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Robust Adaptive Backstepping Control for an Uncertain Nonlinear System with Input Constraint based on Lyapunov Redesign

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Abstract: This paper presents the control problem for a class of n-order semi-strict nonlinear system subjects to unknown parameters, uncertainty, and input constraint. The controller is designed via combining backstepping control and Lyapunov redesign. Firstly, based on the Lyapunov function, a composite adaptive law is constructed to estimate the unknown parameters. To sequel, a robust term is designed to handle the matched and unmatched uncertainties on the basis of Lyapunov redesign technique. The "explosion of terms" problem that inherent in backstepping control is avoided by the robust second-order filters. Thirdly, an auxiliary signal provided by the auxiliary system is employed to handle the influence of the input constraint. It is proved that the closed-loop system is stable in a Lyapunov framework theory and the semi-global uniformly ultimate boundedness of all signals is achieved. Finally, numerical simulations are carried out to evaluate the performance of the proposed control strategy. Numerical example and the application of the hypersonic vehicle (HSV) tracking control are simulated to demonstrate the effectiveness of the proposed control scheme.

Keywords: Backstepping control, input constraint, Lyapunov redesign technique, nonlinear system, matched and unmatched uncertainty, unknown Parameter.

1. INTRODUCTION

The control problem of uncertain nonlinear system has received a lot of attentions in recent years. There are mainly two approaches to solve the uncertainties, one is the robust control and the other is adaptive control. Lyapunov redesign control is one of the robust control methods. It supposes that an uncertainty satisfies the matching condition and special inequality and a feedback control law is designed firstly such that the origin of the nominal closed-loop system is asymptotically stable. Then, the controller is redesigned by adding a feedback controller in a way that the overall controller stabilizes the actual system in the presence of uncertainty [1, 2]. It has been applied in some engineering application. As a typical nonlinear control approach, backstepping methodology has been received more and more attention and widely applied to engineering application [3–9].

Through comparing backsteping control with H-infinity control, passive control (PC) and sliding mode control (SMC), the advantages of backstepping control and the reason for choosing this control method are given as follows.

As we all know, H-infinity control is a robust control method, it is feasible to cope with uncertainty. The main idea of it is to design a controller which makes the internal stabilization of system to minimize the infinity norm of the closed transfer function [10]. Although it can effectively handle the uncertainty, the design procedure is too complex, and the order of the controller is high. So it may induce that it is difficult to be applied in engineering area. The principle idea of backstepping methodology is that the complex system is divided into the lower dimension subsystem at first, then the Lyapunov function and intermediate virtual control inputs are recursively designed for every subsystem. The recursive design proceeds until "backstepping" has been achieved by the whole system, then the actual control input is obtained to achieve satisfactory control performance of the system.

Passive control (PC) is the 'energy shaping' approach, and it considers the energy of the system and gives a clear physical meaning, such that the changes of the overall

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storage function can take a desired form [11, 12]. In fact, passivity provides a competitive tool for studying stability of uncertain or nonlinear systems [13]. The main idea of passivity theory is that the passive properties of systems can keep the systems internal stable [14, 15]. Much research focuses on the smooth state feedback passive problem of the affine nonlinear system. However, feedback can't change the relative degree and zero dynamic, and the system must satisfy the conditions that its relative degree is one and it is a minimum phase system. These conditions are very harsh, it constrains the application of the feedback passive control method. In fact, a passive system may have no relative degree, but most of the passivity control is based on the assumption that the relative degree exists. Backstepping control can eliminate the constraint of relative degree.

SMC is one of the most efficient techniques to cope with external disturbance and parametric uncertainty [16]. However, its application requires that the system has to satisfy "matched condition". What is more, the bounds of uncertainties which may not be easily obtained in practical implementation is an important clue to SMC[R8], since the stability analysis is based on the known upper bounds of the uncertainties. Besides, sign function induces chattering problem caused by that the switching gain of SMC must be chosen larger than the bound of uncertainty [17, 18]. As a feasible and typical nonlinear control approach, backstepping methodology proposed by Kokotovic is a powerful tool to deal with the unmatched uncertainty [19]. It overcomes the limitation on FBL and is a powerful means to handle unmatched uncertainty [20]. And it has been received more and more attention.

Moreover, compared with H-infinity control, passive control and SMC, the backstepping control mainly has the following advantages. (a) It is a powerful and systematic technique that recursively interlaces the choice of a Lyapunov function with the feedback control design. (b) It is the systematic construction of a Lyapunov function for the nonlinear systems, and the control goal can be achieved with reduced control effort. It is worthy to point out that it overcomes the limitation on FBL and is a powerful means to handle unmatched uncertainty [21].

It is noted that the "explosion of terms" problem induced by repeated analytic time derivative of the virtual control input is the limitation of the standard backstepping control. It inevitably leads to a complicated algorithm with heavy computation burden, and the complexity of the controller design increases with the increase of the system order. A first-order filter [22], command filters [23] and robust second order filters [24] are employed to eliminate the complexity computation of time derivative of virtual control inputs to overcome the "explosion of terms" problem. From both theory and engineering aspects, input constraint is great of importance to be taken into consideration. In theory, if input constraint is not considered, the closed-loop system may suffer from important performance limitations or even lose stability [25]. For many practical dynamic systems, physical input saturation on hardware dictates that the magnitude of the control signal is always constrained. Since input constraint affect not only the performance of the system, but also the stability of the system, this problem has received much attention [26–32].

Therefore, in this paper, a class of semi-strict nonlinear system with matched and unmatched uncertainty, input constraint is addressed based on backstepping control and Lyapunov redesign technique. The motivation is shown as follows.

Firstly, it is noted that the nonlinear system in semi-strict feedback form with matched, unmatched uncertainty and input constraint has been received more and more attention, since many practice systems such as robot, Duffing function system and other engineering system can be transformed into this form. To the best of our knowledge, no other works have solved the control design problem of semi-strict feedback systems with parameter uncertainties and input constraint.

Secondly, in the works that investigate the control design problem based on backstepping control or Lyapunov redesign technique only consider parameter uncertainty or only consider matched uncertainty or unmatched uncertainty. And input constraint is not considered. In this paper, the semi-strict system under parameter uncertainty, matched uncertainty or unmatched uncertainty and input constraint is considered, and by effectively combining backstepping control and Lyapunov redesign technique, the advantages of these two methods are played adequate role in control strategy development and achieve the stable tracking control of output. Moreover, the implementation feasibility of the proposed controller is verified by applying in tracking control problem of hypersonic vehicle.

Thirdly, during the controller design procedure, new robust second-order filters are established to avoid the "explosion of terms" problem. An auxiliary signal provided by the auxiliary system is used to handle the influence of input constraint, it is used at the level of controller design and stability analysis. The unknown parameters are estimated by the designed adaptive law. Furthermore, matched and unmatched uncertainty is coped with by the Lyapunov redesign technique. Then the stability of the closed-loop system is analyzed in the framework of Lyapunov theory, and the semi-globally ultimate of all signals are assured.

The other parts of this paper are organized as follows. The problem formulation is stated in section 2. Then, the robust adaptive controller on the basis of the combination of backstepping control and Lyapunov redesign technique is designed and the closed-loop system stability is analyzed in section 3. The effectiveness of the proposed control strategy is testified through simulations in section 4, and this paper ends with the conclusions.

2. PROBLEM FORMULATION

The control problem presented in this paper is for a class of semi-strict nonlinear system with unknown parameters, uncertainty and input constraint, the nonlinear system is generally described as follows:

$$\begin{aligned}
\dot{x}_1 &= f_1(\bar{x}_1) + \varphi_1^T(\bar{x}_1)\theta_1 + g_1(\bar{x}_1)(x_2 + d_1(\bar{x}_1,t)),\\
\dot{x}_2 &= f_2(\bar{x}_2) + \varphi_2^T(\bar{x}_2)\theta_2 + g_2(\bar{x}_2)(x_3 + d_2(\bar{x}_2,t)),\\
&\vdots\\
\dot{x}_i &= f_i(\bar{x}_i) + \varphi_i^T(\bar{x}_i)\theta_i + g_i(\bar{x}_i)(x_{i+1} + d_i(\bar{x}_i,t)),\\
&\vdots\\
\dot{x}_n &= f_n(\bar{x}_n) + \varphi_n^T(\bar{x}_n)\theta_n + g_n(\bar{x}_n)(u + d_n(\bar{x}_n,t)),\\
y &= x_1,
\end{aligned} \tag{1}$$

where $\bar{x}_i = (x_1, \cdots x_i)^T$, $i = 1, ..., n$, $y \in R$ and $u \in R$ are the state vector, system output and input, respectively. $\theta_i \in R^{p_i}$ represent the vectors of unknown parameters, $f_i(\bar{x}_i)$ and $g_i(\bar{x}_i)$ $(i = 1, ..., n)$ are known smooth functions. $d_i(\bar{x}_i,t)$, $i = 1, ..., n$ denote the uncertain nonlinearity. $\varphi_i(\bar{x}_i) \in R^{p_i}$, $i = 1, ..., n$ are known smooth functions.

Considering input constraints, the control input is defined by the following

$$u = \begin{cases} u_{\max} sgn(u_c), |u_c| \geq u_{\max} \\ u_c, 0 < |u_c| < u_{\max} \end{cases} \tag{2}$$

$u_{\max}$is the maximum value of the control input, andu_c is the control input to be designed.

The control objective is to make$x_1$follow a desired trajectory$y_d$in spite of the aforementioned unknown parameters, uncertainty, and input constraint, and guarantee that the closed-loop system is stable under the designed robust adaptive backstepping redesign control lawu_c.

The following assumption and theorem are needed to facilitate control system design.

Assumption 1: y_d, $\dot{y}_d$ are continuous and bound.

Assumption 2: The unknown unmatched uncertainty $d_i(\bar{x}_i,t)$, $(i = 1, ..., n-1)$, and the matched uncertainty $d_n(\bar{x}_n,t)$ are assumed to satisfy the following inequalities

$$\begin{aligned} &|d_i(\bar{x}_i,t)| \leq h_i(\bar{x}_i) + q_i |\bar{x}_{i+1}|, \quad i = 1, \cdots, n-1 \\ &|d_n(\bar{x}_n,t)| \leq h_n(\bar{x}_n) + q_n |u|. \end{aligned} \tag{3}$$

Here $h_i(\bar{x}_i)$, $i = 1, ..., n$ are the continuous and positive functions and $q_i \in [0,1)$, $i = 1, ..., n$.

Assumption 3: The input gain $g_i(\bar{x}_i)$ is nonzero with known sign. Thus, without loss of generality, assume

$$g_i(\bar{x}_i) \geq g_i > 0, i = 1, 2, ..., n,$$

where $g_i > 0$, $i = 1, 2,..., n$ are known positive constants.

Theorem 1 [33]**:** The following inequality holds for any $\varepsilon_0 > 0$ and for any $\eta_0 \in R$

$$|\eta_0| \leq (k_{\eta_0}\varepsilon_0 + \eta_0 \tanh(\frac{\eta_0}{\varepsilon_0})), k_{\eta_0} > 0, \varepsilon_0 > 0, \tag{4}$$

where k_{η_0} is a constant that satisfies $k_{\eta_0} = e^{-k_{\eta_0}+1}$, i.e., $k_{\eta_0} = 0.2758$.

3. CONTROLLER DESIGN AND STABILITY ANALYSIS

In this section, backstepping technique and Lyapunov redesign control are integrated to design the controller for system (1). The controller design procedure starts from the first equation of (1), and the virtual control input is designed. Then this procedure proceeds. The estimation for the uncertain parameters is updated by the adaptive laws. The matched/unmatched uncertainty is tackled by the Lyapunov redesign technique. The novel second-order filters are constructed to overcome the “explosion of terms” problem. The auxiliary system is adopted to provided signal to analyze the input constraints. Then stability of all signals in the closed-loop system is achieved.

3.1. Robust adaptive backstepping Lyapunov redesign control design

According to the backstepping design procedure, the controller is designed according to the following steps. The function arguments are removed in brief.

Step 1: The virtual control input $x_{2d}$is designed for x_1-subsystem.

Considering the first equation in (1), i.e., the output tracking error is defined as

$$z_1 = x_1 - y_d. \tag{5}$$

With the consideration of parameter estimation and tracking error, the Lyapunov candidate function is constructed as

$$V_1 = 0.5z_1^2 + \tilde{\theta}_1^T \Gamma_1^{-1} \tilde{\theta}_1, \tag{6}$$

where $\Gamma_1 = \Gamma_1^T$ is the adaption gain matrix. $\tilde{\theta}_1$ is the estimation error of the unknown parameter vectorθ_1: $\tilde{\theta}_1 = \hat{\theta}_1 - \theta_1$. Since θ_1 is unknown, it needs to be estimated online. The representation $\hat{\theta}_1$ will be applied to denote the estimate of θ_1, and an adaptive law is developed to update the parameter θ_1 based on the Lyapunov stability.

If the uncertainty is not considered and the time derivative of (6)can be expressed as

$$\dot{V}_1 = z_1(f_1 + \varphi_1^T\theta_1 + g_1 x_2 - \dot{y}_d) + \tilde{\theta}_1^T \Gamma^{-1} \dot{\hat{\theta}}_1. \tag{7}$$

The virtual control input is designed as

$$x_{2d0} = 1/g_1(-k_1 z_1 - f_1 - \varphi_1^T \hat{\theta}_1 + \dot{y}_d), \tag{8}$$

where k_1 is the positive design parameter.

Now taking the uncertain nonlinearity into consideration, then (7)is rewritten as

$$\dot{V}_1 = z_1(f_1 + \varphi_1^T\theta_1 + g_1x_2 + g_1d_1 - \dot{y}_d) + \tilde{\theta}_1^T\Gamma^{-1}\dot{\hat{\theta}}_1. \quad (9)$$

So the virtual control input (8) can be redesigned as

$$x_{2d} = x_{2d0} + x_{2dr}, \quad (10)$$

where x_{2dr} is robust feedback for attenuating the effect of uncertain nonlinearity g_1d_1, and the formulation is

$$x_{2dr} = -\frac{\gamma_1^2 z_1 g_1}{\gamma_1|z_1| + \varepsilon e^{-at}}. \quad (11)$$

Furthermore, the adaptive law for $\hat{\theta}_1$ is designed as

$$\dot{\hat{\theta}}_1 = \Gamma_1(\varphi_1 z_1 - \eta_1\hat{\theta}_1), \quad (12)$$

where η_1 is the positive constant.

Based on (8),(11) and (12), (9) becomes

$$\dot{V}_1 = -k_1z_1^2 + z_1g_1x_{2dr} + z_1d_1 - \eta_1\tilde{\theta}_1^T\hat{\theta}_1 + z_1g_1z_2. \quad (13)$$

It is noted that

$$\begin{aligned}
&-\tilde{\theta}_1^T\hat{\theta}_1 \le -0.5\left\|\tilde{\theta}_1\right\|^2 + 0.5\left\|\theta_1\right\|^2, \\
&z_1d_1 \le |z_1|(h_1 + q_1|x_{2d0}| + q_1\gamma_1).
\end{aligned} \quad (14)$$

And

$$\begin{aligned}
&z_1g_1x_{2dr} + z_1d_1 \\
&= -\frac{\gamma_1^2(z_1g_1)^2}{\gamma_1|z_1| + \varepsilon e^{-at}} \\
&\quad + \frac{(\gamma_1|z_1g_1| + \varepsilon e^{-at})|z_1g_1|(h_1 + q_1|x_{2d0}| + q_1\gamma_1)}{\gamma_1|z_1g_1| + \varepsilon e^{-at}} \\
&= \frac{\gamma_1(z_1g_1)^2(-\gamma_1(1-q_1) + h_1 + q_1|x_{2d0}|)}{\gamma_1|z_1g_1| + \varepsilon e^{-at}} \\
&\quad + \frac{\varepsilon e^{-at}|z_1g_1|(h_1 + q_1|x_{2d0}| + q_1\gamma_1)}{\gamma_1|z_1g_1| + \varepsilon e^{-at}}.
\end{aligned} \quad (15)$$

If $\gamma_1 > \frac{h_1+q_1|x_{2d0}|}{1-q_1}$, then

$$z_1g_1x_{2dr} + z_1g_1d_1 \le \frac{\varepsilon e^{-at}|z_1g_1|\gamma_1}{\gamma_1|z_1g_1| + \varepsilon e^{-at}} < \varepsilon e^{-at} \quad (16)$$

From (14)-(16), it can be obtained that (13) satisfies

$$\dot{V}_1 \le -k_1z_1^2 - 0.5\eta_1(\left\|\tilde{\theta}_1\right\|^2 - \left\|\theta_1\right\|^2) + \varepsilon e^{-at} + z_1g_1z_2. \quad (17)$$

The error signal of z_2 is defined as

$$z_2 = x_2 - x_{2d}. \quad (18)$$

Step 2: The virtual control input is designed for thex_2-subsystem.

In the standard backstepping control, the time derivative of$x_{2d}$is needed in the next step, but it is difficult to find the time derivative for the existence of the uncertain nonlinearity, even the time derivative is can be analytic computed, the computation load is very heavy. Therefore, the second-order filter is constructed to avoid the analytic computation of the time derivative of the virtual control x_{2d}.

The following second-order filter that is used

$$\begin{cases}
\dot{\alpha}_{11} = -(\alpha_{11} - x_{2d})/\tau_{11} - \xi_{11}\tanh(\zeta_{11}\xi_{11}(\alpha_{11} - x_{2d})), \\
\dot{\alpha}_{12} = -(\alpha_{12} - \dot{\alpha}_{11})/\tau_{12} - \xi_{12}\tanh(\zeta_{12}\xi_{12}(\alpha_{12} - \dot{\alpha}_{11})),
\end{cases} \quad (19)$$

where τ_{11}, τ_{12} are the time constants of the filter while ξ_{11}, ξ_{12}, ζ_{11}, ζ_{12} are constants.

Considering the second equation in (1), i.e.,

$$\dot{x}_2 = f_2 + \Phi_2^T\theta_2 + g_2(x_3 + d_2). \quad (20)$$

The Lyapunov candidate function is constructed as

$$V_2 = 0.5z_2^2 + \tilde{\theta}_2^T\Gamma_2^{-1}\tilde{\theta}_2 + 0.5e_{11}^2 + 0.5e_{12}^2, \quad (21)$$

where $\tilde{\theta}_2$ is the estimation error of the unknown parameter vector θ_2: $\tilde{\theta}_2 = \hat{\theta}_2 - \theta_2$. $\Gamma_2 = \Gamma_2^T$ is the adaption gain matrix.

The uncertain nonlinearity is not considered. The time derivative of (21) is

$$\begin{aligned}
\dot{V}_2 =& z_2(f_2 + \Phi_2^T\theta_2 + g_2x_3 - \alpha_{22}) + \tilde{\theta}_2^T\Gamma_2^{-1}\dot{\hat{\theta}}_2 \\
&+ e_{11}\dot{e}_{11} + e_{12}\dot{e}_{12},
\end{aligned} \quad (22)$$

where e_{11}, e_{12} are the filter error and defined as $e_{11} = \alpha_{11} - x_{2d}$, $e_{12} = \alpha_{12} - \dot{\alpha}_{11}$.

If the virtual control input is designed as

$$x_{3d0} = 1/g_2(-k_2z_2 - f_2 - \Phi_2^T\hat{\theta}_2 + \alpha_{22} - z_1g_1), \quad (23)$$

where k_2 is the positive design parameter.

Taking the uncertain nonlinearity into consideration, then (22) is rewritten as

$$\begin{aligned}
\dot{V}_2 =& z_2(f_2 + \Phi_2^T\theta_2 + g_2x_3 + g_2d_2 - \alpha_{22}) \\
&+ \tilde{\theta}_2^T\Gamma_2^{-1}\dot{\hat{\theta}}_2 + e_{11}\dot{e}_{11} + e_{12}\dot{e}_{12}.
\end{aligned} \quad (24)$$

The virtual control input (8) is redesigned as

$$x_{3d} = x_{3d0} + x_{3dr}, \quad (25)$$

where x_{3dr} denotes robust feedback to attenuate the effect of uncertain nonlinearity g_2d_2, and it is designed as

$$x_{3dr} = -\frac{\gamma_2^2 z_2 g_2}{\gamma_2|z_2g_2| + \varepsilon e^{-at}}. \quad (26)$$

The adaptive law for$\hat{\theta}_2$is chosen as

$$\dot{\hat{\theta}}_2 = \Gamma_2(\Phi_2(\bar{x}_2)z_2 - \eta_2\hat{\theta}_2), \quad (27)$$

where $\Gamma_2 = \Gamma_2^T$ is the adaption gain matrix and η_2 is the positive constant.

Based on (23), (25)-(27), (24) becomes

$$\begin{aligned}\dot{V}_2 =& -k_2 z_2^2 + z_2 g_2 x_{3dr} + z_2 g_2 d_2 \\ & - \eta_2 \tilde{\theta}_2 \hat{\theta}_2 + z_2 g_2 z_3 + e_{11}\dot{e}_{11} + e_{12}\dot{e}_{12}.\end{aligned} \tag{28}$$

It is noted that

$$\begin{aligned}& -\tilde{\theta}_2^T \hat{\theta}_2 \le -0.5\left\|\tilde{\theta}_2\right\|^2 + 0.5\left\|\theta_2\right\|^2 \\ & z_2 g_2 d_2 \le |z_2 g_2|\,|h_2 + q_2|x_3|| \\ & \qquad \le |z_2 g_2|\,(h_2 + q_2|x_{3d0}| + q_2\gamma_2).\end{aligned} \tag{29}$$

And

$$\begin{aligned}& z_2 g_2 x_{2dr} + z_2 g_2 d_2 \\ & = \frac{\gamma_2 (z_2 g_2)^2(-\gamma_2(1-q_2) + h_2 + q_2|x_{3d0}|)}{\gamma_2|z_2 g_2| + \varepsilon e^{-at}} \\ & \quad + \frac{\varepsilon e^{-at}|z_2 g_2|\,(h_2 + q_2|x_{3d0}| + q_2\gamma_2)}{\gamma_2|z_2 g_2| + \varepsilon e^{-at}}.\end{aligned} \tag{30}$$

If $\gamma_2 > \frac{h_2+q_2|x_{3d0}|(1-q_2)}{1-q_2}$, then

$$z_2 g_2 x_{3dr} + z_2 g_2 d_2 \le \frac{\varepsilon e^{-at}|z_2 g_2|\gamma_2}{\gamma_2|z_2 g_2| + \varepsilon e^{-at}} < \varepsilon e^{-at}. \tag{31}$$

The last two terms of (28) satisfies

$$\begin{aligned}& e_{11}\dot{e}_{11} + e_{12}\dot{e}_{12} \\ & \le -1/\tau_{11}e_{11}^2 - \xi_{11}e_{11}\tanh(\zeta_{11}\xi_{11}e_{11}) + |e_{11}|\,|\dot{x}_{2d}| \\ & \quad -1/\tau_{12}e_{12}^2 - \xi_{12}e_{12}\tanh(\zeta_{12}\xi_{12}e_{12}) + |e_{12}|\,|\ddot{\alpha}_{11}|.\end{aligned} \tag{32}$$

On the basis of (4), if $\xi_{11} > |\dot{x}_{2d}|_{\max}, \xi_{12} > |\ddot{\alpha}_{11}|_{\max}$, the above inequality yields

$$\begin{aligned}& -\xi_{11}e_{11}\tanh(\zeta_{11}\xi_{11}e_{11}) + |e_{11}|\,|\dot{x}_{2d}| \le k_{\eta_0}/\zeta_{11}, \\ & -\xi_{12}e_{12}\tanh(\zeta_{12}\xi_{12}e_{12}) + |e_{12}|\,|\ddot{\alpha}_{11}| \le +k_{\eta_0}/\zeta_{12}.\end{aligned} \tag{33}$$

From (29)-(33), (28) satisfies

$$\begin{aligned}\dot{V}_2 \le& -k_2 z_2^2 - 0.5\eta_2\left\|\tilde{\theta}_2\right\|^2 + 0.5\eta_2\left\|\theta_2\right\|^2 + \varepsilon e^{-at} \\ & + z_2 g_2 z_3 - 1/\tau_{11}e_{11}^2 - 1/\tau_{12}e_{12}^2 + k_{\eta_0}/\zeta_{11} + k_{\eta_0}/\zeta_{12},\end{aligned} \tag{34}$$

where z_3 is the error signal of x_3 and defined as $z_3 = x_3 - x_{3d}$ with x_{3d} is the virtual control of the x_2-subsystem.

Step *i*: The virtual control input$x_{i+1,d}$is designed for the x_i-subsystem.

The second-order filter is constructed to avoid the analytic computation of the time derivative of the virtual control $x_{i,d}$

$$\begin{cases}\dot{\alpha}_{i-1,1} = -\frac{\alpha_{i-1,1}-x_{id}}{\tau_{i-1,1}} \\ \qquad -\xi_{i-1,1}\tanh(\zeta_{i-1,1}\xi_{i-1,1}(\alpha_{i-1,1} - x_{i-1,d})), \\ \dot{\alpha}_{i-1,2} = -\frac{\alpha_{i-1,2}-\dot{\alpha}_{i-1,1}}{\tau_{i-1,2}} \\ \qquad -\xi_{i-1,2}\tanh(\zeta_{i-1,2}\xi_{i-1,2}(\alpha_{i-1,2} - \dot{\alpha}_{i-1,1})),\end{cases} \tag{35}$$

where $\tau_{i-1,1}$, $\tau_{i-1,2}$ are the time constants of the filter while $\xi_{i-1,1}$, $\xi_{i-1,2}$, $\zeta_{i-1,1}$, $\zeta_{i-1,2}$ are constants.

Considering the i-th equation in (1), i.e.,

$$\dot{x}_i = f_i + \Phi_i^T\theta_i + g_i(x_{i+1} + d_i). \tag{36}$$

The Lyapunov candidate function is constructed as

$$V_i = 0.5z_i^2 + \tilde{\theta}_i^T\Gamma_i^{-1}\tilde{\theta}_i + 0.5e_{i-1,1}^2 + 0.5e_{i-1,2}^2, \tag{37}$$

where $\tilde{\theta}_i$ is the estimation error of the unknown parameter vector θ_i:$\tilde{\theta}_i = \hat{\theta}_i - \theta_i$, $e_{i-1,1}, e_{i-1,2}$ are the filter error $e_{i-1,1} = \alpha_{i-1,1} - x_{i-1,d}$, $e_{i-1,2} = \alpha_{i-1,2} - \dot{\alpha}_{i-1,1}$.

The uncertain nonlinearity is not considered. The time derivative of (37) is

$$\begin{aligned}\dot{V}_i =& z_i(f_i + \Phi_i^T\theta_i + g_i x_{i+1} - \alpha_{i2}) \\ & + \tilde{\theta}_i^T\Gamma_i^{-1}\dot{\hat{\theta}}_i + e_{i1}\dot{e}_{i1} + e_{i2}\dot{e}_{i2}.\end{aligned} \tag{38}$$

If the virtual control input is designed as

$$x_{i+1,d0} = g_i^{-1}(-k_i z_i - f_i - \Phi_i^T\hat{\theta}_i + \alpha_{i2} - z_{i-1}g_{i-1}), \tag{39}$$

where k_i is the positive design parameter.

Taking the uncertain nonlinearity into consideration, then (38) is rewritten as

$$\begin{aligned}\dot{V}_i =& z_i(f_i + \Phi_i^T\theta_i + g_i x_{i+1} + g_i d_i - \alpha_{i2}) \\ & + \tilde{\theta}_i^T\Gamma_i^{-1}\dot{\hat{\theta}}_i + e_{i-1,1}\dot{e}_{i-1,1} + e_{i-1,2}\dot{e}_{i-1,2}.\end{aligned} \tag{40}$$

The virtual control input (39)is redesigned as

$$x_{i+1,d} = x_{i+1,d0} + x_{i+1,dr}, \tag{41}$$

where $x_{i+1,dr}$ denotes a robust feedback to attenuate the effect of uncertain nonlinearity $g_i d_i$ and it is designed as

$$x_{i+1,dr} = -\frac{\gamma_{i+1}^2 z_i g_i}{\gamma_{i+1}|z_i g_i| + \varepsilon e^{-at}}. \tag{42}$$

The adaptive law for$\hat{\theta}_i$is designed as

$$\dot{\hat{\theta}}_i = \Gamma_i(\Phi_i z_i - \eta_i\hat{\theta}_i), \tag{43}$$

where $\Gamma_i = \Gamma_i^T$ is the adaption gain matrix and η_i is the positive constant.

Based on (39), (41)-(43), (40) becomes

$$\begin{aligned}\dot{V}_i =& -k_i z_i^2 + z_i g_i x_{i+1,dr} + z_i g_i d_i - \eta_i\tilde{\theta}_i^T\hat{\theta}_i \\ & + z_i g_i z_{i+1} + e_{i-1,1}\dot{e}_{i-1,1} + e_{i-1,2}\dot{e}_{i-1,2}.\end{aligned} \tag{44}$$

It is noted that

$$\begin{aligned}& -\tilde{\theta}_i^T\hat{\theta}_i \le -0.5\left\|\tilde{\theta}_i\right\|^2 + 0.5\left\|\theta_i\right\|^2 \\ & z_i g_i d_i \le |z_i g_i|\,(h_i + q_i|x_{i+1,d0}| + q_i\gamma_i) \\ & z_i g_i x_{idr} + z_i g_i d_i \\ & = \frac{\gamma_i(z_i g_i)^2(-\gamma_i(1-q_i) + h_i + q_i|x_{i+1d0}|)}{\gamma_i|z_i g_i| + \varepsilon e^{-at}}\end{aligned} \tag{45}$$

$$+\frac{\varepsilon e^{-at}\left|z_i g_i\right|\left(h_i+q_i\left|x_{i+1d0}\right|+q_i\gamma_i\right)}{\gamma_i\left|z_i g_i\right|+\varepsilon e^{-at}}.$$

If $\gamma_i > \frac{h_i+q_i|x_{i+1,d0}|}{1-q_i}$, then

$$\gamma_i q_i+h_i+q_i\left|x_{i+1,d0}\right|<\gamma_i, z_i g_i x_{i+1,dr}+z_i g_i d_i \le \varepsilon e^{-at}. \quad (46)$$

The last two terms of (44) satisfies

$$\begin{aligned} & e_{i-1,1}\dot{e}_{i-1,1}+e_{i-1,2}\dot{e}_{i-1,2} \\ & \le -1/\tau_{i-1,1}e_{i-1,1}^2-1/\tau_{i-1,2}e_{i-1,2}^2 \\ & \quad -\xi_{i-1,1}e_{i-1,1}\tanh(\zeta_{i-1,1}\xi_{i-1,1}e_{i-1,1}) \\ & \quad -\xi_{i-1,2}e_{i-1,2}\tanh(\zeta_{i-1,2}\xi_{i-1,2}e_{i-1,2}) \\ & \quad +\left|e_{i-1,1}\right|\left|\dot{x}_{id}\right|+\left|e_{i-1,2}\right|\left|\ddot{\alpha}_{i-1,1}\right|. \end{aligned} \quad (47)$$

Base on (4), if $\xi_{i-1,1} > |\dot{x}_{id}|_{\max}$, $\xi_{i-1,2} > |\ddot{\alpha}_{i-1,1}|_{\max}$,

$$\begin{aligned} & -\xi_{i-1,1}e_{i-1,1}\tanh(\zeta_{i-1,1}\xi_{i-1,1}e_{i-1,1})+\left|e_{i-1,1}\dot{x}_{id}\right| \\ & \quad \le k_{\eta_0}/\zeta_{i-1,1}, \\ & -\xi_{i-1,2}e_{i-1,2}\tanh(\zeta_{i-1,2}\xi_{i-1,2}e_{i-1,2})+\left|e_{i-1,2}\ddot{\alpha}_{i-1,1}\right| \\ & \quad \le k_{\eta_0}/\zeta_{i-1,2}. \end{aligned} \quad (48)$$

From(45)-(48), (44) satisfies

$$\begin{aligned} \dot{V}_i \le & -k_i z_i^2-0.5\eta_i\left\|\tilde{\theta}_i\right\|^2+0.5\eta_i\left\|\theta_i\right\|^2+\varepsilon e^{-at} \\ & +z_i g_i z_{i+1}-1/\tau_{i-1,1}e_{i-1,1}^2-1/\tau_{i-1,2}e_{i-1,2}^2 \\ & +k_{\eta_0}/\zeta_{i-1,1}+k_{\eta_0}/\zeta_{i-1,2}. \end{aligned} \quad (49)$$

z_{i+1} is the error signal of x_{i+1} and defined as

$$z_{i+1}=x_{i+1}-x_{i+1,d}. \quad (50)$$

Here $x_{i+1,d}$ is the virtual control of the ith subsystem.

Step *n*: The control input u_c is designed for system (1).

The second-order filter is constructed to avoid the analytic computation of the time derivative of the virtual control$x_{n-1,d}$

$$\begin{cases} \dot{\alpha}_{n-1,1}=-\frac{\alpha_{n-1,1}-x_{nd}}{\tau_{n-1,1}} \\ \qquad -\xi_{n-1,1}\tanh(\zeta_{n-1,1}\xi_{n-1,1}(\alpha_{n-1,1}-x_{n,d})), \\ \dot{\alpha}_{n-1,2}=-\frac{\alpha_{n-1,2}-\dot{\alpha}_{n-1,1}}{\tau_{n-1,2}} \\ \qquad -\xi_{n-1,2}\tanh(\zeta_{n-1,2}\xi_{n-1,2}(\alpha_{n-1,2}-\dot{\alpha}_{n-1,1})), \end{cases} \quad (51)$$

where $\tau_{n-1,1}$, $\tau_{n-1,2}$ are the time constants of the filter and $\xi_{n-1,1}$, $\xi_{n-1,2}$, $\zeta_{n-1,1}$, $\zeta_{n-1,2}$ are constants.

Considering the nth equation in (1) and input constraint (2)

$$\dot{x}_n=f_n+\Phi_n^T\theta_n+g_n(u_c+\Delta u+d_n), \quad (52)$$

where $\Delta u=u-u_c$.

The Lyapunov candidate function is constructed as

$$V_{n0}=0.5z_n^2+\tilde{\theta}_n^T\Gamma_n^{-1}\tilde{\theta}_n+0.5e_{n-1,1}^2+0.5e_{n-1,2}^2, \quad (53)$$

where $e_{n-1,1}$, $e_{n-1,2}$ are the filter error $e_{n-1,1}=\alpha_{n-1,1}-x_{n,d}$, $e_{n-1,2}=\alpha_{n-1,2}-\dot{\alpha}_{n-1,1}$, $\tilde{\theta}_n$ is the estimation error of the unknown parameter vector θ_n: $\tilde{\theta}_n=\hat{\theta}_n-\theta_n$.

The uncertain nonlinearity is not considered. The time derivative of (53) is

$$\begin{aligned} \dot{V}_{n0}= & z_n(f_n+\Phi_n^T\theta_n+g_n u_0-\alpha_{n-1,2}) \\ & +\tilde{\theta}_n^T\Gamma_n^{-1}\dot{\hat{\theta}}_n+e_{n-1,1}\dot{e}_{n-1,1}+e_{n-1,2}\dot{e}_{n-1,2}. \end{aligned} \quad (54)$$

If the control input is designed as

$$\begin{aligned} u_0= & g_n^{-1}(-k_n(z_n-\sigma_u)-f_n-\Phi_n^T\hat{\theta}_n) \\ & +g_n^{-1}(\alpha_{n-1,2}-z_{n-1}g_{n-1}), \end{aligned} \quad (55)$$

where k_n is the positive design parameter.

Taking the uncertain nonlinearity into account, the augmented Lyapunov function is as

$$V_n=V_{n0}+0.5\sigma_u^2. \quad (56)$$

The time derivative of (56) is

$$\begin{aligned} \dot{V}_n= & z_n(f_n+\Phi_n^T\theta_n+g_n(u_c+\Delta u)+g_n d_n-\alpha_{n-1,2}) \\ & +\tilde{\theta}_n^T\Gamma_n^{-1}\dot{\hat{\theta}}_n+e_{n-1,1}\dot{e}_{n-1,1}+e_{n-1,2}\dot{e}_{n-1,2}+\sigma_u\dot{\sigma}_u. \end{aligned} \quad (57)$$

The control input (50)is redesigned as

$$u_c=u_0+u_r, \quad (58)$$

where u_r presents robust feedback to attenuate the effect of uncertain nonlinearity $g_n d_n$, and it is designed as

$$u_r=-\frac{\gamma_n^2 z_n g_n}{\gamma_n\left|z_n g_n\right|+\varepsilon e^{-at}}. \quad (59)$$

The following auxiliary system is introduced to analyze the effect of the input constraint

$$\dot{\sigma}_u=\begin{cases} -k_{\sigma_u}\sigma_u-\frac{1}{\sigma_u}(|z_n g_n\Delta u|+0.5\mu^2\Delta u^2)-g_n\Delta u, & |\sigma_u|\ge\psi_u \\ 0, & |\sigma_u|<\psi_u, \end{cases} \quad (60)$$

where $\mu > |g_n|$, $\psi_u > 0$ is a small parameter and should be chosen according to the requirement of the tracking performance.

The adaptive law for $\hat{\theta}_n$ is chosen as

$$\dot{\hat{\theta}}_n=\Gamma_n(\Phi_n z_n-\eta_n\hat{\theta}_n), \quad (61)$$

where $\Gamma_n=\Gamma_n^T$ is the adaption gain matrix and η_n is the positive constant.

Remark 1: In control law (58), combined with the input constraint block (60), the role of the term σ_u is as follows:

If $|\sigma_u|\ge\psi_u$, which means there is input saturation:

(a) If $u_c \geq u_{\max}$, σ_u keeps decreasing to decrease u_c until $u_c = u_{\max}$.

(b) If $u_c \leq u_{\min}$, σ_u keeps increasing to increase u_c until $u_c = u_{\min}$.

So $u = u_{\min}$ or $u = u_{\max}$.

If $|\sigma_u| < \psi_u$, $\dot{\sigma}_u = 0$, i.e., there is no input saturation, the influence of the term σ_u is to keep $u_{\min} \leq u_c \leq u_{\max}$, therefore, $u = u_c$.

Based on (55), (58)-(61), (57) becomes

$$\begin{aligned}\dot{V}_n =& -k_n z_n^2 + k_n z_n \sigma_u + z_n g_n u_r + z_n g_n d_n + z_n g_n \Delta u \\ & - \eta_n \tilde{\theta}_n^T \hat{\theta}_n + \sigma_u \dot{\sigma}_u + e_{n-1,1}\dot{e}_{n-1,1} + e_{n-1,2}\dot{e}_{n-1,2}.\end{aligned} \tag{62}$$

It is noted that

$$\begin{aligned}& -\tilde{\theta}_n^T \hat{\theta}_n \leq -0.5\left\|\tilde{\theta}_n\right\|^2 + 0.5\left\|\theta_n\right\|^2, \\ & z_n g_n d_n \leq |z_n g_n| \left|\lambda_n + q_n |u|\right| \leq |z_n g_n| \left(\lambda_n + q_i |u_c| + q_n \gamma_n\right), \\ & z_n g_n v_r + z_n g_n d_n \\ & = \frac{\gamma_n (z_n g_n)^2 (-\gamma_n(1-q_n) + h_n + q_n |u_c|)}{\gamma_n |z_n g_n| + \varepsilon e^{-at}} \\ & \quad + \frac{\varepsilon e^{-at} |z_n g_n| (h_n + |u_c| + q_n \gamma_n)}{\gamma_n |z_n g_n| + \varepsilon e^{-at}}.\end{aligned} \tag{63}$$

If $\gamma_n > \frac{h_n + q_n |u_c|}{1-q_n}$, then

$$\begin{aligned}& -\gamma_n(1-q_n) + h_n + q_n |u_c| < 0, \\ & \gamma_n q_n + h_n + q_n |u_c| < \gamma_n,\end{aligned} \tag{64}$$

and

$$z_n g_n u_r + z_n g_n d_n \leq \varepsilon e^{-at}. \tag{65}$$

The last two terms of (62) satisfies

$$\begin{aligned}& e_{n-1,1}\dot{e}_{n-1,1} + e_{n-1,2}\dot{e}_{n-1,2} \\ & \leq -1/\tau_{n-1,1} e_{n-1,1}^2 - 1/\tau_{n-1,2} e_{n-1,2}^2 + |e_{n-1,1}| |\dot{x}_{nd}| \\ & \quad - \xi_{n-1,1} e_{n-1,1} \tanh(\zeta_{n-1,1}\xi_{n-1,1} e_{n-1,1}) \\ & \quad + |e_{n-1,2}| |\ddot{\alpha}_{n-1,1}| \\ & \quad - \xi_{n-1,2} e_{n-1,2} \tanh(\zeta_{n-1,2}\xi_{n-1,2} e_{n-1,2}).\end{aligned} \tag{66}$$

If $\xi_{n-1,1} > |\dot{x}_{nd}|_{\max}$, $\xi_{n-1,2} > |\ddot{\alpha}_{n-1,1}|_{\max}$,

$$\begin{aligned}& -\xi_{n-1,1} e_{n-1,1} \tanh(\zeta_{n-1,1}\xi_{n-1,1} e_{n-1,1}) + |e_{n-1,1}\dot{x}_{nd}| \\ & \quad \leq k_{\eta_0}/\zeta_{n-1,1}, \\ & -\xi_{n-1,2} e_{n-1,2} \tanh(\zeta_{n-1,2}\xi_{n-1,2} e_{n-1,2}) + |e_{n-1,2}\ddot{\alpha}_{n-1,1}| \\ & \quad \leq k_{\eta_0}/\zeta_{n-1,2}\end{aligned} \tag{67}$$

From (63)-(67), (62) satisfies

$$\begin{aligned}\dot{V}_n \leq & -k_n z_n^2 - 0.5\eta_n \left\|\tilde{\theta}_n\right\|^2 + 0.5\eta_n \left\|\theta_n\right\|^2 + \varepsilon e^{-at} \\ & -1/\tau_{n-1,1} e_{n-1,1}^2 - 1/\tau_{n-1,2} e_{n-1,2}^2 + k_n z_n \sigma_u \\ & + z_n g_n \Delta u + \sigma_u \dot{\sigma}_u - z_n z_{n-1} g_{n-1} \\ & + k_{\eta_0}/\zeta_{n-1,1} + k_{\eta_0}/\zeta_{n-1,2}.\end{aligned} \tag{68}$$

Define $G = k_n z_n \sigma_u + z_n g_n \Delta u + \sigma_u \dot{\sigma}_u$, substituting (60) into it, then it satisfies

$$\begin{aligned}G =& -k_{\sigma_u}\sigma_u^2 - 0.5\mu_n^2 \Delta u^2 + z_n g_n \Delta u \\ & - |z_n g_n \Delta u| + k_n z_n \sigma_u - g_n \Delta u \sigma_u.\end{aligned} \tag{69}$$

Since $k_n \sigma_u z_n - g_n \sigma_u \Delta u \leq 0.5k_n^2 z_n^2 + \sigma_u^2 + 0.5\mu_n^2 \Delta u^2$, $z_n g_n \Delta u - |z_n g_n \Delta u| \leq 0$, the following inequality holds

$$G \leq 0.5k_n^2 z_n^2 + \sigma_u^2 - k_{\sigma_u}\sigma_u^2. \tag{70}$$

Therefore, (68) satisfies

$$\begin{aligned}\dot{V}_n \leq & -(k_n - 0.5k_n^2) z_n^2 - (k_{\sigma_u} - 1)\sigma_u^2 - 0.5\eta_n \left\|\tilde{\theta}_n\right\|^2 \\ & + \varepsilon e^{-at} - 1/\tau_{n-1,1} e_{n-1,1}^2 - 1/\tau_{n-1,2} e_{n-1,2}^2 \\ & + 0.5\eta_n \left\|\theta_n\right\|^2 + k_{\eta_0}/\zeta_{n-1,1} + k_{\eta_0}/\zeta_{n-1,2}.\end{aligned} \tag{71}$$

After comparing the stability analysis parts of [24] and this paper, It can be obtained that without the introduction of state error variablee_i, the stability analysis on the filter error and the closed-loop system become succinctly.

Remark 2: It is known that the hyperbolic tangent function$y = \tanh(x)$is smoother than the saturation function$y = sat(x) = \begin{cases} \operatorname{sgn}(x), & x \geq k \\ x/k, & x < k \end{cases}$. Thus, for the filter with hyperbolic tangent funciton, and the filter with saturation function, even if the estimation ability of filter that includes saturation function component, is equal to the filter that includes hyperbolic tangent function component, the dynamic process of outputs of the former filter is smoother than that of the latter filter. And in this paper, the outputs of the filters are the reference commands of the next controller design steps. When applying the designed control strategy to the control of hypersonic vehicle, under the proposed controller and the smooth reference commands, the responses of the states (γ, α, q) of the hypersonic vehicle are smooth (Moreover, to obtain the smooth response of V and h, the reference commands of V and h are smoothened using a second-order filter with a natural frequency and a damping ratio, which can be seen from the simulation section.). During the flight process of the hypersonic vehicle, it is important and necessary for the smooth change of every state. Based on aforementioned, the hyperbolic tangent function component is superior to the saturation function component, thus the performances of the filters proposed in this paper can be improved.

The stability of the closed-loop system is analyzed. The semi-global stability of the closed-loop system will be shown in the next subsection.

3.2. Stability analysis of the closed-loop system

Theorem 2: Given the uncertain nonlinear system (1), without loss of generality, the system subjects to unknown parameter vector θ_i, ($i = 1$, ..., n), matched/unmatched uncertain nonlinearity d_i, ($i = 1$, ..., n) and control input saturation (2). Then with the application of the control laws (10), (25), (41), (58), the second-order filters (19), (35), (51) and adaptive laws (12), (27), (43), (61). If all Assumptions 1-3 are satisfied and there are exist control parameters, adaptive parameters and filter parameters such that the control scheme guarantees the following properties: all the signals of the system are semi uniformly ultimately bounded (SGUUB). Moreover, the tracking error$z_1$can be ensured to converge to an arbitrarily small neighbour-hood around zero by properly choosing the control parameters.

Proof: The proof includes two parts.

1) The state of the auxiliary system (60) satisfies the condition $|\sigma_u| \geq \psi_u$, i.e., there exists input saturation.

Construct the Lyapunov candidate function as

$$V \leq \sum_{i=1}^{n} V_i + \frac{2n\varepsilon}{a} e^{-at}. \tag{72}$$

Using the derivative respect to time,

$$\dot{V} \leq \sum_{i=1}^{n} V_i - 2n\varepsilon e^{-at}. \tag{73}$$

On the basis of (17), (34), (49) and (71), the following inequality holds

$$\begin{aligned}\dot{V} \leq & -\sum_{i=1}^{n-1} k_i z_i^2 - \sum_{i=1}^{n} 0.5\eta_i \left\|\tilde{\theta}_i\right\|^2 - (k_n - 0.5k_n^2) z_n^2 \\ & - (k_{\sigma_u} - 1)\sigma_u^2 - n\varepsilon e^{-at} - \sum_{j=1}^{n-1}(1/\tau_{j,1}e_{j,1}^2 + 1/\tau_{j,2}e_{j,2}^2) \\ & + \sum_{j=1}^{n-1} k_{\eta_0}(1/\zeta_{j,1} + 1/\zeta_{j,2}) + \sum_{i=1}^{n}(0.5\eta_i \|\theta_i\|^2) \\ \leq & -2cV + \kappa,\end{aligned} \tag{74}$$

where $c = \min\left\{k_i, k_n - 0.5k_n^2, \eta_i/2, \tau_{j,1}, \tau_{j,2}, a/4, k_{\sigma_u} - 1\right\}$, $\kappa = \sum_{i=1}^{n}(0.5\eta_i \|\theta_i\|^2) + \sum_{j=1}^{n-1} k_{\eta_0}(1/\zeta_{j,1} + 1/\zeta_{j,2})$, it can be concluded from (74) that, if $k_{\sigma_u} > 1$, $\kappa > 0$, error signals z_i and $\tilde{\theta}_i$ asymptotically converges to a compact set . In addition, the auxiliary design variable σ_u asymptotically converge to compact set.

Standard arguments can now be applied to solve the differential inequality given in inequality(74) as follows:

$$0 \leq V(t) \leq \kappa/2c + (V(0) - \kappa/2c)\exp(-2ct), \forall t \geq 0. \tag{75}$$

The inequality (75) indicates that $V(t)$ is bounded by$\kappa/2c$, i.e., for any $t \geq 0$, $V(t) \leq \kappa/2c$, which indicates that z_i, θ_i, $e_{j,1}$, $e_{j,2}$, σ_u, ϕ_u are SGUUB. Note that, since$\tilde{\theta}_i = \hat{\theta}_i - \theta_i$, $\hat{\theta}_i$ is SGUUB. Owing to bounded of x_{1d} and $\dot{x}_{1d}$ it can be concluded that x_1 and $\dot{x}_1$ are SGUUB. According to the definition of x_{2d}, x_2 is SGUUB. Similarly, for $i = 3$, ..., n, x_i is SGUUB. So semi-globally uniformly ultimate boundedness of all signals of the closed-loop system is assured.

Based on what was discussed above, the tracking error z_1 converges to an arbitrarily small region will be shown. Because $\dot{V}(t) \leq 0$, $V \geq 0$, V is a decreasing function or constant function.

Inequality (75) indicates that

$$\begin{aligned} z_1^2 &\leq V(t) \\ &\leq \kappa/c + (2V(0) - \kappa/c)\exp(-2ct), \ \ \forall t \geq 0, \end{aligned} \tag{76}$$

$$\begin{aligned} e_{j,l}^2 &\leq V(t) \\ &\leq \kappa/c + (2V(0) - \kappa/c)\exp(-2ct), \forall t \geq 0. \end{aligned} \tag{77}$$

The tracking error obviously yields

$$|z_1| \leq \sqrt{\kappa/c + (2V(0) - \kappa/c)\exp(-2ct)}, \quad \forall t \geq 0, \tag{78}$$

$$|e_{j,l}| \leq \sqrt{\kappa/c + (2V(0) - \kappa/c)\exp(-2ct)}, \quad \forall t \geq 0. \tag{79}$$

Thus

$$\lim_{t\to\infty} |z_1| \leq \sqrt{\kappa/c}, \tag{80}$$

$$\lim_{t\to\infty} |e_{j,l}| \leq \sqrt{\kappa/c}. \tag{81}$$

The convergence region of z_1 can be expressed as the compact set

$$S_{z_1} = \left\{ z_1 \left| |z_1| \leq \sqrt{\kappa/c} \right.\right\} \tag{82}$$

and the filter errors converge to the compact sets

$$S_{e_{j,l}} = \left\{ e_{j,l} \left| |e_{j,l}| \leq \sqrt{\kappa/c} \right.\right\}, \quad j = 1, ..., n-1, \ \ l = 1, 2. \tag{83}$$

It is noted that for any given parameters a_i, k_{η_0}, $\zeta_{j,1}$, $\zeta_{j,2}$, η_i, κ is the unknown but is a bounded constant which is independent of c. By increasing the values of k_i, η_i, $\lambda_{\min}$, Γ_i, $\tau_{j,1}$, $\tau_{j,2}$, a, k_{σ_u}, i.e., increasing the value of c to a big enough value, the value of κ/c can be made arbitrarily small. It can be concluded that κ/c can be made arbitrarily small by choosing c large enough.

Therefore, the control objective is reached when the input constraint occurs, i.e., the desired trajectory of nonlinear system (1) is followed in the presence of unknown, uncertainty under input constraint (2).

To proceed, the stability proof of Theorem 2 is proved as following when there is no saturation.

2) The state of the auxiliary design system (60) satisfies the condition, $|\sigma_u| < \psi_u$, i.e., there does not exist input saturation. So $\Delta u = 0$ and $u = u_c$, the control input u is bounded. Thus, u_c is bounded.

The Lyapunov candidate function for the n-step is constructed as (60), and the time derivative is

$$\begin{aligned}\dot{V}_{n0} =& z_n(f_n + \Phi_n^T\theta_n + g_n u - \alpha_{n-1,2}) \\ &+ \tilde{\theta}_n^T\Gamma_n^{-1}\dot{\hat{\theta}}_n + e_{n-1,1}\dot{e}_{n-1,1} + e_{n-1,2}\dot{e}_{n-1,2}. \quad (84)\end{aligned}$$

The control input u_c is redesigned as

$$u_c = u_0 + u_r, \quad (85)$$

where u_r denotes robust feedback to attenuate the effect of uncertain nonlinearity $g_n d_n$, and it is designed as

$$u_r = -\frac{\gamma_n^2 z_n g_n}{\gamma_n |z_n g_n| + \varepsilon e^{-at}}. \quad (86)$$

The adaptive law for $\hat{\theta}_n$ is same as (61). Similar to the proof of the n step, (84) satisfies

$$\begin{aligned}\dot{V}_{n0} \leq& -k_n z_n^2 - 0.5\eta_n \left\|\tilde{\theta}_n\right\|^2 + \varepsilon e^{-at} - 1/\tau_{n-1,1}e_{n-1,1}^2 \\ &- 1/\tau_{n-1,2}e_{n-1,2}^2 + 0.5\eta_n \left\|\theta_n\right\|^2 + k_{\eta_0}/\zeta_{n-1,1} \\ &+ k_{\eta_0}/\zeta_{n-1,2}. \quad (87)\end{aligned}$$

The Lyapunov function for the whole closed-loop systemis constructed as

$$\bar{V} \leq \sum_{j=1}^{n-1} V_j + V_{n0} + \frac{2n\varepsilon}{a}e^{-at}. \quad (88)$$

The time derivative of it satisfies

$$\begin{aligned}\dot{\bar{V}} \leq& -\sum_{i=1}^{n-1} k_i z_i^2 - \sum_{i=1}^{n} 0.5\eta_i \left\|\tilde{\theta}_i\right\|^2 - (k_n - 0.5k_n^2)z_n^2 \\ &- n\varepsilon e^{-at} - \sum_{j=1}^{n-1}(1/\tau_{j,1}e_{j,1}^2 + 1/\tau_{j,2}e_{j,2}^2) \quad (89) \\ &+ \sum_{j=1}^{n-1} k_{\eta_0}(1/\zeta_{j,1} + 1/\zeta_{j,2}) + \sum_{i=1}^{n}(0.5\eta_i \left\|\theta_i\right\|^2) \\ \leq& -2c_1 V + \kappa,\end{aligned}$$

where $c_1 = \min\{k_i, \eta_i/2, \tau_{j,1}, \tau_{j,2}, a/2\}$, $\kappa = \sum_{i=1}^{n}(0.5\eta_i \left\|\theta_i\right\|^2) + \sum_{j=1}^{n-1} k_{\eta_0}(1/\zeta_{j,1} + 1/\zeta_{j,2})$.

Likely, the convergence region of z_1 can be expressed as the following compact set

$$\bar{S}_{z_1} = \left\{z_1 \middle| |z_1| \leq \sqrt{\kappa/c_1}\right\}, \quad (90)$$

and the filter errors converge to the compact sets

$$\bar{S}_{e_{j,l}} = \left\{e_{j,l} \middle| |e_{j,l}| \leq \sqrt{\kappa/c_1}\right\}. \quad (91)$$

Additionally, semi-globally uniformly ultimate boundedness of all signals of the closed-loop system is guaranteed.

Remark 3: In this paper, the asymptotic stability of the closed-loop system is achieved, and the controller design scheme is applied to the tracking control of hypersonic vehicle. We design the robust second-order filter to solve the "explosion of terms" problem that inherent in backstepping control. In the further work, the finite time or fixed time filter will be designed to achieve the fast response of the filter.

Remark 4: Though the control design method is also based on the combination of backstepping control and Lyapunov redesign technique, the main difference from [2] is given as follows.

First of all, not only unknown parameters and uncertainty but also input constraint is taken into consideration in this paper. Through designing an auxiliary system, an auxiliary signal is provided to handle the influence of input constraint. The auxiliary signal is applied at the level of controller design and stability analysis.

Besides, a new robust second-order filter is developed to solve the "explosion of terms" problem to avoid the complex computation of virtual control inputs inherent in backstepping control design procedure. The advantage of the proposed second-order filters is given in Remark 2.

At last, both numerical example and the application of the hypersonic vehicle tracking control problem are simulated to demonstrate the effectiveness of the proposed control scheme in coping with uncertainty and input constraint.

4. NUMERICAL SIMULATIONS

4.1. Numerical example

In this section, a second-order system is considered and the simulation is carried out to illustrate the performance of the robust adaptive backstepping control scheme developed in the Section 3.

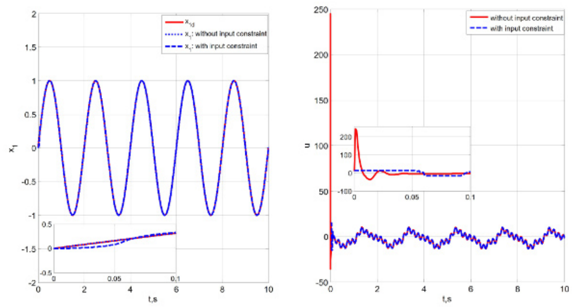
$$\begin{aligned}&\dot{x}_1 = x_1^2 + \varphi_1^T(x_1)\theta_1 + 2x_2 + 2d_1(x_1,x_2,t), \\ &\dot{x}_2 = x_1 + 2x_2^2 + \varphi_2^T(x_1,x_2)\theta_2 + e^{x_2}u + e^{x_2}d_2(x_1,x_2,t), \\ &y = x_1, \quad (92)\end{aligned}$$

where $\varphi_1^T(x_1) = x_1^2\sin(x_1)$, $\varphi_2^T(x_1,x_2) = [x_1x_2, x_2\cos(x_1)]$, $d_1(x_1,x_2,t) = 0.2x_2 + x_1^2\sin(0.01t)$, $d_2(x_1,x_2,t) = 3x_2\sin(10\pi t)$.

In order to do simulation, the nominal values of θ_1 and $\theta_2$are assumed to be that $\theta_1 = 1$, $\theta_2 = [1, 1.5]^T$. The input constraint is $|u| \leq 15$. And the external disturbances satisfies $|d_1(x_1,x_2,t)| \leq 0.2|x_2| + \left|x_1^2\right|, |d_2(x_1,x_2,t)| \leq 3|x_2|$.

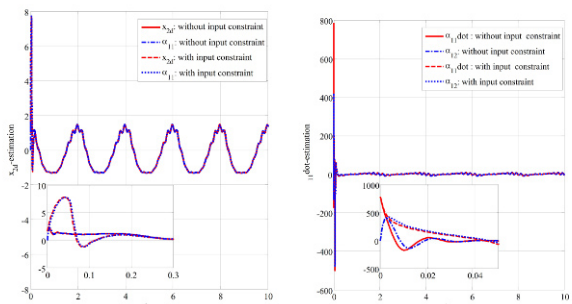
The objective is to design a control input as in the above section to guarantee that the output of the closed-loop system track a reference trajectory $x_{1d} = \sin(\pi t)$.

After many simulations, the control parameters that makes that designed control system achieve relative good tracking performance are adopted as $k_1 = 5$, $k_2 = 12$, $k_3 = 20$, $k_{\sigma_u} = 0.01$, $\mu = 150$, $\psi_u = 0.01$, $\varepsilon = 5$, $a = 0.01$, $\Gamma_1 = \Gamma_2 = 0.1diag(1\ 1)$, $\eta_1 = 0.02$, $\eta_2 = 0.01$, $\tau_1 = \tau_1 =$



(a) (b)

Fig. 1. (a)Time response of output tracking, (b)Time response of control input.



(a) (b)

Fig. 2. (a)Estimation of virtual control input x_{2d}, (b)Estimation of virtual control input $\dot{\alpha}_{11}$.

0.2, $\xi_1 = \xi_2 = 0.01$, $\zeta_1 = \zeta_2 = 100$. To the better demonstration of the capability of the designed control scheme in tackling input constraint, two cases response (with and without input constraints) are presented in one figure, and the local time response of the curves during the dynamic change process are given.The simulation results are with and without input constraint are presented in Figs.1 and 2.

As shown in Fig. 1, it is obvious that the proposed control strategy guarantee the good tracking performance. And the behaviour of input is smooth. Control input saturation is only observed in the initial phase, and it is about 0.08 seconds. After this initial phase, the control moments are within their limits and no saturation is observed. In addition, it is obvious that the proposed control scheme leads to a slower response for system output coverage to reference command than that of the response without input constraints, which is about in 0.01seconds. However, the proposed control approach can still guarantee the stabilization in spite of a longer response and handles the input constraints efficiently. In addition, the second-order has the good capability of estimation from Fig. 2.

It can be concluded from the simulation results that the designed controller satisfies the following two aspects. (a) During the dynamic tuning phase, it guarantees that the control system is still stable when there is input saturation. (b) During the steady-state phase, it assures the control system is stable.

4.2. Aerospace application example

To further show the effectiveness of the designed control strategy , we will consider a HSV with the model described as [34]

$$\begin{aligned}
\dot{V} &= \tfrac{1}{m}(T\cos\alpha - D) - g\sin\gamma,\\
\dot{h} &= V\sin\gamma,\\
\dot{\gamma} &= \tfrac{1}{mV}(L + T\sin\alpha) - \tfrac{1}{V}g\cos\gamma,\\
\dot{q} &= \tfrac{M_{yy}}{I_{yy}},\\
\dot{\alpha} &= q - \dot{\gamma}.
\end{aligned} \tag{93}$$

This model is composed of five state variables: velocity V, altitude h, flight-path angel γ, pitch rate q, angle of attack α.The control inputs are elevator deflection δ_e and throttle setting β, while the outputs to be controlled are selected as the velocity and the altitude. And m denotes vehicle mass, I_{yy} means the moment of inertia, ρ is air density, S is reference area, $\bar{c}$ is mean aerodynamic chord, g represents gravitational acceleration.

The approximations of the forces (thrust T, drag D, lift L) and moments (pitch moment M_{yy}) employed here are given as follows:

$$\begin{aligned}
T &= 0.5\rho V^2 S C_T,\\
D &= 0.5\rho V^2 S C_D,\\
L &= 0.5\rho V^2 S C_L,\\
M_y &= 0.5\rho V^2 S\bar{c}\cdot(C_M(\alpha) + C_M(\delta_e) + C_M(q)),
\end{aligned} \tag{94}$$

and the aerodynamic parameters$C_*$are chosen as:

$$\begin{aligned}
&C_T = C_{T,\beta}\beta + C_{T,0}, C_D = C_{D,\alpha^2}\alpha^2 + C_{D,\alpha}\alpha + C_{D,0},\\
&C_L = C_{L,\alpha}\alpha, C_M(\alpha) = C_{M,\alpha^2}\alpha^2 + C_{M,\alpha}\alpha + C_{M,0},\\
&C_M(\delta_e) = C_{M,\delta_e}(\delta_e - \alpha), C_M(q)\\
&\quad = \frac{\bar{c}q}{2V}(C_{Mq,\alpha^2}\alpha^2 + C_{Mq,\alpha}\alpha + C_{Mq,0}).
\end{aligned} \tag{95}$$

Aerodynamic coefficients, force coefficients and physics parameters are referred to [35].

The control objective is to ensure that the closed-loop system is stable and the state V and h can track a given the reference signals V_{ref} and h_{ref} in the presence of aerodynamic uncertainty and input constraint.

As shown in [36–38], the control input for altitude and velocity can be designed separately based on the backgrounds of hypersonic vehicle.

Substituting (94) and (95) into (93), the dynamic of velocity can be transformed into the following form

$$\dot{V} = F_V + \Phi_{V1}^T\theta_V + \Phi_{V2}\theta_V\beta. \tag{96}$$

And the other states dynamics of (93) can be transformed into the following form

$$\begin{aligned}
\dot{h} &= V\sin\gamma,\\
\dot{\gamma} &= F_\gamma + \Phi_{\gamma 1}\theta_\gamma + \Phi_{\gamma 2}\alpha,\\
\dot{\alpha} &= q - \dot{\gamma},\\
\dot{q} &= \Phi_{q1}^T\theta_q + \Phi_{q2}\delta_e,
\end{aligned} \tag{97}$$

where unknown aerodynamic parameter vectors are

$$\begin{aligned}
\theta_V &= \left[C_{T0}, C_{D\alpha^2}, C_{D\alpha}, C_{D0}\right]^T,\\
\theta_\gamma &= C_{T0},\\
\theta_q &= \left[C_{M\alpha^2}, C_{M\alpha} - c_e, C_{M0}, C_{Mq\alpha^2}, C_{Mq\alpha}, C_{Mq0}\right]^T,
\end{aligned} \tag{98}$$

and functions F_V, F_γ, F_q, Φ_{V1}, Φ_{V2}, $\Phi_{\gamma 1}$, $\Phi_{\gamma 2}$, Φ_{q1}, Φ_{q2} are

$$\begin{aligned}
F_V\left(x, x_{ref}\right) &= -g\sin\gamma,\\
F_\gamma &= -\frac{g}{V}\cos\gamma + \frac{\rho V S C_{T\beta}}{2m}\beta\sin\alpha,\\
\Phi_{V1} &= \frac{\rho V^2 S}{2m}\left[\cos\alpha, -\alpha^2, -\alpha, -1\right]^T,\\
\Phi_{V2} &= \frac{\rho V^2 S C_{T\beta}\cos\alpha}{2m},\\
\Phi_{\gamma 1} &= \frac{\rho V S\sin\alpha}{2m}, \quad \Phi_{\gamma 2} = \frac{\rho V S C_{L\alpha}}{2m},\\
\Phi_{q1} &= \frac{\rho V^2 S\bar{c}}{2I_{yy}}\left[\alpha^2, \alpha, 1, \bar{c}q\alpha^2/2V, \bar{c}q\alpha/2V, \bar{c}q/2V\right]^T,\\
\Phi_{q2} &= \frac{\rho V^2 S\bar{c}c_e}{2I_{yy}}.
\end{aligned} \tag{99}$$

Based on the preceding section, the control inputs are designed for (96) and (97).

The simulation is done to test the designed control strategy. In the simulation, aerodynamic coefficients, force coefficients and physics parameters in the simulation study are referred to [35]. The input constraints are set as $\beta \in [0.1,\ 1.2]$, $\delta_e \in [-0.2618,\ 0.2618]$ rad. The reference signals V_{ref} and are set as 1000 ft/s step-velocity command, and the altitude is kept at the trimmed flight condition, i.e., $h_{ref} = 110000$ ft. The reference commands are generated by a second-order prefilter with a natural frequency 0.03 rad/s and a damping factor 0.95.

Remark 5: To the best of our knowledge, it is important and necessary for the hypersonic vehicle to achieve the stable tracking in short time(in finite time or fixed time). In this paper, by using tuning controller parameters, the stable tracking control is obtained in short time. And we only get the fast response speed in the simulation result without theoretical guarantee. Therefore, in the further work, to achieve the theoretical results and simulation results of fast performance of the designed control scheme, we will do the research based on the results of this paper to obtain the finite time or fixed time of the proposed system and apply it into hypersonic vehicle flight control system.

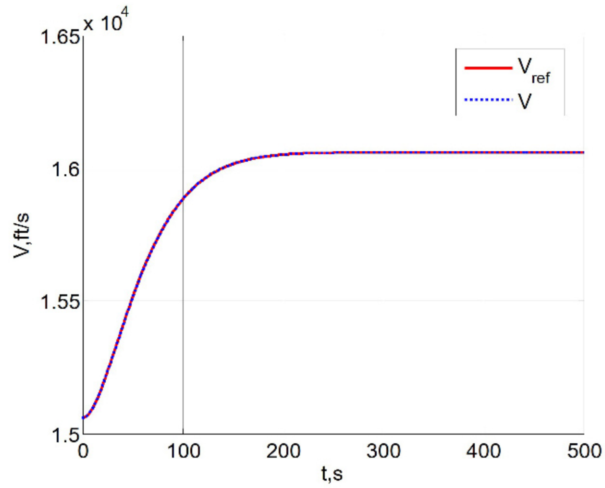


Fig. 3. Time response of outputs velocity.

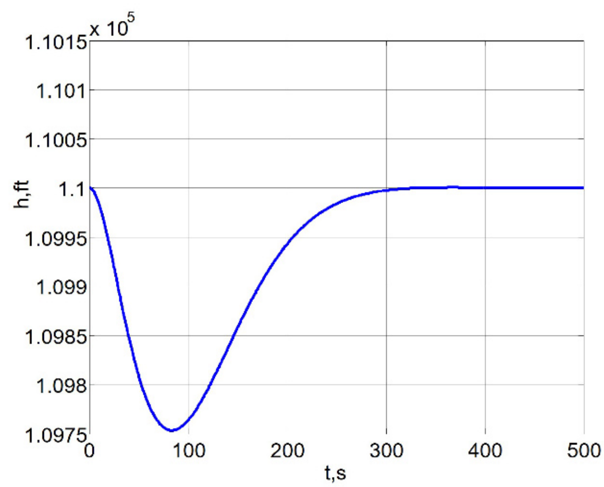


Fig. 4. Time response of outputs altitude.

The simulation results are shown in Figs. 3-6,which demonstrate that the designed controller can achieve the stable tracking of vehicle and altitude in the presence of input constraint and aerodynamic uncertainty.

5. CONCLUSION

In this paper, a Lyapunov redesign approach is employed to synthesize backstepping control to construct the stabilizing control input for a class of nonlinear system with unknown parameters, uncertainty and input constraint. The Lyapunov redesign approach is used to cancel the uncertainty. Robust second-order filters are employed to eliminate the “explosion of terms” problem inherent in the traditional backstepping control. The auxiliary system is constructed to analyze the input constraint, and the state of it is utilized to design control law and the closed-loop system analysis. Simulation results are conducted to show

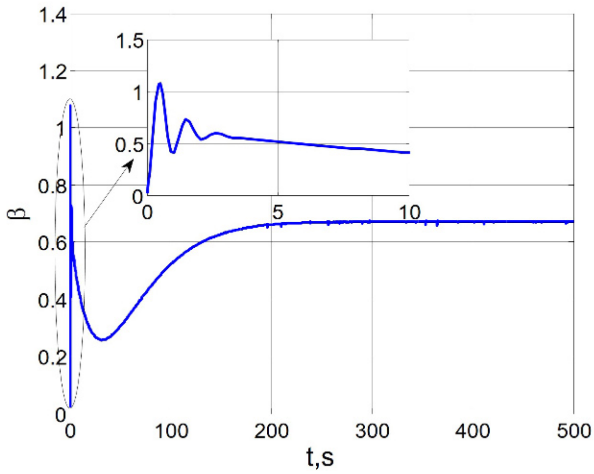


Fig. 5. Time response of control input.

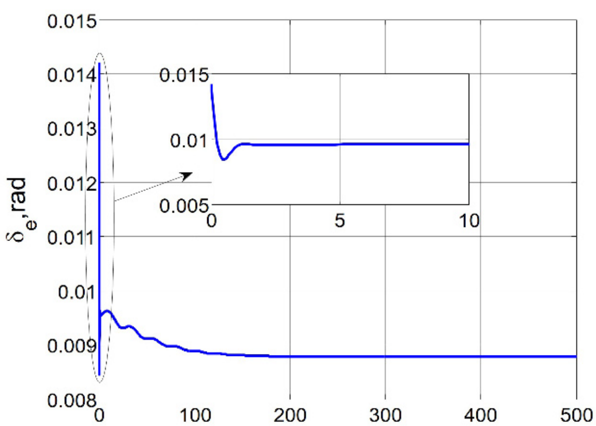


Fig. 6. Time response of control input.

that the developed controller handles the input constraint effectively and the system trajectory tracks its command.

REFERENCES

[1] H. K. Khalil, *Nonlinear systems*, New Jersey: Prentice Hall, 2002.

[2] G. Montaseri and M. J. Yazdanpanah, "Adaptive control of uncertain nonlinear systems using mixed backstepping and Lyapunov redesign techniques," *Commun Nonlinear Sci Numer Simulat*, vol. 17, no.8, pp. 3367-3380, August 2012. [click]

[3] M. A. Hamida and A. G. Jesus de Leon, "Robust integral backstepping control for sensorless IPM synchronous motor controller," *Journal of the Franklin Institute*, vol. 349, no.5, pp.1734-1757, June 2012.

[4] D. J. Lee, "Distributed backstepping control of multiple thrust-propelled vehicles on a balanced graph," *Automatica*, vol. 48, no.11, pp. 2971-2977, November 2012. [click]

[5] F. Zhang, G. R. Duan, and B. Zhou, "Robust adaptive control for a class of cascaded nonlinear systems with applications to space interception," *Internal Journal of Robust and Nonlinear Control*, vol. 24, no.15, pp. 1-31, October 2014.

[6] M. I. El-Hawwary and M. Maggiore, "Reduction theorems for stability of closed sets with application to backstepping control design," *Automatica*, vol. 49, no.1, pp. 214-222, January 2013.

[7] D. Won and W. Kim, "Disturbance observer based backstepping for position control of electro-hydraulic systems," *International Journal of Control, Automation, and Systems*, vol. 13, no.2, pp. 488-493, April 2015. [click]

[8] F. Bribiesca-Argomedo, and M. Krstic, "Backstepping-forwarding control and observation for hyperbolic PDEs with fredholm integrals," *IEEE Trans. on Automatic Control*, vol. 60, no.8, pp. 1-10, February 2015.

[9] Q. K. Shen, and P. Shi, "Distributed command filtered backstepping consensus tracking control of nonlinear multiple-agent systems in strict-feedback form," *Automatica*, no.53, pp. 120-124, March 2015. [click]

[10] Y. Song, X. S. Fang and Q. D. Diao, "Mixed H-2/H-infinity distributed robust model predictive control for polytopic uncertain systems subject to actuator saturation and missing measurements," *Internal Journal of Systems Science,* vol. 47, no. 4, pp.777-790, April 2016. [click]

[11] A. Isidori, *Nonlinear control systems*, Berlin: Springer, 1995.

[12] R. Ortega, A. Vander Schaft, I. Maschke, and E. Gerardo, "Putting energy back in control," *IEEE Control Systems Magazine*, vol. 21, no.2, pp. 18-33, August 2002.

[13] J. Song and S.P. He, "Finite-time robust passive control for a class of uncertain Lipschitz nonlinear systems with time-delays," *Neurocomputing*, vol.159, no.2, pp. 275-281, July 2015. [click]

[14] H. Zeng, Y. He, M. Wu and S.P. Xiao, "Passivity analysis for neural networks with a time-varying delay," *Neurocomputing*, vol, 74, no.5, pp. 730-734, February 2011. [click]

[15] S. He and F. Liu, "Finite-time H-fuzzy control of nonlinear jump systems with time delays via dynamic observer-based state feedback," *IEEE Trans. Fuzzy Syst*. vol. 20, no.4, December2012.

[16] J. Na, M. N. Mahyuddin, G. Herrmann, and P. Barber, "Robust adaptive finite-time parameter estimation and control for robotic systems," *International Journal of Robust and Nonlinear Control*, vol. 25,no. 16, pp. 3045-3071, November 2015. [click]

[17] S.T. Wang and J. T. Fei, "Robust adaptive sliding mode control of MEMS gyroscope using T-S fuzzy model," *Nonlinear Dyn*, vol. 77, no. 1-2, pp. 361-371, February 2014. [click]

[18] Z. W. Qiao, T. N. Shi, Y. D. Wang and C. Xia, "New sliding-mode observer for position sensorless control of permanent-magnet synchronous motor," *IEEE Trans.on Industrial Electronics*, vol. 60, no.2, pp. 710-719, February 2013.

[19] H. J. Xu, H.J., M.D. Mirmirani and P.A. Ioannou, "Adaptive sliding mode control design for a hypersonic flight vehicle," *Journal of Guidance Control and Dynamics* , vol.7, no.5, pp. 829-838, December 2014.

[20] M. Kristic, I. Kanellakopoulos and P. Kosotovic, *Nonlinear and adaptive control design*, Wiley & Sons, New York, 1995:1-598.

[21] P. Kokotovic and M. Arcak, "Survey constructive nonlinear control: a historical perspective," *Automatica*, vol.37, no.5, pp. 637-662, May 2001. [click]

[22] W. J. Dong, J. A. Farrell, M. M. Polycarpou and V. Djapic, "Command filtered adaptive backstepping," *IEEE Trans. on Control Systems Technology*, vol. 20, no.3, pp. 566-580, May 2012.

[23] M. Chen, S. S. Ge, and B. B. Ren, "Adaptive tracking control of uncertain MIMO nonlinear systems with input constraints," *Automatica*, vol. 47, no.3, pp. 452-465, March 2011. [click]

[24] H. Li, L. H. Dou, and Z. Su, "Adaptive dynamic surface based nonsingular fast terminal sliding mode control for semistrict feedback system," *Journal of Dynamics, System, Measurements, and Control*, no.134, pp. 1-9, May2012.

[25] N. O. Perez-Arancibia, T. C. Tsao, and J. S. Gibson, "Saturation induced instability and its avoidance in adaptive control of hard disk drives," *IEEE Trans. on Control System Technology*, vol. 18, no. 2, pp. 368-382, September 2010.

[26] C. Y. Wen, J. Zhou , Z. T. Liu and H Su, "Robust adaptive control of uncertain nonlinear systems in the presence of input saturation and external disturbance," *IEEE Trans. on Automatic Control*, vol. 56, no. 7, pp. 1672-1678, March 2012.

[27] E. G. Gilbert, and C. J. Ong, "Constrained linear systems with hard constraints and disturbances: an extended command governor with large domain of attraction," *Automatica*, no. 47, pp. 334-340, February 2011. [click]

[28] D. Bernardini, and A. Bemporad, "Stabilizing model predictive control of stochastic constrained linear systems," *IEEE Trans. on Automatic Control*, vol. 5, no. 6, pp. 1468-1480, June2012.

[29] Y. J. Maruki, K. Kawano, H. Suemitsua and T. Matsuo, "Adaptive backstepping control of wheeled inverted pendulum with velocity estimator," *International Journal of Control, Automation, and Systems*, vol.12, no. 5, pp.1040-1048, August 2014. [click]

[30] J. S. Huang, C. Y. Wen, W. Wang and Z. P. Jiang, "Adaptive stabilization and tracking control of a nonholonomic mobile robot with input saturation and disturbance," *Systems & Control Letters*, vol. 62, no. 3, pp. 234-241, March 2013. [click]

[31] X. W. Bu, X. Y. Wu, Y. X. Chen and R.Y. Bai, "Design of a class of new nonlinear disturbance observers based on tracking differentiators for uncertain dynamic systems," *International Journal of Control, Automation, and Systems*, vol. 13, no. 3, pp. 595-602, March 2015. [click]

[32] R. Y. Yuan, X. M. Tan, G. L. Fan and J. Yi, "Robust adaptive neural network control for a class of uncertain nonlinear systems with actuator amplitude and rate saturations," *Neurocomputing*, vol. 125, no.3, pp. 72-80, February 2014. [click]

[33] M. M. Polycarpou and P. A. Ioannou, "A robust adaptive nonlinear control design," *Automatica,* vol. 32, no.3, pp. 423-427, March 1996.

[34] Z. D. Wilcox, W. MacKunis, S. Bhat and R Lind, "Robust nonlinear control of a hypersonic aircraft in the presence of aerothermoelastic effects," *Proc. of American Control Conference*, Hyatt Regency Riverfront, St. Louis, MO, USA, 2533-2538,2009.

[35] H. J. Xu and M. Mirmirani, "Adaptive sliding mode control design for a hypersonic flight vehicle," *Journal of Guidance, Control, and Dynamics*, vol. 27, no.5, pp. 829-838, December 2014. [click]

[36] Q. Zong, F. Wang, and B. L. Tian, "Robust adaptive approximate backstepping control design for a flexible airbreathing hypersonic vehicle," *Journal of Aerospace Engineering*, vol. 28, no.4, pp. 1-14, July 2015.

[37] L. Fiorentini, A. Serrani, M. Bolender and DB Doman, "Nonlinear robust adaptive control of flexible air-breathing hypersonic vehicles," *Journal of Guidance, Control, and Dynamics*, vol. 32, no. 2, pp. 401-415, March-April 2009. [click]

[38] Q. Zong, F. Wang, R. Su and S.K. Shao, "Robust adaptive backstepping control design for a flexible air-breathing hypersonic vehicle subject to input constraint," *Proc IMechE Part G: Journal of Aerospace Engineering*, vol. 229, no.1, pp. 10-25, January 2015. [click]

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