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Positive Observer Design for Positive Markovian Jump Systems with Mode-dependent Time-varying Delays and Incomplete Transition Rates

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Abstract: The paper is concerned with positive observer design for positive Markovian jump systems with incomplete transition rates and time delays that are mode-dependent and time-varying. Firstly, by applying an appropriate co-positive type Lyapunov-Krasovskii function and free-connection weighting vectors, sufficient conditions are proposed to ensure stochastic stability of the error positive system and existence of the positive observer. All the proposed conditions are derived in linear programming. Finally, an example is given to demonstrate the validity of the main results.

Keywords: Incomplete transition rates, linear programming, positive Markovian jump systems, stochastic stability.

1. INTRODUCTION

Positive systems, whose states and outputs are positive whenever the initial conditions and inputs are nonnegative, spread almost all over the fields such as communication networks [1], industrial engineering [2], and so on. During the past decades, a lot of attention has been paid to the study of this kind of systems [3–15]. To mention a few, sufficient and necessary conditions for l_1-induced norm of discrete-time positive systems [5] were proposed and the state feedback controller was designed to ensure the closed-loop system positive and asymptotically stable. By implying average dwell time switching method, sufficient conditions for stability of switched positive linear systems [6] were given in the form of linear programming. Positive l_1 filtering for positive Takagi-Sugeno fuzzy systems [7] were designed such that the closed-loop error system was positive and asymptotically stable. The finite-time L_1 state feedback controller of positive switched systems [15] was designed. At the same time, as a class of stochastic hybrid systems, Markovian jump systems have advantages of modeling dynamic systems subject to random abrupt changes such as fault detection systems [16], manufacturing systems [17], networked control systems [18], and control theory [19, 20]. Recently, the study of positive Markovian jump system has attracted considerable attention, such as stability [21, 22], stabilization [23, 24] and filter deign [25].

On the other hand, it is widely recognized that time delay is inherent feature of many practice systems, such as hydraulic pressure systems, networked control systems, chemical engineering process, and so on. The existence of time delay usually causes instability and leads to undesirable performance. The study of positive systems with time delay is categorized into two systems: delay-dependent and delay-independent systems. Delay-dependent systems mean that the information on the size of delay is known while the delay-independent systems are assumed to be time-delayed unknown or possibly unbounded. Since the delay-dependent systems are less conservative that the delay-independent ones, more effort has been paid to positive systems with delay-dependent conditions; for details, see [26–37]. For example, sufficient conditions for stability analysis of positive system or positive switched system with constant time delay were built in [26, 35]. While considering the time-varying delay, finite-time L_1 state feedback controller for positive switched system was designed in [36].

Sometimes, the states of systems are not all measurable in practice. Therefore, it is necessary to consider observer design while designing controller for dynamic systems. Due to the special property of positive system, constructing a positive observer for positive system [38–41] is necessary.

Furthermore, for positive Markovian jump systems, most of the existing results [21–25] are based on nominal system without taking time delay into account. As time delay is frequently encountered in practice, it is necessary

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and significant to further consider positive Markovian jump system with this kind of phenomenon. When taking time delay into account, the problem of choosing an appropriate mode-dependent co-positive Lyapunov function candidate and how to reduce some conservativeness of Lyapunov function will be more complicated and challenging. By the above observations, it should be pointed out that there has not been any result about positive observer design for positive Markovian jump systems with mode-dependent time-varying delays and incomplete transition rates, which is motivation of this paper.

In this paper, positive observer design for positive Markovian jump systems with mode-dependent time-varying delays and incomplete transition rates will be investigated. The main contributions of this paper are listed as follows. First, by employing an appropriate co-positive type Lyapunov function, sufficient conditions for stochastic stability and the existence of positive observer of the considered system are proposed in the form of linear programming. Secondly, the observer gain matrices can be computed by an efficient algorithm.

Notations: $A \succeq (\preceq 0, \succ, \prec)$ represents that all entries of matrix A are nonnegative (non-positive, positive, negative). $A \succ B$ $(A \succeq B)$ means that $A - B \succ 0$ $(A - B \succeq 0)$. $\mathbb{R}$ $(\mathbb{R}_+)$ is the set of all real (positive real) numbers. $\mathbb{R}^n$ $(\mathbb{R}^n_+)$ represents n-dimensional real (positive) vector space. The vector 1-norm is denoted by $||x||_1 = \sum_{k=1}^{n} |x_k|$, where x_k is the kth element of $x \in \mathbb{R}^n$. Matrix A is said to be a Metzler matrix if its off-diagonal elements are all nonnegative real numbers. $E\{\cdot\}$ represents the mathematical expectation.

2. PROBLEM STATEMENT AND PRELIMINARIES

Consider the following positive Markovian jump system with mode-dependent time-varying delays on the probability space (Ξ, Υ, Θ):

$$\begin{aligned} \dot{x}(t) =& A(g_t)x(t) + A_d(g_t)x(t-\tau(g_t,t)), \\ y(t) =& C(g_t)x(t), \\ x(\theta) =& \varphi_1(\theta), \forall \theta \in [-\tau_u, 0], \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $y(t) \in \mathbb{R}^q$ is the controlled out; $\tau(g_t,t)$ denotes mode-dependent time-varying function and satisfies $0 < \tau_d \leq \tau_d(g_t) \leq \tau(g_t,t) \leq \tau_u(g_t) \leq \tau_u$, $\dot{\tau}(g_t,t) \leq h(g_t) \leq h$, where $\tau_d(g_t)$, $\tau_u(g_t)$, $h(g_t)$, τ_d, τ_u, and h are known real constant scalars; $\varphi_1(\theta)$ is a vector-valued initial continuous function which is defined on interval $[-\tau_u, 0]$; $\{g_t, t \geq 0\}$ is a time-homogeneous stochastic Markovian process with right continuous trajectories and takes values in a finite set $S = \{1, 2, \ldots, N\}$ with transition rate matrix $\Pi = \{\pi_{ij}\}$, $i, j \in S$. The transition rate from mode i at time t to mode j at time $t + \Delta t$ is given by:

$$P\{g_{t+\Delta t} = j | g_t = i\} = \begin{cases} \pi_{ij}\Delta t + o(\Delta t), & i \neq j, \\ 1 + \pi_{ij}\Delta t + o(\Delta t), & i = j, \end{cases}$$

where $\Delta t \geq 0$, $\lim_{\Delta t \to 0}(o(\Delta t)/\Delta t) = 0$ and $\pi_{ij} \geq 0$, for $i \neq j$ and $\sum_{j=1, i\neq j}^{N} \pi_{ij} = -\pi_{ii}$.

Throughout the paper, the transition rates are built to be incomplete, that means there are only some elements to be obtained in matrix $\Pi = \{\pi_{ij}\}$. For $\forall i \in S$, the set S^i represents $S^i = S^i_k \bigcup S^i_{uk}$, with

$$\begin{aligned} S^i_k &\triangleq \{j : \pi_{ij} \text{ is known, for } j \in S\}, \\ S^i_{uk} &\triangleq \{j : \pi_{ij} \text{ is unknown, for } j \in S\}. \end{aligned}$$

And if $S^i \neq \varnothing$, it is further given by

$$S^i_k \triangleq \{k^i_1, k^i_2, \ldots, k^i_m\}, 1 \leq m \leq N,$$

where $k^i_m \in S$ means the mth known transition rate of S^i_k in the ith row of the matrix Π. For simplicity, for $g_t = i$, $A(g_t)$, $A_d(g_t)$, $C(g_t)$, $\tau(g_t,t)$, $\tau_d(g_t)$, $\tau_u(g_t)$ and $h(g_t)$ are respectively, denoted as A_i, A_{di}, C_i, $\tau_i(t)$, τ_{di}, τ_{ui} and h_i.

Definition 1 [4]**:** System (1) is said to be positive if, for any initial condition $\varphi_1(\theta) \succeq 0$, $\theta \in [-\tau_u, 0]$, the corresponding trajectory $x(t) \succeq 0$ and $y(t) \succeq 0$.

Lemma 1 [32]**:** System (1) is said to be positive if and only if A_i, for all $i \in S$, are Metzler matrices and $A_{di} \succeq 0$, $C_i \succeq 0$.

Now, we consider the following observer

$$\begin{aligned} \dot{x}_c(t) &= (A_i - L_iC_i)x_c(t) + A_{di}x_c(t-\tau_i(t)) + L_iC_ix(t), \\ y_c(t) &= C_ix_c(t), \\ x_c(\theta) &= \varphi_2(\theta), \forall \theta \in [-\tau_u, 0], \end{aligned} \tag{2}$$

where $x_c(t) \in \mathbb{R}^n$ is the estimated state vector of $x(t)$, $y_c(t) \in \mathbb{R}^q$ is the observer output, and $L_i \in \mathbb{R}^{n\times q}$ are the observer gain matrices to be determined.

Remark 1: For positive Markovian jump system (1), the positivity of the estimated state $x_c(t)$ should be guaranteed according to the literatures [38–41]. Therefore, according to Lemma 1, it is naturally required that $A_i - L_iC_i$ are Metzler matrices and $L_iC_i \succeq 0$, $A_{di} \succeq 0$, $\forall i \in S$.

Define $\tilde{x}(t) = x(t) - x_c(t)$ the estimated error of the system. The following error Markovian jump system is given as follows:

$$\begin{aligned} \dot{\tilde{x}}(t) &= (A_i - L_iC_i)\tilde{x}(t) + A_{di}\tilde{x}(t-\tau_i(t)), \\ \tilde{x}(\theta) &= \varphi(\theta), \forall \theta \in [-\tau_u, 0]. \end{aligned} \tag{3}$$

Remark 2: From lemma 1, the error dynamic system (3) is positive if and only if $A_i - L_iC_i$ are Metzler matrices, $A_{di} \succeq 0$, $\forall i \in S$.

Definition 2 [23]**:** The positive Markovian jump system (3) is said to be stochastically stable if for any initial condition $\varphi(\theta)$ and $g_0 \in S$, the following inequality holds

$$E\left\{\int_0^{\infty} ||\tilde{x}(t)||_1 dt | \varphi(\theta), g_0\right\} < \infty. \tag{4}$$

Definition 3 ([24]): Considering $V(\tilde{x}(t),i,t)$ as the Lyapunov function for the system (3), we define the weak infinitesimal operator as follows:

$$\Gamma V(\tilde{x}(t),i,t)=\lim_{\Delta t\to 0}\frac{1}{\Delta t}[E\{V(\tilde{x}(t+\Delta t),g(t+\Delta t),\\ t+\Delta t)\}-V(\tilde{x}(t),i,t)].$$

In this paper, the positive observer (2) is designed such that the error Markovian jump system (3) with mode-dependent time-varying delays and incomplete transition rates is positive and stochastically stable.

3. MAIN RESULTS

This section will focus on the problem of stochastic stability analysis and observer design.

Theorem 1: The error system (3) with incomplete transition rates is positive, stochastically stable and the observer (2) is positive, if there exist vectors ν_i, σ_{1i}, σ_{2i}, σ_{3i}, σ_1, σ_2, $\sigma_3 \in \mathbb{R}^n_+$, ρ_{1i}, ρ_{2i}, ρ_{3i}, ρ_{4i}, ρ_{5i}, ρ_{6i}, $\rho_{7i} \in \mathbb{R}^n$, $o_i \in \mathbb{R}^q$, for $\forall i \in S$, such that
(i) $A_i - L_iC_i$ are Metzler matrices, $L_iC_i \succeq 0$;
(ii) for a given constant λ,

$$A_i^T\nu_i - C_i^To_i+\sigma_{1i}+\sigma_{2i}+\sigma_{3i}+\tau_u\sigma_1+\tau_u\sigma_2+\tau_u\sigma_3\\ +\sum_{j\in S_k^i}\pi_{ij}(\nu_j-\rho_{1i})+\lambda\nu_i\prec 0, \quad (5)$$

$$A_{di}^T\nu_i-(1-h_i)\sigma_{1i}+\sum_{i\neq j,j\in S_k^i}\pi_{ij}\tau_{uj}\sigma_{1i}-\sum_{j\in S_k^i}\pi_{ij}\rho_{2i}\prec 0, \quad (6)$$

$$\sum_{j\in S_k^i}\pi_{ij}(\tau_{uj}\sigma_{2i}-\rho_{3i})-\sigma_{2i}\prec 0,\\ \sum_{j\in S_k^i}\pi_{ij}(\tau_{dj}\sigma_{3i}-\rho_{4i})-\sigma_{3i}\prec 0, \quad (7)$$

$$\sum_{j\in S_k^i}\pi_{ij}(\sigma_{1j}-\rho_{5i})-\sigma_1\prec 0, \sum_{j\in S_k^i}\pi_{ij}(\sigma_{2j}-\rho_{6i})\\ -\sigma_2\prec 0, \sum_{j\in S_k^i}\pi_{ij}(\sigma_{3j}-\rho_{7i})-\sigma_3\prec 0, \quad (8)$$

$$\nu_j-\rho_{1i}\preceq 0, \tau_{uj}\sigma_{1i}-\rho_{2i}\preceq 0, \tau_{uj}\sigma_{2i}-\rho_{3i}\preceq 0,\\ j\in S_{uk}^i, j\neq i, \quad (9)$$

$$\tau_{dj}\sigma_{3i}-\rho_{4i}\preceq 0, \sigma_{1j}-\rho_{5i}\preceq 0, \sigma_{2j}-\rho_{6i}\preceq 0,\\ \sigma_{3j}-\rho_{7i}\preceq 0, j\in S_{uk}^i, j\neq i, \quad (10)$$

$$\nu_j-\rho_{1i}\succeq 0, \tau_{uj}\sigma_{2i}-\rho_{3i}\succeq 0, \tau_{dj}\sigma_{3i}-\rho_{4i}\succeq 0,\\ j\in S_{uk}^i, j=i, \quad (11)$$

$$\sigma_{1j}-\rho_{5i}\succeq 0, \sigma_{2j}-\rho_{6i}\succeq 0, \sigma_{3j}-\rho_{7i}\succeq 0,\\ j\in S_{uk}^i, j=i, \quad (12)$$

with $o_i = L_i^T\nu_i$.

Proof: Firstly, we prove the positivity of system (3) and observer (2). From condition (i), the system (3) and observer (2) are positive.

Secondly, for positive Markovian jump system (3), choose the co-positive type Lyapunov function candidate as

$$V(\tilde{x}(t),i,t)=V_1(\tilde{x}(t),i,t)+V_2(\tilde{x}(t),i,t)\\ +V_3(\tilde{x}(t),i,t), \quad (13)$$

where

$$V_1(\tilde{x}(t),i,t)=\tilde{x}^T(t)\nu_i,$$
$$V_2(\tilde{x}(t),i,t)=\int_{t-\tau_i(t)}^t\tilde{x}^T(s)\sigma_{1i}ds+\int_{t-\tau_{ui}}^t\tilde{x}^T(s)\sigma_{2i}ds\\ +\int_{t-\tau_{di}}^t\tilde{x}^T(s)\sigma_{3i}ds,$$
$$V_3(\tilde{x}(t),i,t)=\int_{-\tau_u}^0\int_{t+\theta}^t\tilde{x}^T(s)(\sigma_1+\sigma_2+\sigma_3)dsd\theta,$$

where ν_i, σ_{1i}, σ_{2i}, σ_{3i}, σ_1, σ_2, $\sigma_3\in\mathbb{R}^n_+$ and

$$\sum_{j=1}^N\pi_{ij}\sigma_{1j}\preceq\sigma_1, \sum_{j=1}^N\pi_{ij}\sigma_{2j}\preceq\sigma_2, \sum_{j=1}^N\pi_{ij}\sigma_{3j}\preceq\sigma_3. \quad (14)$$

Hence, according to Definition 3, it can be shown that

$$\Gamma\{\int_{t-\tau_i(t)}^t\tilde{x}^T(s)\sigma_{1i}ds\}\\ =\tilde{x}^T(t)\sigma_{1i}-(1-\dot{\tau}_i(t))\tilde{x}^T(t-\tau_i(t))\sigma_{1i}\\ +\sum_{j=1}^N\pi_{ij}\tau_j(t)\tilde{x}^T(t-\tau_i(t))\sigma_{1i}+\int_{t-\tau_i(t)}^t\tilde{x}^T(s)\sum_{j=1}^N\pi_{ij}\sigma_{1j}ds\\ \leq\tilde{x}^T(t)\sigma_{1i}-(1-h_i)\tilde{x}^T(t-\tau_i(t))\sigma_{1i}\\ +\sum_{i\neq j,j=1}^N\pi_{ij}\tau_{uj}\tilde{x}^T(t-\tau_i(t))\sigma_{1i}+\int_{t-\tau_i(t)}^t\tilde{x}^T(s)\sigma_1ds. \quad (15)$$

Similar to the process above, we have

$$\Gamma V_1(\tilde{x}(t),i,t)\\ =\tilde{x}^T(t)(A_i^T\nu_i-C_i^To_i+\sum_{j=1}^N\pi_{ij}\nu_j)+\tilde{x}^T(t-\tau_i(t))A_{di}^T\nu_i,$$

$$\Gamma V_2(\tilde{x}(t),i,t)\\ \leq\tilde{x}^T(t)\sigma_{1i}-(1-h_i)\tilde{x}^T(t-\tau_i(t))\sigma_{1i}\\ +\sum_{i\neq j,j=1}^N\pi_{ij}\tau_{uj}\tilde{x}^T(t-\tau_i(t))\sigma_{1i}+\int_{t-\tau_i(t)}^t\tilde{x}^T(s)\sigma_1ds\\ +\tilde{x}^T(t)\sigma_{2i}-\tilde{x}^T(t-\tau_{ui})\sigma_{2i}\\ +\sum_{j=1}^N\pi_{ij}\tau_{uj}\tilde{x}^T(t-\tau_{ui})\sigma_{2i}+\int_{t-\tau_{ui}}^t\tilde{x}^T(s)\sigma_2ds+\tilde{x}^T(t)\sigma_{3i}\\ -\tilde{x}^T(t-\tau_{di})\sigma_{3i}+\sum_{j=1}^N\pi_{ij}\tau_{dj}\tilde{x}^T(t-\tau_{di})\sigma_{3i}\\ +\int_{t-\tau_{di}}^t\tilde{x}^T(s)\sigma_3ds,$$

$$\begin{aligned}
&\Gamma V_3(\tilde{x}(t),i,t)\\
=&\tau_u\tilde{x}^T(t)(\sigma_1+\sigma_2+\sigma_3)-\int_{t-\tau_u}^{t}\tilde{x}^T(s)(\sigma_1+\sigma_2+\sigma_3)ds\\
\leq&\tau_u\tilde{x}^T(t)(\sigma_1+\sigma_2+\sigma_3)-\int_{t-\tau_i(t)}^{t}\tilde{x}^T(s)\sigma_1 ds\\
&-\int_{t-\tau_{ui}}^{t}\tilde{x}^T(s)\sigma_2 ds-\int_{t-\tau_{di}}^{t}\tilde{x}^T(s)\sigma_3 ds, \qquad (16)
\end{aligned}$$

where $o_i = L_i^T v_i$.

Based on $\sum_{j=1}^{N}\pi_{ij}\rho_{1i} = \sum_{j=1}^{N}\pi_{ij}\rho_{2i} = \sum_{j=1}^{N}\pi_{ij}\rho_{3i} = \sum_{j=1}^{N}\pi_{ij}\rho_{4i} = \sum_{j=1}^{N}\pi_{ij}\rho_{5i} = \sum_{j=1}^{N}\pi_{ij}\rho_{6i} = \sum_{j=1}^{N}\pi_{ij}\rho_{7i} = 0$ for a set of vectors ρ_{1i}, ρ_{2i}, ρ_{3i}, ρ_{4i}, ρ_{5i}, ρ_{6i}, ρ_{7i}, we have

$$\begin{aligned}
&\Gamma V(\tilde{x}(t),i,t)\\
\leq&\tilde{x}^T(t)(A_i^T v_i - C_i^T o_i+\sigma_{1i}+\sigma_{2i}+\sigma_{3i}+\tau_u\sigma_1+\tau_u\sigma_2\\
&+\tau_u\sigma_3+\sum_{j=1}^{N}\pi_{ij}v_j-\sum_{j=1}^{N}\pi_{ij}\rho_{1i})\\
&+\tilde{x}^T(t-\tau_i(t))(A_{di}^T v_i-(1-h_i)\sigma_{1i}+\sum_{i\neq j,j=1}^{N}\pi_{ij}\tau_{uj}\sigma_{1i}\\
&-\sum_{j=1}^{N}\pi_{ij}\rho_{2i})+\tilde{x}^T(t-\tau_{ui})(\sum_{j=1}^{N}\pi_{ij}\tau_{uj}\sigma_{2i}-\sum_{j=1}^{N}\pi_{ij}\rho_{3i}\\
&-\sigma_{2i})+\tilde{x}^T(t-\tau_{di})(\sum_{j=1}^{N}\pi_{ij}\tau_{dj}\sigma_{3i}-\sum_{j=1}^{N}\pi_{ij}\rho_{4i}-\sigma_{3i}),\\
&\sum_{j=1}^{N}\pi_{ij}\sigma_{1j}-\sum_{j=1}^{N}\pi_{ij}\rho_{5i}-\sigma_1\preceq 0,\\
&\sum_{j=1}^{N}\pi_{ij}\sigma_{2j}-\sum_{j=1}^{N}\pi_{ij}\rho_{6i}-\sigma_2\preceq 0,\\
&\sum_{j=1}^{N}\pi_{ij}\sigma_{3j}-\sum_{j=1}^{N}\pi_{ij}\rho_{7i}-\sigma_3\preceq 0. \qquad (17)
\end{aligned}$$

Note that $\pi_{ii} < 0(\forall i,j\in S, i=j)$ and $\pi_{ij}\geq 0(\forall i,j\in S, i\neq j)$, therefore, if $i\in S_k^i$, inequalities (5)-(10) imply that $\Gamma V(\tilde{x}(t),i,t) < -\lambda V_1(\tilde{x}(t),i,t) < 0$. On the other hand, if $i\in S_{uk}^i$, inequalities (5)-(12) also imply that $\Gamma V(\tilde{x}(t),i,t) < -\lambda V_1(\tilde{x}(t),i,t) < 0$. Following the same line of the proof of [23], we can get

$$E\left\{\int_0^\infty ||\tilde{x}(t)||_1 dt|\varphi(\theta),g_0\right\} < \infty.$$

The proof is completed. □

Remark 3: Generally speaking, for Markovian jump system with mode-dependent time-varying delays, Lyapunov function is frequently chosen as follows:

$$\begin{aligned}
V(\tilde{x}(t),i,t)=&\tilde{x}^T(t)v_i+\int_{t-\tau_i(t)}^{t}\tilde{x}^T(s)\sigma_1 ds\\
&+\int_{-\tau_i}^{0}\int_{t+\theta}^{t}\tilde{x}^T(s)\sigma_2 dsd\theta. \qquad (18)
\end{aligned}$$

The parameter in integral term of equality (18) is mode-independent, which may lead to some conservativeness. Here, an appropriate mode-dependent co-positive type Lyapunov function (13) is constructed, and parameter in integral term $V_2(\tilde{x}(t),i,t)$ is mode-dependent, which may reduce some conservativeness.

Remark 4: We cite the selection of the parameter λ from [40, 41]. From Theorem 1, it is easy to see that a smaller λ will be favorable to the solvability of inequalities (5)-(12). First, we can assign a value to λ; if (5)-(12) has no feasible solution for the assigned λ, we can change the parameter λ to be smaller. Following this guideline, a solution to the matrix inequalities (5)-(12) can be found.

Based on Theorem 1, we present an effective algorithm for designing the positive observer which ensures the error Markovian jump system (3) with mode-dependent time-varying delays and incomplete transition rates is positive and stochastically stable.

Algorithm 1: Step 1. Choose a parameter $\lambda > 0$; one can obtain v_i, σ_{1i}, σ_{2i}, σ_{3i}, σ_1, σ_2, σ_3, ρ_{1i}, ρ_{2i}, ρ_{3i}, ρ_{4i}, ρ_{5i}, ρ_{6i}, ρ_{7i}, o_i by solving the Theorem 1;

Step 2. Compute observer gain matrices L_i and check the condition (i) in Theorem 1. If the condition (i) holds, enter the next Step 3; else return to Step 1;

Step 3. The observer gain matrices L_i are obtained.

4. NUMERICAL EXAMPLES

Considering the mathematical model of virus mutation treatment presented in [12] and mode-independent time-varying delay in dynamic systems, we describe the dynamic system as follows:

$$\begin{aligned}
\dot{x}(t)=&(R_i-\delta I+\zeta M)x(t)+A_{di}x(t-\tau_i(t)),\\
y(t)=&C_i x(t), \qquad (19)
\end{aligned}$$

where $x(t)\in\mathbb{R}^2$ indicates two different viral genotypes, i indicates a Markovian process with two different states, ζ is a small parameter representing the mutation rate, δ is the death or decay rate; $M=[M_{mn}]$ denotes the system matrices, $M_{mn}\in\{0,1\}$ represents the genetic connections between genotypes, that is, M_{mn}=1 if and only if it is possible for genotype n to mutate into genotype m, and $y(t)$ denotes the controlled output. The two-mode Markovian jump systems with parameters are given as follows:

$$R_1=\begin{bmatrix}0.05 & 0\\ 0 & 0.25\end{bmatrix},\ R_2=\begin{bmatrix}0.06 & 0\\ 0 & 0.26\end{bmatrix},\ M=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix},$$
$$A_{d1}=\begin{bmatrix}0.2 & 0\\ 0 & 0.2\end{bmatrix},\ A_{d2}=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix},\ C_1=\begin{bmatrix}1.5 & 1.5\end{bmatrix},$$
$$C_2=\begin{bmatrix}0.6 & 0.9\end{bmatrix}.$$

Let $\lambda = 1$. Let $\tau_1(t) = 0.5(1.2-sin(t))$ and $\tau_2(t) = 0.4(1.5-sin(t))$, then $\tau_{d1} = 0.1$, $\tau_{u1} = 1.1$, $\tau_{d2} = 0.2$,

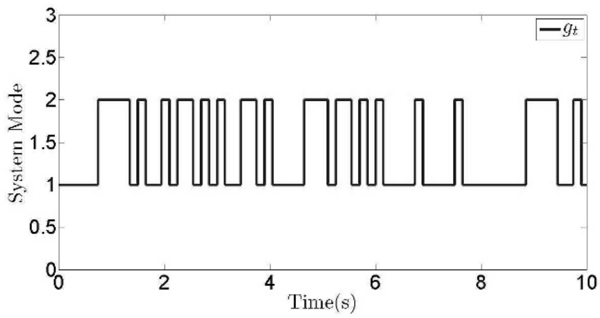

Fig. 1. System modes $g(t)$.

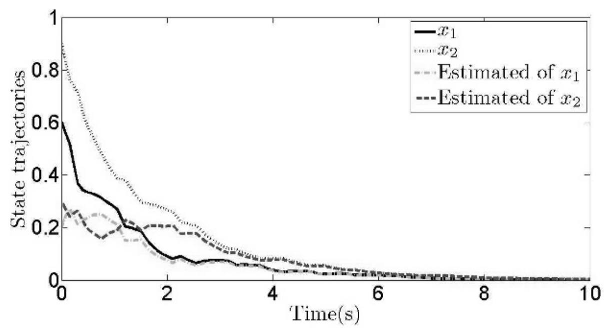

Fig. 2. Actual states $x(t)$ and their estimation $x_c(t)$.

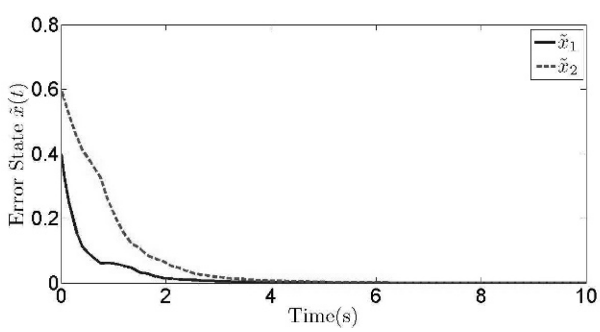

Fig. 3. Estimated error states $\tilde{x}(t)$.

$\tau_{u2} = 1.0$, $\tau_d = 0.1$, $\tau_u = 1.1$, $\dot{\tau}_1(t) = -0.5cos(t)$, $h_1 = 0.5$, $\dot{\tau}_2(t) = -0.4cos(t)$, $h_2 = 0.4$ and $h = 0.5$. The incomplete transition rate matrix is given as follows:

$$\begin{bmatrix} ? & ? \\ 0.1 & -0.1 \end{bmatrix}.$$

Solving Algorithm 1 results in

$$L_1 = \begin{bmatrix} 3.6454 & -1.2248 \\ 3.6454 & -1.2248 \end{bmatrix}, L_2 = \begin{bmatrix} 0.2330 & 2.5358 \\ 0.2330 & 2.5358 \end{bmatrix}.$$

Obviously, the condition (i) in Theorem 1 holds.

The initial mode and state of the system is $g(0) = 1$, $x(0) = \begin{bmatrix} 0.6 & 0.9 \end{bmatrix}^T$, and the initial state of the observer is $x_c(0) = \begin{bmatrix} 0.2 & 0.3 \end{bmatrix}^T$. Fig. 1 shows the system mode $g(t)$. Fig. 2 stands for the actual states $x(t)$ and their estimation $x_c(t)$. Fig. 3 shows the estimated error states $\tilde{x}(t)$. From Fig. 2 and Fig. 3, we can see the states of the designed observer not only possess the positivity, but also approximate those of the original system.

Remark 5: If Lyapunov function is chosen in the form of (18), we can not get the observer gain matrices. This means that the chosen Lyapunov function (13) reduces some conservativeness.

Remark 6: The complete known transition rate matrix is given as follows:

$$\begin{bmatrix} -0.3 & 0.3 \\ 0.1 & -0.1 \end{bmatrix}.$$

Solving Algorithm 1 results in

$$L_1 = \begin{bmatrix} 2.1234 & 1.6243 \\ 2.1234 & 1.6243 \end{bmatrix}, L_2 = \begin{bmatrix} 0.2421 & 2.4321 \\ 0.2421 & 2.4321 \end{bmatrix}.$$

Obviously, the condition (i) in Theorem 1 holds. It means that the error system with complete known transition rates is stochastically stable.

5. CONCLUSIONS

In this paper, we have given an approach design of positive observer for Markovian jump systems with mode-dependent time-varying delays and incomplete transition rates. By employing co-positive type Lyapunov function, sufficient conditions are built to ensure the error system positive and stochastically stable. In the future work, we will study positive observer design for positive Markovian jump systems with mode-dependent distributed time-varying delays and incomplete transition rates.

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