -\mathbf{e^T Qe} - \beta_1\mathbf{e^T PBW_{1e}^T W_{1e} B^T Pe} \\ &\quad - \beta_2\mathbf{e^T PBW_{2e}^T W_{2e} B^T Pe} - \beta_3\mathbf{e^T PBA}(\mathbf{x_d})\mathbf{B^T Pe} \\ &= -\mathbf{e^T Me} \\ &\le 0, \end{aligned} \quad (64)$$

where $\mathbf{M} = \mathbf{Q} + \beta_1\mathbf{PBW_{1e}^T W_{1e} B^T P} + \beta_2\mathbf{PBW_{2e}^T W_{2e} B^T P} + \beta_3\mathbf{PBA}(\mathbf{x_d})\mathbf{B^T P}$ is a positive definite matrix. We can derive $\mathbf{e} \to \mathbf{0}$ as $t \to \infty$ as in the SISO system. Therefore, this closed loop system becomes asymptotically stable.

As in the SISO system, $\mathbf{A}(\mathbf{x_d})\tilde{\mathbf{u}}_\mathbf{l} \to \infty$ as $t \to 0$ in the MIMO system. Because $\mathbf{A}(\mathbf{x_d})$ is a nonsingular matrix, $\tilde{\mathbf{u}}_\mathbf{l} \to 0$ as $\mathbf{A}(\mathbf{x_d})\tilde{\mathbf{u}}_\mathbf{l} \to \mathbf{0}$. □

Remark 6: In the early stage of learning, the linear controller is dominant over the learning input $\mathbf{u_l}$, but role exchange occurs after sufficient learning from the result $\tilde{\mathbf{u}}_\mathbf{l} \to \mathbf{0}$ as $t \to \infty$ in theorem 1. Consequently, inverse dynamics model will be formed in the feedforward controller.

Remark 7: The high-gain feedback technique offers a simple and useful approach to control the uncertain nonlinear systems. However, in practice, the feedback gain cannot be made arbitrarily large due to practical problems such as actuator saturation, unmodelled dynamics and noise vulnerability [8]. The proposed AILC chooses an appropriate feedback gain (i.e. linear controller gain) at the initial stage of learning to ensure that the tracking error stays within a uniform bound. Then, the tracking error converges to zero as the repetition of the learning operation increases, so the proposed AILC is more robust to practical problems than is the high-gain feedback technique.

The AILCs without input learning technique can control uncertain nonlinear systems [15–19]. The difference lies in that the learning input in the proposed AILC can learn the desired input after sufficient learning (Remark 6). After sufficient learning has occurred, the learning input becomes dominant, so the feedback gain can be reset to a small value. Therefore, the proposed AILC is also more robust to practical problems than is the AILCs without input learning.

Remark 8: The proposed AILC can be compared with other AILCs with input learning technique [24, 25]. In [24], they consider the SISO nonlinear systems in the normal form and the cascade form with no parameterization in system uncertainty. In [25], they consider the SISO partially feedback linearizable system, where the system uncertainty is nonparameterizable and the input gain is an unknown constant of known sign. In our paper, we consider the uncertain MIMO nonlinear systems in the normal form with unknown parameters in both the system uncertainty and the input gain uncertainty. Moreover, we consider the convergence of learning input error by means of lemma 1 whereas, in [24], they consider only the existence of the learning input. Further, the input learning rule in the proposed AILC is simpler than that in [25].

4. SIMULATION RESULTS

4.1. Single-link manipulator

A single-link manipulator is used to simulate the performance of the proposed adaptive iterative learning control scheme. The dynamics of this manipulator is given as:

$$\ddot{q} = \frac{0.5m_0 + M_0}{J} gl \sin q + \frac{1}{J}\tau, \tag{65}$$

where the moment of inertia of the joint is $J = M_0 l^2 + m_0 l^2/3$. We set the tip load $m_0 = 2$ kg, the length of the link $l = 0.5$ m, the mass of the link $M_0 = 4$ kg, and the gravitational acceleration $g = 9.8$ m/s^2. We consider the desired trajectory $y_d = \frac{1}{2}\cos\frac{2\pi t}{T}$ with period $T = 1$ s, and set the initial conditions $q(0) = [1/2, 0]^T$.

Denoting $\mathbf{x} = [q, \dot{q}]^T$, $y = h(\mathbf{x}) = q$, and $u = \tau$, we formulate the following dynamic equation

$$\ddot{y} = f(\mathbf{x}) + g(\mathbf{x})u, \tag{66}$$

where

$$f(\mathbf{x}) = \frac{0.5m_0 + M_0}{J} gl \sin x_1, \tag{67}$$

$$g(\mathbf{x}) = \frac{1}{J}. \tag{68}$$

Denoting $\theta_1 = \frac{0.5m_0+M_0}{J}gl$ and $\theta_2 = \frac{1}{J}$, we linearly parameterize $f(\mathbf{x})$ and $g(\mathbf{x})$ as

$$f(\mathbf{x}) = \sin x_1 \cdot \theta_1 = w_1\theta_1, \tag{69}$$

$$g(\mathbf{x}) = 1 \cdot \theta_2 = w_2\theta_2. \tag{70}$$

We implement the control input u based on the control law

$$u = \frac{1}{w_2\hat{\theta}_2}(a_1\dot{e} + a_2 e + (w_{1d} - w_1)\hat{\theta}_1 + w_{2d}\hat{\theta}_2 u_l), \tag{71}$$

where $a_1 = 5$, $a_2 = 50$, and $e = y_d - y$. We obtain $\mathbf{P}$ by solving $\mathbf{PA} + \mathbf{A^T P} = -\mathbf{Q}$ where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -a_2 & -a_1 \end{bmatrix}, \qquad \mathbf{Q} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \tag{72}$$

and

$$\mathbf{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{73}$$

Using $\mathbf{P}$ and $\mathbf{b}$, we choose the parameters $\hat{\theta}_1(t)$, $\hat{\theta}_2(t)$, $u_l(t)$ as

$$\hat{\theta}_1(t) = \hat{\theta}_{1pr}(t-T) + \beta_1 w_{1e}\mathbf{b^T P e}, \tag{74}$$

$$\hat{\theta}_2(t) = \hat{\theta}_{2pr}(t-T) + \beta_2 w_{2e}\mathbf{b^T P e}, \tag{75}$$

$$u_l(t) = u_{lpr}(t-T) + \beta_3\mathbf{b^T P e}, \tag{76}$$

$\forall t \in [T, \infty)$ where $\beta_1 = 1\times 10^{-2}$, $\beta_2 = 1\times 10^{-5}$, $\beta_3 = 7\times 10^{-1}$, $\mathbf{e} = [e, \dot{e}]^T$, $w_{1e} = w_{1d} - w_1$, and $w_{2e} = w_{2d}u_l - w_2 u$. The initial parameter values are set to $\hat{\theta}_1(t) = 0.9\theta_1$, $\hat{\theta}_2(t) = 0.95\theta_2$, and $u_l(t) = 0$, $\forall t \in [0, T)$. For the projections, $\hat{\theta}_1$, $\hat{\theta}_2$, and u_l, bounds are set to $\hat{\theta}_1^{\mathbf{b}} = 1.2\theta_1$, $\hat{\theta}_2^{\mathbf{b}} = 1.2\theta_2$, and $u_l^b = 100$, respectively. The sampling period is set to 5 ms, and we simulate this example for 60 s with Matlab software.

We apply input u in (71) with the parameter learning rules in (74)–(76) to the single-link manipulator system in (66). As shown in Fig. 2, the root mean square (rms) error of the joint position and joint velocity decreases as the repetition of the learning operation increases. In theorem 1, result $\tilde{u}_l \to 0$ is obtained as $t \to \infty$. In this simulation, this result cannot be derived directly, because the actual value of the desired input u_d is unknown. However, $u \simeq u_d$ can be deduced as $t \to \infty$ from the fact that the rms error of the joint position and joint velocity become zero. Therefore, the input error $u_e = u - u_l$ goes to zero as learning proceeds, as shown in Fig. 3. As a result, $\tilde{u}_l \to 0$ is inferred as $t \to \infty$.

The ALC without input learning has been proposed in SISO systems [15, 16] and can also control the SISO systems (Fig. 4). The difference lies in that the learning input in the proposed ALC can learn the desired input after sufficient learning. After sufficient learning occurs, the learning input becomes dominant, and so the feedback gain in linear controller can be reset to a small value. Then, the proposed ALC becomes more robust to practical problems such as actuator saturation, unmodelled dynamics and noise vulnerability than that without input learning.

4.2. Two-link manipulator

A two-link manipulator (Fig. 5) is used to simulate the performance of the proposed learning controller in uncertain MIMO nonlinear system. It is worth highlighting that two-link manipulator is formulated as uncertain MIMO nonlinear system in the normal form for the first time.

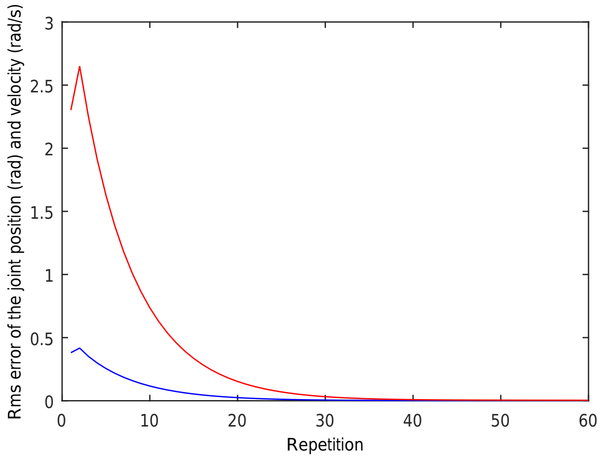

Fig. 2. Rms errors of the joint position and velocity with parameter learning. Solid red line: the rms error of the joint position q; solid red line: the rms error of the joint velocity $\dot{q}$.

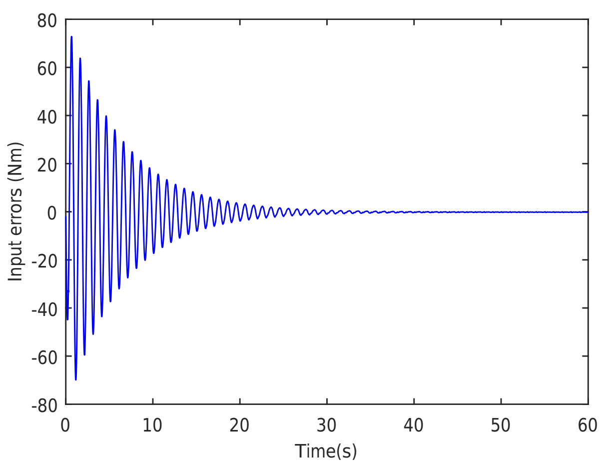

Fig. 3. The input error $u_e = u - u_l$ with parameter learning as the task is repeated.

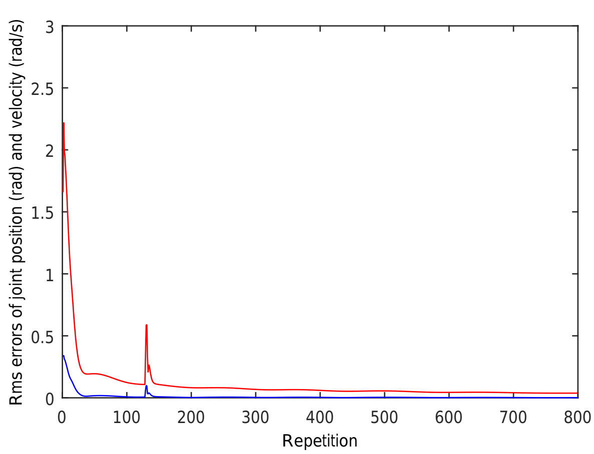

Fig. 4. Rms errors of joint position and velocity without input learning. Solid red line: the rms error of the joint position q; solid red line: the rms error of the joint velocity $\dot{q}$.

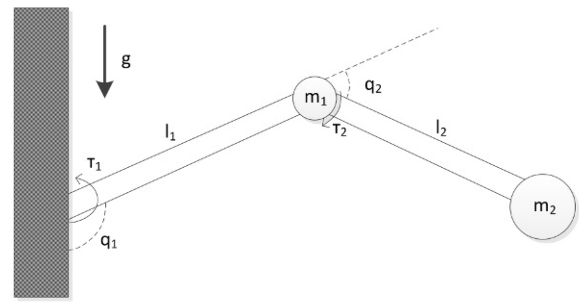

Fig. 5. The two-link manipulator configuration. m_i, l_i, and q_i represent the mass, length, and joint position, respectively.

The dynamic equation of this manipulator is given as [39]:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \tau, \tag{77}$$

where

$$\mathbf{M}(\mathbf{q}) = \begin{pmatrix} m_2 l_2^2 + 2m_2 l_1 l_2 c_2 & m_2 l_2^2 \\ +(m_1+m_2)l_1^2 & +m_2 l_1 l_2 c_2 \\ m_2 l_2^2 + m_2 l_1 l_2 c_2 & m_2 l_2^2 \end{pmatrix}, \tag{78}$$

$$\mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) = \begin{pmatrix} -2m_2 l_1 l_2 s_2 \dot{q}_2 & -m_2 l_1 l_2 s_2 \dot{q}_2 \\ m_2 l_1 l_2 s_2 \dot{q}_1 & 0 \end{pmatrix}, \tag{79}$$

$$\mathbf{G}(\mathbf{q}) = \begin{pmatrix} m_2 l_2 g s_{12} + (m_1+m_2) l_1 g s_1 \\ m_2 l_2 g s_{12} \end{pmatrix}, \tag{80}$$

where $\mathbf{q} = [q_1, q_2]^T$, $s_1 = \sin q_1$, $s_2 = \sin q_2$, $c_2 = \cos q_2$, $s_{12} = \sin(q_1+q_2)$, $m_1 = 20$ kg, $m_2 = 10$ kg, $l_1 = 2$ m, $l_2 = 1$ m, and $g = 9.8$ m/s^2, and the proportional constant $k = 2$ is known when $m_1 = km_2$. We consider the desired trajectory $\mathbf{y_d} = [\frac{1}{2}\sin\frac{2\pi t}{T}, \frac{1}{2}\cos\frac{2\pi t}{T}]^T$ with period $T = 3$ s and set the initial conditions $\mathbf{q}(0) = [0, 1/2]^T$, and $\dot{\mathbf{q}}(0) = [\pi/3, 0]^T$.

Denoting $\mathbf{x} = [q_1, q_2, \dot{q}_1, \dot{q}_2]^T$, $y_1 = h_1(\mathbf{x}) = q_1, y_2 = h_2(\mathbf{x}) = q_2$ and $\mathbf{u} = \tau$, we formulate the following dynamic equation

$$\begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{bmatrix} = \begin{bmatrix} f_1(\mathbf{x}) \\ f_2(\mathbf{x}) \end{bmatrix} + \mathbf{A}(\mathbf{x})\mathbf{u}, \tag{81}$$

where

$$\begin{bmatrix} f_1(\mathbf{x}) \\ f_2(\mathbf{x}) \end{bmatrix} = -\mathbf{M}^{-1}(\mathbf{q})(\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q})), \tag{82}$$

$$\mathbf{A}(\mathbf{x}) = \mathbf{M}^{-1}(\mathbf{q}). \tag{83}$$

Denoting $\theta_{11} = l_2/l_1$, $\theta_{12} = 1$, $\theta_{13} = g/l_1$, $\theta_{14} = 3g/l_1$, $\theta_{15} = 3g/l_2$, $\theta_{16} = 3l_1/l_2$, $\theta_{21} = 1/m_2 l_1^2$, $\theta_{22} = 1/m_2 l_1 l_2$, and $\theta_{23} = 3/m_2 l_2^2$, we linearly parameterize $[f_1(\mathbf{x}), f_2(\mathbf{x})]^T$ and $\mathbf{A}(\mathbf{x})$ as

$$\begin{bmatrix} f_1(\mathbf{x}) \\ f_2(\mathbf{x}) \end{bmatrix} = \frac{l_2}{l_1} \begin{pmatrix} (2s_2\dot{q}_1\dot{q}_2 + s_2\dot{q}_2^2 + s_2\dot{q}_1^2)/(3-c_2^2) \\ -(2s_2\dot{q}_1\dot{q}_2 + s_2\dot{q}_2^2 + s_2\dot{q}_1^2)/(3-c_2^2) \end{pmatrix}$$

$$+\begin{pmatrix}(s_2c_2\dot{q}_1^2)/(3-c_2^2)\\ -(2s_2c_2\dot{q}_1\dot{q}_2+s_2c_2\dot{q}_2^2+2s_2c_2\dot{q}_1^2)\\ /(3-c_2^2)\end{pmatrix}$$
$$+\frac{g}{l_1}\begin{pmatrix}(c_2s_{12})/(3-c_2^2)\\ -(c_2s_{12})/(3-c_2^2)\end{pmatrix}$$
$$+\frac{3g}{l_1}\begin{pmatrix}-s_1/(3-c_2^2)\\ s_1/(3-c_2^2)\end{pmatrix}$$
$$+\frac{3g}{l_2}\begin{pmatrix}0\\ -(s_{12}-s_1c_2)/(3-c_2^2)\end{pmatrix}$$
$$+\frac{3l_1}{l_2}\begin{pmatrix}0\\ -(s_2\dot{q}_1^2)/(3-c_2^2)\end{pmatrix}$$
$$=\Sigma_{i=1}^{6}\mathbf{w}_{1i}^{\mathbf{T}}\theta_{1i}=\mathbf{W}_1^{\mathbf{T}}\theta_1, \tag{84}$$

$$\mathbf{A}(\mathbf{x})=\frac{1}{m_2l_1^2}\begin{pmatrix}1/(3-c_2^2) & -1/(3-c_2^2)\\ -1/(3-c_2^2) & 1/(3-c_2^2)\end{pmatrix}$$
$$+\frac{1}{m_2l_1l_2}\begin{pmatrix}0 & -c_2/(3-c_2^2)\\ -c_2/(3-c_2^2) & 2c_2/(3-c_2^2)\end{pmatrix}$$
$$+\frac{3}{m_2l_2^2}\begin{pmatrix}0 & 0\\ 0 & 1/(3-c_2^2)\end{pmatrix}$$
$$=\Sigma_{i=1}^{3}\mathbf{A_i}(\mathbf{x})\theta_{2i}. \tag{85}$$

We implement the control input $\mathbf{u}$ based on the control law

$$\begin{bmatrix}u_1\\ u_2\end{bmatrix}=\hat{\mathbf{A}}(\mathbf{x})^{-1}\left(\begin{bmatrix}a_{11}\dot{e}_1+a_{12}e_1\\ +((\mathbf{W_{1d}}-\mathbf{W_1})^{\mathbf{T}}\hat{\theta}_1)(1)\\ a_{21}\dot{e}_2+a_{22}e_2\\ +((\mathbf{W_{1d}}-\mathbf{W_1})^{\mathbf{T}}\hat{\theta}_1)(2)\end{bmatrix}+\hat{\mathbf{A}}(\mathbf{x_d})\begin{bmatrix}u_{1l}\\ u_{2l}\end{bmatrix}\right), \tag{86}$$

where $a_{11}=1$, $a_{12}=4$, $a_{21}=2$, $a_{22}=16$, $e_1=y_{1d}-y_1$, $e_2=y_{2d}-y_2$ and $\hat{\mathbf{A}}(\mathbf{x})=\Sigma_{i=1}^{3}\mathbf{A_i}(\mathbf{x})\hat{\theta}_{\mathbf{2i}}$. We obtain $\mathbf{P_i}$ by solving $\mathbf{P_iA_i}+\mathbf{A_i^TP_i}=-\mathbf{Q_i}$ for all $i=1,2$ where

$$\mathbf{A_1}=\begin{bmatrix}0 & 1\\ -a_{12} & -a_{11}\end{bmatrix},\mathbf{A_2}=\begin{bmatrix}0 & 1\\ -a_{22} & -a_{21}\end{bmatrix}, \tag{87}$$

$$\mathbf{Q_1}=\mathbf{Q_2}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, \tag{88}$$

and

$$\mathbf{b_1}=\mathbf{b_2}=\begin{bmatrix}0\\ 1\end{bmatrix}. \tag{89}$$

By diagonalizing $\mathbf{P_1}$, $\mathbf{P_2}$, $\mathbf{b_1}$, and $\mathbf{b_2}$, we obtain $\mathbf{P}$ and $\mathbf{B}$. We choose parameters $\hat{\theta}_1(t)$, $\hat{\theta}_2(t)$, $\mathbf{u_l}(t)$ using $\mathbf{P}$ and $\mathbf{B}$ with adaptive learning laws as

$$\hat{\theta}_1(t)=\hat{\theta}_{\mathbf{1pr}}(t-T)+\beta_1\mathbf{W_{1e}}\mathbf{B^T}\mathbf{Pe}, \tag{90}$$
$$\hat{\theta}_2(t)=\hat{\theta}_{\mathbf{2pr}}(t-T)+\beta_2\mathbf{W_{2e}}\mathbf{B^T}\mathbf{Pe}, \tag{91}$$
$$\mathbf{u_l}(t)=\mathbf{u_{lpr}}(t-T)+\beta_3\mathbf{B^T}\mathbf{Pe}, \tag{92}$$

$\forall t\in[T,\infty)$ where $\beta_1=2\times10^{-4}$, $\beta_2=2\times10^{-6}$, $\beta_3=7\times10^{-1}$, $\mathbf{e}=[e_1,\dot{e}_1,e_2,\dot{e}_2]^T$, $\mathbf{W_{1e}^T}=(\mathbf{W_{1d}}-\mathbf{W_1})^{\mathbf{T}}$, and $\mathbf{W_{2e}^T}=[\mathbf{A_1}(\mathbf{x_d})\mathbf{u_l}-\mathbf{A_1}(\mathbf{x})\mathbf{u},\ \mathbf{A_2}(\mathbf{x_d})\mathbf{u_l}-\mathbf{A_2}(\mathbf{x})\mathbf{u},\ \mathbf{A_3}(\mathbf{x_d})\mathbf{u_l}-\mathbf{A_3}(\mathbf{x})\mathbf{u}]$. The initial parameter values were set to $\hat{\theta}_1(t)=0.95\theta_1$, $\hat{\theta}_2(t)=0.95\theta_2$, and $\mathbf{u_l}(t)=0$ $\forall t\in[0,T)$. For the projections, the $\hat{\theta}_1$, $\hat{\theta}_2$, and $\mathbf{u_l}$ bounds are set to $\hat{\theta}_1^{\mathbf{b}}=1.2\theta_1$, $\hat{\theta}_2^{\mathbf{b}}=1.2\theta_2$, $\mathbf{u_l^b}=[200,200]^T$, respectively. The sampling period was set to 5 ms, and we simulated this example for 900 s with Matlab software.

We applied input $\mathbf{u}$ in (86) with the parameter learning rules to the two-link manipulator system in (81). The proposed learning controller performed poorly at the initial stage because of parameter mismatch (Figs. 6 and 7). However, as the number of repetitions of the learning operation increased, rms errors for the joint positions and joint velocities both decreased. In addition, the input error $\mathbf{u_e}=\mathbf{u}-\mathbf{u_l}$ became zero as learning proceeded (Fig. 8). Namely, $\tilde{\mathbf{u}}_\mathbf{l}\to\mathbf{0}$ as $t\to\infty$.

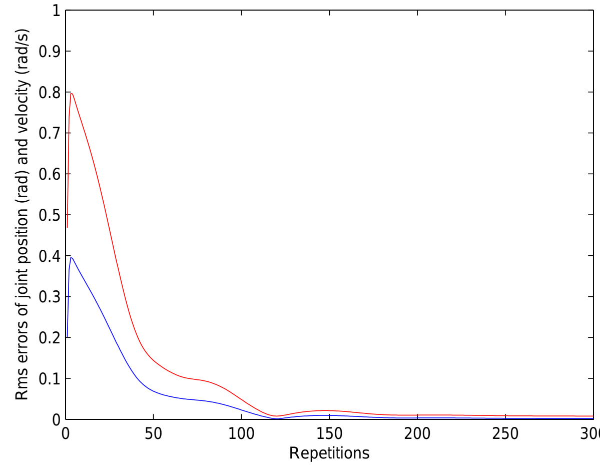

Fig. 6. Rms errors of joint 1 with parameter learning. Solid blue line: the rms error of the joint position q_1; solid red line: the rms error of the joint velocity $\dot{q}_1$.

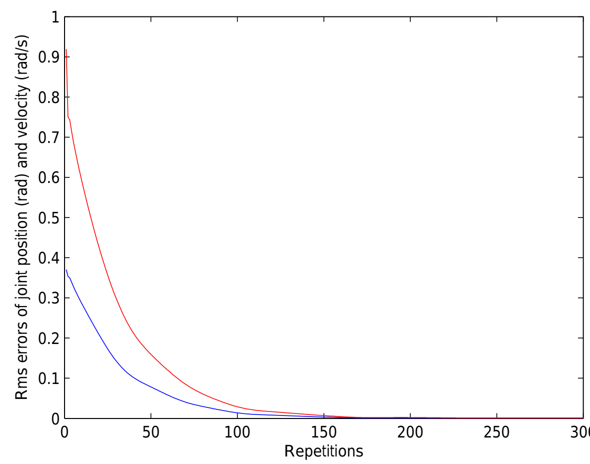

Fig. 7. Rms errors of joint 2 with parameter learning. Solid blue line: the rms error of the joint position q_2; solid red line: the rms error of the joint velocity $\dot{q}_2$.

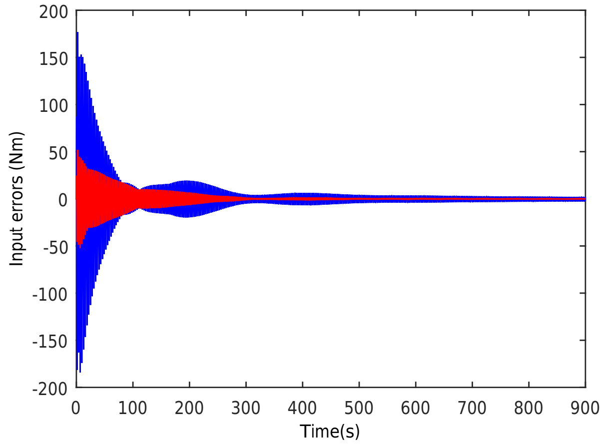

Fig. 8. The input error $\mathbf{u_e} = \mathbf{u} - \mathbf{u_l}$ with parameter learning as the task is repeated. Solid blue line: the input error $u_{1e} = u_1 - u_{1l}$; solid red line: the input error $u_{2e} = u_2 - u_{2l}$.

5. CONCLUSION

A new AILC approach has been proposed for uncertain MIMO nonlinear systems in the normal form. The proposed AILC learns the system and input gain parameters as well as the desired input. Compared with the existing results in AILC, the AILC with input learning is first developed for the uncertain MIMO nonlinear systems in the normal form; the input learning rule is simple, and so it can be easily implemented in industrial applications. The tracking error and desired input error signal asymptotically converge to zero, and the error signals are bounded in the learning control system. Single-link and two-link manipulators are presented as simulation examples to demonstrate the validity of the proposed controller.

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