International Journal of Control, Automation and Systems 14(4) (2016) 948-956
http://dx.doi.org/10.1007/s12555-015-0228-3
ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Free-Matrix-based Integral Inequality for Stability Analysis of Uncertain T-S Fuzzy Systems with Time-varying Delay

Wen-Pin Luo*, Jun Yang, and Xin Zhao

Abstract: This paper focuses on further improved stability criteria for uncertain T-S fuzzy systems with time-varying delay by delay-partitioning approach and Free-Matrix-based integral inequality. A modified augmented Lyapunov-Krasovskii functional (LKF) is established by partitioning the delay in all integral terms. Then, on the basis of taking into account the independent upper bounds of the delay derivative in various delay intervals, some new results on tighter bounding inequalities, such as Peng-Park's integral inequality and the Free-Matrix-based integral inequality are employed to effectively reduce the enlargement in bounding the derivative of LKF, therefore, less conservative results can be expected in terms of e_s and LMIs. Finally, three numerical examples are included to show that the proposed method is less conservative than existing ones.

Keywords: Delay-partitioning approach, free-matrix-based integral inequality, linear matrix inequalities (LMIs), Lyapunov-Krasovskii functional (LKF), stability, time-varying delay, T-S fuzzy systems.

1. INTRODUCTION

Ever since Takagi-Sugeno (T-S) fuzzy model was firstly introduced in [1], considerable attentions have been paid to stability analysis and control synthesis of this model during the past three decades, due to the fact that it can combine the flexibility of fuzzy logic theory and fruitful linear system theory into a unified framework to approximate complex nonlinear systems (especially those with incomplete information) such as truck-trailer system, TORA system, inverted pendulum system and chaotic systems [1–4]. Furthermore, time-delay phenomenon widely exists in practical systems and is often a source of instability and poor performance, therefore, stability analysis for T-S fuzzy systems with (time-varing) delay has attracted much attentions over the last two decades, see, e.g., [5–10] and references therein. As far as the recent techniques adopted in the stability analysis of T-S fuzzy systems with time-varying delay are concerned, the delay-partitioning approach is the most noteworthy since it has been proved that less conservative results can be expected with increasing delay-partitioning segments [11, 12, 14, 15]. On the other hand, as [16] pointed out that, for the delay-partitioning approach, further less conservative stability conditions for time-varying delay systems can be achieved by taking into account of independent upper bounds of the delay derivative in various delay-partitioning intervals.

Recently, [15] has developed less conservative stability criteria than those in [6, 14, 17] for the uncertain T-S fuzzy systems with interval time-varying delay via delay-partitioning approach and a tighter bounding inequality (Peng-Park's integral inequality) established by reciprocally convex approach [18]. More recently, on the basis of a novel LKF and reciprocally convex approach [18], [11] has achieved less conservative results than those in [6, 15, 20–23, 25] for the uncertain T-S fuzzy systems with time-varying delay. Most recently, for the uncertain T-S fuzzy systems with time-varying delay, by means of delay-partitioning approach, Finsler's lemma and an appropriate LKF established in the framework of state vector augmentation, [24] employs some tighter bounding inequalities to effectively estimate the derivative of LKF, thus less conservative stability criteria than those in [6, 11, 21, 23] have obtained in [24]. However, when revisiting this problem, we find that the aforementioned works still leave plenty of room for improvement since that the challenges of deriving a further less conservative result is to construct an appropriate LKF that includes more useful augmented state

Manuscript received May 27, 2015; revised July 31, 2015; accepted September 1, 2015. Recommended by Associate Editor Tae-Hyoung Kim under the direction of Editor Zengqi Sun. This work was partially supported by the National Natural Science Foundation of China (Grant nos. 11501390, 11501392), the Key Natural Science Foundation of Sichuan Province Education Department (Grant No. 15ZA0234), the Opening Project of Sichuan Province University Key Laboratory of Bridge Non-destruction Detecting and Engineering Computing (Grant nos. 2014QZJ02, 2015QZJ02) and the scientific research fund project of SUSE (Grant no. 2012KY09). The authors would like to thank the editor and the anonymous reviewers for their constructive comments and suggestions to improve the quality of the paper.

Wen-Pin Luo is with the College of Science, Sichuan University of Science and Engineering (SUSE), Zigong, Sichuan, 643000, P. R. China (e-mail: luowenp@126.com). Jun Yang is with the College of Computer Science, Civil Aviation Flight University of China (CAFUC), Guanghan, Sichuan, 618307, P. R. China (e-mail: yj_uestc@126.com). Xin Zhao is with the Postgraduate Department, Civil Aviation Flight University of China (CAFUC), Guanghan, Sichuan, 618307, P. R. China (e-mail: cafuczx@126.com).
* Correspondign author.

information and to reduce the enlargement in bounding the derivative of LKF as much as possible.

Motivated by the above discussion, the main purpose of this paper is to develop less conservative stability criteria for uncertain T-S fuzzy systems with time-varying delay on the basis of delay-partitioning approach and Finsler's lemma. Firstly, inspired by the work [11], an appropriate augmented LKF is established by partitioning time delay into all integral terms, and the $\tau(t)$-dependent and $[T_{ij}]_{m\times m}$-dependent sub-LKFs are included in the augmented LKF, which make the appropriate LKF include more useful augmented state information. Secondly, the independent upper bounds of the delay derivative in the various delay intervals have been taken into full account. Thirdly, some new results on tighter bounding inequalities, i.e., Peng-Park's integral inequality (reciprocally convex approach) and the Free-Matrix-based integral inequality (which includes the well-known Wirtinger-based inequality as its special case) have been employed to effectively reduce the enlargement in bounding the derivative of LKF. Finally, three numerical examples are provided to show the merits of the proposed results.

The rest of this paper is organized as follows. The main problem is formulated in Section 2 and the improved stability criteria for the uncertain T-S fuzzy systems with time-varying delay are derived in Section 3. In Section 4, three numerical examples are provided; and a concluding remark is given in Section 5.

Notations: Through this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n\times m$ real matrices; the notation $A>(\geq)B$ means that $A-B$ is positive (semi-positive) definite; I (0) is the identity (zero) matrix with appropriate dimension; A^{T} denotes the transpose; $\mathrm{He}\{A\}$ represents the sum of A and A^{T}; $\|\bullet\|$ denotes the Euclidean norm in $\mathbb{R}^n$; "$*$" denotes the elements below the main diagonal of a symmetric block matrix; $C([-\tau,0],\mathbb{R}^n)$ is the family of continuous functions ϕ from interval $[-\tau,0]$ to $\mathbb{R}^n$ with the norm $\|\phi\|_\tau=\sup_{-\tau\leq\theta\leq 0}\|\phi(\theta)\|$; let $x_t(\theta)=x(t+\theta)$, $\theta\in[-\tau,0]$.

2. PROBLEM FORMULATION

In this section, a class of uncertain T-S fuzzy systems with time-varying delay is concerned. For each $i=1,2,\cdots,r$ (r is the number of plant rules), the ith rule of this T-S fuzzy model is represented as follows:

Plant Rule i: IF $\theta_1(t)$ **is** M_{i1}, $\theta_2(t)$ **is** M_{i2}, $\cdots$, $\theta_p(t)$ **is** M_{ip}, **THEN**

$$\begin{cases}\dot{x}(t)=A_i(t)x(t)+A_{di}(t)x(t-\tau(t)),\ t\geq 0,\\ x(t)=\phi(t),\quad t\in[-\tau,0],\end{cases}\tag{1}$$

where $\theta_1(t)$, $\theta_2(t)$, $\cdots$, $\theta_p(t)$ are the premise variables, and each $M_{il}(i=1,2,\cdots,r;l=1,2,\cdots,p)$ is a fuzzy set; $x(t)\in\mathbb{R}^n$ is the state vector; $\phi(t)\in C([-\tau,0],\mathbb{R}^n)$ is the initial function; the delay $\tau(t)$ is a time-varying functional satisfying

$$0\leq\tau(t)\leq\tau,\tag{2}$$
$$\dot{\tau}(t)\leq\mu,\tag{3}$$

where τ and μ are constants; $A_i(t)=A_i+\Delta A_i(t)$, $A_{di}(t)=A_{di}+\Delta A_{di}(t)$, A_i and A_{di} are constant real matrices with appropriate dimensions. The matrices $\Delta A_i(t)$ and $\Delta A_{di}(t)$ denote the uncertainties in the system which are defined as

$$[\Delta A_i(t),\Delta A_{di}(t)]=HF(t)[E_i,E_{di}],\tag{4}$$

where H, E_i and E_{di} are known constant matrices and $F(t)$ is an unknown matrix function satisfying

$$F^{\mathrm{T}}(t)F(t)\leq I.\tag{5}$$

By a center-average defuzzier, product inference and singleton fuzzifier, the dynamic fuzzy model in (1) can be represented by

$$\begin{cases}\dot{x}(t)=\sum_{i=1}^{r}h_i(\theta(t))\{A_i(t)x(t)+A_{di}(t)x(t-\tau(t))\},\\ x(t)=\phi(t),\quad t\in[-\tau,0],\end{cases}\tag{6}$$

where

$$h_i(\theta(t))=\frac{\prod_{l=1}^{p}M_{il}(\theta_l(t))}{\sum_{i=1}^{r}\prod_{l=1}^{p}M_{il}(\theta_l(t))},\ \ i=1,\cdots,r,\tag{7}$$

in which $M_{il}(\theta_l(t))$ is the grade of membership of $\theta_l(t)$ in M_{il}, and $\theta(t)=(\theta_1(t),\cdots,\theta_r(t))$; By definition, the fuzzy weighting functions $h_i(\theta(t))$ satisfy $h_i(\theta(t))\geq 0$ and $\sum_{i=1}^{r}h_i(\theta(t))=1$. For notational simplicity, h_i is used to represent $h_i(\theta(t))$ in the following description.

Before proceeding, recall the following lemmas which will be used throughout the proofs.

Lemma 1 (Finsler's lemma [27])**:** Let $\zeta\in\mathbb{R}^n$, $\Phi=\Phi^{\mathrm{T}}\in\mathbb{R}^{n\times n}$, and $B\in\mathbb{R}^{m\times n}$ such that $rank(B)<n$. Then the following statements are equivalent:

(i) $\zeta^{\mathrm{T}}\Phi\zeta<0,\ \forall B\zeta=0,\ \zeta\neq 0$;
(ii) ${B^{\perp}}^{\mathrm{T}}\Phi B^{\perp}<0$;
(iii) $\exists L\in\mathbb{R}^{n\times m}:\ \Phi+\mathrm{He}(LB)<0$,

where $B^{\perp}\in\mathbb{R}^{n\times(n-rank(B))}$ is the right orthogonal complement of B.

Lemma 2 [28]**:** Let x be a differentiable function: $[\alpha,\beta]\to\mathbb{R}^n$. For symmetric matrices $Z\in\mathbb{R}^{n\times n}$, $X\in\mathbb{R}^{2n\times 2n}$ and $Y\in\mathbb{R}^{2n\times n}$ with $\begin{bmatrix}X & Y\\ * & Z\end{bmatrix}\geq 0$, if the integrals concerned are well defined, then the following inequality holds:

$$-\int_{\alpha}^{\beta}\dot{x}^{\mathrm{T}}(s)Z\dot{x}(s)ds \le \widehat{\varpi}^{\mathrm{T}}(t)[(\beta-\alpha)X+\mathrm{He}(Y\Pi)]\widehat{\varpi}(t), \tag{8}$$

where $\Pi=[I,\ -I]$ and $\widehat{\varpi}(t)=[x^{\mathrm{T}}(\beta),\ x^{\mathrm{T}}(\alpha)]^{\mathrm{T}}$.

Lemma 3 (Peng-Park's integral inequality [15, 18]): For any matrix $\begin{bmatrix} Z & S \\ * & Z\end{bmatrix}\ge 0$, positive scalars τ and $\tau(t)$ satisfying $0<\tau(t)<\tau$, vector function $\dot{x}:[-\tau,0]\to\mathbb{R}^n$ such that the concerned integrations are well defined, then

$$-\tau\int_{t-\tau}^{t}\dot{x}^{\mathrm{T}}(s)Z\dot{x}(s)ds \le \varpi^{\mathrm{T}}(t)\Omega\varpi(t),$$

where

$$\varpi(t)=[x^{\mathrm{T}}(t),x^{\mathrm{T}}(t-\tau(t)),x^{\mathrm{T}}(t-\tau)]^{\mathrm{T}},$$

$$\Omega=\begin{bmatrix} -Z & Z-S & S \\ * & -2Z+\mathrm{He}(S) & -S+Z \\ * & * & -Z\end{bmatrix}.$$

Lemma 4 (Free-Matrix-based integral inequality [29]): Let x be a differentiable function: $[\alpha,\beta]\to\mathbb{R}^n$. For symmetric matrices $Z\in\mathbb{R}^{n\times n}$ and $W_1,W_3\in\mathbb{R}^{3n\times 3n}$, and any matrices $W_2\in\mathbb{R}^{3n\times 3n}$ and $N_1,N_2\in\mathbb{R}^{3n\times n}$ satisfying

$$\begin{bmatrix} W_1 & W_2 & N_1 \\ * & W_3 & N_2 \\ * & * & Z\end{bmatrix}\ge 0,$$

then, the following inequality holds:

$$-\int_{\alpha}^{\beta}\dot{x}^{\mathrm{T}}(s)Z\dot{x}(s)ds \le \bar{\varpi}^{\mathrm{T}}(t)\bar{\Omega}\bar{\varpi}(t), \tag{9}$$

where $\bar{\varpi}(t)=[x^{\mathrm{T}}(\beta),\ x^{\mathrm{T}}(\alpha),\ \frac{1}{\beta-\alpha}\int_{\alpha}^{\beta}x^{\mathrm{T}}(s)ds]^{\mathrm{T}}$ and

$$\bar{\Omega}=(\beta-\alpha)(W_1+\tfrac{1}{3}W_3)+\mathrm{He}\{N_1\bar{\Pi}_1+N_2\bar{\Pi}_2\},$$
$$\bar{\Pi}_1=\bar{e}_1-\bar{e}_2,\bar{\Pi}_2=2\bar{e}_3-\bar{e}_1-\bar{e}_2,$$
$$\bar{e}_1=[I,\ 0,\ 0],\ \bar{e}_2=[0,\ I,\ 0],\ \bar{e}_3=[0,\ 0,\ I].$$

Remark 1: By setting $N_1=\frac{1}{\beta-\alpha}[-Z,\ Z,\ 0]^{\mathrm{T}}$, $N_2=\frac{3}{\beta-\alpha}[Z,\ Z,\ -2Z]^{\mathrm{T}}$, $W_1=N_1Z^{-1}N_1^{\mathrm{T}}$, $W_2=N_1Z^{-1}N_2^{\mathrm{T}}$ and $W_3=N_2Z^{-1}N_2^{\mathrm{T}}$, the Free-Matrix-based integral inequality (9) reduces to the well-known Wirtinger-based inequality [26]. Since the Free-Matrix-based integral inequality is composed of a set of adjustable slack variables, it can provide extra freedom in reducing the conservativeness of the inequality and yield less conservative conditions than the use of the Wirtinger-based inequality does [29].

Lemma 5 [30]: Let $Q=Q^{\mathrm{T}}$, H, E and $F(t)$ satisfying $F^{\mathrm{T}}(t)F(t)\le I$ are appropriately dimensional matrices, then the following inequality

$$Q+\mathrm{He}\{HF(t)E\}<0$$

is true, if and only if the following inequality holds for any $\varepsilon>0$,

$$Q+\varepsilon^{-1}HH^{\mathrm{T}}+\varepsilon E^{\mathrm{T}}E<0.$$

3. MAIN RESULTS

This section aims to develop further improved stability criteria for uncertain fuzzy system (6) with time-varying delay by delay-partitioning approach.

For any integer $m\ge 1$, define $\delta=\frac{\tau}{m}$, then $[0,\tau]$ can be divided into m segments, i.e.,

$$[0,\tau]=\bigcup_{j=1}^{m}[(j-1)\delta,j\delta]. \tag{10}$$

For any $t\ge 0$, there should exist an integer $k\in\{1,\cdots,m\}$, such that $\tau(t)\in[(k-1)\delta,\ k\delta]$.

Remark 2: In previous works such as [11, 13, 24, 31] and references therein, considerable attention has been paid to the case that the derivative of the time-varying delay $\dot{\tau}(t)$ satisfies (3). In fact, $\dot{\tau}(t)$ may have different upper bounds in various delay intervals, that is,

$$\dot{\tau}(t)\le\mu_k,\ \tau(t)\in[(k-1)\delta,\ k\delta],\ k=1,2,\cdots,m. \tag{11}$$

In this case, the treatment in [11,13,24,31] means that $\dot{\tau}(t)$ in (11) is enlarged to $\dot{\tau}(t)\le\mu=\max\{\mu_1,\mu_2,\cdots,\mu_m\}$, which may inevitably lead to conservativeness [16].

For notational simplification, motivated by [15], let

$$\begin{cases} e_s=\Big[\underbrace{0,\cdots,0}_{s-1},I,\underbrace{0,\cdots,0}_{m-s+4}\Big]^{\mathrm{T}},\ s=1,2,\cdots,m+4 \\ \zeta(t)=[x^{\mathrm{T}}(t-\tau(t)),\ \dot{x}^{\mathrm{T}}(t),\ \frac{1}{\delta}\int_{t-\delta}^{t}x^{\mathrm{T}}(s)ds,\ \zeta_1^{\mathrm{T}}(t), \\ \qquad x^{\mathrm{T}}(t-m\delta)]^{\mathrm{T}},\end{cases} \tag{12}$$

where

$$\zeta_1(t)=[x^{\mathrm{T}}(t),x^{\mathrm{T}}(t-\delta),\cdots,x^{\mathrm{T}}(t-(m-1)\delta)]^{\mathrm{T}}.$$

Based on Lyapunov-Krasovskii stability theorem [32], we firstly state the following stability criterion for the nominal system (6), i.e., the system (6) without parameter uncertainties.

Theorem 1: Given a positive integer m, scalars $\tau\ge 0$, $\delta=\frac{\tau}{m}$ and μ_k $(k=1,\cdots,m)$, then the nominal system (6) with a time-delay $\tau(t)$ satisfying (2) and (11) is asymptotically stable if there exist symmetric positive matrices Z_0, Z_j, $Q_j\in\mathbb{R}^{n\times n}(j=1,\ ...,\ m)$, $T=[T_{ij}]_{m\times m}\in\mathbb{R}^{mn\times mn}$, $P\in\mathbb{R}^{2n\times 2n}$, $R_l\in\mathbb{R}^{2n\times 2n}$ $(l=1,\ ...,\ m-1)$, symmetric matrices $X_{ij}\in\mathbb{R}^{2n\times 2n}$, $W_1,W_3\in\mathbb{R}^{3n\times 3n}$, and any matrices $Y_{ij}\in\mathbb{R}^{2n\times n}$, $W_2\in\mathbb{R}^{3n\times 3n}$, $N_1,N_2\in\mathbb{R}^{3n\times n}$, $L\in\mathbb{R}^{(m+4)n\times n}$

and $S_{ij} \in \mathbb{R}^{n\times n}$ $(i = 1,\cdots,r; j = 1,\cdots,m)$, such that the following LMIs hold for $i = 1,\cdots,r$ and $k = 1,\cdots,m$:

$$\Psi = \begin{bmatrix} W_1 & W_2 & N_1 \\ * & W_3 & N_2 \\ * & * & Z_0 \end{bmatrix} \geq 0, \tag{13}$$

$$\Lambda(i,k) = \begin{bmatrix} X_{ij} & Y_{ij} \\ * & Z_j \end{bmatrix} \geq 0, \ \ j = 1,\cdots,m, \tag{14}$$

$$\Upsilon(i,k) = \begin{bmatrix} Z_k & S_{ik} \\ * & Z_k \end{bmatrix} \geq 0, \tag{15}$$

$$\Xi(i,k) + \mathrm{He}\{L\Gamma_i\} < 0, \tag{16}$$

where

$$\Xi(i,k) = \sum_{j=0}^{3} \Xi_j + \Xi_4(k) + \Xi_5(i,k) + e_2\bar{Z}e_2^{\mathrm{T}},$$

$$\Gamma_i = A_i e_4^{\mathrm{T}} + A_{di}e_1^{\mathrm{T}} - e_2^{\mathrm{T}},$$

$$\Xi_0 = \begin{bmatrix} e_4^{\mathrm{T}} \\ e_5^{\mathrm{T}} \\ e_3^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \Omega_0 \begin{bmatrix} e_4^{\mathrm{T}} \\ e_5^{\mathrm{T}} \\ e_3^{\mathrm{T}} \end{bmatrix},$$

$$\Xi_1 = \mathrm{He}\left\{\begin{bmatrix} e_4^{\mathrm{T}} \\ \delta e_3^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} P \begin{bmatrix} e_2^{\mathrm{T}} \\ e_4^{\mathrm{T}} - e_5^{\mathrm{T}} \end{bmatrix}\right\},$$

$$\Xi_2 = \begin{bmatrix} e_4^{\mathrm{T}} \\ e_5^{\mathrm{T}} \\ \vdots \\ e_{m+3}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} T \begin{bmatrix} e_4^{\mathrm{T}} \\ e_5^{\mathrm{T}} \\ \vdots \\ e_{m+3}^{\mathrm{T}} \end{bmatrix} - \begin{bmatrix} e_5^{\mathrm{T}} \\ e_6^{\mathrm{T}} \\ \vdots \\ e_{m+4}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} T \begin{bmatrix} e_5^{\mathrm{T}} \\ e_6^{\mathrm{T}} \\ \vdots \\ e_{m+4}^{\mathrm{T}} \end{bmatrix},$$

$$\Xi_3 = \sum_{j=1}^{m-1}\left(\begin{bmatrix} e_{j+3}^{\mathrm{T}} \\ e_{j+4}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} R_j \begin{bmatrix} e_{j+3}^{\mathrm{T}} \\ e_{j+4}^{\mathrm{T}} \end{bmatrix} - \begin{bmatrix} e_{j+4}^{\mathrm{T}} \\ e_{j+5}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} R_j \begin{bmatrix} e_{j+4}^{\mathrm{T}} \\ e_{j+5}^{\mathrm{T}} \end{bmatrix}\right),$$

$$\Xi_4(k) = \sum_{j=1}^{k-1}\left[e_{j+3}Q_je_{j+3}^{\mathrm{T}} - e_{j+4}Q_je_{j+4}^{\mathrm{T}}\right] + e_{k+3}Q_ke_{k+3}^{\mathrm{T}} - (1-\mu_k)e_1Q_ke_1^{\mathrm{T}},$$

$$\Xi_5(i,k) = \sum_{j=1,j\neq k}^{m} \begin{bmatrix} e_{j+3}^{\mathrm{T}} \\ e_{j+4}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \left[\delta^2 X_{ij} + \mathrm{He}\{\delta Y_{ij}\Pi\}\right] \begin{bmatrix} e_{j+3}^{\mathrm{T}} \\ e_{j+4}^{\mathrm{T}} \end{bmatrix} + \begin{bmatrix} e_{k+3}^{\mathrm{T}} \\ e_1^{\mathrm{T}} \\ e_{k+4}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -Z_k & Z_k - S_{ik} & S_{ik} \\ * & -2Z_k + \mathrm{He}(S_{ik}) & Z_k - S_{ik} \\ * & * & -Z_k \end{bmatrix} \begin{bmatrix} e_{k+3}^{\mathrm{T}} \\ e_1^{\mathrm{T}} \\ e_{k+4}^{\mathrm{T}} \end{bmatrix},$$

with $\bar{Z} = \delta^2 \sum_{j=0}^{m} Z_j$, $\Omega_0 = \delta[\delta(W_1 + \frac{1}{3}W_3) + \mathrm{He}\{N_1\bar{\Pi}_1 + N_2\bar{\Pi}_2\}]$, and Π, $\bar{\Pi}_1$, $\bar{\Pi}_2$ are defined in Lemma 2 and Lemma 4.

Proof: For any $t \geq 0$, there should exist an integer $k \in \{1,\cdots,m\}$, such that $\tau(t) \in [(k-1)\delta,\ k\delta]$. Then, choose the following LKF candidate:

$$V(t,x_t)|_{\{\tau(t)\in[(k-1)\delta,\,k\delta]\}} = \sum_{i=1}^{5} V_i(x_t), \tag{17}$$

where

$$V_1(x_t) = \eta_0^{\mathrm{T}}(t)P\eta_0(t),$$

$$V_2(x_t) = \int_{t-\delta}^{t} \zeta_1^{\mathrm{T}}(s)T\zeta_1(s)ds,$$

$$V_3(x_t) = \sum_{j=1}^{m-1}\int_{t-\delta}^{t} \eta_j^{\mathrm{T}}(s)R_j\eta_j(s)ds,$$

$$V_4(x_t) = \sum_{j=1}^{k-1}\int_{t-j\delta}^{t-(j-1)\delta} x^{\mathrm{T}}(s)Q_jx(s)ds + \int_{t-\tau(t)}^{t-(k-1)\delta} x^{\mathrm{T}}(s)Q_kx(s)ds,$$

$$V_5(x_t) = \sum_{j=1}^{m}\delta\int_{-j\delta}^{-(j-1)\delta}\int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s)Z_j\dot{x}(s)dsd\theta + \delta\int_{-\delta}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)Z_0\dot{x}(s)dsd\theta,$$

with $\eta_0(t) = [x^{\mathrm{T}}(t), \int_{t-\delta}^{t} x^{\mathrm{T}}(s)ds]^{\mathrm{T}}$ and $\eta_j(s) = [x^{\mathrm{T}}(s-(j-1)\delta), x^{\mathrm{T}}(s-j\delta)]^{\mathrm{T}}$, $j = 1,\cdots,m-1$.

Taking derivative of $V(t,x_t)|_{\{\tau(t)\in[(k-1)\delta,\,k\delta]\}}$ along the trajectory of the nominal system (6) yields:

$$\dot{V}(t,x_t)|_{\{\tau(t)\in[(k-1)\delta,\,k\delta]\}} = \sum_{i=1}^{5}\dot{V}_i(x_t), \tag{18}$$

where

$$\dot{V}_1(x_t) = 2\eta_0^{\mathrm{T}}(t)P\dot{\eta}_0(t) = \zeta^{\mathrm{T}}(t)\Xi_1\zeta(t), \tag{19}$$

$$\dot{V}_2(x_t) = \zeta_1^{\mathrm{T}}(t)T\zeta_1(t) - \zeta_1^{\mathrm{T}}(t-\delta)T\zeta_1(t-\delta) = \zeta^{\mathrm{T}}(t)\Xi_2\zeta(t), \tag{20}$$

$$\dot{V}_3(x_t) = \sum_{j=1}^{m-1}[\eta_j^{\mathrm{T}}(t)R_j\eta_j(t) - \eta_j^{\mathrm{T}}(t-\delta)R_j\eta_j(t-\delta)] = \zeta^{\mathrm{T}}(t)\Xi_3\zeta(t), \tag{21}$$

$$\begin{aligned}\dot{V}_4(x_t) \leq& \sum_{j=1}^{k-1}[x^{\mathrm{T}}(t-(j-1)\delta)Q_jx(t-(j-1)\delta) \\ &- x^{\mathrm{T}}(t-j\delta)Q_jx(t-j\delta] \\ &+ x^{\mathrm{T}}(t-(k-1)\delta)Q_kx(t-(k-1)\delta) \\ &- (1-\mu_k)x^{\mathrm{T}}(t-\tau(t))Q_kx(t-\tau(t)) \\ =& \zeta^{\mathrm{T}}(t)\Xi_4(k)\zeta(t),\end{aligned} \tag{22}$$

$$\dot{V}_5(x_t) = \dot{x}^{\mathrm{T}}(t)\bar{Z}\dot{x}(t) - \delta\sum_{j=1}^{m}\int_{t-j\delta}^{t-(j-1)\delta}\dot{x}^{\mathrm{T}}(s)Z_j\dot{x}(s)ds - \delta\int_{t-\delta}^{t}\dot{x}^{\mathrm{T}}(s)Z_0\dot{x}(s)ds. \tag{23}$$

Applying Lemma 2 and Lemma 3 (Peng-Park's integral inequality) to bound the second item in (23), it can be deduced for $\begin{bmatrix} \widehat{X}_j & \widehat{Y}_j \\ * & Z_j \end{bmatrix} \geq 0$ $(j = 1,\cdots,m, j\neq k)$ and

$\begin{bmatrix} Z_k & \widehat{S}_k \\ * & Z_k \end{bmatrix} \geq 0$ (where $\widehat{X}_j = \sum_{i=1}^{r} h_i X_{ij}$, $\widehat{Y}_j = \sum_{i=1}^{r} h_i Y_{ij}$ and $\widehat{S}_k = \sum_{i=1}^{r} h_i S_{ik}$) that

$$\begin{aligned} &-\delta \sum_{j=1}^{m} \int_{t-j\delta}^{t-(j-1)\delta} \dot{x}^{\mathrm{T}}(s) Z_j \dot{x}(s) ds \\ &\leq \sum_{j=1, j\neq k}^{m} \varpi_1^{\mathrm{T}}(t) \left[\delta^2 \widehat{X}_j + \mathrm{He}\{\delta \widehat{Y}_j \Pi\} \right] \varpi_1(t) \\ &+ \varpi_2^{\mathrm{T}}(t) \begin{bmatrix} -Z_k & Z_k - \widehat{S}_k & \widehat{S}_k \\ * & -2Z_k + \mathrm{He}(\widehat{S}_k) & Z_k - \widehat{S}_k \\ * & * & -Z_k \end{bmatrix} \varpi_2(t) \\ &= \sum_{i=1}^{r} h_i \zeta^{\mathrm{T}}(t) \Xi_5(i,k) \zeta(t), \end{aligned} \tag{24}$$

where $\varpi_1(t) = \left[x^{\mathrm{T}}(t-(j-1)\delta), x^{\mathrm{T}}(t-j\delta) \right]^{\mathrm{T}}$ and $\varpi_2(t) = [x^{\mathrm{T}}(t-(k-1)\delta), x^{\mathrm{T}}(t-\tau(t)), x^{\mathrm{T}}(t-k\delta)]^{\mathrm{T}}$.

On the other hand, it follows from the Free-Matrix-based integral inequality (Lemma 4) that

$$-\delta \int_{t-\delta}^{t} \dot{x}^{\mathrm{T}}(s) Z_0 \dot{x}(s) ds \leq \varpi_3^{\mathrm{T}}(t) \Omega_0 \varpi_3(t) = \zeta^{\mathrm{T}}(t) \Xi_0 \zeta(t). \tag{25}$$

where $\varpi_3(t) = \left[x^{\mathrm{T}}(t), x^{\mathrm{T}}(t-\delta), \frac{1}{\delta} \int_{t-\delta}^{t} x^{\mathrm{T}}(s) ds \right]^{\mathrm{T}}$.

By (18)-(25), the following inequality holds

$$\dot{V}(t,x_t)|_{\{\tau(t)\in[(k-1)\delta,\, k\delta]\}} \leq \sum_{i=1}^{r} h_i \zeta^{\mathrm{T}}(t) \Xi(i,k) \zeta(t), \tag{26}$$

where $\Xi(i,k)$ is defined in Theorem 1.

Furthermore, the nominal system (6) with the augmented vector $\zeta(t)$ can be rewritten as:

$$0 = \sum_{i=1}^{r} h_i \Gamma_i \zeta(t),$$

where Γ_i is defined in Theorem 1.

Therefore, the asymptotic stability conditions for the nominal system (6) can be represented by

$$\begin{aligned} &\sum_{i=1}^{r} h_i \zeta^{\mathrm{T}}(t) \Xi(i,k) \zeta(t) < 0, \\ &subject\ to: \ \ 0 = \sum_{i=1}^{r} h_i \Gamma_i \zeta(t). \end{aligned} \tag{27}$$

By Finsler's lemma, for $L \in \mathbb{R}^{(m+4)n\times n}$, the conditions in (27) are equivalent to

$$\sum_{i=1}^{r} h_i \zeta^{\mathrm{T}}(t) [\Xi(i,k) + \mathrm{He}\{L\Gamma_i\}] \zeta(t) < 0. \tag{28}$$

Then, it follows from (26), (27), (28) and LMIs (16) that $\dot{V}(t,x_t)|_{\{\tau(t)\in[(k-1)\delta,\ k\delta]\}} < 0$. This means

$$\dot{V}(t,x_t)|_{\{\tau(t)\in[(k-1)\delta,\, k\delta]\}} < -\gamma \|x(t)\|^2$$

for a sufficiently small $\gamma > 0$. Therefore, by Lyapunov-Krasovskii stability theorem [32], the nominal system (6) with any delay $\tau(t)$ satisfying (2) and (11) is globally asymptotically stable. This completes the proof. □

For the uncertain T-S fuzzy system (6), replacing A_i and A_{di} with $A_i + HF(t)E_i$ and $A_{di} + HF(t)E_{di}$ in (16), then, the following result can be readily derived by applying Lemma 5 and Schur complement [33]. Thus, it is omitted here.

Theorem 2: Given a positive integer m, scalars $\tau \geq 0$, μ_k ($k = 1, \ldots, m$) and $\delta = \frac{\tau}{m}$, then the uncertain T-S system (6) with the time-delay $\tau(t)$ satisfying (2) and (11) is asymptotically stable if there exist scalars $\varepsilon_{ik} > 0$ ($i = 1, \ldots, r$; $k = 1, \ldots, m$), symmetric positive matrices $Z_0, Z_j, Q_j \in \mathbb{R}^{n\times n}$ ($j = 1, \ldots, m$), $T = [T_{ij}]_{m\times m} \in \mathbb{R}^{mn\times mn}$, $P \in \mathbb{R}^{2n\times 2n}$, $R_l \in \mathbb{R}^{2n\times 2n}$ ($l = 1, \ldots, m-1$), symmetric matrices $X_{ij} \in \mathbb{R}^{2n\times 2n}$, $W_1, W_3 \in \mathbb{R}^{3n\times 3n}$, and any matrices $Y_{ij} \in \mathbb{R}^{2n\times n}$, $W_2 \in \mathbb{R}^{3n\times 3n}$, $N_1, N_2 \in \mathbb{R}^{3n\times n}$, $L \in \mathbb{R}^{(m+4)n\times n}$ and $S_{ij} \in \mathbb{R}^{n\times n}$ ($i = 1, \cdots, r$; $j = 1, \cdots, m$), such that the following LMIs hold for $i = 1, \cdots, r$ and $k = 1, \cdots, m$:

$$\begin{aligned} &\Psi \geq 0,\ \Lambda(i,k) \geq 0,\ \Upsilon(i,k) \geq 0, \\ &\begin{bmatrix} \Xi(i,k) + \mathrm{He}\{L\Gamma_i\} & LH & \varepsilon_{ik}(e_4 E_i^{\mathrm{T}} + e_1 E_{di}^{\mathrm{T}}) \\ * & -\varepsilon_{ik} I & 0 \\ * & * & -\varepsilon_{ik} I \end{bmatrix} < 0, \end{aligned} \tag{29}$$

where Ψ, $\Lambda(i,k)$, $\Upsilon(i,k)$, $\Xi(i,k)$ and Γ_i are defined in Theorem 1.

Remark 3: Based on delay-partitioning approach, the augmented LKF (17) is much different from those in [6, 15, 20, 22, 23, 25, 29] from the following respects: i) The modified augmented LKF (17) is established by partitioning time delay in all integral terms; ii) the $[T_{ij}]_{m\times m}$-dependent sub-LKF is introduced, so the relationships between the augmented state vectors $[x^{\mathrm{T}}(t), x^{\mathrm{T}}(t-\delta), \cdots, x^{\mathrm{T}}(t-m\delta)]^{\mathrm{T}}$ have been taken a full consideration; iii) by employing the $\tau(t)$-dependent sub-LKF, the independent upper bounds of the delay derivative in various delay segments have been taken a full account. With these differences and advantages above-mentioned, less conservative results than those in [6, 15, 20, 22, 23, 25, 29] can be expected, which will be demonstrated later by numerical examples.

Remark 4: The Free-Matrix-based integral inequality (Lemma 4) [29] is employed to effectively bound the integral term $-\delta \int_{t-\delta}^{t} \dot{x}^{\mathrm{T}}(s) Z_0 \dot{x}(s) ds$. Since this integral inequality is composed of a set of adjustable slack variables, it can provide extra freedom in reducing the conservativeness of the inequality and further lead to less conservative conditions than the use of the Wirtinger-based inequality does in [11, 24]. This will be verified by numerical simulations in the next section.

Finally, in the case of the time-varying delay $\tau(t)$ being non-differentiable or unknown $\dot{\tau}(t)$, setting $Q_k = 0 (Q_j \neq 0, j = 1, \cdots, k-1)$ in Theorem 2, one has the following corollary.

Corollary 1: Given a positive integer m, scalars $\tau \geq 0$ and $\delta = \frac{\tau}{m}$, then the uncertain T-S system (6) with a time-delay $\tau(t)$ satisfying (2) is asymptotically stable if there exist scalars $\varepsilon_{ik} > 0$ ($i = 1, ..., r$; $k = 1, ..., m$), symmetric positive matrices Z_0, Z_j, $Q_j \in \mathbb{R}^{n\times n}$ ($j = 1, ..., m$), $T = [T_{ij}]_{m\times m} \in \mathbb{R}^{mn\times mn}$, $P \in \mathbb{R}^{2n\times 2n}$, $R_l \in \mathbb{R}^{2n\times 2n}$ ($l = 1, ..., m-1$), symmetric matrices $X_{ij} \in \mathbb{R}^{2n\times 2n}$, $W_1, W_3 \in \mathbb{R}^{3n\times 3n}$, and any matrices $Y_{ij} \in \mathbb{R}^{2n\times n}$, $W_2 \in \mathbb{R}^{3n\times 3n}$, N_1, $N_2 \in \mathbb{R}^{3n\times n}$, $L \in \mathbb{R}^{(m+4)n\times n}$ and $S_{ij} \in \mathbb{R}^{n\times n}$ ($i = 1, ..., r$; $j = 1, ..., m$), such that the following LMIs hold for $i = 1, 2, \cdots, r$ and $k = 1, 2, \cdots, m$:

$$\Psi \geq 0,\ \Lambda(i,k) \geq 0,\ \Upsilon(i,k) \geq 0,$$
$$\begin{bmatrix} \tilde{\tilde{\Xi}}(i,k) + \mathrm{He}\{L\Gamma_i\} & LH & \varepsilon_{ik}(e_4 E_i^{\mathrm{T}} + e_1 E_{di}^{\mathrm{T}}) \\ * & -\varepsilon_{ik} I & 0 \\ * & * & -\varepsilon_{ik} I \end{bmatrix} < 0, \tag{30}$$

where $\tilde{\tilde{\Xi}}(i,k) = \sum_{j=0}^{3} \Xi_j + \tilde{\tilde{\Xi}}_4(k) + \Xi_5(i,k) + e_2 \bar{Z} e_2^{\mathrm{T}}$ with $\tilde{\tilde{\Xi}}_4(k) = \sum_{j=1}^{k-1} \left[e_{j+3} Q_j e_{j+3}^{\mathrm{T}} - e_{j+4} Q_j e_{j+4}^{\mathrm{T}}\right]$, and Ψ, $\Lambda(i,k)$, $\Upsilon(i,k)$, Γ_i, Ξ_j, $\Xi_5(i,k)$ and $\bar{Z}$ are defined in Theorem 1.

4. NUMERICAL EXAMPLE

This section gives three examples to demonstrate the effectiveness of the proposed approach.

Example 1: Consider the system (6) with time-varying delay and the following parameters [19, 29, 34]:

$$A_1(t) = A_2(t) = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix},$$
$$A_{d1}(t) = A_{d2}(t) = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}.$$

For comparison, we assume that $\mu_1 = \cdots = \mu_m = \mu$ (similarly hereinafter) in Theorem 1. For various values of μ, the maximum admissible upper bounds (**MAUB**s) of time-varying delay $\tau(t)$ derived from [19, 29, 34] and Theorem 1 proposed in this paper are tabulated in Table 1, where "–" denotes that the results are not provided in these papers, similarly hereinafter. As shown in the table, the criteria derived in this paper are less conservative than those in [19, 29, 34].

Table 1. The achieved MAUBs of $\tau(t)$ for various values of μ– Example 1.

Methods \ μ	0	0.05	0.10	0.50	3.00
[19]	1.99	1.81	1.75	1.61	1.60
[34]	2.52	2.17	2.02	1.62	1.60
[29] Cor. 1	3.03	2.55	2.37	1.71	1.66
[29] Th. 1	3.03	2.57	2.41	1.93	–
Th. 1 ($m = 2$)	5.95	5.22	4.65	2.43	2.11
Th. 1 ($m = 3$)	6.09	5.32	4.72	2.46	2.20

Table 2. The achieved MAUBs of $\tau(t)$ for various values of μ– Example 2.

Methods \ μ	0	0.1	≥ 1
[35]	1.60	–	0.72
[6]	1.60	1.48	0.83
[22]	1.60	1.48	0.98
[23]	1.60	1.49	1.26
[25]	1.80	–	0.99
[20]	1.66	1.53	1.27
[11] ($m = 3$)	2.00	1.81	1.36
[24] ($m = 3$)	2.33	2.17	1.64
Th. 1 ($m = 3$)	2.45	2.21	1.67
Th. 1 ($m = 4$)	2.53	2.27	1.72

Example 2: Consider the T-S fuzzy system (6) with time-varying delay and the following rules [15, 25, 35]:

$$R^1: \text{ If } \theta(t) \text{ is } \pm\pi/2, \text{ then } x(t) = A_1 x(t) + A_{d1} x(t-\tau(t));$$
$$R^2: \text{ If } \theta(t) \text{ is } 0, \text{ then } x(t) = A_2 x(t) + A_{d2} x(t-\tau(t)). \tag{31}$$

where

$$A_1 = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, A_{d1} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -1 & 0.5 \\ 0 & -1 \end{bmatrix}, A_{d2} = \begin{bmatrix} -1 & 0 \\ 0.1 & -1 \end{bmatrix}.$$

The membership functions for above rules 1 and 2 are

$$h_1(\theta(t)) = \sin^2(\theta(t)),\ h_2(\theta(t)) = \cos^2(\theta(t)), \tag{32}$$

where $\theta(t) = x_1(t)$.

For various values of μ, the MAUBs of $\tau(t)$ derived from [6, 11, 20, 22–25, 35] and Theorem 1 proposed in this paper are tabulated in Table 2. As shown in Table 2, the criteria derived in this paper improve some previous ones [6, 11, 20, 22–25, 35]. With initial state conditions $[1, -1]^{\mathrm{T}}$, Fig. 1 shows the simulation results of the state responses of the T-S fuzzy system (31) with $\mu = 0.1$, $0 \leq \tau(t) \leq 1.53$, 1.81, 2.17, 2.27 listed respectively in Table 2; and the phase portraits of system (31) is given in

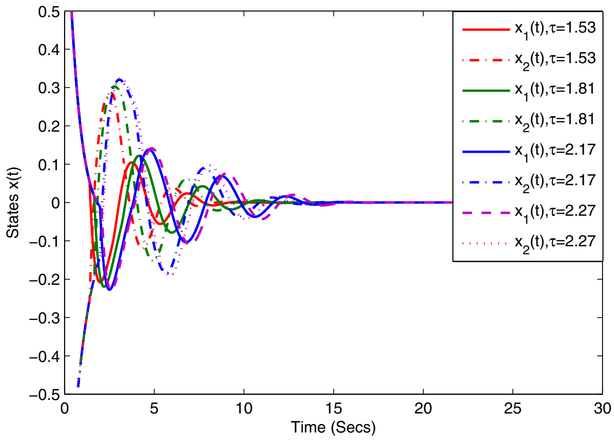


Fig. 1. The state responses of system (31).

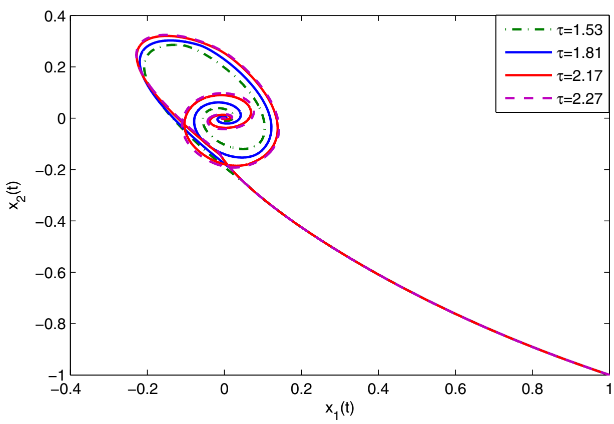


Fig. 2. The phase portraits of system (31).

Fig. 2. It shows from the simulation results (Figs. 1 and 2) that the MAUBs of $\tau(t)$ listed in Table 2 are capable of guaranteeing asymptotical stability of (31).

Example 3: Consider the following uncertain T-S fuzzy system [6, 11, 21, 23, 24]:

$$\dot{x}(t) = \sum_{i=1}^{2} h_i\{(\theta(t))[A_i(t)x(t) + A_{di}(t)x(t-\tau(t))\}, \quad (33)$$

where

$$A_1 = \begin{bmatrix} -2 & 1 \\ 0.5 & -1 \end{bmatrix}, A_{d1} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}, A_{d2} = \begin{bmatrix} -1.6 & 0 \\ 0 & -1 \end{bmatrix},$$
$$E_1 = \begin{bmatrix} 1.6 & 0 \\ 0 & 0.05 \end{bmatrix}, E_{d1} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix},$$
$$E_2 = \begin{bmatrix} 1.6 & 0 \\ 0 & -0.05 \end{bmatrix}, E_{d2} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix},$$
$$H = \begin{bmatrix} 0.03 & 0 \\ 0 & -0.03 \end{bmatrix},$$

Table 3. The achieved MAUBs of $\tau(t)$ for various values of μ– Example 3.

Methods \ μ	0	0.1	0.5	Unknown
[21]	0.95	0.89	0.63	–
[6]	1.17	1.12	0.93	0.50
[23]	1.19	1.15	1.10	1.05
[11] ($m=2$)	1.40	1.32	1.13	1.12
[24] ($m=2$)	1.47	1.42	1.29	1.23
[24] ($m=3$)	1.64	1.60	1.49	1.42
Th. 2 / Cor. 1 ($m=3$)	1.71	1.65	1.52	1.44

Table 4. The achieved MAUBs of τ for various values of μ_1, μ_2– Example 3.

$\mu_1 \setminus \mu_2$	0	0.1	0.5	0.9
0	1.490	1.419	1.243	1.243
0.1	1.479	1.418	1.241	1.230
0.5	1.471	1.405	1.232	1.228
0.9	1.237	1.231	1.230	1.227

and the membership functions are defined in (32).

For various values of μ, the MAUBs of $\tau(t)$ derived from [6, 11, 21, 23, 24] and Theorem 2 and Corollary 1 proposed in this paper are are summarized in Table 3. It can be concluded that the result proposed in this paper is less conservative than those in [6, 11, 21, 23, 24]. With initial state conditions $[1, -1]^{\mathrm{T}}$ and the unknown matrix function $F(t) = \mathrm{diag}\{\sin t, \cos t\}$, Fig. 3 shows the simulation results of the state responses of the system (33) with $\mu = 0.5$, $0 \le \tau(t) \le 1.10$, 1.13, 1.49, 1.52 listed respectively in Table 3; and the phase portraits of (33) is given in Fig.4. It also shows from the simulation results (Figs.3 and 4) that the MAUBs of $\tau(t)$ listed in Table 3 are capable of guaranteeing asymptotically robust stability of the uncertain system (33).

Finaly, in order to further verify Remark 2, by Theorem 2, the MAUBs of $\tau(t)$ for $m=2$ and various values of μ_1 and μ_2 are calculated in Table 4. From this table, it can be concluded that the achieved MAUB is 1.405 when $\mu_1 = 0.5$ and $\mu_2 = 0.1$. However, if $\dot{\tau}(t)$ is bounded by (3) with $\dot{\tau}(t) \le \mu = \max\{\mu_1, \mu_2\}$=0.5, the achieved MAUB is only 1.232. Then, the effectiveness of the proposed method is further illustrated.

5. CONCLUSION

Taking advantage of delay-partitioning approach, LMI approach and Finsler's lemma, this paper is mainly concerned with the further improved stability criteria for uncertain T-S fuzzy systems with time-varying delay. A modified augmented LKF is established by partitioning

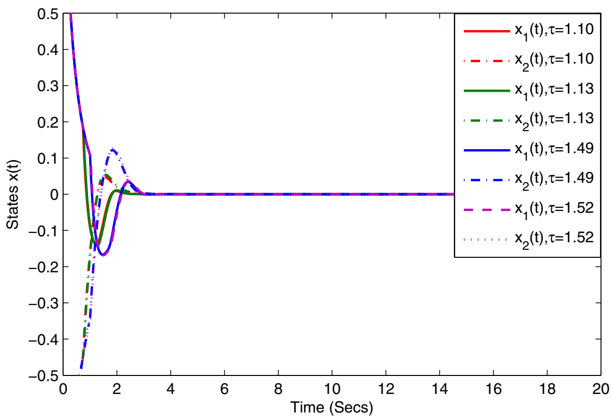


Fig. 3. The state responses of system (33).

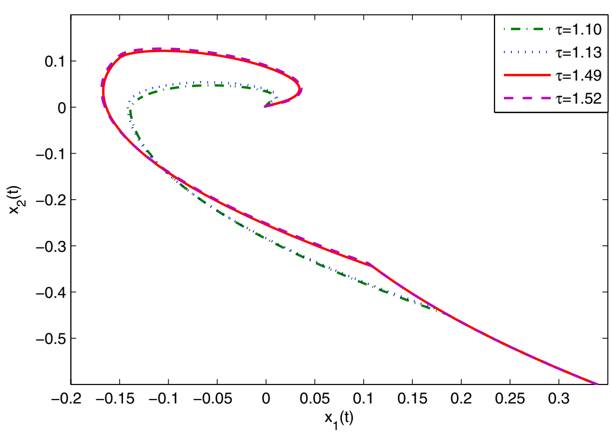


Fig. 4. The phase portraits of system (33).

the delay in all integral terms, and the $\tau(t)$-dependent and $[T_{ij}]_{m\times m}$-dependent sub-LKFs are introduced to the augmented LKF, which make the LKF include more useful augmented state information. Meanwhile, the independent upper bounds of the delay derivative in various delay intervals have been taken a full consideration. Then, Peng-Park's integral inequality and the Free-Matrix-based integral inequality (which yields less conservative stability criteria than the use of Wirtinger-based inequality does) are utilized to effectively bound the derivative of LKF. Therefore, less conservative stability criteria in terms of e_s and LMIs can be achieved. Finally, three numerical examples are included to show the merits of the proposed results.

REFERENCES

[1] T. Takagi and M. Sugeno,"Fuzzy identification of systems and its application to modeling and control," *IEEE Transactions On Systems, Man, And Cybernetics*, vol. 15, no. 1, pp. 116-132, Feb. 1985. [click]

[2] K. Tanaka and M. Sano, "A robust stabilization problem of fuzzy control systems and its application to backing up control of a truck-trailer," *IEEE Trans. Fuzzy Syst.*, vol. 2, no. 2, pp. 119-134, May 1994. [click]

[3] M.C. Teixeira and S.H. Zak, "Stabilizing controller design for uncertain nonlinear systems using fuzzy models," *IEEE Trans. Fuzzy Syst.*, vol. 7, no.2, pp. 133-142, Apr. 1999. [click]

[4] K. Tanaka and H. O. Wang, "Fuzzy control systems design and analysis fuzzy control systems design and analysis: A linear matrix inequality approach," New York: John Wiley & Sons, 2001.

[5] H. K. Lam and F. H. F. Leung, "Sampled-data fuzzy controller for time-delay nonlinear systems: fuzzy-based LMI approach," *IEEE Trans Syst Man Cybern B Cybern*, vol. 37, no. 3, pp. 617-629, Jun. 2007. [click]

[6] C. H. Lien, K. W. Yu, W. D. Chen, Z. L. Wan, and Y .J. Chung, "Stability criteria for uncertain Takagi-Sugeno fuzzy systems with interval time-varying delay," *IET Control Theory Appl.*, vol. 1, no. 3, pp. 746-769, May 2007. [click]

[7] W. P. Luo, J. Yang, L. Xiang, and S. M. Zhong. "Robust H_∞ DOF control for uncertain T-S fuzzy neutral systems," *Int. J. of Control, Automation, and Systems*, vol. 9, no. 3, pp. 525-533, Jun 2011. [click]

[8] C. Peng and T. C. Yang, "Communication delay distribution dependent networked control for a class of T-S fuzzy system," *IEEE Trans. Fuzzy Syst.*, vol. 18, no. 2, pp. 326-335, Apr. 2010. [click]

[9] J. Yang, S. M. Zhong, and L. L. Xiong, "A descriptor system approach to non-fragile H_∞ control for uncertain neutral fuzzy systems," *Fuzzy Sets and Systems*, vol. 160, no. 4, pp. 423-438, Feb. 2009. [click]

[10] J. Yang, S. M. Zhong, W. P. Luo, and G. H. Li, "Delay-dependent stabilization for stochastic delayed fuzzy systems with impulsive effects," *Int. J. of Control, Automation, and Systems*, vol. 8, no. 1, pp. 127-134, 2010. [click]

[11] H. B. Zeng, J. H. Park, J. W. Xi, and S. P. Xiao, "Improved delay-dependent stability criteria for T-S fuzzy systems with time-varying delay," *Appl. Math. Comput.*, vol. 235, pp. 492-501, May 2014. [click]

[12] F. Gouaisbaut and D. Peaucelle, "Delay-dependent stability analysis of linear time delay systems," *IFAC Workshop on Time Delay Systems*, 2006.

[13] X. M. Zhang and Q. L. Han, "A delay decomposition approach to delay-dependent stability for linear systems with time-varying delays," *Int. J. of Robust & Nonlinear Control*, vol 19, no. 17, pp. 1922-1930, Mar. 2009. [click]

[14] S. J. S. Theesar and P. Balasubramaniam, "Robust stability analysis of uncertain T-S fuzzy systems with time- varying delay," *IEEE Int. Conf. Commu. Control and Comput. Tech. (ICCCCT)*, pp. 707-712, 2010. [click]

[15] C. Peng and M. R. Fei, "An improved result on the stability of uncertain T-S fuzzy systems with interval time-varying delay," *Fuzzy Sets and Systems*, vol. 212, pp. 97-109, Feb. 2013. [click]

[16] H. B. Zeng, Y. He, M. Wu, and S.-P. Xiao, "Less conservative results on stability for linear systems with a time-varying delay," *Optim. Control Appl. Meth.*, vol. 34, pp. 670-679, Jul. 2013. [click]

[17] E. G. Tian, D. Yue, and Z. Gu, "Robust H_∞ control for nonlinear system over network: a piecewise analysis method," *Fuzzy Sets Syst.*, vol. 161, no. 21, pp. 2731-2745, Nov. 2010. [click]

[18] P. G. Park, J. W. Ko, and C. K. Jeong, "Reciprocally convex approach to stability of systems with time-varying delays," *Automatica*, vol. 47, no. 1, pp. 235-238, Jan. 2011. [click]

[19] P. Park and J. W. Ko, "Stability and robust stability for systems with a time-varying delay," *Automatica*, vol. 43, no. 10, pp. 1855-1858, Oct. 2007. [click]

[20] O. M. Kwon, M. J. Park, S. M. Lee, and J. H. Park, "Augmented Lyapunov-Krasovskii functional approaches to robust stability criteria for uncertain Takagi-Sugeno fuzzy systems with time-varying delays," *Fuzzy Sets Syst.*, vol. 201, pp. 1-19, Aug. 2012.

[21] C. G. Li, H. J. Wang, and X. F. Liao, "Delay-dependent robust stability of uncertain fuzzy systems with time-varying delays," *IEE Proc. Control Theory Appl.*, vol. 151, no. 4, pp. 417-421, Jul. 2004. [click]

[22] L. Li, X. Liu, and T. Chai, "New approaches on H_∞ control of T-S fuzzy systems with interval time-varying delay," *Fuzzy Sets Syst.*, vol. 160, no. 12, pp. 1669-1688, Jun. 2009. [click]

[23] F. Liu, M. Wu, Y. He, and R. Yokoyama, "New delay-dependent stability criteria for T-S fuzzy systems with time-varying delay," *Fuzzy Sets Syst.*, vol. 161, no. 15, pp. 2033-2042, Aug. 2010. [click]

[24] J. Yang, W. P. Luo, Y. H. Wang, and C. S. Duan, "Improved stability criteria for T-S fuzzy systems with time-varying delay by delay-partitioning approach," *Int. J. of Control, Automation, and Systems*,vol. 13, no. 6, pp. 1521-1529, Sept. 2015.

[25] C. Peng and L. Y. Wen, "On delay-dependent robust stability criteria for uncertain T-S fuzzy systems with interval time-varying delay," *Int. J. Fuzzy Syst.*, vol. 13, no. 1, pp. 35-44, 2011.

[26] A. Seuret and F. Gouaisbaut,"Wirtinger-based integral inequality: application to time-delay systems," *Automatica*, vol. 49, no. 9, pp. 2860-2866, Sep. 2013. [click]

[27] M. C. de Oliveira and R. E. Skelton, *Stability Tests for Constrained Linear Systems*, Springer-Verlag, Berlin, 2001.

[28] X. M. Zhang, M. Wu, J. H. She, and Y. He, "Delay-dependent stabilization of linear systems with time-varying state and input delays," *Automatica*, vol. 41, no. 8, pp. 1405-1412, Aug. 2005. [click]

[29] H. B. Zeng, Y. He, M. Wu, and J. H. She, "Free-matrix-based integral inequality for stability analysis of systems with time-varying delay," *IEEE Tran. on Automatic Control*, vol. 60, no.10, pp. 2768-2772, Oct. 2015.

[30] I. R. Petersen and C. V. Hollot, "A Riccati equation approach to the stabilization of uncertain linear systems," *Automatica*, vol. 22, no.4, pp. 397-411, Jul. 1986. [click]

[31] X. L. Zhu and G. H. Yang, "New results of stability analysis for systems with time-varying delay," *Int. J. of Robust and Nonlinear Control*, vol. 20, no. 5, pp. 596-606, May 2010. [click]

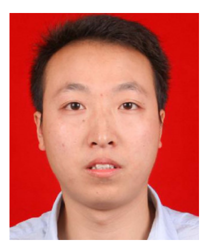
[32] J. Hale, *Theory of Functional Differential Equation*, Springer, New York, 1977.

[33] S. Boyd, L. E. Ghaoui, and E. Feron, *Linear Matrix Inequality in System and Control Theory, SIAM Studies in Applied Mathematics*, SIAM, Philadelphia, 1994.

[34] J .H. Kim, "Note on stability of linear systems with time-varying delay," *Automatica*, vol. 47, no. 9, pp. 2118-2121, Sep. 2011. [click]

[35] E. G. Tian and C. Peng, "Delay-dependent stability analysis and synthesis of uncertain T-S fuzzy systems with time-varying delay," *Fuzzy Sets Syst.*, vol. 157, no.4, pp. 544-559, Feb. 2006.

Wen-Pin Luo received the B.S. degree from Sichuan Normal University, Chengdu, China, in 2004 and the M.S. degree from University of Electronic Science and Technology of China, Chengdu, China, in 2007, all in Applied Mathematics. She is currently a Lecturer with Sichuan University of Science and Engineering, Zigong, China. Her current research interests include fuzzy control systems, impulsive systems and neural networks.



Jun Yang received the B.S. degree from Leshan Normal University, Leshan, China, in 2004 and the Ph.D. degree from University of Electronic Science and Technology of China, Chengdu, China, in 2009, all in Applied Mathematics. He is currently an Associate Professor with Civil Aviation Flight University of China, Guanghan, China. His current research interests include system and control theory, fuzzy control systems and functional differential equations. He is an active reviewer for many international journals.

Xin Zhao received the Ph.D. degree from Sichuan University, Chengdu, China, in 2009, majored in Material Physics and Chemistry. He is currently an Associate Professor with Civil Aviation Flight University of China (CAFUC). His current research interests include aeronautic structural materials and optoelectronic information materials, etc.