International Journal of Control, Automation and Systems 15(2) (2017) 603-610
http://dx.doi.org/10.1007/s12555-015-0437-9

ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Fuzzy Filter for Nonlinear Sampled-data Systems: Intelligent Digital Redesign Approach

Ho Jun Kim, Jin Bae Park*, and Young Hoon Joo

Abstract: This paper presents a fuzzy filter design method for nonlinear sampled-data systems using an intelligent digital redesign (IDR) technique. Based on a Takagi–Sugeno (T–S) fuzzy model, discretized closed-loop systems with pre-designed analog fuzzy and digital fuzzy filters are presented. An IDR problem is given to guarantee both state-matching condition and asymptotic stability. Sufficient conditions for solving the IDR problem are proposed and are derived in terms of linear matrix inequalities (LMIs). Finally, a simulation example is given to show the effectiveness of the proposed method.

Keywords: Fuzzy filter, intelligent digital redesign (IDR), linear matrix inequality (LMI), sampled-data systems, Takagi-Sugeno (T-S) fuzzy model.

1. INTRODUCTION

In recent years, since many practical systems have high nonlinearities, there has been much attention paid to the analysis of nonlinear systems [1, 2]. Among the many techniques for the nonlinear system analysis, the Takagi–Sugeno (T–S) fuzzy method is an interesting approach because of its capability to represent the nonlinear dynamics. Using the T–S fuzzy method, the nonlinear dynamics were represented by a set of linear models interpolated by membership functions [2, 3]. For this reason, it can effectively bridge the gap between nonlinear systems and various linear system theories [4, 5]. Although many studies for the T–S fuzzy system have been conducted, there still remain many issues, especially sampled-data system issues, to be solved.

On the other hand, as most engineering applications have both analog plant and digital computer-based implementation, sampled-data systems have gained much attention in various fields such as transportation systems, communication networks, and mobile robotics [6–8]. Because the continuous and discrete-time signals coexist simultaneously in the sampled-data system, the traditional analysis methods for a homogeneous signal system can not be directly used. To conquer the above problem, various digital techniques have been researched and can be classified into three categories: direct digital design techniques [9], sampled-data techniques [10–13], and digital redesign (DR) techniques [14, 15]. In the first method, the continuous-time system is discretized, and then discrete-time controller or filter is designed for the discretized system. The second technique represents the direct design method of the discrete-time controller or filter for the continuous system. In the third method, the analog controller or filter is first designed and then approximately converted to an equivalent digital ones in the sense of state-matching. Especially compared to other methods, the DR technique can not only maintain the performance of the continuous-time controller or filter, but also apply various conventional continuous-time techniques.

For these reasons, many DR methods have been proposed [15–17] and can be successfully extended to the nonlinear systems using the intelligent digital redesign (IDR) technique, which is merged with the DR technique and the T–S fuzzy theory. In [18], the IDR technique was presented in terms of linear matrix inequalities (LMIs), ensuring not only stability but also global state matching. In [19], using the guaranteed cost control method, the performance of the IDR was improved. An observer-based controller using the IDR technique has been developed with a non-measurable premise variable [20], parametric uncertainties [21], and output-feedback tracking controller [22]. Very recently, the IDR technique of the tracking controller using delta operator and piecewise linear-

Manuscript received November 20, 2015; revised February 17, 2016 and March 13, 2016; accepted March 22, 2016. Recommended by Associate Editor Do Wan Kim under the direction of Editor Duk-Sun Shim. This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MEST, MSIP) (NRF-2015R1A2A2A05001610, NRF-2015R1A2A2A01007545) and the Human Resources Development program (No. 20144030200590) of the Korea Institute of Energy Technology Evaluation and Planning (KETEP) grant funded by the Korea government Ministry of Knowledge Economy.

Ho Jun Kim and Jin Bae Park are with the Department of Electrical and Electronic Engineering, Yonsei University, Seoul 120-749, Korea (e-mails: {khj08121, jbpark}@yonsei.ac.kr). Young Hoon Joo is with the Department of Control and Robot Engineering, Gunsan National University, Gunsan, Chonbuk 573-701, Korea (e-mail: yhjoo@kunsan.ac.kr).
* Corresponding author.

approximation has been proposed in [23]. However, research for the IDR technique has mainly focused on the control issue, and the design of the fuzzy filter based on the IDR technique has not yet been addressed.

Motivated by the aforementioned studies, this paper presents the T–S fuzzy filter design method for the nonlinear sampled-data system using the IDR method. The nonlinear continuous system is represented by the T–S fuzzy model. For development of the IDR technique, discretized closed-loop systems with pre-designed analog fuzzy and digital fuzzy filters are presented. Based on the discretized systems, the IDR problem is given to guarantee both the state-matching condition and the asymptotic stability. To resolve the IDR problem, sufficient conditions, which are derived in terms of LMIs, are given based on Lyapunov theory. Finally, a simulation example is given to evaluate the feasibility of the proposed method.

2. T-S FUZZY MODEL

Consider a T–S fuzzy system that is represented by the following ith IF-THEN rule:

$$R_i : \text{IF } z_1(t) \text{ is } \Gamma_1^i \text{ and } \cdots \text{ and } z_q(t) \text{ is } \Gamma_q^i,$$
$$\text{THEN} \begin{cases} \dot{x}(t) = A_i x(t) + B_i w(t) \\ s(t) = C_{1i} x(t) \\ y(t) = C_{2i} x(t) + D_i v(t), \end{cases} \tag{1}$$

where $z_p(t) \in \{\mathcal{I}_q := \{1,2,\ldots,q\}\}$ is the premise variable, $x(t) \in \mathbb{R}^n$ is the state, $w(t) \in \mathbb{L}_2^{q_1}$ is the disturbance, $v(t) \in \mathbb{R}^{q_2}$ is the measurement noise, $s(t) \in \mathbb{R}^{m_1}$ is the output to be estimated, $y(t) \in \mathbb{R}^{m_2}$ is the measurement output; Γ_p^i, $(i,p) \in \{\mathcal{I}_r := \{1,2,\ldots,r\}\} \times \{\mathcal{I}_q := \{1,2,\ldots,q\}\}$ is the fuzzy set for z_p; and A_i, B_i, C_{1i}, C_{2i}, and D_i are nominal system matrices of appropriate dimensions for the ith IF–THEN rule.

Using the singleton fuzzifier, product inference, and center-average deffuzifier, the global dynamics of (1) are inferred as

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r} h_i(z(t)) \Big\{ A_i x(t) + B_i w(t) \Big\}, \\ s(t) &= \sum_{i=1}^{r} h_i(z(t)) C_{1i} x(t), \\ y(t) &= \sum_{i=1}^{r} h_i(z(t)) \Big\{ C_{2i} x(t) + D_i v(t) \Big\}, \end{aligned} \tag{2}$$

where

$$h_i(z(t)) := \frac{\omega_i(z(t))}{\sum_{i=1}^{r} \omega_i(z(t))}, \quad \omega_i(z(t)) := \prod_{p=1}^{q} \Gamma_p^i(z_p(t)),$$

and $\Gamma_p^i : U_{z_p} \subset \mathbb{R} \to \mathbb{R}_{[0,1]}$ is the membership function of z_p on compact set U_{z_p}.

First, a pre-designed analog fuzzy filter for the fuzzy system (2) is supposed to have the following form:

$$\begin{aligned} \dot{\hat{x}}_c(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) \Big\{ \hat{A}_i^c \hat{x}_c(t) + \hat{B}_i^c y(t) \Big\}, \\ \hat{s}_c(t) &= \sum_{i=1}^{r} h_i(z(t)) \hat{C}_i^c \hat{x}_c(t), \end{aligned} \tag{3}$$

where $\hat{x}_c(t) \in \mathbb{R}^n$ is the state for the filter, $\hat{s}(t) \in \mathbb{R}^{m_1}$ is the filter output, the subscript 'c' refers to the analog signal, and $\hat{A}_i^c, \hat{B}_i^c$, and $\hat{C}_i^c$ are filter gain matrices, which are to be predesigned.

Remark 1: In the T–S fuzzy model, the filter is divided into two parts: the fuzzy-rule-dependent filter and the fuzzy-rule-independent filter. In the fuzzy-rule-dependent filter, the filter and the system share the same premise variable, and thus, the assumption that the premise variable of the fuzzy system is known in advance is needed. On the other hand, in the fuzzy-rule-independent filter, the premise variable is supposed to be unavailable in the filter design and more conservativeness can be induced than for the fuzzy-rule-dependent one. However, the filter can be designed regardless of the complexity of the premise variable of the system. In this paper, only the fuzzy-rule-dependent filter in the form of (3) is considered, and these results will be extended to the fuzzy-rule-independent filter in the future works.

Defining the analog filter error $e_c(t) := s(t) - \hat{s}_c(t)$ and substituting (3) into (2), the error system of the analog filter system is represented by the following form:

$$\begin{aligned} \dot{\varepsilon}_c(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) \\ &\quad \times \Big\{ \bar{A}_{ij}^c \varepsilon_c(t) + \bar{B}_{1i}^c w(t) + \bar{B}_{2ij}^c v(t) \Big\}, \\ e_c(t) &= \sum_{i=1}^{r} h_i(z(t)) \bar{C}_i^c \varepsilon_c(t), \end{aligned} \tag{4}$$

where

$$\begin{aligned} \varepsilon_c(t) &= \begin{bmatrix} x(t) \\ x(t) - \hat{x}_c(t) \end{bmatrix}, \\ \bar{A}_{ij}^c &= \begin{bmatrix} A_i & 0 \\ A_i - \hat{A}_i^c - \hat{B}_i^c C_{2j} & \hat{A}_i^c \end{bmatrix}, \\ \bar{B}_{1i}^c &= \begin{bmatrix} B_i \\ B_i \end{bmatrix}, \ \bar{B}_{2ij}^c = \begin{bmatrix} 0 \\ -\hat{B}_i^c D_j \end{bmatrix}, \\ \bar{C}_i^c &= \begin{bmatrix} C_{1i} - \hat{C}_i^c & \hat{C}_i^c. \end{bmatrix}. \end{aligned} \tag{5}$$

Before discretization of the continuous error system (4), the following assumption is needed in order to maintain the polytopic structure of the discretized T–S fuzzy system for the construction of the fuzzy model based digital filter.

Assumption 1 [18]**:** Assume that the firing strength of the ith rule is approximated as $h_i(z(t)) \approx h_i(z(kT))$

where $T \in \mathbb{R}_{>0}$ for $t \in [kT, kT+T)$, $k \in \mathbb{Z}_{\geq 0}$. Consequently, the nonlinear matrix functions $\sum_{i=1}^{r} h_i(z(t))A_i$ and $\sum_{i=1}^{r} h_i(z(t))B_i$ can be approximated by constant matrices $\sum_{i=1}^{r} h_i(z(kT))A_i$ and $\sum_{i=1}^{r} h_i(z(kT))B_i$, respectively, over any interval [kT, kT+T).

Using Assumption 1, the error system (4) is approximately discretized in the following form:

$$\begin{aligned} \varepsilon_c(kT+T) \approx &\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(kT))h_j(z(kT)) \\ &\times \left\{ \bar{G}_{ij}^{c}\varepsilon_c(kT) + \bar{T}_{ij}^{c} d(kT) \right\}, \\ e_c(kT) \approx &\sum_{i=1}^{r} h_i(z(kT))\bar{C}_i^{c}\varepsilon_c(kT), \end{aligned} \tag{6}$$

where

$$\begin{aligned} \bar{G}_{ij}^{c} =& e^{\bar{A}_{ij}^{c}T} = \begin{bmatrix} \Phi_{ij}^{11} & 0 \\ \Phi_{ij}^{21} & \Phi_{ij}^{22} \end{bmatrix}, \\ \bar{T}_{ij}^{c} =& [\bar{H}_{1ij}^{c} \;\; \bar{H}_{2ij}^{c}] = \begin{bmatrix} T_{ij}^{11} & T_{ij}^{12} \\ T_{ij}^{21} & T_{ij}^{22} \end{bmatrix}, \\ \bar{H}_{1ij}^{c} =& (\bar{G}_{ij}^{c} - I)(\bar{A}_{ij}^{c})^{-1}\bar{B}_{1i}^{c},\; \bar{H}_{2ij}^{c} = (\bar{G}_{ij}^{c} - I)(\bar{A}_{ij}^{c})^{-1}\bar{B}_{2ij}^{c}, \\ d(kT) =& [w^T(kT) \;\; v^T(kT)]^T. \end{aligned}$$

Next, the fuzzy system (2) is approximately discretized as:

$$\begin{aligned} x(kT+T) \approx &\sum_{i=1}^{r} h_i(z(kT)) \left\{ G_i x(kT) + H_i w(kT) \right\}, \\ s(kT) \approx &\sum_{i=1}^{r} h_i(z(kT)) C_{1i} x(kT), \\ y(kT) \approx &\sum_{i=1}^{r} h_i(z(kT)) \left\{ C_{2i} x(kT) + D_i v(kT) \right\}, \end{aligned} \tag{7}$$

where

$$G_i = e^{A_i T}, \qquad H_i = (G_i - I)A_i^{-1}B_{1i}.$$

Remark 2: The discretization process is performed by using the Taylor series expansion method and the discretization error is tolerable under the sufficiently small sampling period T. The detail of this issue is addressed in [18].

Remark 3: The approximately discretized models (6)–(7) contain the discretization error with order of $\mathcal{O}(T^2)$.

Then, a digital fuzzy filter for the fuzzy system (7) is described as:

$$\begin{aligned} \hat{x}_d(kT+T) = &\sum_{i=1}^{r} h_i(z(kT)) \left\{ \hat{A}_i^{d}\hat{x}_d(kT) + \hat{B}_i^{d} y(kT) \right\}, \\ \hat{s}_d(kT) = &\sum_{i=1}^{r} h_i(z(kT)) \hat{C}_i^{d}\hat{x}_d(kT), \end{aligned} \tag{8}$$

where $\hat{x}_d(kT) \in \mathbb{R}^n$ is the state for the digital filter, $\hat{s}_d(kT) \in \mathbb{R}^{m_1}$ is the digital filter output, the subscript 'd' refers to the digital signal, and $\hat{A}_i^{d}, \hat{B}_i^{d}$, and $\hat{C}_i^{d}$ are filter gain matrices that should be determined.

Defining the digital filter error $e_d(kT) := s(kT) - \hat{s}_d(kT)$ and substituting (8) into (7), the error system of the digital filter system is represented by the following form:

$$\begin{aligned} \varepsilon_d(kT+T) \approx &\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(kT))h_j(z(kT)) \\ &\times \left\{ \bar{G}_{ij}^{d}\varepsilon_d(kT) + \bar{T}_{ij}^{d} d(kT) \right\}, \\ e_d(kT) \approx &\sum_{i=1}^{r} h_i(z(kT))\bar{C}_i^{d}\varepsilon_d(kT), \end{aligned} \tag{9}$$

where

$$\begin{aligned} &\varepsilon_d(kT) = \begin{bmatrix} x(kT) \\ x(kT) - \hat{x}_d(kT) \end{bmatrix}, \; \bar{T}_{ij}^{d} = [\bar{H}_{1i}^{d} \;\; \bar{H}_{2ij}^{d}] \\ &\bar{G}_{ij}^{d} = \begin{bmatrix} G_i & 0 \\ G_i - \hat{A}_i^{d} - \hat{B}_i^{d} C_{2j} & \hat{A}_i^{d} \end{bmatrix}, \; \bar{H}_{1i}^{d} = \begin{bmatrix} H_i \\ H_i \end{bmatrix}, \\ &\bar{H}_{2ij}^{d} = \begin{bmatrix} 0 \\ -\hat{B}_i^{d} D_j \end{bmatrix}, \; \bar{C}_i^{d} = \begin{bmatrix} C_{1i} - \hat{C}_i^{d} & \hat{C}_i^{d}. \end{bmatrix}, \end{aligned} \tag{10}$$

The main objective of this paper is to design the digital fuzzy filter (8) for the T–S fuzzy system (2) to stabilize the error system (9) and to minimize the state-matching error trajectory between the analog fuzzy filter (3) and the digital fuzzy filter (8).

3. DIGITAL FUZZY FILTER BASED ON THE IDR METHOD

The main problem of this paper is stated as follows.

Problem 1: Given well-constructed analog filter matrices $\hat{A}_i^{c}$, $\hat{B}_i^{c}$, and $\hat{C}_i^{c}$ for the stabilizing analog error system (4), find the digital filter gain matrices $\hat{A}_i^{d}$, $\hat{B}_i^{d}$, and $\hat{C}_i^{d}$ such that the following objectives are satisfied.

- The state-matching error $\hat{x}_c(kT) - \hat{x}_d(kT)$ and $e_c(kT) - e_d(kT)$ are minimized for any $k \in \mathbb{Z}_{\geq 0}$.
- The discretized error system (9) is globally asymptotically stable under the zero disturbance conditions, $d(kT) = 0$.

To minimize the state-matching error, the most intuitive method is to obtain $\hat{x}_c(kT+T) = \hat{x}_d(kT+T)$ under the assumption $\hat{x}_c(0) = \hat{x}_d(0)$. Then, if there exist $\hat{A}_i^{d}$, $\hat{B}_i^{d}$, and $\hat{C}_i$ such that the following equalities are satisfied

$$\bar{G}_{ij}^{c} = \bar{G}_{ij}^{d}, \tag{11}$$

$$\bar{T}_{ij}^{c} = \bar{T}_{ij}^{d}, \tag{12}$$

$$\bar{C}_i^{c} = \bar{C}_i^{d}, \tag{13}$$

for all pairs $(i,j) \in \mathcal{I}_r \times \mathcal{I}_r$, the states $\hat{x}_d(kT)$ and $e_d(kT)$ closely match the states $\hat{x}_c(kT)$ and $e_c(kT)$, respectively. The equality (13) is satisfied when the analog filter matrix $\hat{C}_i^c$ and digital filter matrix $\hat{C}_i^d$ have the same value. However, it is difficult to find $\hat{A}_i^d$ and $\hat{B}_i^d$ because the equations (11)-(12) for $(i,j) \in \mathcal{I}_r \times \mathcal{I}_r$ are usually inconsistent for practical engineering. In order to solve these problems, the norm minimization method is given as follows:

$$||\bar{G}_{ij}^c - \bar{G}_{ij}^d|| \prec \hat{\gamma}_1, \tag{14}$$

$$||\bar{T}_{ij}^c - \bar{T}_{ij}^d|| \prec \hat{\gamma}_2, \tag{15}$$

where $\hat{\gamma}_1$ and $\hat{\gamma}_2$ are possibly small positive scalars.

Before proceeding to our main results, the following proposition is introduced to be used in the proof.

Proposition 1: Under zero disturbance, there exists some constant $\eta \in \mathbb{R}_{>0}$ such that

$$||\xi(t)|| \leq \eta ||\varepsilon_d(kT)|| \tag{16}$$

where

$$\begin{aligned} \xi(t) =& x(t) - \hat{x}_d(kT), \\ \eta =& e^{\sup_{i\in\mathcal{I}_r}\|A_i\|T}\sqrt{(\sup_{i\in\mathcal{I}_r}\|TA_i\|)^2 + (I + \sup_{i\in\mathcal{I}_r}\|TA_i\|)^2}. \end{aligned}$$

Proof: Under Assumption 1, the solution of (2) can be obtained by integrating from kT to t for $t \in [kT, kT + T)$, $k \in \mathbb{R}_{\geq 0}$

$$\begin{aligned} x(t) \approx& x(kT) + \int_{kT}^{t} \sum_{i=1}^{r} h_i(z(kT)) A_i x(\tau) d\tau \\ =& x(kT) + \int_{kt}^{t} \sum_{i=1}^{r} h_i(z(kT)) \left\{ A_i \xi(\tau) + A_i \hat{x}_d(kT) \right\} d\tau \\ \leq& x(kT) + \sum_{i=1}^{r} h_i(z(kT)) TA_i \hat{x}_d(kT) \\ &+ \int_{kT}^{kT+T} \sum_{i=1}^{r} h_i(z(kT)) A_i \xi(\tau) d\tau. \end{aligned}$$

The above inequality can be further derived as:

$$\begin{aligned} &\xi(t) \\ &\leq x(kT) + \left(\sum_{i=1}^{r} h_i(z(kT)) TA_i - I \right) \hat{x}_d(kT) \\ &\quad + \int_{kT}^{kT+T} \sum_{i=1}^{r} h_i(z(kT)) A_i \xi(\tau) d\tau \\ &= \sum_{i=1}^{r} h_i(z(kT)) TA_i x(kT) + \left(I - \sum_{i=1}^{r} h_i(z(kT)) TA_i \right) \xi(kT) \\ &\quad + \int_{kT}^{kT+T} \sum_{i=1}^{r} h_i(z(kT)) A_i \xi(\tau) d\tau. \end{aligned} \tag{17}$$

Taking the norms on both sides (17) yields

$$\begin{aligned} \|\xi(t)\| \leq& \sum_{i=1}^{r} h_i(z(kT)) \|TA_i\| \|x(kT)\| \\ &+ \left(\|I - \sum_{i=1}^{r} h_i(z(kT)) TA_i\| \right) \|\xi(kT)\| \\ &+ \int_{kT}^{kT+T} \sum_{i=1}^{r} h_i(z(kT)) \|A_i\| \|\xi(\tau)\| d\tau \\ \leq& \sup_{i\in\mathcal{I}_r} \|TA_i\| \|x(kT)\| \\ &+ \left(I + \sup_{i\in\mathcal{I}_r} \|TA_i\| \right) \|\xi(kT)\| \\ &+ \int_{kT}^{kT+T} \sup_{i\in\mathcal{I}_r} \|A_i\| \|\xi(\tau)\| d\tau \\ \leq& \sqrt{(\sup_{i\in\mathcal{I}_r}\|TA_i\|)^2 + (I + \sup_{i\in\mathcal{I}_r}\|TA_i\|)^2} \|\varepsilon_d(kT)\| \\ &+ \int_{kT}^{kT+T} \sup_{i\in\mathcal{I}_r} \|A_i\| \|\xi(\tau)\| d\tau. \end{aligned}$$

Finally, an application of the Gronwall-Bellman inequality results in

$$\begin{aligned} \|\xi(t)\| \leq& e^{\sup_{i\in\mathcal{I}_r}\|A_i\|T}\sqrt{(\sup_{i\in\mathcal{I}_r}\|TA_i\|)^2 + (I + \sup_{i\in\mathcal{I}_r}\|TA_i\|)^2} \\ &\times \|\varepsilon_d(kT)\| \\ =& \eta \|\varepsilon_d(kT)\|. \end{aligned}$$

□

Also, following assumption is needed to analyze the error system (9).

Assumption 2: The equilibrium point of the discretized T–S fuzzy system (7) is asymptotically stable under the zero disturbance condition, $w(kT) = 0$.

Remark 4: From Proposition 1 and Assumption 2, it is concluded that, under zero disturbance, if state $\varepsilon_d(kT)$ converges to the origin, then the digital filter state $\hat{x}_d(kT)$ tends to the system state $x(t)$. This allows the stability analysis of the error system of the digital filter system (9) to guarantee the stability of the error system between the T–S fuzzy system (2) and the digital filter (8).

Then, the design method of the digital fuzzy filter based on the IDR method is summarized as follows:

Theorem 1: If there exist some symmetric positive matrices Q_1, Q_2, some matrices S_i, U_i, and some scalars γ_1, γ_2 such that the following LMIs are satisfied, then $x_d(kT)$ and $e_d(kT)$ closely match $x_c(kT)$ and $e_c(kT)$, respectively, and the equilibrium point of (9) is asymptotically stable under the zero disturbance conditions, $d(kT) = 0$,

$$\begin{bmatrix} -\gamma_1 I & * \\ \Xi_{ij} & -\gamma_1 Q \end{bmatrix} \prec 0, \; (i,j) \in \mathcal{I}_r \times \mathcal{I}_r, \tag{18}$$

$$\begin{bmatrix} -\gamma_2 I & * & * & * \\ 0 & -\gamma_2 I & * & * \\ Q_1 T_{ij}^{11} - Q_1 H_i & Q_1 T_{ij}^{12} & -\gamma_2 Q_1 & * \\ Q_2 T_{ij}^{21} - Q_2 H_i & Q_2 T_{ij}^{22} + U_i D_j & 0 & -\gamma_2 Q_2 \end{bmatrix}$$

$$\prec 0,\ (i,j)\in\mathcal{I}_r\times\mathcal{I}_r, \tag{19}$$

$$\begin{bmatrix} -Q_1 & * & * & * \\ 0 & -Q_2 & * & * \\ Q_1G_i & 0 & -Q_1 & * \\ Q_2G_i-S_i-U_iC_{2j} & S_i & 0 & -Q_2 \end{bmatrix} \prec 0,$$

$$(i,j)\in\mathcal{I}_r\times\mathcal{I}_r, \tag{20}$$

where

$$\Xi_{ij}=\begin{bmatrix} Q_1\Phi_{ij}^{11}-Q_1G_i & 0 \\ Q_2\Phi_{ij}^{21}-Q_2G_i+S_i+U_iC_{2j} & Q_2\Phi_{ij}^{22}-S_i \end{bmatrix},$$

$$Q=\begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix}.$$

Also, the matrices of the filter gains are given by $\hat{A}_i^d=(Q_2)^{-1}S_i$, $\hat{B}_i^d=(Q_2)^{-1}U_i$, and $\hat{C}_i^d=\hat{C}_i^c$.

Proof: Let us consider the first objective of Problem 1 and the norm minimization inequalities (14) and (15). Then, we can obtain following inequalities:

$$(14)\Leftrightarrow(\bar{G}_{ij}^c-\bar{G}_{ij}^d)^T(\bar{G}_{ij}^c-\bar{G}_{ij}^d)\prec\gamma_1^2P,$$

$$(15)\Leftrightarrow(\bar{T}_{ij}^c-\bar{T}_{ij}^d)^T(\bar{T}_{ij}^c-\bar{T}_{ij}^d)\prec\gamma_2^2P,$$

where $P=Q^{-1}$. Using the Schur complement and applying the congruence transformation with $diag\{I,\ Q\}$, the following inequalities are satisfied.

$$\begin{bmatrix} -\gamma_1 I & * \\ Q\bar{G}_{ij}^c-Q\bar{G}_{ij}^d & -\gamma_1 Q \end{bmatrix}\prec 0, \tag{21}$$

$$\begin{bmatrix} -\gamma_2 I & * \\ Q\bar{T}_{ij}^c-Q\bar{T}_{ij}^d & -\gamma_2 Q \end{bmatrix}\prec 0. \tag{22}$$

Finally, (5) and (10) are substituted into the above inequalities, and then (21) and (22) are equivalent to (18) and (19), respectively. Next, the second objective of Problem 1 can be solved using the following Lyapunov candidate function

$$V(\varepsilon_d(kT))=\varepsilon_d^T(kT)Q\varepsilon_d(kT). \tag{23}$$

The first forward difference of (23) becomes

$$\begin{aligned}
&\triangle V(\varepsilon_d(kT))\\
\approx&\varepsilon_d^T(kT+T)Q\varepsilon_d(kT+T)-\varepsilon_d^T(kT)Q\varepsilon_d(kT)\\
=&\left\{\sum_{i=1}^r\sum_{j=1}^r h_i(z(kT))h_j(z(kT))\bar{G}_{ij}^d\varepsilon_d(kT)\right\}^T Q\\
&\times\left\{\sum_{i=1}^r\sum_{j=1}^r h_i(z(kT))h_j(z(kT))\bar{G}_{ij}^d\varepsilon_d(kT)\right\}\\
&-\varepsilon_d^T(kT)Q\varepsilon_d(kT)\\
=&\sum_{i=1}^r\sum_{j=1}^r\sum_{k=1}^r\sum_{l=1}^r h_i(z(kT))h_j(z(kT))h_l(z(kT))h_m(z(kT))\\
&\times\varepsilon_d^T(kT)\left\{(\bar{G}_{ij}^d)^TQ\bar{G}_{lm}^d-Q\right\}\varepsilon_d(kT).
\end{aligned} \tag{24}$$

The inequality (24) can be further formulated as follows:

$$\begin{aligned}
&\triangle V(\varepsilon_d(kT))\\
\leq&\sum_{i=1}^r\sum_{j=1}^r h_i(z(kT))h_j(z(kT))\varepsilon_d^T(kT)\\
&\times\left\{(\bar{G}_{ij}^d)^TQ\bar{G}_{ij}^d-Q\right\}\varepsilon_d(kT).
\end{aligned} \tag{25}$$

Thus, it is obvious that, if equation (25) is less than zero, then $\triangle V(\varepsilon_d(kT))$ is also less than zero. In addition, the following inequality is valid using the Schur complement and applying the congruence transformation with $diag\{I,\ Q\}$:

$$(\bar{G}_{ij}^d)^TQ\bar{G}_{ij}^d-Q\prec 0\Leftrightarrow\begin{bmatrix} -Q & * \\ Q\bar{G}_{ij}^d & -Q \end{bmatrix}\prec 0. \tag{26}$$

Finally, (5) and (10) are substituted into above inequality, and hence, (26) is equivalent to (20). □

Remark 5: The major contributions of this paper can be summarized as follows:

- This is the first time to our best knowledge that the IDR technique for the T–S fuzzy filter for the nonlinear sampled-data system is handled. Using this approach, various analog filter studies can be applied and the performance of the analog filter is maintained at the sampled-data system within a certain range.
- Using Proposition 1, it is shown that the stability analysis for the error system of the digital filter (9) guarantees the performance of the digital filter for the continuous system.

4. SIMULATION EXAMPLE

To verify the proposed technique, we consider a nonlinear mass-spring-damper mechanical system in the following form:

$$m\ddot{\theta}(t)+d(\dot{\theta}(t))\dot{\theta}(t)+\kappa\theta(t)=0,$$

$$y(t)=g\dot{\theta}(t),$$

where $y(t)$ is the output; $\theta(t)$ is the relative position of the mass, $\theta(t)=[\theta_1(t)^T\ \ \theta_2(t)^T]^T$; and m, κ, and g are the mass, stiffness of the springs, and input coefficient, respectively. The damping coefficients of the nonlinear dampers are assumed to be $d(\dot{\theta}(t))=d_1+d_2(\dot{\theta}(t))^2$. If $\dot{\theta}(t)\in[-\Omega\ \ \Omega]$ with $\Omega>0$, then the membership functions are:

$$h_1(z(t))=\Gamma_1(\dot{\theta}(t))=1-\frac{(\dot{\theta}(t))^2}{(\Omega)^2},$$

$$h_2(z(t))=\Gamma^2(\dot{\theta}(t))=\frac{(\dot{\theta}(t))^2}{(\Omega)^2}.$$

Then, the T–S fuzzy system composed of two rules can be constructed as follows:

$$\dot{x}(t) = \sum_{i=1}^{2} h_i(z(t))\left\{A_i x(t) + B_i w(t)\right\},$$
$$s(t) = \sum_{i=1}^{2} h_i(z(t)) C_{1i} x(t),$$
$$y(kT) = \sum_{i=1}^{2} h_i(z(kT))\left\{C_{2i} x(kT) + D_i v(kT)\right\},$$

where

$$A_1 = \begin{bmatrix} -d_1/m_1 & -\kappa_1/m_1 \\ 1 & 0 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -d_1/m_1 - d_2(\Omega)^2 & -\kappa_1/m_1 \\ 1 & 0 \end{bmatrix},$$
$$B_1 = B_2 = \begin{bmatrix} 0.3 \\ 0 \end{bmatrix},\ C_{11} = \begin{bmatrix} 0.3 & 0.05 \end{bmatrix},$$
$$C_{12} = \begin{bmatrix} 0.4 & 0.04 \end{bmatrix},\ C_{21} = \begin{bmatrix} 1 & 0.1 \end{bmatrix},$$
$$C_{21} = \begin{bmatrix} 1 & 0.2 \end{bmatrix},\ D_1 = D_2 = 0.02,\quad m_1 = m_2 = 1,$$
$$\kappa_1 = 0.2,\quad \kappa_2 = 0.3,\quad d_1 = 0.6,\quad d_2 = 0.8,\quad \Omega = 1,$$
$$x(t) = [x_1^T(t)\ x_2^T(t)]^T = [\dot{\theta}^T(t)\ \theta^T(t)]^T.$$

We assume the initial state conditions $x(0) = [0.2\ -0.1]^T$, and $\hat{x}(0) = [0\ 0]^T$. To develop the IDR technique, well-constructed analog filter gains are obtained using the simple LMIs for the analog filtering technique:

$$\hat{A}_1^c = \begin{bmatrix} -6.6933 & -1.0961 \\ 0.0601 & -0.1504 \end{bmatrix},$$
$$\hat{A}_2^c = \begin{bmatrix} -7.4785 & -0.9979 \\ 0.0956 & -0.1383 \end{bmatrix},$$
$$\hat{B}_1^c = \begin{bmatrix} 6.7368 \\ 0.9487 \end{bmatrix},\ \hat{B}_2^c = \begin{bmatrix} 5.8809 \\ 0.8648 \end{bmatrix},$$
$$\hat{C}_1^c = \begin{bmatrix} 0.3000 & 0.0500 \end{bmatrix},\ \hat{C}_2^c = \begin{bmatrix} 0.3998 & 0.0400 \end{bmatrix}.$$

Based on Theorem 1 with the sampling period $T = 0.1$ and well-constructed analog filter gains, the digital filter gains are given as follows:

$$\hat{A}_1^d = \begin{bmatrix} 0.5121 & -0.0810 \\ 0.0044 & 0.9845 \end{bmatrix},$$
$$\hat{A}_2^d = \begin{bmatrix} 0.4730 & -0.0699 \\ 0.0066 & 0.9858 \end{bmatrix},$$
$$\hat{B}_1^d = \begin{bmatrix} 0.4740 \\ 0.0935 \end{bmatrix},\ \hat{B}_2^d = \begin{bmatrix} 0.3830 \\ 0.0829 \end{bmatrix},$$
$$\hat{C}_1^d = \begin{bmatrix} 0.3000 & 0.0500 \end{bmatrix},\ \hat{C}_2^d = \begin{bmatrix} 0.3998 & 0.0400 \end{bmatrix}.$$

The disturbances are set as $w(t) = e^{-0.2t}\sin(3t)$ and $v(kT) = 0.5e^{-0.2kT}\sin(3kT)$, and then the output responses of the system, analog filter, and digital filter are shown in Fig. 1. From the results of Fig. 1, the output

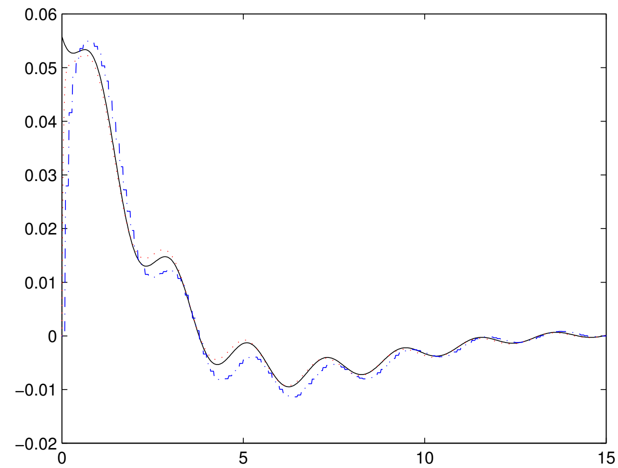

Fig. 1. Time responses of the output of mass-spring-damper with $T = 0.1$: $s(t)$ (solid), $\hat{s}_c(t)$ (dotted), and $\hat{s}_d(kT)$ (dash-dotted).

errors between the system and digital filter, $s(t) - \hat{s}_d(kT)$, and the analog filter and digital filter, $\hat{s}_c(t) - \hat{s}_d(kT)$, are bounded in some neighborhood of the origin. Finally, we take the sampling times $T = 0.2$ and $T = 0.5$ to show the state-matching performances according to the different sampling times. As one can see in Fig. 2 and Fig. 3, the state-matching performances of the proposed method are somewhat degraded, yet the output trajectories have a strong resemblance to the original one.

5. CONCLUSION

In this paper, the fuzzy filter design method has been proposed for the nonlinear sampled-data systems using the IDR technique. Using the T–S fuzzy model, discretized closed-loop systems with pre-designed analog fuzzy and digital fuzzy filters have been presented. The IDR problem was given to guarantee both the state-matching condition and asymptotic stability. The proposed sufficient conditions for solving the IDR problem were derived and formulated in terms of LMIs format. Finally, the simulation example illustrated the efficiency and feasibility of the proposed method.

REFERENCES

[1] H. K. Khalil, *Nonlinear Systems*, 3rd edition, Upper Saddle River, NJ: Prentice-Hall, 2000.

[2] K. Tanaka, and M. Segeno, "Stability analysis and design of fuzzy control of nonlinear systems: Stability and the design issues," *Fuzzy Sets Syst.*, vol. 45, no. 2, pp. 135-156, 1992. [click]

[3] H. O. Wang, K. Tanaka, and M. F. Griffin, "An approach to fuzzy control of nonlinear systems: stability and design issues," *IEEE Trans. Fuzzy Syst.*, vol. 4, no. 1, pp. 14-23, 1996.

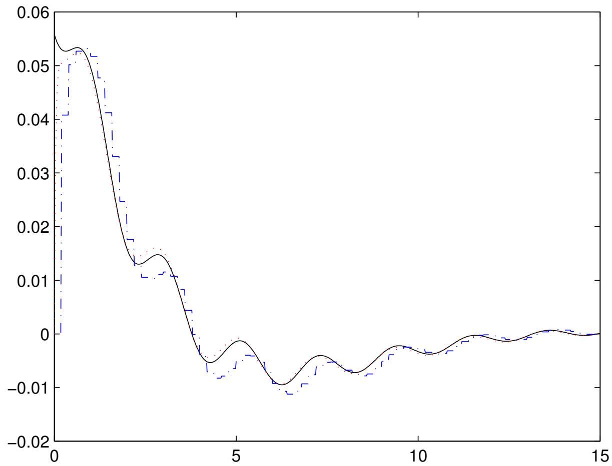

Fig. 2. Time responses of the output of mass-spring-damper with $T = 0.2$: $s(t)$ (solid), $\hat{s}_c(t)$ (dotted), and $\hat{s}_d(kT)$ (dash-dotted).

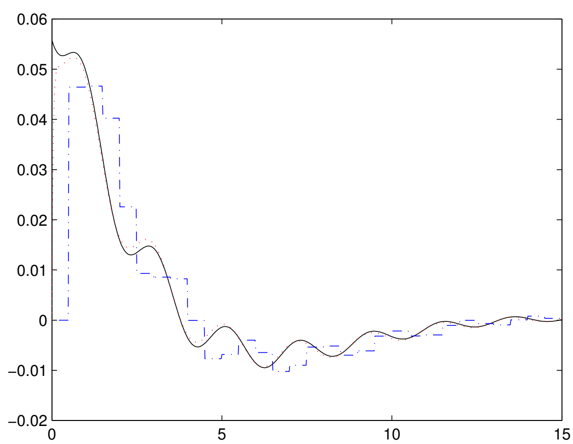

Fig. 3. Time responses of the output of mass-spring-damper with $T = 0.5$: $s(t)$ (solid), $\hat{s}_c(t)$ (dotted), and $\hat{s}_d(kT)$ (dash-dotted).

[4] C. S. Tseng, B. S. Chen, and H. J. Uang, "Fuzzy tracking control design for nonlinear dynamic systems via T–S fuzzy model. Fuzzy Systems," *IEEE Trans. Fuzzy Syst.*, vol. 9, no. 3, pp. 381-392, 2001.

[5] L. Xiaodong, and Z. Qingling, "New approaches to H_∞ controller designs based on fuzzy observers for T–S fuzzy systems via LMI," *Automatica*, vol. 39, no. 9, pp. 1571-1582, 2003. [click]

[6] L. S. Hu, T. Bai, P. Shi, and Z. Wu, "Sampled-data control of networked linear control systems," *Automatica*, vol. 43, no. 5, pp. 903-911, 2007. [click]

[7] H. Gao, W. Sun, and P. Shi, "Robust sampled-data control for vehicle active suspension systems," *IEEE Trans. Control Syst. Technology*, vol. 18, no. 1, pp. 238-245, 2010.

[8] X. L. Zhu, and Y. Wang, "Stabilization for sampled-data neural-network-based control systems," *IEEE Trans. Syst., Man, and Cybern. B, Cybern.*, vol. 41, no. 1, pp. 210-221, 2011.

[9] B. C. Kuo, *Digital Control Systems*, Oxford University Press, New York, NY, 1992.

[10] E. Fridman, "A refined input delay approach to sampled-data control," *Automatica*, vol. 46, no. 2, pp. 421-427 2010. [click]

[11] D. W. Kim, H. J. Lee, and M. Tomizuka, "Fuzzy stabilization of nonlinear systems under sampled-data feedback: An exact discrete-time model approach," *IEEE Trans. Fuzzy Syst.*, vol. 18, no. 2, pp. 251-260, 2010.

[12] J. Yoneyama, "Robust H_∞ filtering for sampled-data fuzzy systems," *Fuzzy Sets Syst.*, vol. 217, pp. 110-129, 2013. [click]

[13] H. J. Kim, G. B. Koo, J. B. Park, and Y. H. Joo, "Decentralized sampled-data H_∞ fuzzy filter for nonlinear large-scale systems," *Fuzzy Sets Syst.*, vol. 273, pp.68-86, 2015. [click]

[14] N. Rafee, T. Chen, and O. P. Malik, "A technique for optimal digital redesign of analog controllers," *IEEE Trans. Control Syst. Technol.*, vol. 5, no. 1, pp. 89-99, 1996.

[15] S. M. Guo, L. S. Shieh, G. Chen, and C. F. Lin, "Effective chaotic orbit tracker: a prediction-based digital redesign approach," *IEEE Trans. Circ. Syst. I*, vol. 47, no. 11, pp. 1557-1570, 2000.

[16] H. J. Lee, J. B. Park, and Y. H. Joo, "An efficient observer-based sampled-data control: digital redesign approach," *IEEE Trans. Circ. Syst. I*, vol. 50, no. 12, pp. 1595-1600, 2003.

[17] H. J. Lee, L. S. Shieh, H. A. Malki, and D. W. Kim, " Pulse-width-modulated feedback control of nonlinear systems: optimal linearization-based digital redesign approach," *International Journal of General Systems*, vol. 40, no. 3, pp. 283-300, 2011.

[18] H. J. Lee, H. Kim, Y. H. Joo, W. Chang, and J. B. Park, "A new intelligent digital redesign for T–S fuzzy systems: global approach," *IEEE Trans. Fuzzy Syst.*, vol. 12, no. 2, pp. 274-284, 2004.

[19] G. B. Koo, J. B. Park, and Y. H. Joo, "Intelligent digital redesign for non-linear systems using a guaranteed cost control method," *Int. J. Control, Autom. Syst.*, vol. 11, no. 6, pp. 1075-1083, 2013. [click]

[20] J. H. Moon, and H. J. Lee, "Performance recovery of intelligent digital redesign for observer-based output feedback under immeasurable premise variables," *IET Control Theory Appl.*, vol. 9, no. 12, pp. 1846-1856, 2015.

[21] H. C. Sung, D. W. Kim, J. B. Park, and Y. H. Joo, "Robust digital control of fuzzy systems with parametric uncertainties: LMI-based digital redesign approach," *Fuzzy Sets Syst.*, vol. 161 , no. 1, pp. 919-933, 2010. [click]

[22] J. H. Moon, and H. J. Lee, "Digital redesign of observer-based output-feedback tracking controllers: enhancement," *International Journal of Control*, vol. 87, no. 11, pp. 2420-2429, 2014. [click]

[23] H. J. Lee, "Performance-recoverable intelligent digital redesign for fuzzy tracking controllers," *Information Sciences*, vol. 326, no. 1, pp. 350-367, 2016. [click]

Ho Jun Kim received the B.S. and M.S. degrees in electrical engineering from Yonsei University, Seoul, Korea, in 2011 and 2013, respectively. Since 2013, he has been pursuing the Ph.D. degree with Yonsei University. His current research interests include large-scale systems, sampled-data systems, decentralized filters, digital redesign, and fuzzy systems.

JinBaePark received the B.E. degree in Electrical Engineering from Yonsei University, Seoul, Korea, and the M.S. and Ph.D. degrees in Electrical Engineering from Kansas State University, Manhattan, in 1977, 1985, and 1990, respectively. Since 1992, he has been with the Department of Electrical and Electronic Engineering, Yonsei University, Seoul, Korea, where he is currently a professor. His research interests include robust control and filtering, nonlinear control, drone, mobile robot, fuzzy logic control, neural networks, and genetic algorithms. He served as the Editor-in-Chief for the Intelligent Journal of Control, Automation, and Systems (IJCAS) (2006-2010), the President for the Institute of Control, Robot, and Systems Engineers (ICROS) (2013), and the Senior Vice-President for Yonsei University(2015-2016).

Young Hoon Joo received his B.S., M.S., and Ph.D. degrees in electrical engineering from Yonsei University, Seoul, Korea, in 1982, 1984, and 1995, respectively.

He worked with Samsung Electronics Company, Seoul, Korea, from 1986 to 1995, as a project manager. He was with the University of Houston, Houston, TX, from 1998 to 1999, as a visiting professor in the Department of Electrical and Computer Engineering. He is currently a professor in the Department of Control and Robotics Engineering, Gunsan National University, Gunsan, Korea. His major interest is mainly in the field of intelligent robot, intelligent control, human-robot interaction, robot vision, and wind farm control. He served as President for Korea Institute of Intelligent Systems (KIIS) (2008-2009) and is serving as Editor-in-Chief for the International Journal of Control, Automation, and Systems (IJCAS) (2014-present) and the Vice-President for the Korean Institute of Electrical Engineers (KIEE) (2016-present) and the Vice-President for the Institute of Control, Robotics, and Systems (ICROS) (2016-present).