International Journal of Control, Automation and Systems 14(6) (2016) 1455-1463
http://dx.doi.org/10.1007/s12555-015-0256-z
ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Iterative Learning Control for a Class of Mixed Hyperbolic-parabolic Distributed Parameter Systems

Qin Fu*, Wei-Guo Gu, Pan-Pan Gu, and Jian-Rong Wu

Abstract: This paper deals with the problem of iterative learning control algorithm for a class of mixed distributed parameter systems. Here, the considered distributed parameter systems are composed of mixed hyperbolic-parabolic partial differential equations. The domain of the system is divided into two parts in which the system is hyperbolic and parabolic, respectively, with transmission conditions at the interface. According to the characteristics of the systems, iterative learning control laws are proposed for such mixed hyperbolic-parabolic distributed parameter systems based on P-type learning scheme. Using the contraction mapping method, it is shown that the scheme can guarantee the output tracking errors on L^2 space converge along the iteration axis. A simulation example illustrates the effectiveness of the proposed method.

Keywords: Hyperbolic-parabolic partial differential equations, iterative learning control, L^2 space, mixed distributed parameter systems, P-type learning scheme.

1. INTRODUCTION

Since the complete algorithm of iterative learning control (ILC) was first proposed by Arimoto *et al*. [1], it has become the hot issues of cybernetics and has attracted broad attention in recent years [2–5]. The basic idea of ILC is to improve the control signal for the present operation cycle by feeding back the control error in the previous cycle. And the classical formulation of ILC design problem is as follows: find an update mechanism for the output trajectory of a new cycle based on the information from previous cycles so that the output trajectory converges asymptotically to the desired reference trajectory. Owing to its simplicity and effectiveness, ILC has been found to be a good alternative in many areas and applications, e.g., see [6] for detailed results. Nowadays, ILC is playing a more and more important role in controlling repeatable processes.

Due to many practical problems can be described by distributed parameter systems (DPSs) governed by partial differential equations (PDEs), the applications of DPSs have been involved in many fields in the last few years, and a series of the research achievements have been obtained [7–9]. In the field of control for DPSs, hitherto, there are two methods often used: one is the boundary control [10–12], the other is the distributed control [13, 14]. This paper deals with the distributed control problems of DPSs.

Since the variables of the DPSs are related to infinite dimensional space, studies of ILC for infinite dimensional processes are limited and there have been only a few works reported on ILC for DPSs, while ILC has been widely investigated for finite dimensional systems. Furthermore, most of them focus on parabolic DPSs. Papers [15–17] designed ILC algorithms for parabolic DPSs by using P-type learning scheme. Paper [18] discussed both P-type and D-type ILC schemes for a parabolic DPS, which was transformed into a linear system on Hilbert space. In [19], ILC was applied to a temporal-spatial discretized first order hyperbolic PDE, guaranteeing stability of the closed loop system and satisfying the requirements of performance. Recently, paper [20] proposed a ILC algorithm for a distributed parameter system which is governed by a second order hyperbolic PDE, and the corresponding convergence condition was given. In [21], a D-type ILC algorithm for irregular DPSs was introduced with the aid of the weak convergences in functional analysis.

Mixed partial differential equations (MPDEs), one class of partial differential equations, arise in many mathematical-physical models (e.g., fluid dynamics, ground water flow, semiconductor equations) and have

Manuscript received June 29, 2015; revised November 15, 2015; accepted December 11, 2015. Recommended by Associate Editor Young Ik Son under the direction of Editor Yoshito Ohta. This work was supported by the National Natural Science Foundation of China (No. 11371013). The authors would like to express their gratitude to the editor and the anonymous referees for their valuable suggestions that have greatly improved the quality of the paper.

Qin Fu, Wei-Guo Gu, Pan-Pan Gu, and Jian-Rong Wu are with the School of Mathematics and Physics, Suzhou University of Science and Technology, Suzhou 215009, P. R. China (e-mails: fuqin925@sina.com, guwg879@139.com, 1005131776@qq.com, jrwu@mail.usts.edu.cn).
* Corresponding author.

been used in the literature to model a wide phenomena of practical applications in science and engineering [22–29]. The so-called 'mixed', means that this kind of equations have different types in different domains. For instance, while a PDE is hyperbolic in one domain and parabolic in another domain, it is called the mixed hyperbolic-parabolic PDE [24–29]. Hitherto, the related research works about MPDEs have mainly focused on its well-posedness [24–26] and numerical solutions [22, 23, 27–29]. Papers [24–26] discussed the uniqueness, existence and stability for MPDEs. In [27–29], the numerical methods were proposed for solving multi-dimensional hyperbolic-parabolic differential equations by difference schemes. As a matter of course, a system governed by MPDEs is referred to as mixed distributed parameter system (MDPS). How to apply iterative learning control scheme to MDPS and conduct corresponding control design, to the best of our knowledge, there is no relevant literature about this.

The problem of iterative learning control algorithm for MDPSs will be first put forward in this paper. The considered MDPSs are composed of mixed hyperbolic-parabolic PDEs just as that in [29]. According to the characteristics of the systems, iterative learning control laws are proposed for such mixed hyperbolic-parabolic DPSs based on P-type learning scheme. Using the contraction mapping method, it is shown that the scheme can guarantee the output tracking errors on L^2 space converge along the iteration axis.

In this paper, the following notational conventions are adopted: for function $Q(t,x)$: $[-1,1]\times[0,1]\to R$, take the norm: $\|Q(t,\cdot)\|_{L^2}=\sqrt{\int_0^1 Q^2(t,x)\mathrm{d}x}$, and define $\|Q\|_{L^2,s}=\max\limits_{t\in[-1,1]}\|Q(t,\cdot)\|_{L^2}^2$.

2. PROBLEM DESCRIPTION

Firstly, we give a brief description of the following multi-dimensional hyperbolic-parabolic PDE given in [29], with Dirichlet and Neumann conditions.

$$\begin{cases} \dfrac{\partial^2 u(t,\mathbf{x})}{\partial t^2}-\sum\limits_{r=1}^{n}(a_r(\mathbf{x})u_{x_r})_{x_r}+\sigma u(t,\mathbf{x}) \\ \quad =f(t,\mathbf{x}), 0\le t\le 1, \mathbf{x}=(x_1,x_2,\cdots,x_n)\in\Omega, \\ \dfrac{\partial u(t,\mathbf{x})}{\partial t}-\sum\limits_{r=1}^{n}(a_r(\mathbf{x})u_{x_r})_{x_r}+\sigma u(t,\mathbf{x}) \\ \quad =g(t,\mathbf{x}), -1\le t\le 0, \mathbf{x}=(x_1,x_2,\cdots,x_n)\in\Omega, \\ u(-1,\mathbf{x})=\sum\limits_{j=1}^{K}\alpha_j u(\mu_j,\mathbf{x})+\sum\limits_{j=1}^{L}\beta_j u_t(\lambda_j,\mathbf{x})+\varphi(\mathbf{x}), \\ \quad \mathbf{x}\in\bar{\Omega}, \\ \sum\limits_{j=1}^{K}|\alpha_j|, \sum\limits_{j=1}^{L}|\beta_j|\le 1, 0<\mu_j,\lambda_j\le 1, \\ u(t,\mathbf{x})=0, \mathbf{x}\in S_1, \dfrac{\partial u(t,\mathbf{x})}{\partial\vec{n}}=0, \\ \quad \mathbf{x}\in S_2, -1\le t\le 1, \end{cases} \tag{1}$$

where Ω be the unit open cube in the n-dimensional Euclidean space R^n, that is,

$$\Omega=\{\mathbf{x}|\mathbf{x}=(x_1,x_2,\cdots,x_n), 0<x_k<1, 1\le k\le n\},$$

while

$$\begin{aligned} S&=\{\mathbf{x}|\mathbf{x}=(x_1,x_2,\cdots,x_n), x_m=0 \text{ or } 1, \\ &\qquad 0\le x_k\le 1, k\ne m, 1\le m\le n\}, \\ S_1&=\{\mathbf{x}|\mathbf{x}=(x_1,x_2,\cdots,x_n), x_m=0 \text{ or } 1, \\ &\qquad 0\le x_k\le 1, k\ne m, 1\le m\le r\} \\ S_2&=\{\mathbf{x}|\mathbf{x}=(x_1,x_2,\cdots,x_n), x_m=0 \text{ or } 1, \\ &\qquad 0\le x_k\le 1, k\ne m, r+1\le m\le n\}, \end{aligned}$$

with boundary $S=S_1\cup S_2$ and $\bar{\Omega}=\Omega\cup S$. Here, $a_r(\mathbf{x})(\mathbf{x}\in\Omega)$, $\varphi(\mathbf{x})(\mathbf{x}\in\bar{\Omega})$, $f(t,\mathbf{x})(t\in[0,1], \mathbf{x}\in\Omega)$, $g(t,\mathbf{x})(t\in[-1,0], \mathbf{x}\in\Omega)$ are smooth functions, $\vec{n}$ is the normal vector to Ω, σ is a positive number and $a_r(\mathbf{x})\ge a>0$.

Remark 1: The uniqueness and existence of solution of (1) have been given in [29].

In this paper, we will expand the ILC framework to the mixed hyperbolic-parabolic DPSs governed by (1). For convenience, we take $n=1$, $\alpha_j=0(j=1,2,\cdots,K)$, $\beta_j=0(j=1,2,\cdots,L)$. For the requirement of ILC design, we replace both $f(t,x)$ and $g(t,x)$ given in (1) with control variable $u(t,x)$, and replace $u(t,x)$ given in (1) with state variable $Q(t,x)$, respectively. By adding an output variable $y(t,x)$ with general form, the following mixed hyperbolic-parabolic DPS governed by (1) is given:

$$\begin{cases} \dfrac{\partial^2 Q(t,x)}{\partial t^2}-(a(x)Q_x(t,x))_x+\sigma Q(t,x) \\ \quad =u(t,x), 0\le t\le 1, x\in(0,1), \\ \dfrac{\partial Q(t,x)}{\partial t}-(a(x)Q_x(t,x))_x+\sigma Q(t,x) \\ \quad =u(t,x), -1\le t\le 0, x\in(0,1), \\ y(t,x)=C(t)Q(t,x)+D(t)u(t,x), \\ \quad -1\le t\le 1, x\in(0,1), \end{cases} \tag{2}$$

with initial-boundary conditions: $Q(-1,x)=\varphi(x)$, $x\in[0,1]$; $Q(t,0)=0$ (or $\left.\dfrac{\partial Q(t,x)}{\partial x}\right|_{x=0}=0$), $Q(t,1)=0$ (or $\left.\dfrac{\partial Q(t,x)}{\partial x}\right|_{x=1}=0$), $-1\le t\le 1$, where $Q(t,x)$, $u(t,x)$, $y(t,x)\in R$ represent the state, control input and output of the system, respectively, and $a(x)\ge a>0$.

Remark 2: It is easy to see that, the system (2) is parabolic in $[-1,0]\times(0,1)$, and hyperbolic in $[0,1]\times(0,1)$.

The system (2) is assumed to satisfy the following assumptions:

Assumption 1: $0<D_1\le D(t)\le D_2$, where D_1, D_2 are known constants. That is the system (1) has direct

transmission from inputs to outputs. $|C(t)| \le C$, where C is an unknown constant.

Assumption 2: For the given trajectory $y_r(t,x)$, there exists a unique $u_r(t,x)$ such that

$$\begin{cases} \dfrac{\partial^2 Q_r(t,x)}{\partial t^2} - \dfrac{\partial}{\partial x}\left(a(x)\dfrac{\partial Q_r(t,x)}{\partial x}\right) + \sigma Q_r(t,x) \\ \quad = u_r(t,x), 0 \le t \le 1, x \in (0,1), \\ \dfrac{\partial Q_r(t,x)}{\partial t} - \dfrac{\partial}{\partial x}\left(a(x)\dfrac{\partial Q_r(t,x)}{\partial x}\right) + \sigma Q_r(t,x) \\ \quad = u_r(t,x), -1 \le t \le 0, x \in (0,1), \\ y_r(t,x) = C(t)Q_r(t,x) + D(t)u_r(t,x), \\ \quad -1 \le t \le 1, x \in (0,1). \end{cases}$$

It is assumed that the system (2) is repeatable over $t \in [-1,1]$. Rewrite the system (2) at each iteration as:

$$\begin{cases} \dfrac{\partial^2 Q_k(t,x)}{\partial t^2} - \dfrac{\partial}{\partial x}\left(a(x)\dfrac{\partial Q_k(t,x)}{\partial x}\right) + \sigma Q_k(t,x) \\ \quad = u_k(t,x), 0 \le t \le 1, x \in (0,1), \\ \dfrac{\partial Q_k(t,x)}{\partial t} - \dfrac{\partial}{\partial x}\left(a(x)\dfrac{\partial Q_k(t,x)}{\partial x}\right) + \sigma Q_k(t,x) \\ \quad = u_k(t,x), -1 \le t \le 0, x \in (0,1), \\ y_k(t,x) = C(t)Q_k(t,x) + D(t)u_k(t,x), \\ \quad -1 \le t \le 1, x \in (0,1). \end{cases} \tag{3}$$

Assumption 3: The initial-boundary resetting conditions hold for all iterations, i.e., $Q_k(-1,x) = \varphi(x), x \in [0,1]$; $Q_k(t,0) = 0(\text{or} \dfrac{\partial Q_k(t,x)}{\partial x}|_{x=0} = 0)$, $Q_k(t,1) = 0(\text{or} \dfrac{\partial Q_k(t,x)}{\partial x}|_{x=1} = 0)$, $-1 \le t \le 1$. $k = 0,1,2,\cdots$.

The learning control target is to find an appropriate learning scheme, so that the iterative learning sequence $y_k(t,x)$ uniformly converges to the desired trajectory $y_r(t,x)$ on L^2 space, that is

$$\lim_{k\to\infty} \|e_k\|_{L^2,s} = 0,$$

where $e_k(t,x) = y_r(t,x) - y_k(t,x)$.

Lemma 1 [4]**:** Suppose $\{a_k\}$, $\{b_k\}$ are two nonnegative real sequences satisfying

$$a_{k+1} \le \rho a_k + b_k,\ 0 \le \rho < 1,$$

if $\lim_{k\to\infty} b_k = 0$, then $\lim_{k\to\infty} a_k = 0$.

3. MAIN RESULTS

For D_1, D_2 given in Assumption 1, take a positive number ε satisfying

$$\frac{D_2}{D_1} < \frac{\sqrt{1+\varepsilon}+1}{\sqrt{1+\varepsilon}-1}. \tag{4}$$

Constructing the learning scheme for the system (3) as follows:

$$u_{k+1}(t,x) = u_k(t,x) + q e_k(t,x), \tag{5}$$

where $q > 0$ is the learning gain, then we have the following theorem:

Theorem 1: Let Assumptions 1-3 are satisfied . If

$$\rho = \max_{t\in[-1,1]} |1 - qD(t)| < \frac{1}{\sqrt{1+\varepsilon}}, \tag{6}$$

then the iterative process of the system (3) is convergent, under the effect of the control law (5) , i.e., $\lim_{k\to\infty} \|e_k\|_{L^2,s} = 0$.

Proof: Denote $\delta Q_k(t,x) = Q_{k+1}(t,x) - Q_k(t,x)$, $\delta u_k(t,x) = u_{k+1}(t,x) - u_k(t,x)$. It follows from (3) and (5) that

$$\begin{aligned} e_{k+1}(t,x) =& e_k(t,x) + y_k(t,x) - y_{k+1}(t,x) \\ =& e_k(t,x) - C(t)\delta Q_k(t,x) - D(t)\delta u_k(t,x) \\ =& e_k(t,x) - C(t)\delta Q_k(t,x) - qD(t)e_k(t,x) \\ =& (1 - qD(t))e_k(t,x) - C(t)\delta Q_k(t,x). \end{aligned}$$

From (6) and Assumption 1, we have

$$|e_{k+1}(t,x)| \le \rho\,|e_k(t,x)| + C\,|\delta Q_k(t,x)|.$$

Using the basic inequality, it yields

$$\begin{aligned} (e_{k+1}(t,x))^2 \le& (1+\varepsilon)\rho^2 (e_k(t,x))^2 \\ &+ (1+\frac{1}{\varepsilon})C^2(\delta Q_k(t,x))^2. \end{aligned}$$

Integrating both sides with respect to x from 0 to 1, we get

$$\begin{aligned} \|e_{k+1}(t,\cdot)\|_{L^2}^2 \le& (1+\varepsilon)\rho^2 \|e_k(t,\cdot)\|_{L^2}^2 \\ &+ (1+\frac{1}{\varepsilon})C^2 \|\delta Q_k(t,\cdot)\|_{L^2}^2. \end{aligned}$$

For $\lambda > 0$, we have

$$\begin{aligned} &\max_{t\in[-1,0]} \left\{ \mathrm{e}^{-\lambda(t+1)} \|e_{k+1}(t,\cdot)\|_{L^2}^2 \right\} \le \\ &\quad (1+\varepsilon)\rho^2 \max_{t\in[-1,0]} \left\{ \mathrm{e}^{-\lambda(t+1)} \|e_k(t,\cdot)\|_{L^2}^2 \right\} \\ &\quad + (1+\frac{1}{\varepsilon})C^2 \max_{t\in[-1,0]} \left\{ \mathrm{e}^{-\lambda(t+1)} \|\delta Q_k(t,\cdot)\|_{L^2}^2 \right\}, \end{aligned} \tag{7}$$

$$\begin{aligned} &\max_{t\in[0,1]} \left\{ \mathrm{e}^{-\lambda t} \|e_{k+1}(t,\cdot)\|_{L^2}^2 \right\} \le \\ &\quad (1+\varepsilon)\rho^2 \max_{t\in[0,1]} \left\{ \mathrm{e}^{-\lambda t} \|e_k(t,\cdot)\|_{L^2}^2 \right\} \\ &\quad + (1+\frac{1}{\varepsilon})C^2 \max_{t\in[0,1]} \left\{ \mathrm{e}^{-\lambda t} \|\delta Q_k(t,\cdot)\|_{L^2}^2 \right\}. \end{aligned} \tag{8}$$

It follows from (3) and (5) that

$$\begin{aligned} &\frac{\partial(\delta Q_k(t,x))}{\partial t} - \frac{\partial}{\partial x}\left(a(x)\frac{\partial(\delta Q_k(t,x))}{\partial x}\right) + \sigma\delta Q_k(t,x) \\ &= q e_k(t,x), \quad -1 \le t \le 0, x \in (0,1), \end{aligned} \tag{9}$$

$$\frac{\partial^2(\delta Q_k(t,x))}{\partial t^2}-\frac{\partial}{\partial x}\left(a(x)\frac{\partial(\delta Q_k(t,x))}{\partial x}\right)+\sigma\delta Q_k(t,x)$$
$$=qe_k(t,x),\ \ 0\le t\le 1, x\in(0,1). \tag{10}$$

Multiplying both sides of (9) by $\delta Q_k(t,x)$ and integrating with respect to x from 0 to 1, we can get

$$\begin{aligned}&\int_0^1\left\{\delta Q_k(t,x)\frac{\partial(\delta Q_k(t,x))}{\partial t}\right\}\mathrm{d}x\\&\quad-\int_0^1\left\{\delta Q_k(t,x)\frac{\partial}{\partial x}\left(a(x)\frac{\partial(\delta Q_k(t,x))}{\partial x}\right)\right\}\mathrm{d}x\\&\quad+\sigma\int_0^1(\delta Q_k(t,x))^2\mathrm{d}x\\&=q\int_0^1\delta Q_k(t,x)e_k(t,x)\mathrm{d}x,\ \ -1\le t\le 0,\end{aligned}\tag{11}$$

while

$$\int_0^1\left\{\delta Q_k(t,x)\frac{\partial(\delta Q_k(t,x))}{\partial t}\right\}\mathrm{d}x$$
$$=\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int_0^1(\delta Q_k(t,x))^2\mathrm{d}x=\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\delta Q_k(t,\cdot)\|_{L^2}^2, \tag{12}$$

$$\begin{aligned}&\int_0^1\delta Q_k(t,x)e_k(t,x)\mathrm{d}x\le\frac{1}{2}\int_0^1(\delta Q_k(t,x))^2\mathrm{d}x\\&\quad+\frac{1}{2}\int_0^1(e_k(t,x))^2\mathrm{d}x\\&=\frac{1}{2}\|\delta Q_k(t,\cdot)\|_{L^2}^2+\frac{1}{2}\|e_k(t,\cdot)\|_{L^2}^2.\end{aligned}\tag{13}$$

Integrating by parts and combining with the boundary resetting conditions given in Assumptions 3, we can derive

$$\begin{aligned}&\int_0^1\left\{\delta Q_k(t,x)\frac{\partial}{\partial x}\left(a(x)\frac{\partial(\delta Q_k(t,x))}{\partial x}\right)\right\}\mathrm{d}x\\&=\left\{\delta Q_k(t,x)a(x)\frac{\partial(\delta Q_k(t,x))}{\partial x}\right\}\Bigg|_0^1\\&\quad-\int_0^1\left\{\frac{\partial(\delta Q_k(t,x))}{\partial x}a(x)\frac{\partial(\delta Q_k(t,x))}{\partial x}\right\}\mathrm{d}x\\&=-\int_0^1\left\{\frac{\partial(\delta Q_k(t,x))}{\partial x}a(x)\frac{\partial(\delta Q_k(t,x))}{\partial x}\right\}\mathrm{d}x\le 0.\end{aligned}\tag{14}$$

Note that $\sigma>0$. Substituting (12)-(14) into (11), it yields

$$\frac{\mathrm{d}}{\mathrm{d}t}\|\delta Q_k(t,\cdot)\|_{L^2}^2\le q\|\delta Q_k(t,\cdot)\|_{L^2}^2+q\|e_k(t,\cdot)\|_{L^2}^2,$$
$$-1\le t\le 0.$$

Applying Gronwall lemma and combining with the initial resetting conditions given in Assumptions 3, we have

$$\begin{aligned}&\|\delta Q_k(t,\cdot)\|_{L^2}^2\le q\int_{-1}^t\mathrm{e}^{q(t-\eta)}\|e_k(\eta,\cdot)\|_{L^2}^2\mathrm{d}\eta\\&\le q\mathrm{e}^q\int_{-1}^t\|e_k(\eta,\cdot)\|_{L^2}^2\mathrm{d}\eta\\&=q\mathrm{e}^q\int_{-1}^t\mathrm{e}^{\lambda(\eta+1)}\mathrm{e}^{-\lambda(\eta+1)}\|e_k(\eta,\cdot)\|_{L^2}^2\mathrm{d}\eta\\&\le q\mathrm{e}^q\int_{-1}^t\mathrm{e}^{\lambda(\eta+1)}\mathrm{d}\eta\max_{t\in[-1,0]}\left\{\mathrm{e}^{-\lambda(t+1)}\|e_k(t,\cdot)\|_{L^2}^2\right\}\\&=q\mathrm{e}^q\frac{\mathrm{e}^{\lambda(t+1)}-1}{\lambda}\max_{t\in[-1,0]}\left\{\mathrm{e}^{-\lambda(t+1)}\|e_k(t,\cdot)\|_{L^2}^2\right\}.\end{aligned}\tag{15}$$

Therefore

$$\begin{aligned}&\max_{t\in[-1,0]}\left\{\mathrm{e}^{-\lambda(t+1)}\|\delta Q_k(t,\cdot)\|_{L^2}^2\right\}\\&\le q\mathrm{e}^q\max_{t\in[-1,0]}\left\{\frac{1-\mathrm{e}^{-\lambda(t+1)}}{\lambda}\right\}\\&\quad\times\max_{t\in[-1,0]}\left\{\mathrm{e}^{-\lambda(t+1)}\|e_k(t,\cdot)\|_{L^2}^2\right\}\\&=q\mathrm{e}^q\frac{1-\mathrm{e}^{-\lambda}}{\lambda}\max_{t\in[-1,0]}\left\{\mathrm{e}^{-\lambda(t+1)}\|e_k(t,\cdot)\|_{L^2}^2\right\}.\end{aligned}$$

Substituting the above expression into (7), it yields

$$\begin{aligned}&\max_{t\in[-1,0]}\left\{\mathrm{e}^{-\lambda(t+1)}\|e_{k+1}(t,\cdot)\|_{L^2}^2\right\}\\&\le\left\{(1+\varepsilon)\rho^2+(1+\frac{1}{\varepsilon})C^2q\mathrm{e}^q\frac{1-\mathrm{e}^{-\lambda}}{\lambda}\right\}\\&\quad\times\max_{t\in[-1,0]}\left\{\mathrm{e}^{-\lambda(t+1)}\|e_k(t,\cdot)\|_{L^2}^2\right\}.\end{aligned}\tag{16}$$

It follows from (6) that $(1+\varepsilon)\rho^2<1$. Therefore, there exists a sufficiently large λ such that

$$(1+\varepsilon)\rho^2+(1+\frac{1}{\varepsilon})C^2q\mathrm{e}^q\frac{1-\mathrm{e}^{-\lambda}}{\lambda}<1. \tag{17}$$

From (16), (17), it can be concluded that

$$\lim_{k\to\infty}\left\{\max_{t\in[-1,0]}\left\{\mathrm{e}^{-\lambda(t+1)}\|e_k(t,\cdot)\|_{L^2}^2\right\}\right\}=0. \tag{18}$$

We have

$$\mathrm{e}^{-\lambda(t+1)}\|e_k(t,\cdot)\|_{L^2}^2\ge\mathrm{e}^{-\lambda}\|e_k(t,\cdot)\|_{L^2}^2,\ t\in[-1,0],$$

which implies

$$\|e_k(t,\cdot)\|_{L^2}^2\le\mathrm{e}^{\lambda}\mathrm{e}^{-\lambda(t+1)}\|e_k(t,\cdot)\|_{L^2}^2, t\in[-1,0]. \tag{19}$$

It follows from (18), (19) that

$$\lim_{k\to\infty}\left\{\max_{t\in[-1,0]}\|e_k(t,\cdot)\|_{L^2}^2\right\}=0. \tag{20}$$

From (15), (18), we can get

$$\lim_{k\to\infty}\left\{\max_{t\in[-1,0]}\|\delta Q_k(t,\cdot)\|_{L^2}^2\right\}=0.$$

Then

$$\lim_{k\to\infty}\|\delta Q_k(0,\cdot)\|_{L^2}^2=0. \tag{21}$$

Multiplying both sides of (10) by $\frac{\partial(\delta Q_k(t,x))}{\partial t}$ and integrating with respect to x from 0 to 1, we can obtain

$$\begin{aligned}&\int_0^1\left\{\frac{\partial(\delta Q_k(t,x))}{\partial t}\frac{\partial^2(\delta Q_k(t,x))}{\partial t^2}\right\}\mathrm{d}x\\&\quad-\int_0^1\left\{\frac{\partial(\delta Q_k(t,x))}{\partial t}\frac{\partial}{\partial x}\left(a(x)\frac{\partial(\delta Q_k(t,x))}{\partial x}\right)\right\}\mathrm{d}x\\&\quad+\sigma\int_0^1\frac{\partial(\delta Q_k(t,x))}{\partial t}\delta Q_k(t,x)\mathrm{d}x\\&=q\int_0^1\frac{\partial(\delta Q_k(t,x))}{\partial t}e_k(t,x)\mathrm{d}x, 0\le t\le 1,\end{aligned}\tag{22}$$

while

$$\begin{aligned}&\int_0^1\left\{\frac{\partial(\delta Q_k(t,x))}{\partial t}\frac{\partial^2(\delta Q_k(t,x))}{\partial t^2}\right\}\mathrm{d}x\\&=\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int_0^1\left(\frac{\partial(\delta Q_k(t,x))}{\partial t}\right)^2\mathrm{d}x\\&=\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left\|\frac{\partial(\delta Q_k(t,\cdot))}{\partial t}\right\|_{L^2}^2,\end{aligned}\tag{23}$$

$$\begin{aligned}&\left|\int_0^1\frac{\partial(\delta Q_k(t,x))}{\partial t}\delta Q_k(t,x)\mathrm{d}x\right|\\&\le\frac{1}{2}\int_0^1\left(\frac{\partial(\delta Q_k(t,x))}{\partial t}\right)^2\mathrm{d}x+\frac{1}{2}\int_0^1(\delta Q_k(t,x))^2\mathrm{d}x\\&=\frac{1}{2}\left\|\frac{\partial(\delta Q_k(t,\cdot))}{\partial t}\right\|_{L^2}^2+\frac{1}{2}\|\delta Q_k(t,\cdot)\|_{L^2}^2,\end{aligned}\tag{24}$$

$$\begin{aligned}&\int_0^1\frac{\partial(\delta Q_k(t,x))}{\partial t}e_k(t,x)\mathrm{d}x\\&\le\frac{1}{2}\left\|\frac{\partial(\delta Q_k(t,\cdot))}{\partial t}\right\|_{L^2}^2+\frac{1}{2}\|e_k(t,\cdot)\|_{L^2}^2.\end{aligned}\tag{25}$$

Integrating by parts and combining with the boundary resetting conditions given in Assumptions 3, we can derive

$$\begin{aligned}&\int_0^1\left\{\frac{\partial(\delta Q_k(t,x))}{\partial t}\frac{\partial}{\partial x}\left(a(x)\frac{\partial(\delta Q_k(t,x))}{\partial x}\right)\right\}\mathrm{d}x\\&=\left\{\frac{\partial(\delta Q_k(t,x))}{\partial t}a(x)\frac{\partial(\delta Q_k(t,x))}{\partial x}\right\}\Big|_0^1\\&\quad-\int_0^1\left\{\frac{\partial^2(\delta Q_k(t,x))}{\partial t\partial x}a(x)\frac{\partial(\delta Q_k(t,x))}{\partial x}\right\}\mathrm{d}x\\&=-\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int_0^1\left\{a(x)\left(\frac{\partial(\delta Q_k(t,x))}{\partial x}\right)^2\right\}\mathrm{d}x.\end{aligned}\tag{26}$$

Substituting (23)-(26) into (22), it yields

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}\left\|\frac{\partial(\delta Q_k(t,\cdot))}{\partial t}\right\|_{L^2}^2\\&\quad+\frac{\mathrm{d}}{\mathrm{d}t}\int_0^1\left\{a(x)\left(\frac{\partial(\delta Q_k(t,x))}{\partial x}\right)^2\right\}\mathrm{d}x\\&\le(\sigma+q)\left\|\frac{\partial(\delta Q_k(t,\cdot))}{\partial t}\right\|_{L^2}^2+\sigma\|\delta Q_k(t,\cdot)\|_{L^2}^2\\&\quad+q\|e_k(t,\cdot)\|_{L^2}^2.\end{aligned}$$

It follows from $a(x)\ge a>0$ that

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}\left\{\left\|\frac{\partial(\delta Q_k(t,\cdot))}{\partial t}\right\|_{L^2}^2\right.\\&\quad\left.+\int_0^1\left\{a(x)\left(\frac{\partial(\delta Q_k(t,x))}{\partial x}\right)^2\right\}\mathrm{d}x\right\}\\&\le(\sigma+q)\left\|\frac{\partial(\delta Q_k(t,\cdot))}{\partial t}\right\|_{L^2}^2+\sigma\|\delta Q_k(t,\cdot)\|_{L^2}^2\\&\quad+q\|e_k(t,\cdot)\|_{L^2}^2\\&\le(\sigma+q)\left\|\frac{\partial(\delta Q_k(t,\cdot))}{\partial t}\right\|_{L^2}^2+\sigma\|\delta Q_k(t,\cdot)\|_{L^2}^2\\&\quad+q\|e_k(t,\cdot)\|_{L^2}^2\\&\quad+(\sigma+q)\int_0^1\left\{a(x)\left(\frac{\partial(\delta Q_k(t,x))}{\partial x}\right)^2\right\}\mathrm{d}x,\\&0\le t\le 1.\end{aligned}$$

Applying Gronwall lemma and combining with the initial resetting conditions given in Assumptions 3, we have

$$\begin{aligned}&\left\|\frac{\partial(\delta Q_k(t,\cdot))}{\partial t}\right\|_{L^2}^2\\&\quad+\int_0^1\left\{a(x)\left(\frac{\partial(\delta Q_k(t,x))}{\partial x}\right)^2\right\}dx\\&\le\int_0^t\mathrm{e}^{(\sigma+q)(t-\eta)}\left\{\sigma\|\delta Q_k(\eta,\cdot)\|_{L^2}^2+q\|e_k(\eta,\cdot)\|_{L^2}^2\right\}\mathrm{d}\eta\\&\le\mathrm{e}^{\sigma+q}\frac{\mathrm{e}^{\lambda t}-1}{\lambda}\left\{\sigma\max_{t\in[0,1]}\left\{\mathrm{e}^{-\lambda t}\|\delta Q_k(t,\cdot)\|_{L^2}^2\right\}\right.\\&\quad\left.+q\max_{t\in[0,1]}\left\{\mathrm{e}^{-\lambda t}\|e_k(t,\cdot)\|_{L^2}^2\right\}\right\}.\end{aligned}$$

So

$$\begin{aligned}&\max_{t\in[0,1]}\left\{\mathrm{e}^{-\lambda t}\left\|\frac{\partial(\delta Q_k(t,\cdot))}{\partial t}\right\|_{L^2}^2\right\}\\&\le\max_{t\in[0,1]}\left\{\mathrm{e}^{-\lambda t}\left\{\left\|\frac{\partial(\delta Q_k(t,\cdot))}{\partial t}\right\|_{L^2}^2\right.\right.\\&\quad\left.\left.+\int_0^1\left\{a(x)\left(\frac{\partial(\delta Q_k(t,x))}{\partial x}\right)^2\right\}\mathrm{d}x\right\}\right\}\\&\le\mathrm{e}^{\sigma+q}\max_{t\in[0,1]}\left\{\frac{1-\mathrm{e}^{-\lambda t}}{\lambda}\right\}\\&\quad\times\left\{\sigma\max_{t\in[0,1]}\left\{\mathrm{e}^{-\lambda t}\|\delta Q_k(t,\cdot)\|_{L^2}^2\right\}\right.\\&\quad\left.+q\max_{t\in[0,1]}\left\{\mathrm{e}^{-\lambda t}\|e_k(t,\cdot)\|_{L^2}^2\right\}\right\}\\&=\mathrm{e}^{\sigma+q}\frac{1-\mathrm{e}^{-\lambda}}{\lambda}\left\{\sigma\max_{t\in[0,1]}\left\{\mathrm{e}^{-\lambda t}\|\delta Q_k(t,\cdot)\|_{L^2}^2\right\}\right.\end{aligned}$$

$$+q\max_{t\in[0,1]}\left\{\mathrm{e}^{-\lambda t}\|e_k(t,\cdot)\|_{L^2}^2\right\}\bigg\}. \tag{27}$$

On the other hand, by using the basic inequality, we have

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}\|\delta Q_k(t,\cdot)\|_{L^2}^2=\frac{\mathrm{d}}{\mathrm{d}t}\int_0^1(\delta Q_k(t,x))^2\mathrm{d}x\\&=2\int_0^1\delta Q_k(t,x)\frac{\partial\delta Q_k(t,x)}{\partial t}\mathrm{d}x\\&\le\int_0^1(\delta Q_k(t,x))^2\mathrm{d}x+\int_0^1\left(\frac{\partial\delta Q_k(t,x)}{\partial t}\right)^2\mathrm{d}x\\&=\|\delta Q_k(t,\cdot)\|_{L^2}^2+\left\|\frac{\partial\delta Q_k(t,\cdot)}{\partial t}\right\|_{L^2}^2,\ 0\le t\le 1.\end{aligned}$$

Applying Gronwall lemma, we can obtain

$$\begin{aligned}&\|\delta Q_k(t,\cdot)\|_{L^2}^2\le\int_0^t\left\{\mathrm{e}^{t-\eta}\left\|\frac{\partial\delta Q_k(\eta,\cdot)}{\partial\eta}\right\|_{L^2}^2\right\}\mathrm{d}\eta\\&\quad+\mathrm{e}^t\|\delta Q_k(0,\cdot)\|_{L^2}^2\\&=\mathrm{e}^t\int_0^t\left\{\mathrm{e}^{(\lambda-1)\eta}\mathrm{e}^{-\lambda\eta}\left\|\frac{\partial\delta Q_k(\eta,\cdot)}{\partial\eta}\right\|_{L^2}^2\right\}\mathrm{d}\eta\\&\quad+\mathrm{e}^t\|\delta Q_k(0,\cdot)\|_{L^2}^2\\&\le\mathrm{e}^t\int_0^t\mathrm{e}^{(\lambda-1)\eta}\mathrm{d}\eta\max_{t\in[0,1]}\left\{\mathrm{e}^{-\lambda t}\left\|\frac{\partial\delta Q_k(t,\cdot)}{\partial t}\right\|_{L^2}^2\right\}\\&\quad+\mathrm{e}\|\delta Q_k(0,\cdot)\|_{L^2}^2\\&=\frac{\mathrm{e}^{\lambda t}-\mathrm{e}^t}{\lambda-1}\max_{t\in[0,1]}\left\{\mathrm{e}^{-\lambda t}\left\|\frac{\partial\delta Q_k(t,\cdot)}{\partial t}\right\|_{L^2}^2\right\}\\&\quad+\mathrm{e}\|\delta Q_k(0,\cdot)\|_{L^2}^2.\end{aligned}$$

Take $\lambda>1$, then

$$\begin{aligned}&\max_{t\in[0,1]}\left\{\mathrm{e}^{-\lambda t}\|\delta Q_k(t,\cdot)\|_{L^2}^2\right\}\\&\le\max_{t\in[0,1]}\left\{\frac{1-\mathrm{e}^{-(\lambda-1)t}}{\lambda-1}\right\}\max_{t\in[0,1]}\left\{\mathrm{e}^{-\lambda t}\left\|\frac{\partial\delta Q_k(t,\cdot)}{\partial t}\right\|_{L^2}^2\right\}\\&\quad+\max_{t\in[0,1]}\left\{\mathrm{e}^{-\lambda t}\mathrm{e}\|\delta Q_k(0,\cdot)\|_{L^2}^2\right\}=\frac{1-\mathrm{e}^{-(\lambda-1)}}{\lambda-1}\\&\quad\times\max_{t\in[0,1]}\left\{\mathrm{e}^{-\lambda t}\left\|\frac{\partial\delta Q_k(t,\cdot)}{\partial t}\right\|_{L^2}^2\right\}+\mathrm{e}\|\delta Q_k(0,\cdot)\|_{L^2}^2.\end{aligned} \tag{28}$$

Substituting (28) into (27) and selecting λ large enough such that

$$\sigma\mathrm{e}^{\sigma+q}\frac{1-\mathrm{e}^{-\lambda}}{\lambda}\frac{1-\mathrm{e}^{-(\lambda-1)}}{\lambda-1}<1,$$

we have

$$\begin{aligned}&\max_{t\in[0,1]}\left\{\mathrm{e}^{-\lambda t}\left\|\frac{\partial(\delta Q_k(t,\cdot))}{\partial t}\right\|_{L^2}^2\right\}\\&\le\frac{q\mathrm{e}^{\sigma+q\frac{1-\mathrm{e}^{-\lambda}}{\lambda}}}{1-\sigma\mathrm{e}^{\sigma+q\frac{1-\mathrm{e}^{-\lambda}}{\lambda}\frac{1-\mathrm{e}^{-(\lambda-1)}}{\lambda-1}}}\max_{t\in[0,1]}\left\{\mathrm{e}^{-\lambda t}\|e_k(t,\cdot)\|_{L^2}^2\right\}\\&\quad+\frac{\sigma\mathrm{e}^{\sigma+q+1\frac{1-\mathrm{e}^{-\lambda}}{\lambda}}}{1-\sigma\mathrm{e}^{\sigma+q\frac{1-\mathrm{e}^{-\lambda}}{\lambda}\frac{1-\mathrm{e}^{-(\lambda-1)}}{\lambda-1}}}\|\delta Q_k(0,\cdot)\|_{L^2}^2.\end{aligned}$$

Substituting the above expression into (28), it yields

$$\begin{aligned}&\max_{t\in[0,1]}\left\{\mathrm{e}^{-\lambda t}\|\delta Q_k(t,\cdot)\|_{L^2}^2\right\}\\&\le\frac{1-\mathrm{e}^{-(\lambda-1)}}{\lambda-1}\frac{q\mathrm{e}^{\sigma+q\frac{1-\mathrm{e}^{-\lambda}}{\lambda}}}{1-\sigma\mathrm{e}^{\sigma+q\frac{1-\mathrm{e}^{-\lambda}}{\lambda}\frac{1-\mathrm{e}^{-(\lambda-1)}}{\lambda-1}}}\\&\quad\times\max_{t\in[0,1]}\left\{\mathrm{e}^{-\lambda t}\|e_k(t,\cdot)\|_{L^2}^2\right\}\\&\quad+\left(\mathrm{e}+\frac{\sigma\mathrm{e}^{\sigma+q+1\frac{1-\mathrm{e}^{-\lambda}}{\lambda}\frac{1-\mathrm{e}^{-(\lambda-1)}}{\lambda-1}}}{1-\sigma\mathrm{e}^{\sigma+q\frac{1-\mathrm{e}^{-\lambda}}{\lambda}\frac{1-\mathrm{e}^{-(\lambda-1)}}{\lambda-1}}}\right)\|\delta Q_k(0,\cdot)\|_{L^2}^2.\end{aligned}$$

Substituting the above expression into (8) and denoting

$$\alpha=\frac{1-\mathrm{e}^{-(\lambda-1)}}{\lambda-1}\frac{q\mathrm{e}^{\sigma+q\frac{1-\mathrm{e}^{-\lambda}}{\lambda}}}{1-\sigma\mathrm{e}^{\sigma+q\frac{1-\mathrm{e}^{-\lambda}}{\lambda}\frac{1-\mathrm{e}^{-(\lambda-1)}}{\lambda-1}}},$$

$$\beta=\mathrm{e}+\frac{\sigma\mathrm{e}^{\sigma+q+1\frac{1-\mathrm{e}^{-\lambda}}{\lambda}\frac{1-\mathrm{e}^{-(\lambda-1)}}{\lambda-1}}}{1-\sigma\mathrm{e}^{\sigma+q\frac{1-\mathrm{e}^{-\lambda}}{\lambda}\frac{1-\mathrm{e}^{-(\lambda-1)}}{\lambda-1}}},$$

we can obtain

$$\begin{aligned}&\max_{t\in[0,1]}\left\{\mathrm{e}^{-\lambda t}\|e_{k+1}(t,\cdot)\|_{L^2}^2\right\}\\&\le\left\{(1+\varepsilon)\rho^2+(1+\frac{1}{\varepsilon})C^2\alpha\right\}\max_{t\in[0,1]}\left\{\mathrm{e}^{-\lambda t}\|e_k(t,\cdot)\|_{L^2}^2\right\}\\&\quad+(1+\frac{1}{\varepsilon})C^2\beta\|\delta Q_k(0,\cdot)\|_{L^2}^2.\end{aligned} \tag{29}$$

It is clear that $\alpha\to 0$ as $\lambda\to\infty$, so we can select λ large enough such that

$$(1+\varepsilon)\rho^2+(1+\frac{1}{\varepsilon})C^2\alpha<1. \tag{30}$$

Applying lemma 1 to (29) and combining with (21),(30), we have

$$\lim_{k\to\infty}\left\{\max_{t\in[0,1]}\left\{\mathrm{e}^{-\lambda t}\|e_k(t,\cdot)\|_{L^2}^2\right\}\right\}=0.$$

Therefore

$$\lim_{k\to\infty}\left\{\max_{t\in[0,1]}\|e_k(t,\cdot)\|_{L^2}^2\right\}=0. \tag{31}$$

From (20), (31), it can be concluded that

$$\lim_{k\to\infty}\left\{\max_{t\in[-1,1]}\|e_k(t,\cdot)\|_{L^2}^2\right\}=0.$$

Then

$$\lim_{k\to\infty}\|e_k\|_{L^2,s}=0.$$

This completes the proof. □

Remark 3: From Assumption 1, when we choose the learning gain q such that $\frac{\sqrt{1+\varepsilon}-1}{D_1\sqrt{1+\varepsilon}} < q < \frac{\sqrt{1+\varepsilon}+1}{D_2\sqrt{1+\varepsilon}}$, the convergence condition (6) holds. And from (4), the learning gain q satisfying the convergence condition (6) always exists.

Remark 4: From (3), (5) and Remark 1, as long as the initial control $u_0(t,x)$ is selected a piecewise smooth function ($(t,x)\in[-1,0]\times(0,1)$,$(t,x)\in[0,1]\times(0,1)$), the solutions of the system (3) at k^{th} iteration are always existing and unique ($k=0,1,2,\cdots$).

4. SIMULATION EXAMPLE

Taking $a(x)=1$, $\sigma=1$, $C(t)=D(t)=1$, then the system (2) is as follows:

$$\begin{cases} \frac{\partial^2 Q(t,x)}{\partial t^2}-\frac{\partial^2 Q(t,x)}{\partial x^2}+Q(t,x) \\ \quad =u(t,x), 0\le t\le 1, x\in(0,1), \\ \frac{\partial Q(t,x)}{\partial t}-\frac{\partial^2 Q(t,x)}{\partial x^2}+Q(t,x) \\ \quad =u(t,x), -1\le t\le 0, x\in(0,1), \\ y(t,x)=Q(t,x)+u(t,x), \\ \quad -1\le t\le 1, x\in(0,1). \end{cases}$$

For the given desired trajectory: $y_r(t,x)=3\mathrm{e}^t x(x-1)-2\mathrm{e}^t$, we have $Q_r(t,x)=\mathrm{e}^t x(x-1)$, $u_r(t,x)=2\mathrm{e}^t x(x-1)-2\mathrm{e}^t$.

Construct the k^{th} iteration

$$\begin{cases} \frac{\partial^2 Q_k(t,x)}{\partial t^2}-\frac{\partial^2 Q_k(t,x)}{\partial x^2}+Q_k(t,x) \\ \quad =u_k(t,x), 0\le t\le 1, x\in(0,1), \\ \frac{\partial Q_k(t,x)}{\partial t}-\frac{\partial^2 Q_k(t,x)}{\partial x^2}+Q_k(t,x) \\ \quad =u_k(t,x), -1\le t\le 0, x\in(0,1), \\ y_k(t,x)=Q_k(t,x)+u_k(t,x), \\ \quad -1\le t\le 1, x\in(0,1). \end{cases}$$

Combining with the initial-boundary resetting conditions in Assumption 3, we take the initial-boundary (Dirichlet) values at k^{th} iteration:

$Q_k(-1,x)=Q_r(-1,x)=\mathrm{e}^{-1}x(x-1)$,

$Q_k(t,0)=0, \quad Q_k(t,1)=0.$

Take the initial control $u_0(t,x)=1$, and construct the following iterative learning control:

$u_{k+1}(t,x)=u_k(t,x)+qe_k(t,x)$,

$k=0,1,2,\cdots$. Taking $\varepsilon=\frac{9}{16}$, it follows from (6) that the iteration is convergent for $0.2<q<1.8$. Therefore, we take $q=1.5$. By using the mathematical software Mathematica, it is easy to see that the output tracking errors on L^2 space tend to zero as $k\to\infty$ (shown in Fig. 1). Furthermore, the simulation results of the output tracking errors $e_k(t,x)$ with the change of the iteration index k are shown in Figs. 2-4. From Figs. 2-4, we can know that $|e_k(t,x)|$ becomes small gradually as k increases.

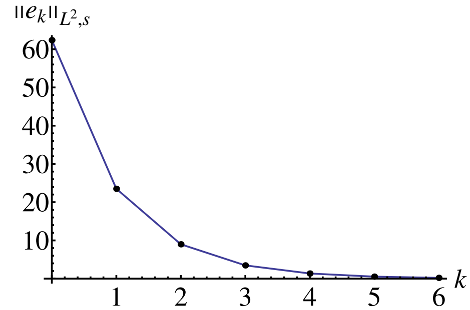


Fig. 1. Iterations for the output tracking errors.

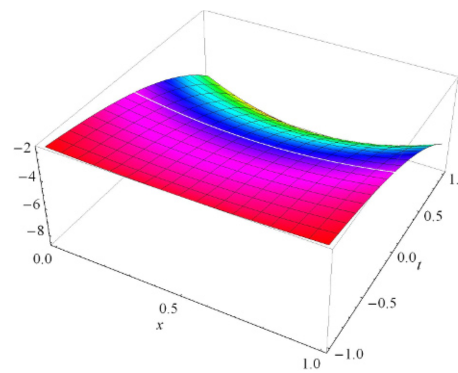


Fig. 2. Trajectory of $e_0(t,x)$ over $[-1,1]\times[0,1]$.

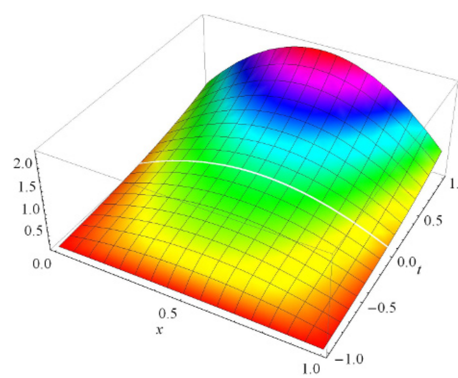


Fig. 3. Trajectory of $e_3(t,x)$ over $[-1,1]\times[0,1]$.

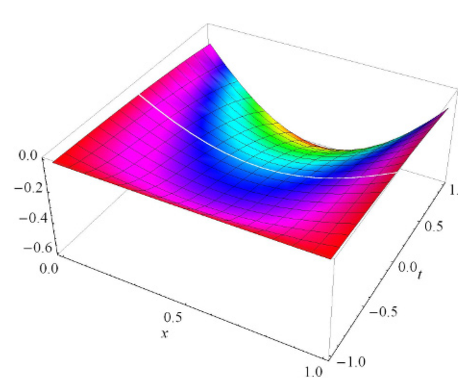


Fig. 4. Trajectory of $e_6(t,x)$ over $[-1,1]\times[0,1]$.

5. CONCLUSIONS

This paper considers the iterative learning control problem for a class of mixed distributed parameter systems which are composed of mixed hyperbolic-parabolic partial differential equations. By using P-type learning scheme, the convergence theorem of the output tracking errors on L^2 space is established based on the contraction mapping method. The simulation result is consistent with theoretical analysis. How to apply iterative learning control scheme to other MDPSs as mentioned in the references of this paper, it remains further research.

REFERENCES

[1] S. Arimoto, S. Kawamura, and F. Miyazaki, "Bettering operation of robots by learning," *Journal of Robotic Systems*, vol. 1, no. 2, pp.123-140, Summer 1984. [click]

[2] J. X. Xu, "Analysis iterative learning control for a class of nonlinear discrete-time systems," *Automatica*, vol. 33, no. 10, pp. 1905-1907, October 1997. [click]

[3] D. H. Owens, "Multivariable norm optimal and parameter optimal iterative learning control: a unified formulation," *International Journal of Control*, vol. 85, no. 8, pp. 1010-1025, August 2012. [click]

[4] M. Sun and D. Wang. "Sampled-data iterative learning control for nonlinear systems with arbitrary relative degree," *Automatica*, vol. 37, no. 2, pp. 283-289, February 2001. [click]

[5] M. X. Sun, D. W. Wang, and Y. Y. Wang, "Varying-order iterative learning control against perturbed initial conditions," *Journal of The Franklin Institute*, vol. 347, no. 8, pp. 1526-1549, October 2010.

[6] D. A. Bristow, M. Tharayil, and A. G. Alleyne, "A survey of iterative learning control: A learning- method for high-performance tracking control," *IEEE Control Syst.Mag.*, vol. 26, no. 3, pp. 96-114, June 2006. [click]

[7] G. Bastin and J. M. Coron, "On boundary feedback stabilization of non-uniform linear 2×2 hyperbolic systems over a bounded interval," *Systems Control Letters*, vol. 60, no. 11, pp. 900-906, November 2011. [click]

[8] S. X. Tang and C. K. Xie, "State and output feedback boundary control for a coupled PDE-ODE system," *Systems Control Letters*, vol. 60, no. 8, pp. 540-545, August 2011. [click]

[9] R. Vazquez and M. Krstic, "A closed-form feedback controller for stabilization of the linearized 2-D Navier-Stokes poiseuille system," *IEEE Transactions on Automatic Control*, vol. 52, no. 12, pp. 2298-2312, December 2007.

[10] Z. H. Qu, "An iterative learning algorithm for boundary control of a stretched moving string," *Automatica*, vol. 38, no. 5, pp. 821-827, May 2002. [click]

[11] D. Q. Huang, J. X. Xu, X. F. Li, C. Xu, and M. Yu, "D-type anticipatory iterative learning control for a class of inhomogeneous heat equations," *Automatica*, vol. 49, no. 8, pp. 2397-2408, August 2013. [click]

[12] D. Q. Huang, X. F. Li, J. X. Xu, C. Xu, and W. He, "Iterative learning control of inhomogeneous distributed parameter systems-frequency domain design and analysis," *Systems Control Letters*, vol. 72, pp. 22-29, October 2014. [click]

[13] Y. V. Orlov, "Discontinuous unit feedback control of uncertain infinite-dimensional systems," *IEEE Transactions on Automatic Control*, vol. 45, no. 5, pp. 834-843, May 2000.

[14] A. Pisano, Y. Orlov, and E. Usai, "Tracking control of the uncertain heat and wave equation via power-fractional and sliding-mode techniques," *SIAM J. Control Optim*, vol. 49, no. 2, pp. 363-382, March 2011. [click]

[15] X. J. Fan, S. P Tian, and H. P. Tian, "Iterative learning control of distributed parameter system based on geometric analysis," *Proc. of the Eighth International Conference on Machine Learning and Cybernetics*, pp. 3673-3677, 2009.

[16] J. L. Kang, "A Newton-type iterative learning algorithm of output tracking control for uncertain nonlinear distributed parameter systems," *Proc. of the 33rd Chinese Control Conference*, pp. 8901-8905, 2014.

[17] X. S. Dai and S. P. Tian, "Iterative learning control for distributed parameter systems with time-delay," *Proc. of 2011 Chinese Control and Decision Conference*, pp. 2304 -2307,2011.

[18] C. Xu, R. Arastoo, and E. Schuster, "On Iterative learning control of parabolic distributed parameter systems," *Proc. of 17th Mediterranean Conference on Control and Automation*, pp. 510-515, 2009.

[19] J. Choi, B. J. Seo, and K. S. Lee, "Constrained digital regulation of hyperbolic PDE: a learning control approach," *Korean Journal of Chemical Engineering*, vol. 18, no. 5, pp. 606-611, September 2001. [click]

[20] Q. Fu, "Iterative learning control for second order nonlinear hyperbolic distributed parameter systems," *Journal of Systems Science and Mathematical Sciences* (in Chinese), vol. 34, no. 3, pp. 284-293, March 2014.

[21] Q. Fu, "Iterative learning control for irregular distributed parameter systems," *Control and Decision* (in Chinese), vol. 31, no. 1, pp. 114-122, January 2016.

[22] G. P. Boswell, H. Jacobs, F. A. Davidson, G. M. Gadd, and K. Ritz, "A positive numerical scheme for a mix-type partial differential equation model for fungal growth," *Applied Mathematics and Computation*, vol. 138, no. 2,3, pp. 321-340, June 2003. [click]

[23] R. K. Kenneyd and D. Lee, "A finite-difference solution to a mix-type partial differential equation: an ocean dynamic motion model," *Mathl Comput.Modelling*, vol. 10, no. 2, pp. 75-85, February 1988.

[24] O. Terlyga, H. Bellout, and F. Bloom, "Global existence, uniqueness, and stability for a nonlinear hyperbolic-parabolic problem in pulse combustion," *Acta Mathematica Scientia*, vol. 32B, no. 1, pp. 41-74, January 2012.

[25] M. Raoofi and K. Zumbrun, "Stability of undercompressive viscous shock profiles of hyperbolic-parabolic systems," *Journal of Differential equations*, vol. 246, no. 4, pp. 1539-1567, February 2009. [click]

[26] T. Nguyen and K. Zumbrun, "Long time stability of large-amplitude noncharacteristic boundary layers for hyperbolic-parabolic systems," *J. Math. Pures Appl.*, vol. 92, no. 6, pp. 547-598, December 2009. [click]

[27] A. Ashyralyev and H. A. Yurtsever, "A note on the second order of accuracy difference schemes for hyperbolic-parabolic equations," *Applied Mathematics and Computation*, vol. 165, no. 3, pp. 517-537, July 2005.

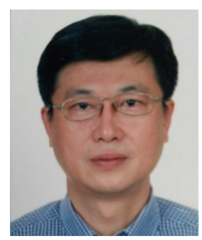
[28] A. Ashyralyev and Y. Ozdemir, "Stability of difference schemes for hyperbolic-parabolic equations," *Computers and Mathematics with Applications*, vol. 50, no. 8-9, pp. 1443-1476, October-November 2005. [click]

[29] A. Ashyralyev and Y. Ozdemir, "On numerical solutions for hyperbolic-parabolic equations with multipoint nonlocal boundary condition," *Journal of The Franklin Institute*, vol. 351, no. 2, pp. 602-630, February 2014. [click]

Qin Fu received his Ph.D degree in Control Theory and Control Engineering from Nanjing University of Science and Technology, China in 2009. He is currently an associate professor of Suzhou University of Science and Technology. His research interests include decentralized control, robust control, and iterative learning control.

Wei-Guo Gu received his M.S. degree in fundamental mathematics from Suzhou University of Science and Technology, China in 2015. He is currently a computer programmer. His research interests include iterative learning control and feedback control.

Pan-Pan Gu is currently pursuing a M.S. in School of Mathematics and Physics, Suzhou University of Science and Technology, China. His research interest is iterative learning control.



Jian-Rong Wu received his B.S. degree in Mathematics from Nanjing Normal University, China in 1985, and his M.S. and Ph.D degrees in Mathematics from Harbin Institue Technology, China, in 1988 and 2000, respectively. He is currently a professor of Suzhou University of Science and Technology. His research interests are in the areas of fuzzy systems and singular systems etc.