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Design of Sliding Mode Guidance Law with Dynamic Delay and Impact Angle Constraint

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Abstract: A sliding mode guidance law with dynamic delay and impact angle constraints is designed for the relative motion between the missile and the target in the intercepting plane. First of all, the missile's first order dynamic delay is involved into the system model to design the guidance law based on sliding mode variable dynamic method. Secondly, the target's maneuvering is taken as the system disturbance, and a non-homogeneous disturbance observer is applied to estimate such maneuvering in finite time rapidly, which, through dynamic compensation, realizes the missiles precision attack to targets of different maneuvering at a desired line-of-sight (LOS) angle. Finally, numerical simulations are performed to demonstrate the effectiveness of the designed guidance law.

Keywords: Autopilot, guidance law, impact angle, observer, sliding mode.

1. INTRODUCTION

With the constant development of precision-guided munition, the missile is required to hit a target at a specific impact angle, for the maximum efficacy of the missile's warhead and the best damage effect. For example, anti-ship missile is expected to hit a ship's ammunition section and power section at a specific angle; anti-tank missile to hit a tank's weak part on its top at a large impact angle; and penetrating bomb to get into or through the target vertically when it attacks a multi-story building or a underground bunker. Therefore, it is necessary to make further study on the guidance law with terminal impact angle constraints, for this special guidance task.

In 1973, Kim and Grider [1] proposed optimal guidance laws with impact angle constraint for the motion of reentry vehicle in the vertical plane based on a linear model. Since then, the design of guidance law with impact angle constraint has attracted broad attention of the scholars, and there have been certain achievements. In [2,3], a guidance law to overcome the ship's short range defense system was designed, and applied it to salvo attack of multiple missiles, while it was for still targets or those in uniform motion. Shima [4] studied sliding mode variable structure control to study the impact angle constraint in three scenarios of interception. During the realization of guidance law, the saturation function is used to substitute the sign function to reduce the chattering behavior of sliding mode manifold; however, the sliding mode manifold converges to the boundary layer of the saturation function, but not to zero. In [5, 6], a terminal sliding mode manifold that includes LOS angle and LOS angle rate was proposed, to design a finite-time convergent sliding mode guidance law, but it does not include the issue of singularity of the terminal sliding mode. Rao *et al*. [7] designed two guidance laws corresponding to two linear sliding mode manifolds for switching between two control laws, to realize all-round attack to the target. All the literatures above are based on bounded estimation of the target's maneuvering, and compensate the target's unknown maneuvering in the system by switching function. However, there tends to be excessive constraints when the boundary of the target's maneuvering is too large, while, the system is unstable when it is too small. Another method to deal with the target's maneuvering in the system is to employ an observer for tracking and observation. For example, integral sliding mode manifold in combination with observer to design guidance law was presented in [8, 9], which avoided estimating the target's maneuvering in the system.

During terminal guidance, in addition to the target's escape maneuvering, the dynamic delay of missile autopilot is another main factor influencing the precision of guidance. The time constant of the dynamic response process of missile autopilot determines the response time of the missile to the acceleration command given by the guidance law. A too large time constant will cause an exces-

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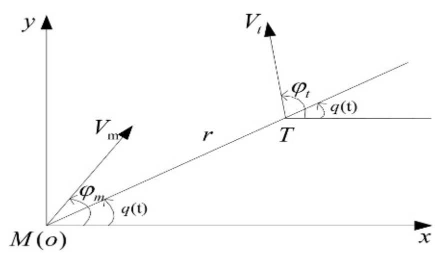

Fig. 1. Relative motion geometry of missile and target.

sively long response time of the missiles acceleration tracking guidance command, which will significantly affect the guidance performance in the critical terminal phase of guidance. Therefore, it is of great engineering significance to consider the missile's dynamic delay in designing guidance law. At present, the main methods of this study include dynamic surface control [10], backstepping [11], and directly considering the influence of the missile's dynamic behavior in designing guidance law [8, 12].

Due to the preferable robustness of sliding mode variable structure control to parameter perturbation and external disturbance, it has been widely applied in missile guidance control. Shtessel *et al.* [13] proposed a sliding mode variable dynamic method, which allows the system state and its derivative to converge to zero in finite time. Inspired by this method, we select a linear sliding mode manifold that includes LOS angle and LOS angle rate, to design a sliding mode guidance law with dynamic delay and impact angle constraint. During the realization of this guidance law, a non-homogeneous disturbance observer is applied to perform finite-time tracking and estimation of the target's total disturbance in the system. The results of numerical stimulation show that the missile can intercept a target with high speed and great maneuvering at a desired LOS angle by the designed guidance law.

2. PROBLEM DESCRIPTION

Consider the relative motion of the missile and the target in the intercepting plane, both of which are regarded as point masses, represented by M and T respectively, and their connecting line is the line-of-sight (LOS), as shown in Fig. 1.

Based on Fig. 1, we can get the equation description of the relative motion between the missile and the target [14]:

$$\dot{r} = V_t \cos(q-\varphi_t) - V_m \cos(q-\varphi_m), \tag{1}$$

$$r\dot{q} = -V_t \sin(q-\varphi_t) + V_m \sin(q-\varphi_m). \tag{2}$$

Furthermore, we can get

$$\ddot{q} = -\frac{2\dot{r}}{r}\dot{q} - \frac{1}{r}a_m + \frac{1}{r}a_t, \tag{3}$$

where $a_t = V_t\cos(q-\varphi_t)\dot{\varphi}_t$, $a_m = V_m\cos(q-\varphi_m)\dot{\varphi}_m$, r and $\dot{r}$ are the relative distance and the relative velocity from the missile to the target, which are regarded as known time-varying parameters; q, $\dot{q}$ represent the LOS angle and the LOS angle rate respectively; φ_m, φ_t denote the missile's and the target's speed direction angles respectively; and a_m, a_t represent the component of the acceleration of the missile and that of the target in the direction normal to the LOS, which are regarded as the system's control input and unknown external disturbance input respectively.

According to the principle of quasi-parallel approaching method, the key to guidance law design is to control the LOS angle rate $\dot{q}$ to approach zero, to ensure hitting the target precisely. For the missile flight control system, if the autopilot's dynamics model is a linear first order inertia equation, then

$$\dot{a}_m = -\frac{1}{\tau}a_m + \frac{1}{\tau}u, \tag{4}$$

where τ is the time constant of missiles autopilot, and u denotes the guidance command acceleration given to missile autopilot.

As it is restrained by acceleration capability, the maximum lateral acceleration that can be actually provided by the missile and the target is limited. During terminal guidance, as it is restrained by the power of the angle tracking system, receiver acceleration and other factors, the seeker has a minimum operating range r_0. When the relative distance between the missile and the target is no more than r_0, the guidance circuit will be open [15]. The start time of terminal guidance is noted as 0, and without loss of generality, we can assume the conditions as follows for the variables in the system.

Assumption 1: There exist constants $A_m > 0$, $A_t > 0$, $A_1 > 0$ and $A_2 > 0$, which make

$$|a_m| \le A_m, \ \ |a_t| \le A_t, \ \ |\dot{a}_t| \le A_t, \ \ |\ddot{a}_t| \le A_2. \tag{5}$$

Assumption 2: The time-varying parameter $r(t)$ in system (3) satisfies:

$$r(t) \ge r_0. \tag{6}$$

To further design the guidance law, relevant definitions and lemmas are given as follows:

Definition 1 [16]**:** Let $f(x) : \mathbb{R}^n \to \mathbb{R}^n$ be a vector function. For any $\varepsilon > 0$, if there is $(r_1, r_2, ..., r_n)^{\mathrm{T}} \in \mathbb{R}^n$ that makes $f(x)$ satisfy

$$f_i(\varepsilon^{r_1}x_1, \varepsilon^{r_2}x_2, \cdots, \varepsilon^{r_n}x_n) = \varepsilon^{k+r_i} f_i(x), i = 1, 2, \cdots, n, \tag{7}$$

where $r_i > 0$, $(i = 1, 2, ..., n)$, $k \ge -\max\{r_i,\ i = 1, 2, ..., n\}$, then $f(x)$ is said to be homogeneous of degree k for $(r_1, r_2, ..., r_n)$. Thus, system $\dot{x} = f(x)$ is a homogeneous system.

Lemma 1 [16]**:** Suppose that the system $\dot{x} = f(x)$, $(x \in \mathbb{R}^n)$ is homogeneous of degree $k < 0$ for extension coefficients $(r_1, r_2, \ldots, r_n)$, and $f(x)$ is continuous with $x = 0$ as its asymptotically stable equilibrium point, then the system's equilibrium point is globally finite time stable.

Lemma 2 [17] (Lassalle's Invariant Principle)**:** Suppose a system $\dot{x} = f(x)$, where $f : \mathbf{D} \to \mathbb{R}^n$ is a local Lipschitz mapping of the domain $\mathbf{D} \subset \mathbb{R}^n$ onto $\mathbb{R}^n$; suppose that $\mathbf{\Omega} \subset \mathbf{D}$ is a positively invariant compact set of the system equation, and $V(x) : \mathbf{D} \to \mathbb{R}^n$ a continuous and differentiable function, which satisfies $\dot{V}(x) \leq 0$ within $\mathbf{\Omega}$; and suppose that M is the set of all the points that satisfies $\dot{V}(x) = 0$ in $\mathbf{\Omega}$, and $\mathbf{M}'$ the maximum invariant set in $\mathbf{M}$, then, when $t \to \infty$, any solution starting from $\mathbf{\Omega}$ will approach $\mathbf{M}'$.

3. DESIGN AND REALIZATION OF GUIDANCE LAW

Define $x_1 = q - q_d$, $x_2 = \dot{x}_1$ and $x_3 = \dot{x}_2 - \ddot{q}$, where q_d is the desired LOS angle during the guidance of missile, then the guidance (3) can be rewritten as:

$$x_3 = \dot{x}_2 = -\frac{2\dot{r}}{r}x_2 - \frac{1}{r}a_m + \frac{1}{r}a_t, \tag{8}$$

$$\frac{1}{r}a_t = \frac{1}{r}a_m + \frac{2\dot{r}}{r}x_2 + x_3. \tag{9}$$

Taking the derivative of (8), and substituting (4), we can get:

$$\begin{aligned}\dot{x}_3 =& \left(-\frac{2\ddot{r}}{r} + \frac{2\dot{r}^2}{r^2}\right)x_2 - \frac{2\dot{r}}{r}x_3 + \left(\frac{\dot{r}}{r^2} + \frac{1}{r\tau}\right)a_m \\ & - \frac{1}{r\tau}u - \frac{\dot{r}}{r^2}a_t + \frac{1}{r}\dot{a}_t.\end{aligned} \tag{10}$$

Substituting (9) into (10), we can rewrite it as

$$\dot{x}_3 = -\frac{2\ddot{r}}{r}x_2 - \frac{3\dot{r}}{r}x_3 + \frac{1}{r\tau}a_m - \frac{1}{r\tau}u + \frac{1}{r}\dot{a}_t. \tag{11}$$

Let $f_0(x_3, a_m) = -\frac{3\dot{r}}{r}x_3 + \frac{1}{r\tau}a_m$, $f_1(x_2, \dot{a}_t) = -\frac{2\ddot{r}}{r}x_2 + \frac{1}{r}\dot{a}_t$, $b = -\frac{1}{r\tau}$. According to (8) and (11), the system equation can be rewritten as

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = x_3, \\ \dot{x}_3 = f_0(x_3, a_m) + f_1(x_2, \dot{a}_t) + bu. \end{cases} \tag{12}$$

For system (12), we select the linear sliding mode manifold

$$s = k_1 x_1 + k_2 x_2 + x_3, \tag{13}$$

where parameter k_1, k_2 satisfies the polynomial $p^2 + k_2 p + k_1$ as Hurwitz determinant, in which p is the Laplace operator.

Taking the derivative of (13), we can get:

$$\begin{aligned}\dot{s} =& k_1\dot{x}_1 + k_2\dot{x}_2 + \dot{x}_3 \\ =& k_1 x_2 + k_2 x_3 + f_0(x_2, a_m) + f_1(x_1, \dot{a}_t) + bu.\end{aligned} \tag{14}$$

Inspired by literature [13], we select

$$\begin{cases} \dot{s} = -\alpha_1 |s|^{\frac{p-1}{p}} \operatorname{sgn}(s) + w(t), \\ \dot{w}(t) = -\alpha_2 |s|^{\frac{p-2}{p}} \operatorname{sgn}(s), \quad p \geq 2, \end{cases} \tag{15}$$

where $\operatorname{sgn}(\cdot)$ is the sign function.

Combined with (14) and (15), we design the guidance law as

$$\begin{cases} u = -\frac{1}{b}\Big(f_0(x_3, a_m) + f_1(x_2, \dot{a}_t) + k_1 x_2 \\ \qquad + k_2 x_3 + \alpha_1 |s|^{\frac{p-1}{p}} \operatorname{sgn}(s) - w(t)\Big), \\ \dot{w}(t) = -\alpha_2 |s|^{\frac{p-2}{p}} \operatorname{sgn}(s), \quad p \geq 2. \end{cases} \tag{16}$$

Theorem 1: For guidance system (12), a sliding mode guidance law (16) with impact angle constraints is designed, so that and converge to zero asymptotically in case of an autopilot with first order delay.

Proof: For system (15), we select Lyapunov function

$$V(t) = \frac{p}{2(p-1)}\alpha_2 |s|^{\frac{2(p-1)}{p}} + \frac{1}{2}w(t)^2. \tag{17}$$

It is not difficult to verify that $V(t)$ is positive definite and continuously differentiable. Taking the derivative of (17), and substituting the guidance law (16), we can get:

$$\begin{aligned}\dot{V}(t) =& \alpha_2 |s|^{\frac{p-2}{p}} \dot{s}\operatorname{sgn}(s) + w(t)\dot{w}(t) \\ =& \alpha_2 |s|^{\frac{p-2}{p}} \operatorname{sgn}(s)(k_1 x_2 + k_2 x_3 + f_0(x_2, a_m) \\ & + f_1(x_1, \dot{a}_t) + bu) + w(t)\dot{w}(t) \\ =& \alpha_2 |s|^{\frac{p-2}{p}} \operatorname{sgn}(s)(-\alpha_1 |s|^{\frac{p-1}{p}} \operatorname{sgn}(s) + w(t)) \\ & + w(t)(-\alpha_2 |s|^{\frac{p-2}{p}} \operatorname{sgn}(s)) \\ =& -\alpha_1\alpha_2 |s|^{\frac{2p-3}{p}}.\end{aligned} \tag{18}$$

According to Lemma 2, $\dot{V}(t) = 0$ contains the unique solution $s = 0$, therefore, system (18) gradually converges to zero. Based on Definition 2, it is easy to verify that the system (15) is homogeneous of degree-1. According to Lemma 1, s and$\dot{s}$ in system (15) converge to zero in finite time. When $s = k_1 x_1 + k_2 x_2 + x_3 = 0$, we can get$k_1 x_1 + k_2 \dot{x}_1 + \ddot{x}_2 = 0$, and the corresponding equation $\lambda^2 + k_2\lambda + k_1 = 0$. Since k_1 and k_2 satisfy the polynomial $p^2 + k_2 p + k_1$ as Hurwitz determinant, we can get $x_1(t) = c_1 e^{-\lambda_1 t} + c_2 e^{-\lambda_2 t}$, and $k_1 = \lambda_1\lambda_2$, $k_2 = \lambda_1 + \lambda_2$, $\lambda_1 > 0$, $\lambda_2 > 0$ only when two different latent roots are selected. Therefore, when $t \to \infty$, $x_1(t) \to 0$, which means that the error of desired LOS angle asymptotically converges to zero; and $x_2 = \dot{x}_1 = -\lambda_1 c_1 e^{-\lambda_1 t} - \lambda_2 c_2 e^{-\lambda_2 t}$, $x_3 = \dot{x}_2 = \lambda_1^2 c_1 e^{-\lambda_1 t} + \lambda_2^2 c_2 e^{-\lambda_2 t}$, which means that the

LOS angle rate and the LOS angle acceleration exponentially converge to zero .

Remark 1: In guidance law (16), the system's total disturbance $f_1(x_1,a) = -\frac{2\ddot{r}}{r}x_1 + \frac{1}{r}\dot{a}_t$ is unknown. A method in previous literatures is to use a switching function "$\eta\,\mathrm{sgn}(s)$" for disturbance attenuation, where η is greater than the boundary of total disturbance. However, there will be more variables to control when η is too great; while it is possible to result in an unstable control system when it is too small. Another method is to use observer to track and estimate the disturbance in the system. This paper uses non-homogeneous disturbance observer to track and estimate the system's total disturbance in finite time, in order to realize the guidance law (16).

Considering the first order SISO non-linear system

$$\dot{s} = g(t) + u, \quad s \in \mathbb{R}. \tag{19}$$

Equation (19) describes the dynamic characteristics of the sliding mode along the trajectory of the system. $s = 0$ defines the system's motion on the sliding mode manifold, $u \in \mathbb{R}$ is the continuous control input, and $g(t)$ is a sufficiently smooth indeterminate function. This control aims at designing a continuous control u, to make s and $\dot{s}$ converges to zero in finite time.

If the sliding mode variable s and the control input u can be obtained in real time, $g(t)$ is $m-1$ order differentiable, and $g^{m-1}(t)$ has a known Lipschitz constant L. Based on Levant's non-homogeneous differentiator [18], literature [19] proposes non-homogeneous disturbance observer to fasten the transient process, whose form is as follows:

$$\begin{cases} \dot{z}_0 = v_0 + u, v_0 = h_0(z_0 - s) + z_1, \\ \dot{z}_1 = v_1, v_1 = h_1(z_1 - v_0) + z_2, \\ \vdots \\ \dot{z}_{m-1} = v_{m-1}, v_{m-1} = h_{m-1}(z_{m-1} - v_{m-2}) + z_m, \\ \dot{z}_m = h_m(z_m - v_{m-1}), \end{cases} \tag{20}$$

where h_i is a function with a form as follows:

$$h_i(\sigma) = -\lambda_{m-i} L^{\frac{1}{m-i+1}} |\sigma|^{\frac{m-i}{m-i+1}} \mathrm{sgn}(\sigma) - \mu_{m-i}\sigma, \tag{21}$$

where λ_i, $\mu_i > 0$, $i = 0, 1, ..., m$.

Lemma 3 [19]**:** Suppose that $s(t)$ and $u(t)$ in the system of (19) are measurable without measurement noise, and the parameter λ_i, μ_i are sufficiently great in reverse order, the equations below will be valid after a transient process within finite time.

$$z_0 = s(t), z_1 = g(t), \cdots, z_i = v_{i-1} = g^{(i-1)}, \quad i = 1, \cdots, m.$$

During the terminal guidance, the change of $\dot{r}$ is not significant. Therefore, suppose $\dot{r} \approx$const, then $\ddot{r} = 0$, $f_1(x_1,\dot{a}_t) = -\frac{2\ddot{r}}{r}x_1 + \frac{1}{r}\dot{a}_t \approx \frac{1}{r}\dot{a}_t$. And from Assumption 1 and Assumption 2 we can obtain:

$$\begin{aligned} \left|\dot{f}_1(x_1,\dot{a}_t)\right| &= \left|\frac{\ddot{a}_t r - \dot{a}_t \dot{r}}{r^2}\right| \\ &= \left|\frac{\ddot{a}_t}{r} - \frac{\dot{a}_t(V_t\cos(q-\phi_t) - V_m\cos(q-\phi_m))}{r^2}\right| \\ &\le \frac{A_2}{r_0} + \frac{A_1 V_t^{\max}}{r_0^2} + \frac{A_1 V_m^{\max}}{r_0^2} \\ &= L, \end{aligned} \tag{22}$$

where $V_m^{\max}$ and $V_t^{\max}$ are the missile's maximum speed and the target's maximum speed respectively.

For this purpose, we track and estimate the total disturbance of system (14) within finite time, and we select a non-homogeneous disturbance observer as

$$\begin{cases} \dot{z}_0 = v_0 + k_1 x_2 + k_2 x_3 + f_0(x_2, a_m) + bu, \\ v_0 = -\lambda_2 L^{\frac{1}{3}} |z_0 - s|^{\frac{2}{3}} \mathrm{sgn}(z_0 - s) - \mu_2(z_0 - s) + z_1, \\ \dot{z}_1 = v_1, \\ v_1 = -\lambda_1 L^{\frac{1}{2}} |z_1 - v_0|^{\frac{1}{2}} \mathrm{sgn}(z_1 - v_0) - \mu_1(z_1 - v_0) + z_2, \\ \dot{z}_2 = -\lambda_0 L\mathrm{sgn}(z_2 - v_1) - \mu_0(z_2 - v_1), \end{cases} \tag{23}$$

then, in finite time,

$$z_0 = s, z_1 = \hat{f}_1(x_1, \dot{a}_t), \tag{24}$$

$\hat{f}(x_1,\dot{a}_t)$ is the estimation of $f_1(x_1,\dot{a}_t)$.

Therefore, based on Theorem 1, the realizable form of the guidance of system (12) is (25).

$$\begin{cases} u = -\frac{1}{b}(f_0(x_3, a_m) + \hat{f}_1(x_2, \dot{a}_t) + k_1 x_2 + k_2 x_3 \\ \qquad + \alpha_1 |s|^{\frac{p-1}{p}} \mathrm{sgn}(s) - w(t)), \\ \dot{w}(t) = -\alpha_2 |s|^{\frac{p-2}{p}} \mathrm{sgn}(s), \quad p \ge 2. \end{cases} \tag{25}$$

Remark 2: In (25), when is substituted into (14), we can get:

$$\begin{cases} \dot{s} = -\alpha_1 |s|^{\frac{1}{2}} \mathrm{sgn}(s) + w(t), \\ \dot{w}(t) = -\alpha_2 \mathrm{sgn}(s). \end{cases} \tag{26}$$

This is the so-called super-twisting algorithm, while it is non-smooth, and the sliding mode manifold approaches zero with chattering around zero, which impacts the guidance performance. The simulation below indicates that when $p > 2$, the performance of the guidance law designed in this paper is much better.

4. SIMULATIONS

Suppose that a missile flies at a specific height with a Mach number of 3.5, the sound velocity is 295.07 m/s, the

Table 1. Miss distances and interception time.

		$p=2$	$p=3$	$p=4$	$p=5$
$q_d=-20°$	miss distance (m)	0.076	0.088	0.049	0.080
	intersection times (s)	9.108	9.151	9.182	9.204
$q_d=60°$	miss distance (m)	0.086	0.036	0.050	0.071
	intersection times (s)	8.346	8.366	8.375	8.380

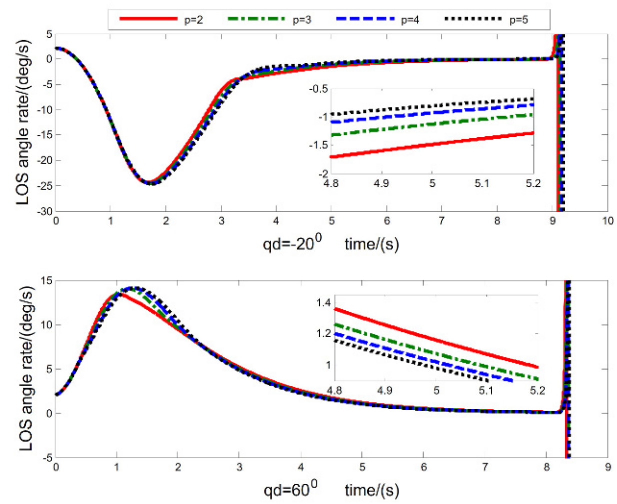

Fig. 2. Curve of LOS angle rate.

target's flight velocity is 900 m/s, and both the target and the missile are moving in a vertical plane. Suppose that at the initial time of terminal guidance, the missile's position in inertial system is $x_m(0)=0.5$ km, $y_m(0)=16$ km, the missile's and the target's initial trajectory deflection angles are $\varphi_m(0)=\varphi_t(0)=10°$, the target's initial position is $x_t(0)=1.5$ km, $y_t(0)=16.5$ km, the guidance distance of the seeker is $r_0=100$ m, and the limit of the missile's normal acceleration is $50g$, $g=9.8$ m/s^2. We consider two different scenarios of the target's maneuvering.

Scenario 1: the target performs cosine maneuvering $a_t=4g\cos(\pi t/4)$ in the normal direction. In guidance law of (25), the parameters selected are $\alpha_1=2$, $\alpha_2=3$, $k_1=2$, $k_2=3$ and $\tau=0.5$. The parameters selected for the observer are $\lambda_0=1.1$, $\lambda_1=1.5$, $\lambda_2=2$, $\mu_1=6$, $\mu_2=8$ and $L=10$. When $p=2$, 3, 4, 5, and the desired LOS angle are either $q_d=-20°$ or $q_d=60°$, the corresponding LOS angle rates, desired LOS angles, sliding mode manifolds, the missile's normal accelerations and the observer's estimate errors are shown in Figs. 2-7, and the corresponding miss distances and interception times given in Table 1.

As shown in Table 1, for different values of p, there is no significant difference of flight time or accuracy of miss distance between the two different LOS angles, and the designed guidance law (25) can make the missile hit the target precisely.

Fig. 2 shows the change rule of LOS angle rate, and in case of cosine maneuvering, the change curve of LOS angle rate converges to zero rapidly, ensuring that the missile can hit the target precisely. However, according to the two charts in Fig. 2, when $p=2$, in spite of the fact that the miss distance is very small and the desired LOS angle meets the requirement of guidance, there is slight saw-tooth shaped flutter in the corresponding curve of LOS angle rate. As shown in Figs. 3 to 6, when $p=2$, there is slight flutter in the corresponding curves of desired LOS angle, sliding mode manifold, the missile's normal acceleration and the observer's tracking error. Especially for the curve of disturbance estimate error, the initial error is quite great, resulting in the degradation of guidance performance. Moreover, for different values of the parameters selected for the guidance law, it is possible to result in greater chattering. Therefore, when $p=2$, the guidance law resulting from super-twisting algorithm is inferior to that when $p>2$. Meanwhile, for the other three values of $p>2$, the curve of sliding mode manifold and that of disturbance estimate error converge to zero in a stable, smooth and rapid manner, which indicates the effectiveness of the selected non-homogeneous disturbance observer. The curve of the missile's normal acceleration shows no saturation at the initial time of terminal guidance, and rapidly become stable within a small range around a fixed value. According to Fig. 3, for different values of p, the curve of desired LOS angle can rapidly converge to the corresponding desired value, with an error of desired LOS angle less than $0.1°$. Meanwhile, under the influence of the designed guidance law (25), the interval of desired LOS angle can be determined to be $[-20°, 60°]$ for the simulation condition in this paper, and the missile can hit the target precisely at any angle in this interval. Fig. 7 shows the curve of the guidance law. It can see that when $p=2$, the guidance law resulting from super-twisting algorithm is inferior to that when $p>2$.

Scenario 2: When $t=3s$, the target performs step maneuvering $a_t=4g$ in the normal direction, only $p=3$ is selected in guidance law (25), and three angles $q_d=-20°$, $30°$, $60°$ are selected as the desired LOS angle, with the simulation curves as below. The corresponding curves of LOS angle rate, LOS angle, sliding mode manifold and the missile's normal acceleration are shown in Fig. 8 and Fig. 9. The miss distances and interception times are given in Table 2.

According to Table 2, for the three different desired LOS angle when $p=2$, the missile can hit the target precisely at the corresponding desired LOS angle. Under this maneuvering, it is possible to perform simulation to con-

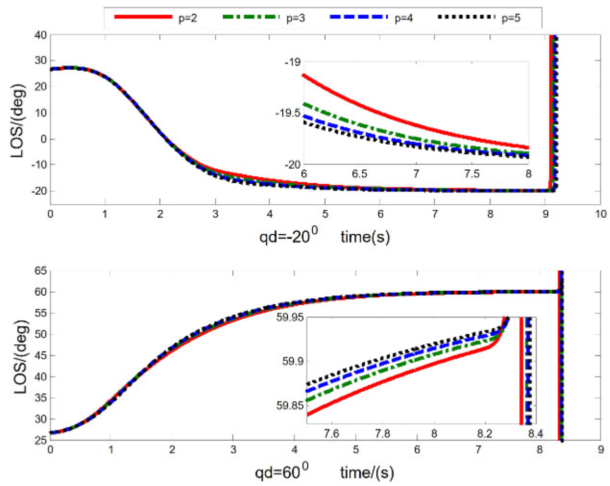

Fig. 3. Curve of LOS angle.

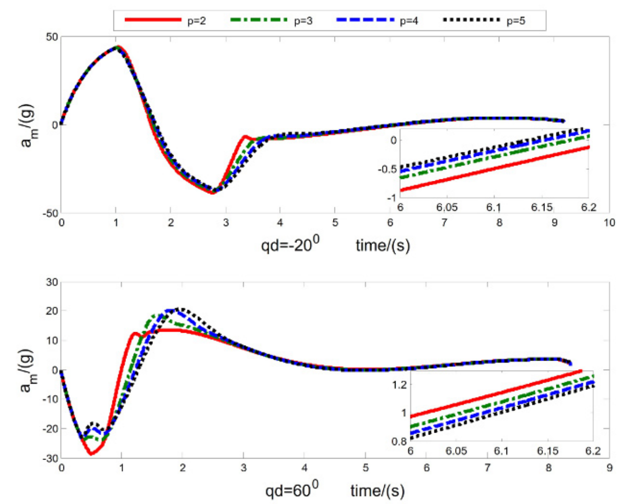

Fig. 5. Curve of missile normal acceleration.

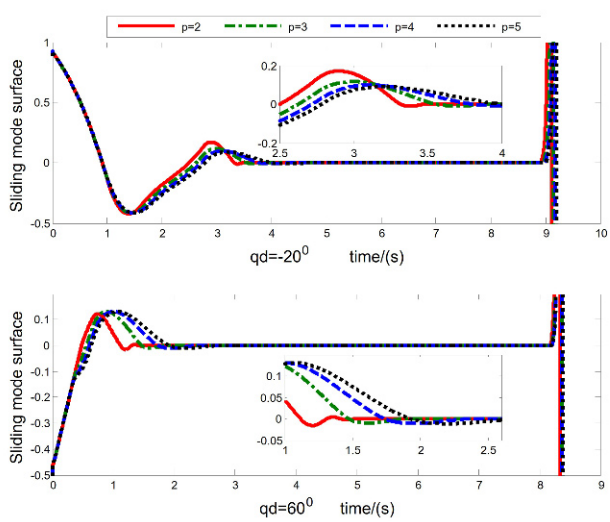

Fig. 4. Curve of sliding mode surface.

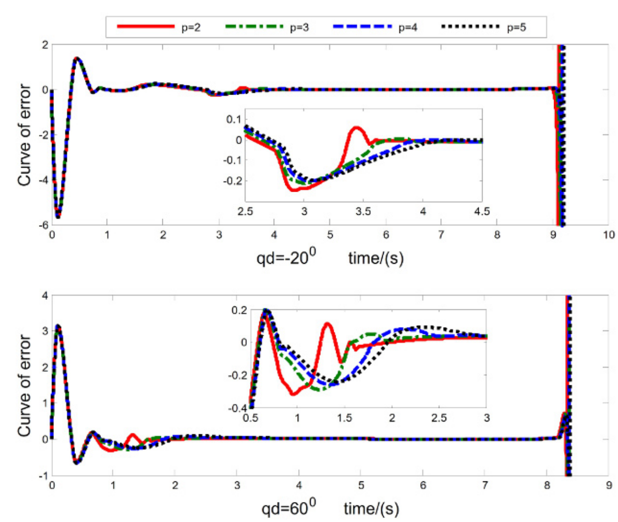

Fig. 6. Curve of tracking error.

Table 2. Miss distances and interception time.

		$q_d = -20^\circ$	$q_d = 30^\circ$	$q_d = 60^\circ$
$p = 3$	miss distances (m)	0.057	0.139	0.074
	intersection times (s)	8.434	8.267	9,188

firm that the interval of the desired LOS angle for the missile is still $[-20^\circ,\ 60^\circ]$.

According to Fig.8, the LOS angle rate (Fig. 8(a)) converges to zero, and the LOS angle (Fig. 8(b)) converge to corresponding desired value. As the target performs step maneuvering at 3s, the curve of sliding mode manifold shows a rapid step at 3s, but approaches zero rapidly, as shown in Fig. 9(a). Accordingly, the curve of the missile's normal acceleration shows a step reactivity, but with no significant change, and approaches a stable value later when it becomes smooth, as shown in Fig. 9(b). According to the curve of the observer's error shown in Fig. 9(c), it is obvious that there is a great step at 3s, however, the observer realizes tracking the target's maneuvering in a short time, and the curve of the observer's error converges to zero rapidly. To sum up, the results of simulation indicate that the designed guidance law and observer feature rapid responding, high precision guidance and tracking, and high adaption to the target's different maneuvering.

Scenario 3: The simulation compared with other guidance law

The guidance law (25) with is shorthand for LSMG. In the literature [8], by choosing integral sliding mode sur-

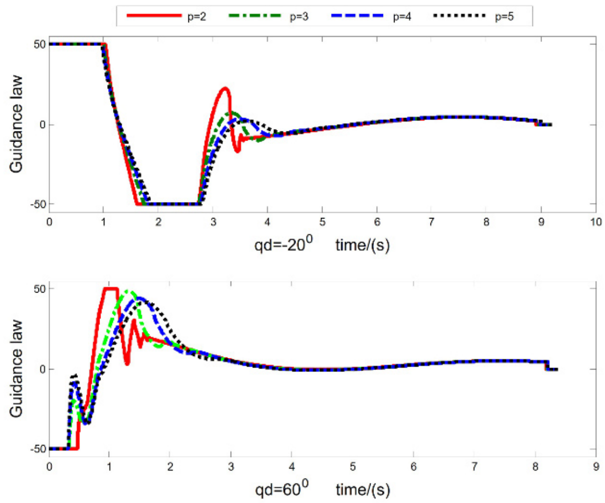

Fig. 7. Curve of the guidance law.

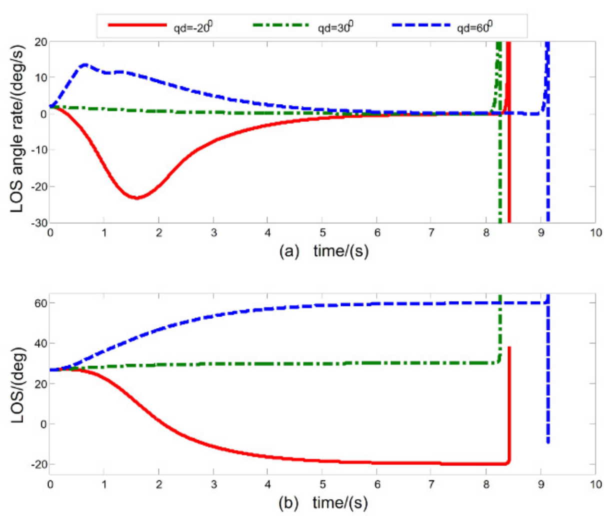

Fig. 8. Curve of LOS angle rate and LOS angle.

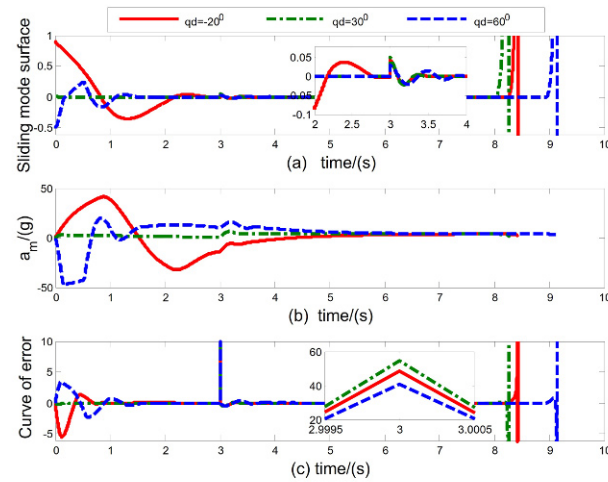

Fig. 9. Curve of sliding mode surface, missile normal acceleration and observer error.

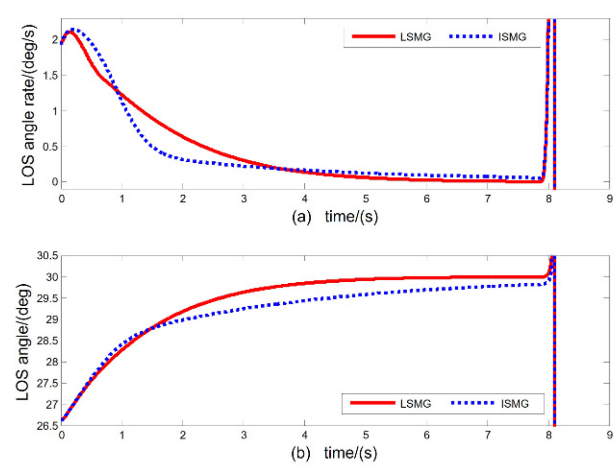

Fig. 10. Curve of LOS angle rate and LOS angle.

face

$$
\begin{aligned}
s = & x_3 - x_3(t_0) \\
& + \int_{t_0}^{t} (\beta_1 sig^{\alpha_1}(x_1) + \beta_2 sig^{\alpha_2}(x_2) + \beta_3 sig^{\alpha_3}(x_3))dt,
\end{aligned}
$$

the integral sliding mode guidance law (ISMG) with dynamic delay and impact angle constraints is designed with finite time convergence

$$
\begin{aligned}
u = & -2\tau\dot{r}x_2 + r\tau(\beta_1 sig^{\alpha_1}(x_1) + \beta_2 sig^{\alpha_2}(x_2) + \beta_3 sig^{\alpha_3}(x_3)) \\
& -3\tau\dot{r}x_3 + a_m + \tau\eta sgn(s)).
\end{aligned}
$$

In ISMG, $\alpha_1 = 2/11$, $\alpha_2 = 1/4$, $\alpha_3 = 2/5$, $\beta_1 = 0.2$, $\beta_2 = 0.5$, $\beta_3 = 0.5$, $\eta = 40$. The target performs cosine maneuvering $a_t = 4g\cos(\pi t/4)$ in the normal direction. The desired LOS angle is $q_d = 30°$ and the limit of the missile's normal acceleration is $20g$. Other guidance conditions are the same as Scenario1 and Scenario 2. The corresponding simulation curves are shown in Figs. 10-12.

Fig.10 shows the curves of LOS angle rate and LOS angle. It can see that LSMG has higher convergence precision than ISMG. Fig.11 shows the curves of sliding mode surface and missile normal acceleration. It can see that as ISMG uses the sign function, the chattering is larger which directly influences on the performance of the missile guidance. Fig. 12 shows the curves of two guidance laws. It can see that the curve of LSMG is smooth, while, the curve of ISMG is intense chattering. The above comparison shows that the designed guidance law has more stable performance, wider adaptability and convenient for the application in the engineering than ISMG.

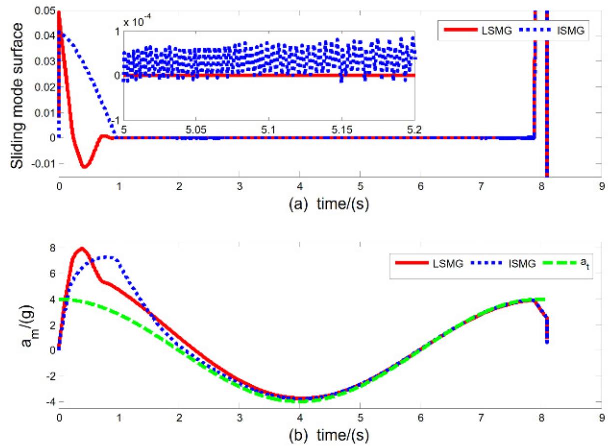

Fig. 11. Curve of sliding mode surface and missile normal acceleration.

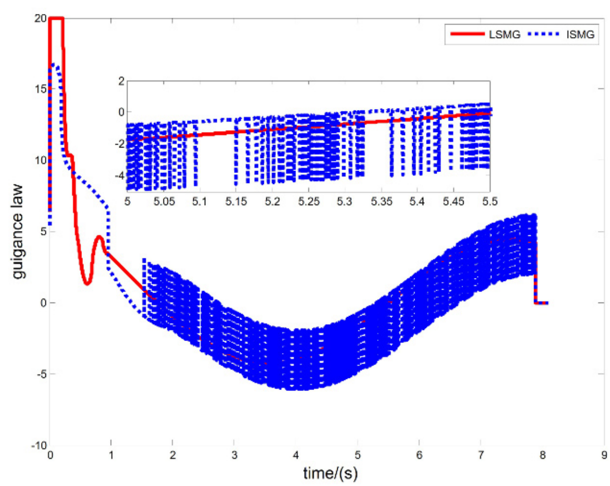

Fig. 12. Curve of the guidance law under LSMG and ISMG.

5. CONSLUSIONS

In order to meet the requirement of guidance for the missile to intercept target with high speed and great maneuvering at a certain impact angle, a sliding mode manifold guidance law is designed based on annihilation of LOS angle rate. The application of sliding mode variable dynamic method allows the system to converge to the sliding mode manifold in finite time, and the selected linear sliding mode manifold that includes desired LOS angle and LOS angle rate is simple and convenient for practical engineering application. During the realization of this guidance law, a non-homogeneous disturbance observer is applied to perform finite-time rapid tracking and estimation of the target's total disturbance in the system. This guidance law overcomes the dynamic delay of the missile's autopilot and the influence of the target's maneuvering on the precision of guidance. The results of numerical simulation demonstrate the effectiveness of the guidance law.

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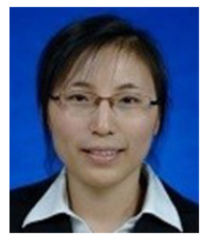

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