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Pinning Exponential Synchronization of Nonlinearly Coupled Neural Networks with Mixed Delays via Intermittent Control

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Abstract: This paper is concerned with the exponential synchronization problem of nonlinearly coupled neural networks with mixed delays. By employing the intermittent control strategy, several appropriate linear and adaptive pinning controllers are designed in each control period. With the help of a new differential inequality, some conditions are proposed to guarantee that the coupled networks can realize pinning synchronization exponentially. The minimum number of pinned nodes is determined by using high-degree pinning scheme. Two numerical examples are provided finally to demonstrate the effectiveness of the theoretical results.

Keywords: Coupled neural networks, exponential synchronization, intermittent control, mixed delays, pinning control.

1. INTRODUCTION

In the past decades, due to the successful applications in various engineering fields, neural networks have drawn close attention [1,2]. The practical applications mainly depend on the dynamic behavior of neural networks. Time delay is unavoidably encountered be-cause of the finite signal propagation time, which may destroy dynamical behaviors or exhibit some complex dynamics and even chaotic behaviors [3–6]. When the signal propagation is instantaneous, it can be modeled by discrete delay. When the signal propagation exists during a certain time period, the distributed delay is a useful description. Therefore, mixed delays (both discrete delay and distributed delay) should be considered when modeling a realistic neural network [7].

Recently, as an important and interesting phenomenon in complex networks, synchronization has been widely investigated by many researchers. Sometimes, coupled networks can achieve synchronization by the interaction of intrinsic dynamical behavior of nodes, topological structure and communication way [8–10]. But in most cases, synchronization cannot be obtained by themselves. Accordingly, some control injections should be applied to force the coupled systems to synchronize [11–20]. It is impossible to add controllers to all nodes in a large-scale coupled network. To reduce the number of controlled nodes, pinning control is a natural idea and has been employed in the study of synchronization for coupled dynamical networks [21–24]. On the other hand, period-ically intermittent control has been broadly explored due to the advantages in practical applications and reducing economic cost. In this kind of control strategy, each period usually contains two types of time: work time and rest time. The controllers are activated in each work time and is off in the rest time. To further reduce the control cost and amount of transmitted information, combining pinning control and intermittent control together is thus of meaningful, and a great many achievements have been acquired in [25–28].

As a special case of complex networks, delayed neural networks can also exhibit some chaotic behaviors. Hence, synchronization of delayed chaotic neural networks has become a research focus in control field nowadays, see [29–31] for some recent works. Furthermore, an array of coupled neural networks has been found to exhibit more complicated behaviors and its synchronization has been one of the attractive and challenging research. For the synchronization stability problems of coupled neural net-works, some sufficient conditions based on linear matrix inequality were developed in [32–36]. Some control strategies are also introduced to realize synchronization if the coupled neural network cannot synchronize by itself. The impulsive control was utilized to investigate the synchronization of coupled neural networks in [37]. In [38], the randomly occurring control was introduced to deal with the distributed synchronization of stochastic coupled neural networks. The pinning synchronization of linearly coupled neural networks with delayed coupling

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was investigated in [39]. The synchronization problems of linearly coupled reaction-diffusion neural networks were discussed in [40, 41]. On the basis of T-S fuzzy theory, chaotic synchronization for coupled neural net-works was studied in [42].

Up to now, some researches have been carried out in the synchronization of coupled neural networks via pinning control. However, there are few theoretical results focusing on pinning intermittent synchronization of coupled neural networks. Recently, in the work of [43], the authors have first concerned with the pinning syn-chronization of linear coupled neural networks with mixed delays via linear intermittent control. Due to the main results derived in [43] based on the generalized Halanay inequality, two restrictive conditions were imposed, one is the control width should be larger than the time delay, and the other is the time delay should be smaller than the non-control width. The disadvantage of this approach is the difficulty of choosing the control period and control width, which would lead to restrict its scope of real applications. These two conditions also involved in the intermittent synchronization of delayed dynamical networks [27]. Although there were some results for relaxing these two conservative conditions [25, 26], they can not be used to deal with the systems with distributed delay. On the other hand, to give a more precise and realistic description of coupled networks, it is necessary and important to take the nonlinear coupling into account in practical applications [10,28]. It is still an interesting but difficult task to investigate the synchronization of nonlinearly coupled neural networks with by using the intermittent pinning control.

Motivated by the above discussions, this paper devotes to investigate the pinning synchronization problem of nonlinearly coupled neural networks with mixed delays by using periodically intermittent control. By adding linear and adaptive periodically intermittent controllers to a small fraction of nodes respectively, some conditions are developed based on a novel differential inequality. Moreover, we can find that the minimum number of pinned nodes is determined by using high-degree pinning scheme. In addition, for the linear feedback pinning scheme, we can obtain the domain of the feedback gains. Two numerical examples are given to demonstrate the effectiveness of the proposed methods.

2. PROBLEM FORMULATION

Consider an array of nonlinearly coupled neural networks with mixed delays, which is characterized by

$$\begin{aligned}\dot{x}_i(t) =& -Dx_i(t)+Af(x_i(t))+Bf(x_i(t-\tau_1(t)))\\&+E\int_{t-\tau_2(t)}^{t} f(x_i(v))dv+J(t)\\&+c\sum_{j=1}^{N} g_{ij}\Gamma h(x_j(t))+u_i(t),\ \ i=1,2,\cdots,N,\end{aligned} \tag{1}$$

where $x_i=(x_{i1},x_{i2},\cdots,x_{in})^T\in R^n$ is the state variable of the ith neural network. $D=diag\{d_1,d_2,...,d_n\}>0$, $A=(a_{ij})\in R^{n\times n}$, $B=(b_{ij})\in R^{n\times n}$, and $E=(e_{ij})\in R^{n\times n}$ are, respectively, the connection weight matrix, the discretely delayed connection weight matrix and the distributively delayed connection weight matrix. $f(x_i(t))=(f_1(x_{i1}(y)),f_2(x_{i2}(t)),...,f_n(x_{in}(t))^T$ is the neuron activation function. $J(t)=(J_1(t),J_2(t),...,J_n(t))^T$ is the external input vector of each neural network. $\tau_1(t)\in[0,\tau_1]$ and $\tau_2(t)\in[0,\tau_2]$ denote the discrete delay and distributed delay, respectively. $u_i(t)\in R^n$ is the control input to be designed later. $c>0$ is the coupling strength, and $\Gamma=diag\{\gamma_1,\gamma_2,...,\gamma_n\}>0$ is the inner-coupling matrix between neural network nodes. $h(x_i(t))=(g(x_{i1}(t)),g(x_{i2}(t)),...,g(x_{in}(t))^T$ is the nonlinear coupling function. $G=(g_{ij})\in R^{N\times N}$ denotes the coupling configuration matrix. If there is a connection between node i and node j $(i\neq j)$, then $g_{ij}=g_{ji}>0$; otherwise, $g_{ij}=g_{ji}=0$. The diagonal elements of G is defined as $g_{ii}=-\sum_{j=1,j\neq i}^{N} g_{ij}$.

Remark 1: In the work of [43], a neural network with mixed delays (discrete delay and distributed delay) was considered as node. The two delays were assumed to be the same constant. In this paper, the mixed delays are not only time-varying but also different. What is more, we consider the nonlinear coupling phenomenon, which has not been considered in [43]. In this regards, the coupled neural networks (1) is an extension of [43].

In this paper, we assume that s(t) be the isolated neural network of system (1), which is described by

$$\begin{aligned}\dot{s}(t) =& -Ds(t)+Af(s(t))+Bf(s(t-\tau_1(t)))\\&+E\int_{t-\tau_2(t)}^{t} f(s(v))dv+J(t),\end{aligned} \tag{2}$$

where $s(t)=(s_1(t),s_2(t),\cdots,s_n(t))^T\in R^n$.

Denote $\tau=\max\{\tau_1,\ \tau_2\}$. Let $C([-\tau,0],\ R^n)$ be the Banach space of continuous functions mapping the interval $[-\tau,0]$ into R^n with the norm $\|\phi\|=\sup_{-\tau\leq\theta\leq 0}\|\phi(\theta)\|$, where $\|\cdot\|$ is the Euclidean norm. Then the rigorous mathematical definition of exponential synchronization of coupled networks (1) is defined as follows:

Definition 1: Let $x_i(t;t_0;\phi)$, $i=1,...,N$ be a solution of coupled network (1), where $\phi=(\phi_1^T,\phi_2^T,\cdots,\phi_N^T)^T$, $\phi_i=\phi_i(\theta)\in C([-\tau,0],\ R^n)$ are the initial conditions. If there exist constants $\varepsilon>0$, $\lambda>0$ and a non-empty subset $\Lambda\subseteq R^n$ such that ϕ_i take values in Λ and $x_i(t;t_0;\phi)\in R^n$ for all $t\geq t_0$ and

$$\begin{aligned}&\lim_{t\to\infty}\|x_i(t;t_0;\phi)-s(t;t_0;s_0)\|\\&\leq\varepsilon e^{-\lambda t}\sup_{-\tau\leq\theta\leq 0}\|\phi_i(\theta)-s_0\|.\end{aligned} \tag{3}$$

Then coupled neural networks (1) is said to realize exponential synchronization.

To proceed further, the following assumptions and useful lemmas are needed.

Assumption 1 [43]**:** Assume that there exist positive constants $l_i > 0$ $(i = 1, 2, \cdots, n)$, such that

$$|f_i(x) - f_i(y)| \le l_i |x - y| \tag{4}$$

hold for any $x, y \in R$.

Assumption 2 [10]**:** Assume that there exist positive constants α and β, such that

$$\alpha \le \frac{h(x) - h(y)}{x - y} \le \beta \tag{5}$$

holds for any $x, y \in R$.

Remark 2: Assumption 1 is the usual Lipschitz-type condition [43] for activation function, and Assumption 2 is as same as that in [10]. These two assumptions are reasonable and frequently used in the study of synchronization of coupled neural networks.

Lemma 1 [23]**:** Assume that A, B are N by N Hermitian matrices. Let $\alpha_1 \ge \alpha_2 \ge \cdots \ge \alpha_N$, $\beta_1 \ge \beta_2 \ge \cdots \ge \beta_N$, and $\gamma_1 \ge \gamma_2 \ge \cdots \ge \gamma_N$ be eigenvalues of A, B and $A+B$, respectively. Then, one has $\alpha_i + \beta_N \le \gamma_i \le \alpha_i + \beta_1$, $i = 1, 2, \cdots, N$.

Lemma 2 [23]**:** For a diagonal matrix $K = diag(k_1, ..., k_l, \underbrace{0, ..., 0}_{N-l})$ with $k_i > 0$ $(i = 1, ..., l, 1 \le l \le N)$ and a symmetric matrix $M \in R^{N\times N}$, let $M - K = \begin{pmatrix} E - \bar{K} & S \\ S^T & M_l \end{pmatrix}$, where M_l is the minor matrix of M by removing its first l $(1 \le l < N)$ row-column pairs, E and S are matrices with appropriate dimensions, $\bar{K} = diag(k_1, ..., k_l)$. If $k_i > \lambda_{\max}(E - SM_l^{-1}S^T)$, then $M - K < 0$ is equival-ent to $M_l < 0$.

Lemma 3 [17]**:** If $G = (g_{ij}) \in R^{N\times N}$ satisfies $g_{ij} = g_{ji}$ and $g_{ii} = -\sum_{j=1, j\ne i}^{N} g_{ij}$, $i, j = 1, 2, \cdots, N$, then for any two vectors $x = [x_1, x_2, \cdots, x_N]^T$ and $y = [y_1, y_2, \cdots, y_N]^T$, we have

$$x^T G y = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i g_{ij} y_j = -\sum_{j>1} g_{ij}(x_i - x_j)(y_i - y_j). \tag{6}$$

Lemma 4 Suppose that function $y(t)$ is continuous and non-negative when $t \in [-\tau, \infty]$ and satisfies the following condition

$$\begin{cases} \dot{y}(t) = -r_1 y(t) + r_3 y(t - \tau_1(t)) + r_4 \displaystyle\int_{t-\tau_2(t)}^{t} y(s)ds, \\ \qquad mT \le t < mT + \theta T, \\ \dot{y}(t) = r_2 y(t) + r_3 y(t - \tau_1(t)) + r_4 \displaystyle\int_{t-\tau_2(t)}^{t} y(s)ds, \\ \qquad mT + \theta T \le t < (m+1)T, \end{cases} \tag{7}$$

where r_1, r_2, r_3, r_4, T and $0 < \theta < 1$ are some positive constants, and $m = 0, 1, 2, \cdots$. If

$$\begin{aligned} &r_1 > r_3, \\ &\rho = \zeta - \gamma(1-\theta) > 0, \end{aligned} \tag{8}$$

then $y(t) \le \sup_{-\tau \le s \le 0} y(s)\exp\{-\rho t\}, t \ge 0$, where $\gamma = r_1 + r_2$ and $\zeta > 0$ is the unique positive solution of the equation $\zeta - r_1 + r_3 \exp\{\zeta\tau\} - \frac{r_4}{\zeta} + \frac{r_4}{\zeta}\exp\{\zeta\tau\} = 0$.

The explicit proof of Lemma 4 is given in the Appendix. It is noted that, Lemmas 3 and 4 in [43] provided the solution for different integral inequalities with distributed delay, respectively. There were two restrictive conditions attached while deriving the main results under intermittent strategy in [43]. In comparison with the lemmas used in [43], the proposed Lemma 4 makes two working states of intermittent strategy to unify in together. When the distributed time-delay is not involved in (8), Lemma 4 reduces to Lemma 1 of [26]. Therefore, the conclusion of Lemma 4 contains some previous results as special case.

3. MAIN RESULTS

In this section, we will investigate the exponential synchronization of coupled neural networks (1) via different pinning intermittent control schemes: linear feedback control and adaptive control, which will be stated below in different subsections. Before giving the main results, for the sake of presentation simplicity, we denote

$$\begin{aligned} &Q = -D + \frac{AA^T + BB^T + EE^T}{2} + \frac{1}{2}L^2 I_n, \\ &L = \max_{1\le i\le n}\{l_i\}, \ \ \rho_1 = \frac{\lambda_{\max}(Q) + \frac{1}{2}a_1}{\lambda_{\min}(\Gamma)}, \\ &\rho_2 = \frac{\lambda_{\max}(Q) - \frac{1}{2}(a_2 - a_1)}{\lambda_{\min}(\Gamma)}, \\ &M = \rho_1 I_N + c\alpha G, \ \ \tilde{M} = \rho_2 I_N + c\alpha G, \\ &M - K = \begin{pmatrix} F - \bar{K} & S \\ S^T & M_l \end{pmatrix}, \end{aligned}$$

where I_N be an N-dimensional identity matrix, $\lambda_{\max}(Q)$ is the maximal eigenvalue of Q, $\lambda_{\min}(\Gamma)$ is the minimum eigenvalue of Γ, a_1 and a_2 are positive constants to be designed latter, $K = diag(k_1, ..., k_l, \underbrace{0, ..., 0}_{N-l})$, $\bar{K} = diag(k_1, ..., k_l)$, M_l is the minor matrix of M by removing its first l $(1 \le l < N)$ row-column pairs, F and S are matrices with appropriate dimensions.

3.1. Pinning exponential synchronization via the linear intermittent control

Let $e_i(t) = x_i(t) - s(t)$ be the synchronization error. Without loss of generality, assume that the first l nodes are selected and pinned with the linear intermittent controllers, which are defined as follows:

$$u_i(t)=\begin{cases}-k_i\Gamma e_i(t),\ t\in[mT,\ mT+T_1),\ 1\le i\le l,\\ 0,\ t\in[mT,\ mT+T_1),\ l+1\le i\le N,\\ 0,\ t\in[mT+T_1,\ (m+1)T),\ 1\le i\le N,\end{cases}\tag{9}$$

where $k_i>0$ are control gains, $T>0$ is the control period, $0<T_1<T$ is the control width, and $m=0,\ 1,\ 2,\ ....$ Let $\theta=T_1/T$ be the ratio between T_1 and T. The error dynamics is governed by

$$\begin{cases}\dot e_i(t)=-De_i(t)+A\hat f(e_i(t))+B\hat f(e_i(t-\tau_1(t)))\\ \qquad +E\int_{t-\tau_2(t)}^{t}\hat f(e_i(v))dv+c\sum_{j=1}^{N}g_{ij}\Gamma h(e_j(t))\\ \qquad -k_ie_i(t),\ t\in[mT,\ mT+\theta T),\ 1\le \mathrm{i}\le l,\\ \dot e_i(t)=-De_i(t)+A\hat f(e_i(t))+B\hat f(e_i(t-\tau_1(t)))\\ \qquad +E\int_{t-\tau_2(t)}^{t}\hat f(e_i(v))dv+c\sum_{j=1}^{N}g_{ij}\Gamma h(e_j(t)),\\ \qquad t\in[mT,\ mT+\theta T),\ l+1\le i\le N,\\ \dot e_i(t)=-De_i(t)+A\hat f(e_i(t))+B\hat f(e_i(t-\tau_1(t)))\\ \qquad +E\int_{t-\tau_2(t)}^{t}\hat f(e_i(v))dv+c\sum_{j=1}^{N}g_{ij}\Gamma h(e_j(t)),\\ \qquad t\in[mT+\theta T,\ (m+1)T),\ 1\le i\le N,\end{cases}\tag{10}$$

where $\hat f(e_i(t))=f(x_i(t))-f(s(t))$.

Theorem 1: If the control gains can be sufficiently large and there exist positive constants $a_2>a_1>0$ such that

$$\rho_1+c\alpha\lambda_{\max}(G_l)<0,\tag{11}$$
$$\rho_2<0,\tag{12}$$
$$a_1>L^2,\tag{13}$$
$$\rho=\zeta-a_2(1-\theta)>0,\tag{14}$$
$$k_i>\lambda_{\max}(F-SM_l^{-1}S^T),\tag{15}$$

where $\zeta>0$ is the unique positive solution of the equation $\zeta-a_1+L^2\exp\{\zeta\tau\}-\frac{L^2}{\zeta}+\frac{L^2}{\zeta}\exp\{\zeta\tau\}=0$. Then the synchronization of coupled networks (1) can be achieved under the linear pinning intermittent controllers (9).

Proof: Construct the following standard Lyapunov function

$$V(t)=\frac{1}{2}\sum_{i=1}^{N}e_i^T(t)e_i(t).\tag{16}$$

When $mT\le t<mT+\theta T$, taking the derivative of V(t) with respect to time t along the solutions of error system (10) yields

$$\begin{aligned}\dot V(t)=&\sum_{i=1}^{N}e_i^T(t)[-De_i(t)+A\hat f(e_i(t))\\ &+B\hat f(e_i(t-\tau_1(t)))+E\int_{t-\tau_2(t)}^{t}\hat f(e_i(v))dv]\\ &+c\sum_{i=1}^{N}\sum_{j=1}^{N}e_i^T(t)g_{ij}\Gamma h(e_j(t))-\sum_{i=1}^{l}k_ie_i^T(t)\Gamma e_i(t).\end{aligned}\tag{17}$$

Let $W(t)=\sum_{i=1}^{N}e_i^T(t)[-De_i(t)+A\hat f(e_i(t))+B\hat f(e_i(t-\tau_1(t)))+E\int_{t-\tau_2(t)}^{t}\hat f(e_i(v))dv]$. In view of assumption 1 and the Jensen inequality, we have

$$\begin{aligned}W(t)\le&-\sum_{i=1}^{N}e_i^T(t)De_i(t)\\ &+\frac{1}{2}\sum_{i=1}^{N}[e_i^T(t)AA^Te_i(t)+\hat f^T(e_i(t))\hat f(e_i(t))]\\ &+\frac{1}{2}\sum_{i=1}^{N}[e_i^T(t)BB^Te_i(t)\\ &+\hat f^T(e_i(t-\tau_1(t)))\hat f(e_i(t-\tau_1(t)))]\\ &+\frac{1}{2}\sum_{i=1}^{N}[e_i^T(t)EE^Te_i(t)\\ &+(\int_{t-\tau_2(t)}^{t}\hat f(e_i(v))dv)^T\int_{t-\tau_2(t)}^{t}\hat f(e_i(v))dv]\\ \le&-\sum_{i=1}^{N}e_i^T(t)De_i(t)\\ &+\frac{1}{2}\sum_{i=1}^{N}[e_i^T(t)AA^Te_i(t)+L^2e_i^T(t)e_i(t)]\\ &+\frac{1}{2}\sum_{i=1}^{N}[e_i^T(t)BB^Te_i(t)\\ &+L^2e_i^T(t-\tau_1(t))e_i(t-\tau_1(t))]\\ &+\frac{1}{2}\sum_{i=1}^{N}[e_i^T(t)EE^Te_i(t)\\ &+L^2\int_{t-\tau_2(t)}^{t}e_i^T(v)e_i(v)dv]\\ \le&\lambda_{\max}(Q)e^T(t)e(t)\\ &+\frac{1}{2}L^2e^T(t-\tau_1(t))e(t-\tau_1(t))\\ &+\frac{1}{2}L^2\int_{t-\tau_2(t)}^{t}e^T(v)e(v)dv,\end{aligned}\tag{18}$$

where $e(t)=(e_1^T(t),e_2^T(t),...,e_n^T(t))^T$.

Denote $\tilde e_k(t)=[e_{1k}(t),\ e_{2k}(t),\ \cdots,\ e_{Nk}(t)]^T$, and $\tilde h(\tilde e_k(t))=[g(x_{1k}(t))-g(s_k(t)),\ g(x_{2k}(t))-g(s_k(t)),\ \cdots,\ g(x_{Nk}(t))-g(s_k(t))]^T$. It follows from Lemma 3 that

$$\begin{aligned}&\sum_{i=1}^{N}\sum_{j=1}^{N}e_i^T(t)g_{ij}\Gamma h(e_j(t))\\ &=\sum_{k=1}^{n}\gamma_k\tilde e_k^T(t)G\tilde h(e_k(t))\\ &=-\sum_{k=1}^{n}\gamma_k\sum_{j>i}G_{ij}(x_{ik}(t)-x_{jk}(t))(g(x_{ik}(t))-g(x_{jk}(t)))\end{aligned}$$

$$\leq -\alpha \sum_{k=1}^{n} \gamma_k \sum_{j>i} G_{ij}(x_{ik}(t)-x_{jk}(t))(g(x_{ik}(t))-g(x_{jk}(t)))$$
$$= \alpha \sum_{k=1}^{n} \gamma_k \tilde{e}_k^T(t) G \tilde{e}_k(t) = \alpha \sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T(t) g_{ij} \Gamma e_j(t). \tag{19}$$

From (18) and (19), we have

$$\begin{aligned}\dot{V}(t) \leq & \frac{\lambda_{\max}(Q)+\frac{1}{2}a_1}{\lambda_{\min}(\Gamma)} e^T(t)(I_N \otimes \Gamma)e(t) \\ & - \frac{a_1}{2} e^T(t)e(t) + \frac{1}{2} L^2 e^T(t-\tau_1(t))e(t-\tau_1(t)) \\ & + \frac{1}{2} L^2 \int_{t-\tau_2(t)}^{t} e^T(v)e(v)dv \\ & + e^T(t)((c\alpha G - K)\otimes \Gamma)e(t) \\ = & e^T(t)((M-K)\otimes\Gamma)e(t) - a_1 V(t) \\ & + L^2 V(t-\tau_1(t)) + L^2 \int_{t-\tau_2(t)}^{t} V(v)dv \end{aligned} \tag{20}$$

It is obvious to see that M is symmetric. According to Lemma 2, if one can select $k_i > \lambda_{\max}(F - SM_l^{-1}S^T)$, then $M-K<0$ is equivalent to $M_l<0$. Based on the condition (11) and Lemma 1, we have $\lambda_{\max}(M_l) \leq \rho_1 + c\alpha\lambda_{\max}(G_l) < 0$, which implies that $M_l < 0$. Then, we can obtain

$$\dot{V}(t) \leq -a_1 V(t) + L^2 V(t-\tau_1(t)) + L^2 \int_{t-\tau_2(t)}^{t} V(v)dv. \tag{21}$$

When $mT+\theta T \leq t < (m+1)T$, by differentiating V(t) along the trajectories of error system (10), we have

$$\begin{aligned}\dot{V}(t) \leq & e^T(t)(\tilde{M}\otimes\Gamma)e(t) + (a_2-a_1)V(t) \\ & + L^2 V(t-\tau_1(t)) + L^2 \int_{t-\tau_2(t)}^{t} V(v)dv. \end{aligned} \tag{22}$$

By using Lemma 1 and Lemma 1 in [8], we have $\lambda_{\max}(\tilde{M}) \leq \rho_2 + c\alpha\lambda_{\max}(G) = \rho_2$. If the given condition in (12) is satisfied, it is easy to have $\tilde{M}<0$. Then we can obtain

$$\dot{V}(t) \leq (a_2-a_1)V(t) + L^2 V(t-\tau_1(t)) + L^2 \int_{t-\tau_2(t)}^{t} V(v)dv. \tag{23}$$

Together with (21) and (23), we have

$$\begin{cases} \dot{V}(t) \leq -a_1 V(t) + L^2 V(t-\tau_1(t)) + L^2 \int_{t-\tau_2(t)}^{t} V(v)dv, \\ \qquad mT \leq t < mT+\theta T, \\ \dot{V}(t) \leq (a_2-a_1)V(t) + L^2 V(t-\tau_1(t)) \\ \qquad + L^2 \int_{t-\tau_2(t)}^{t} V(v)dv, \\ \qquad mT+\theta T \leq t < (m+1)T. \end{cases} \tag{24}$$

By using Lemma 4 and the conditions (12) and (13), we have $\dot{V}(t) \leq (\sup_{-\tau\leq s\leq 0} V(s))\exp\{-\rho t\},\ t\geq 0$. Hence, the zero solution of the error system (10) is globally exponentially stable according to Definition 1, this ends the proof. □

Remark 3: In the work of [43], two restrictive conditions, $T_1 \geq \tau$ and $T - T_1 \geq \tau$, were involved to derive the main results. By employing the useful Lemma 4, these two restrictions are removed. From (13) of Theorem 1, we find that the proposed conditions relies on the rate between the control widthT_1 and the control period T, but not the actual values of T_1 or T. If the control rate is fixed, we can choose randomly the control period T for achieving synchronization. Therefore, our results are less conservative and more practically applicable than [43].

3.2. Pinning exponential synchronization via the adaptive intermittent control

In this subsection, considering that the theoretical linear feedback gains $k_i > \lambda_{\max}(F - SM_l^{-1}S^T)$ may be much larger than the practical needed values, we introduce an adaptive strategy to adjust the feedback gains. The adaptive periodically intermittent controllers are designed to be

$$u_i(t) = \begin{cases} -k_i(t)\Gamma e_i(t), & t\in[mT,\ mT+\theta T), \\ & 1\leq i\leq l, \\ 0, & t\in[mT,\ mT+\theta T),\ l+1\leq i\leq N, \\ 0, & t\in[mT+\theta T,\ (m+1)T),\ 1\leq i\leq N, \end{cases} \tag{25}$$

with the updating laws

$$\dot{k}_i(t) = \begin{cases} \alpha_i \exp(a_1 t) e_i^T(t)\Gamma e_i(t), & t\in[mT,\ m+\theta T), \\ 0, & t\in[mT+\theta T_1,\ (m+1)T), \end{cases} \tag{26}$$

where α_i are positive constants to be designed latter.

From (1) and (25), the following error state equations can be obtained

$$\begin{cases} \dot{e}_i(t) = -De_i(t) + A\hat{f}(e_i(t)) + B\hat{f}(e_i(t-\tau_1(t))) \\ \qquad + E\int_{t-\tau_2(t)}^{t} \hat{f}(e_i(v))dv + c\sum_{j=1}^{N} g_{ij}\Gamma h(e_j(t)) \\ \qquad - k_i(t)e_i(t),\ t\in[mT,\ mT+\theta T),\ 1\leq \text{i} \leq l, \\ \dot{e}_i(t) = -De_i(t) + A\hat{f}(e_i(t)) + B\hat{f}(e_i(t-\tau_1(t))) \\ \qquad + E\int_{t-\tau_2(t)}^{t} \hat{f}(e_i(v))dv + c\sum_{j=1}^{N} g_{ij}\Gamma h(e_j(t)), \\ \qquad t\in[mT,\ mT+\theta T),\ l+1\leq i\leq N, \\ \dot{e}_i(t) = -De_i(t) + A\hat{f}(e_i(t)) + B\hat{f}(e_i(t-\tau_1(t))) \\ \qquad + E\int_{t-\tau_2(t)}^{t} \hat{f}(e_i(v))dv + c\sum_{j=1}^{N} g_{ij}\Gamma h(e_j(t)), \\ \qquad t\in[mT+\theta T,\ (m+1)T),\ 1\leq i\leq N. \end{cases} \tag{27}$$

Theorem 2: If there exist positive constants $a_2 > a_1 > 0$ such that (11)-(14) are satisfied, then the pinning synchronization of coupled neural networks (1) can be achieved under the adaptive intermittent controllers (25) and updating laws (26).

Proof: Consider a Lyapunov function as follows:

$$V(t)=\frac{1}{2}\sum_{i=1}^{N}e_i^T(t)e_i(t)+\frac{1}{2}\sum_{i=1}^{l}\exp(-a_1t)\frac{(k_i(t)-k_i^*)^2}{\alpha_i}, \tag{28}$$

where $k_i^* > 0$ are some positive constants to be determined below.

When $mT \le t < mT+\theta T$, taking the derivative of V(t) with respect to time t along the solutions of error system (27) yields

$$\begin{aligned}\dot{V}(t) \le& e^T(t)((M-K^*)\otimes\Gamma)e(t)-a_1V(t)\\&+L^2V(t-\tau_1(t))+L^2\int_{t-\tau_2(t)}^{t}V(v)dv,\end{aligned} \tag{29}$$

where $K^* = diag(k_1^*,...,k_l^*,\underbrace{0,...,0}_{N-l})$. Using the similar analysis of Theorem 1, if the given condition (11) is satisfied, we can obtain

$$\dot{V}(t)\le -a_1V(t)+L^2V(t-\tau_1(t))+L^2\int_{t-\tau_2(t)}^{t}V(v)dv. \tag{30}$$

When $mT+\theta T \le t < (m+1)T$, using (12) and the similar peocess of Theorem 1, we have

$$\begin{aligned}\dot{V}(t) \le&(a_2-a_1)V(t)+L^2V(t-\tau_1(t))\\&+L^2\int_{t-\tau_2(t)}^{t}V(s)ds.\end{aligned} \tag{31}$$

Thus we have

$$\begin{cases}\dot{V}(t) \le -a_1V(t)+L^2V(t-\tau_1(t))\\ \qquad +L^2\int_{t-\tau_2(t)}^{t}V(v)dv,\ \ mT\le t< mT+\theta T,\\ \dot{V}(t)\le(a_2-a_1)V(t)+L^2V(t-\tau_1(t))\\ \qquad +L^2\int_{t-\tau_2(t)}^{t}V(v)dv,\\ \qquad mT+\theta T\le t<(m+1)T.\end{cases} \tag{32}$$

In virtue of Lemma 4 and the conditions (13) and (14), we have $\dot{V}(t) \le (\sup_{-\tau\le s\le 0} V(s))\exp\{-\rho t\}$, $t \ge 0$. This ends the proof. □

Remark 4: Theorems 1 and 2 give some sufficient conditions to ensure pinning synchronization of coupled neural networks (1) via linear or adaptive intermittent control, respectively. The first condition (11) can be regarded as the pinning condition, because it provides a theoretical answer to the question of how many nodes should be pinned. It is worth nothing that the value of $\lambda_{\max}(G_l)$ relies on the pinning scheme. For a given undirected network with N nodes, a simple high-degree pinning scheme is presented based on [20] as bellows.

Step 1: Let $d_i = \sum_{j=1,j\ne i}^{N} G_{ij}$ be the total weights between node i and all the other nodes, where d_i is the degree of node i. Define a degree vector: $\text{Deg}(i) = d_i$, $i = 1,\cdots,N$.

Step 2: Rearrange the network nodes according to the decrease of degree. For the nodes with the same degree, we sort them in descending order according to their initial order. Let $l = 1$.

Step 3: Select the first l network nodes as pinned candidates. Evaluate and check if pinning condition (11) is satisfied.

Step 4: If pinning condition (11) is not satisfied, let $l = l+1$, go to Step 3. Otherwise, end.

Remark 5: In Theorems 1 and 2, there are many parameters to be determined. In the following, a simple algorithm is presented for how to choose appropriate parameters such that the nonlinearly coupled neural networks can realize exponential synchronization under linear intermittent pinning control strategy.

Algorithm 1 (A parameter selection algorithm for linear intermittent pinning controller design)**:**

Step 1: For a given network model, rearrange nodes by using the high-degree selection scheme, and calculate $\lambda_{\max}(Q)$, $\lambda_{\max}(\Gamma)$, and ρ_1, respectively.

Step 2: Choose a_1 and a_2 such that $a_1 > L^2$ and $\rho_2 < 0$, respectively.

Step 3: Solving the equation $\zeta - a_1 + L^2\exp\{\zeta\tau\} - \frac{L^2}{\zeta} + \frac{L^2}{\zeta}\exp\{\zeta\tau\} = 0$ to obtain the unique positive solution ζ.

Step 4: Choose θ such that $\rho = \zeta - a_2(1-\theta) > 0$.

Step 5: Determine the minimum number of pinned nodes l such that $\lambda_{\max}(G_{l-1}) \ge -\rho_1/c\alpha$ and $\lambda_{\max}(G_l) < -\rho_1/c\alpha$.

Step 6: Choose the control period T and control width T_1 according to θ.

Step 7: Choose k_i such that $k_i > \lambda_{\max}(F - SM_l^{-1}S^T)$.

Once these parameters are determined, the proposed synchronization criterions are satisfied. Moreover, the parameter selection algorithm 1 is also applicable for the adaptive intermittent pinning controller design by just choosing steps 1-6.

Remark 6: Recently, many researches have appeared on the $H\infty$ synchronization for delayed system [29–31, 46, 47], $H\infty$ control for Markovian jump systems [44, 45], or output feedback control problems [48–50]. For a coupled network with external disturbance or the states are note available, the proposed results are not applicable. A meaningful question is how to achieve synchronization for coupled dynamical networks with more complicated circumstance under intermittent control.

4. NUMERICAL EXAMPLES

In this section, we consider the following delayed neural network with mixed delays as the node of coupled networks (1),

$$\begin{aligned}\dot{x}(t) =& -Dx(t)+Af(x(t))+Bf(x(t-\tau_1(t)))\\ &+E\int_{t-\tau_2(t)}^{t} f(x(v))dv+J(t),\end{aligned} \quad (33)$$

where $x(t) = (x_1(t),\ x_2(t))^T$, $f(x(t)) = (\tanh(x_1(t)), \tanh(x_2(t)))^T$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $A = \begin{bmatrix} 1.8 & -0.15 \\ -5.2 & 3.5 \end{bmatrix}$, $B = \begin{bmatrix} -1.7 & -0.12 \\ -0.26 & -2.5 \end{bmatrix}$, $E = \begin{bmatrix} 0.6 & 0.15 \\ -2 & -0.12 \end{bmatrix}$. It is assumed that $J(t) = [0,\ 0]^T$, $\tau_1(t) = e^t/(1+e^t)$ and $\tau_2(t) = 0.5\sin^2 t$. A straightforward calculation gives that $\tau_1 = 1$, $\tau_2 = 0.5$, and $L = 1$. The chaotic attractor of neural network (33) is shown in Fig. 1 with the initial state $x(0) = [0.5\ \ 0.6]^T$.

We assume that the coupling configuration matrix G is determined by the free-scale network with $N = 100$ and $m_0 = m = 3$, and the inner coupling matrix is taken as $\Gamma = diag\{2,\ 2\}$. The orbits of $\lambda_{\max}(G_l)$ as functions of the number of pinned nodes by high-degree, low-degree and random pinning schemes are shown in Fig. 2.

To simplify the analysis, the nonlinear coupling function is chosen as $h(x) = 3x+\sin x$, which implies $\alpha = 2$ and $\beta = 4$. Choosing $\theta = 0.95$, $c = 8$, $a_1 = 5$ and $a_2 = 60$, we can check that the conditions (12)-(14) are satisfied. Some simple calculations give that $-\rho_1/\alpha c = -0.8748$. From Fig. 2, we only need to pin 28, 22 and 7 nodes by using low-degree, random and high-degree pinning schemes, respectively. In what follows, we will adopt the linear and adaptive feedback controllers to achieve synchronization under the high-degree pinning scheme, respectively.

To measure the extent to which synchronization of nonlinearly coupled neural networks (1) is achieved, we introduce a quantity, $E(t) = \max\{||x_i(t) - s(t)||,\ i = 1,2,...,100\}$, for $t \in [0,+\infty)$.

4.1. Pinning synchronization via the linear feedback control

For $c = 8$, we can obtain the linear feedback gain $k_i > 3991.8$ ($i = 1,\ \cdots,\ 7$) by testing the condition (11). For convenience, we take $k_i = 3992$ ($i = 1,\cdots,7$).

According to $\theta = 0.95$, one can choose $T = 0.2$and $T_1 = 0.19$ in the simulation analysis. The initial values of the dynamical coupled networks are set to be $x_i(0) = (2+0.3i, 3+0.5i)^T$, $1 \le i \le 100$, and $s(0) = (2,\ 3)^T$. The evolution of synchronization error quantity is illustrated in Fig. 3. Obviously, the synchronization of coupled neural networks (1) is achieved under the pinning intermittent control scheme with $l = 7$.

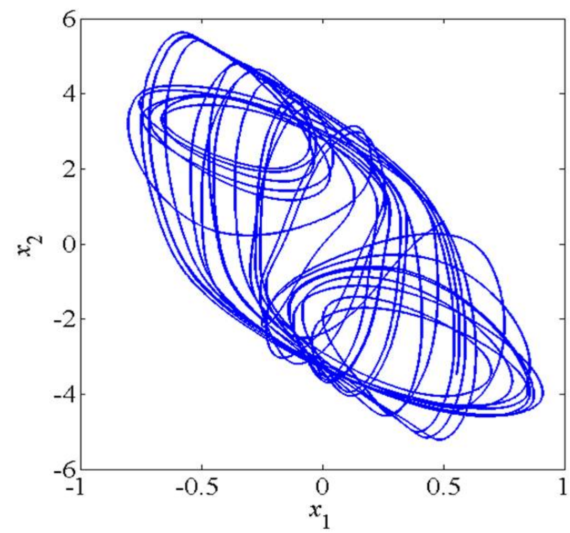

Fig. 1. Chaotic attractor of neural network (33).

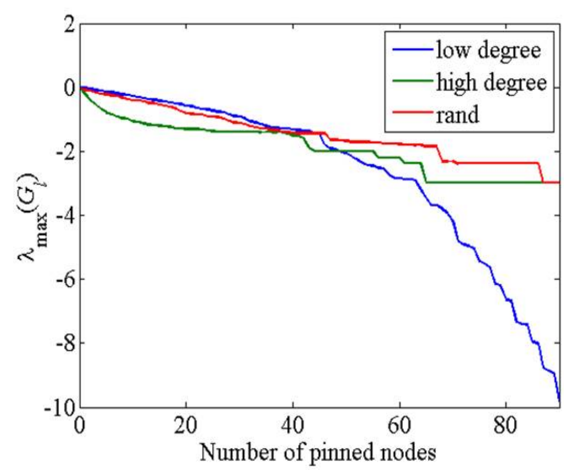

Fig. 2. Orbits of $\lambda_{\max}(G_l)$ as functions of the number of pinned nodes by high-degree, low-degree and random pinning schemes.

5. PINNING SYNCHRONIZATION VIA THE ADAPTIVE FEEDBACK CONTROL

We take the same parameters as the above subsection and $k_i(0) = 2+i$, where $1 \le i \le 7$. The evolution of the synchronization error quantity $E(t)$ is illustrated in Fig. 4. The adaptive feedback gains are shown in Fig. 5, which are much smaller than the linear feedback gain. The numerical simulation results demonstrate the validity of our theoretical analysis.

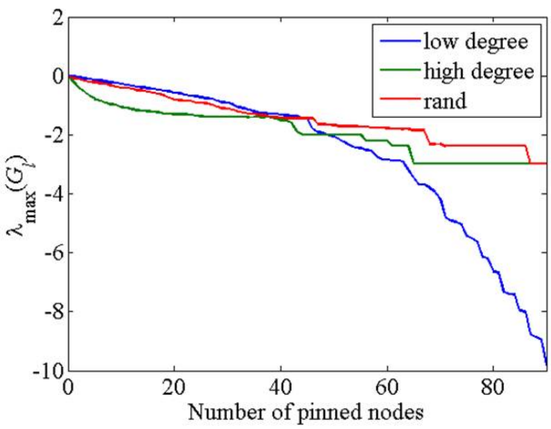

Fig. 3. Synchronization error quantity $E(t)$ via linear feedback control.

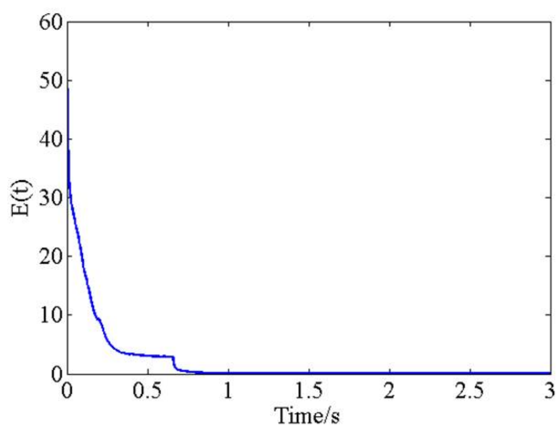

Fig. 4. Synchronization error quantity $E(t)$ via adaptive feedback control.

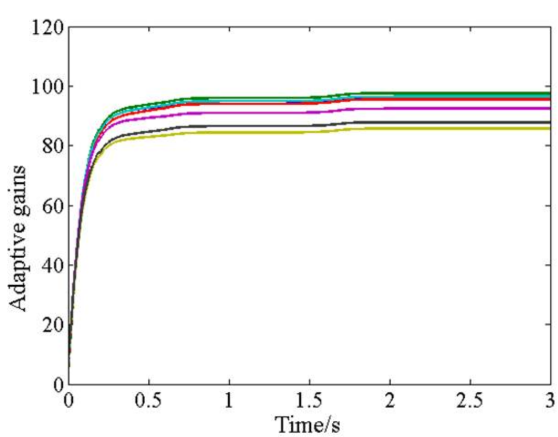

Fig. 5. Evolution of adaptive feedback gains.

6. CONCLUSION

In this paper, the exponential synchronization problem has been investigated for an array of nonlinearly coupled neural networks with mixed time-varying delays. By using the intermittent control approach, linear and adaptive pinning controllers are added to a fraction of network nodes, respectively. Based on a new differential inequality, several sufficient conditions are obtained to ensure that the coupled nerual networks can achieve synchronization exponentially. Numerical simulations show the validity of the proposed methodology.

It is known that Markovian jump is a suitable mathematical pattern to represent a class of complex networks subject to random abrupt variations in the structures. The extensions of the current results to the underlying systems with Markovian jumping parameters are also interesting, which constitutes a future research direction.

APPENDIX A

A.1. Proof of Lemma 4

Proof: Denote

$$f(\zeta)=\begin{cases}\zeta-r_1+r_3\exp\{\zeta\tau\}-\dfrac{r_4}{\zeta}+\dfrac{r_4}{\zeta}\exp\{\zeta\tau\}, \\ \qquad \zeta>0, \\ -r_1+r_3, \quad \zeta=0. \end{cases} \tag{A.1}$$

Since $r_1>r_3$, we have $f(0)<0$, $f(+\infty)>0$, and $f^{'}(\zeta)>0$ for $\zeta>0$. Using the continuity and monotonicity of $f(\zeta)$, the equation $\zeta-r_1+r_3\exp\{\zeta\tau\}-\frac{r_4}{\zeta}+\frac{r_4}{\zeta}\exp\{\zeta\tau\}=0$ has an unique positive solution for $\zeta>0$. Take $M=\sup_{-\tau\le t\le 0} y(t)$, $W(t)=\exp\{\zeta t\}y(t)$, where $t\ge 0$. Let $Q(t)=W(t)-hM$, where $h>1$ is a constant. It is easy to see that for all $t\in[-\tau,\ 0]$,

$$Q(t)<0, \tag{A.2}$$

In the following, we will prove that for all $t\in[0,\ \theta T)$,

$$Q(t)<0, \tag{A.3}$$

If it is not true, by (A.1) and the continuity of $y(t)$ as $t\in[-\tau,\ \infty)$, then there exist a $t_0\in[0,\ \theta T)$ such that

$$Q(t_0)=0,\ \ \dot{Q}(t_0)\ge 0, \tag{A.4}$$

$$Q(t)<0,\quad -\tau\le t<t_0. \tag{A.5}$$

According to (A.3) and (A.4), it is easy to verify that

$$\begin{aligned}\dot{Q}(t_0)\le&\zeta W(t_0)-r_1W(t_0)\\&+r_3\exp\{\zeta\tau_1(t_0)\}W(t_0-\tau_1(t_0))\\&+r_4\exp\{\zeta t_0\}\int_{t_0-\tau_2(t_0)}^{t_0}y(s)ds.\end{aligned} \tag{A.6}$$

Note that $W(t_0)=hM$. From (A.4), we have $W(t_0-\tau_1(t_0))<W(t_0)$, and $y(t)<W(t_0)\exp\{-\zeta t\}$ for all $t\in[t_0-\tau_2(t_0),\ t_0]$. It follows from (A.5) that

$$\dot{Q}(t_0)\le\zeta W(t_0)-r_1W(t_0)+r_3\exp\{\zeta\tau_1(t_0)\}W(t_0)$$

$$+r_4 W(t_0)\exp\{\zeta t_0\}\int_{t_0-\tau_2(t_0)}^{t_0}\exp\{-\zeta s\}ds$$
$$\leq(\zeta - r_1 + r_3\exp\{\zeta\tau\} - \frac{r_4}{\zeta} + \frac{r_4}{\zeta}\exp\{\zeta\tau\})W(t_0)$$
$$=0. \tag{A.7}$$

This contradicts the second inequality in (A.3), and so (A.2) holds. Together with (A.1), we can obtain that for all $t \in [-\tau,\ \theta T)$,

$$y(t) < hM\exp\{-\zeta t\}. \tag{A.8}$$

Now, we prove that for $t \in [\theta T,\ T)$

$$H(t) = W(t) - hM\exp\{\gamma(t-\theta T)\} < 0. \tag{A.9}$$

Otherwise, there exists $t_1 \in [\theta T,\ T)$ such that

$$H(t_1) = 0,\ \dot{H}(t_1) \geq 0, \tag{A.10}$$
$$H(t) < 0,\ \theta T \leq t < t_1. \tag{A.11}$$

If $\theta T \leq t_1 - \tau < t_1$, it follows from (A.9) and (A.10) that

$$y(t_1 - \tau_1(t_1)) < \exp\{\zeta\tau\}y(t_1), \tag{A.12}$$

and for all $t \in [t_1 - \tau_2(t_1),\ t_1)$

$$y(t) < hM\exp\{\gamma(t_1-\theta T)\}\exp\{-\zeta t$$
$$= W(t_1)\exp\{-\zeta t\}. \tag{A.13}$$

If $-\tau \leq t_1 - \tau < \theta T$, from (A.7), (A.9) and (A.10), we get

$$y(t_1 - \tau_1(t_1)) < \exp\{\zeta\tau\}y(t_1), \tag{A.14}$$

and for all $t \in [t_1 - \tau_2(t_1),\ t_1)$

$$y(t) < W(t_1)\exp\{-\zeta t\}. \tag{A.15}$$

Hence, for $\tau > 0$, we always have

$$y(t_1 - \tau_1(t_1)) < \exp\{\zeta\tau\}y(t_1), \tag{A.16}$$

and for $t \in [t_1 - \tau_2(t_1),\ t_1)$

$$y(t) < W(t_1)\exp\{-\zeta t\}. \tag{A.17}$$

Then we have

$$\dot{H}(t_1) \leq \left(\zeta - r_1 + r_3\exp\{\zeta\tau\} - \frac{r_4}{\zeta} + \frac{r_4}{\zeta}\exp\{\zeta\tau\}\right)W(t_1)$$
$$=0, \tag{A.18}$$

which contradicts the second inequality in (A.9). Hence (A.8) holds. Consequently, for $t \in [\theta T,\ T)$,

$$W(t) < hM\exp\{\gamma(t-\theta T)\} \leq hM\exp\{\gamma(1-\theta)T\}.$$

Together with (A.1) and (A.2), we get that for all $t \in [-\tau,\ T)$,

$$W(t) < hM\exp\{\gamma(1-\theta)T\}. \tag{A.19}$$

Similarly, we can prove that for $t \in [T,\ (1+\theta)T)$,

$$W(t) < hM\exp\{\gamma(1-\theta)T\},$$

and for $t \in [(1+\theta)T,\ 2T)$,

$$W(t) < hM\exp\{\gamma(t-2\theta T)\}.$$

By using mathematical induction, we can derive the following estimation of $W(t)$ for any integer m.

For $t \in [mT,\ (m+\theta)T)$,

$$W(t) < hM\exp\{m\gamma(1-\theta)T\}, \tag{A.20}$$

and for $t \in [(m+\theta)T,\ (m+1)T)$,

$$W(t) < hM\exp\{\gamma(t-(m+1)\theta T)\}. \tag{A.21}$$

Since for any $t \geq 0$, there exists a nonnegative integer k, such that $kT \leq t < (k+1)T$, we can deduce the following estimation of $W(t)$ for any t by (A.19) and (A.20).

For $t \in [kT,\ (k+\theta)T)$,

$$W(t) < hM\exp\{k\gamma(1-\theta)T\} \leq hM\exp\{\gamma(1-\theta)t\},$$

and for $t \in [(k+\theta)T,\ (k+1)T)$,

$$W(t) < hM\exp\{\gamma(t-(k+1)\theta T)\}$$
$$\leq hM\exp\{\gamma(1-\theta)t\}.$$

Let $h \to 1$, from the definition of $W(t)$, we obtain

$$y(t) < M\exp\{-(\zeta - \gamma(1-\theta))t\} = M\exp\{-\rho t\}.$$

This completes the proof. □

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