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Stochastic Admissibility and Stabilization of Singular Markovian Jump Systems with Multiple Time-varying Delays

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Abstract: This paper is concerned with stochastic admissibility and state feedback stabilization for a class of singular Markovian jump systems with multiple time-varying delays. The singular matrix E with both mode-dependent and mode-independent is considered in the system. Firstly, based on Lyapunov functional method and free-weighting matrix method, sufficient condition is presented in the form of linear matrix inequalities (LMIs) to guarantee the considered system to be stochastically admissible. Secondly, by state feedback controller, sufficient condition is derived in terms of strict LMIs to ensure the closed-loop system to be stochastically stabilizable. Finally, numerical examples are provided to illustrate the effectiveness of the proposed approaches.

Keywords: Markovian jump systems, singular systems, stochastic admissibility, time-varying delays.

1. INTRODUCTION

Singular systems, which are also referred to as generalized systems, descriptor systems, differential algebraic systems or semi-state systems, give better description of a large class of physical systems than state-space systems, such as economic systems, power systems and circuits systems [1, 2]. A number of important and interesting results have been established for singular systems [3–7]. It is well known that the stability analysis of singular systems is much more complicated than that of regular ones due to not only the asymptotic stability is needed to be checked, but also regularity and absence of impulses (for continuous singular systems) [4] or causality (for discrete singular systems) [5] are needed to be considered.

On the other hand, Markovian jump systems, as a special class of stochastic hybrid systems, have attracted considerable attention in the past few decades since they are very appropriate to model various physical systems with abrupt structural changes. For example, fault-tolerant control systems, repairs of machines in manufacturing systems and states of transition in maneuvered target tracking. Systems of this family, in which the mode-process is a continuous-time discrete-state Markov process taking values in a finite set, have been studied for years and many fundamental results have been established, such as stability analysis and (finite-time) H_∞ analysis given in [8–12], stabilization problems discussed in [13–16].

Very recently, singular Markovian jump systems with time-delay also have received extensive attention. As we know, time-delay is frequently encountered in physical process, chemical process, and it is considered as the major cause of instability and poor performance of dynamic systems, see, e.g., [14, 17–19]. In [18], the solution to the problem of stability and stabilization of continuous-time singular hybrids systems is presented. In [19], the author considered the stability and stabilizability of singular stochastic systems with delays. Also, in [14], robust stabilization of Markovian delay systems with delay-dependent exponential estimates is investigated. However, the singular matrix E in the considered systems is mode-independent, noting that the singular Markovian jump systems with mode-dependent singular matrices $E(r_t)$ also have wide application in the practical systems, such as the DC motor in position control servomechanisms [20] and in other practical systems [21]. So the further study of general singular Markovian jump systems with mode-dependent singular matrices $E(r_t)$ is of both theoretical and practical importance. Recently, in [22], H_∞ filtering for a class of singular Markoivan jump systems with time-varying delay is studied and the considered singular matrix is mode-dependent. To date and to our knowledge, the control problems of singular Markovian jump systems with multiply time-varying delay have not been fully investigated, especially with mode-dependent singular matrix $E(r_t)$. Based on the above analysis, this motivates us

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to study the stochastic admissibility and stabilization of singular Markovian jump systems with multiple time-varying delays and the considered singular matrix is mode-dependent.

This main contribution of this paper is that a new approach based on novel augmented Lyapunov functional is proposed for singular Markovian jump systems with multiple time-varying delays. Moreover, in order to reduce the possible conservativeness and computation burden, slack matrices are introduced and strict LMI condition is derived. The rest of this paper is organized as follows. In Section 2, problem formulations with some definitions are presented. In section 3, the stochastic admissibility of the considered system with both mode-dependent singular matrix $E(r_t)$ and mode-independent singular matrix E is studied first; Then, by state feedback controller, sufficient condition is established to guarantee the closed-loop system to be stochastically admissible. Numerical examples are offered to show the effectiveness of the proposed approaches in Section 4. Conclusions are provided in Section 5.

Notions: Throughout this paper, matrices, if not explicitly stated, are assumed to have compatible dimensions. The notion $X>0$ means that X is symmetric positive definite matrix. I and 0 are used to represent an identity matrix and zero matrix of appropriate dimensions, respectively. $\|\cdot\|$ refers to the Euclidean vector norm or spectral matrix norm. Let $\lambda_{\max}(\cdot)$ be the maximum eigenvalue of a matrix. $(\Omega,\mathbb{F},\mathbb{P})$ is a probability space, Ω is the sample space, $\mathbb{F}$ is the σ-algebra of Ω and $\mathbb{P}$ be the probability measure on $\mathbb{F}$. $\mathbf{E}(\cdot)$ denotes the expectation operator with respect to some probability measure $\mathbb{P}$. The symmetric elements of the symmetric matrix are denoted by $*$.

2. PRELIMINARIES

Let $\{r_t, t\geq 0\}$ be a continuous-time Markov process with right continuous trajectories and takes values in a finite set $\mathcal{L}=\{1,2,\cdots,N\}$ with transition probability matrix $\Pi\triangleq\{\pi_{ij}\},(i,j\in\mathcal{L})$ given by

$$\mathbf{Pr}\{r_{t+\triangle}=j|r_t=i\} = \begin{cases} \pi_{ij}\triangle+o(\triangle), & if \quad i\neq j,\\ 1+\pi_{ii}\triangle+o(\triangle), & if \quad i=j,\end{cases}$$

where $\triangle>0$ and $\lim_{\triangle\to 0}o(\triangle)/\triangle=0$, $\pi_{ij}>0(i\neq j)$ is the transition rate from mode i at time t to mode j at time $t+\triangle$, and $\pi_{ii}=-\sum_{j\neq i}\pi_{ij}<0$ for each $i\in\mathcal{L}$.

Consider the following singular Markovian jump system on the probability space:

$$\begin{cases} E(r_t)\dot{x}(t)=A(r_t)x(t)+\sum_{k=1}^{p}A_k(r_t)x(t-d_k(t)) \\ \qquad\qquad +B(r_t)u(t), \\ x(t)=\varphi(t), \quad t\in[-d,0], \end{cases} \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the system state vector, $u(t)\in\mathbb{R}^m$ is the control input and $\varphi(t)$ is a continuous-time vector valued initial function. For each $r_t\in\mathcal{L}$, the singular matrix $E(r_t)\in\mathbb{R}^{n\times n}$ is assumed $\mathrm{rank}(E(r_t))=r(r_t)\leq n$, $A(r_t)$, $A_k(r_t)$ and $B(r_t)$ are matrices of the random jump process with appropriate dimensions, and $d_k(t)$, $k=1,\cdots,p$, is the time-varying delay satisfies

$$0\leq d_k(t)\leq d_k, \qquad \dot{d}_k(t)\leq u_k<1, \tag{2}$$

with d_k and u_k are known scalars, $d\triangleq max\{d_1,d_2\cdots,d_p\}$.

Definition 1 [23]**:** The singular Markovian jump time delay system (1) with $u(t)=0$ is said to be

1) regular and impulse-free, if the pair $(E(r_t),A(r_t))$ is regular and impulse-free for every $r_t\in\mathcal{L}$.
2) stochastically stable, if there exists a scalar $M(r_0,\varphi(\cdot))$ such that

$$\lim_{T\to\infty}\mathbf{E}\left\{\int_0^T\|x(t)\|^2\mathrm{d}t|r_0,x(s)=\phi(s),s\in[d,0]\right\} \leq M(r_0,\varphi(\cdot)). \tag{3}$$

3) stochastically admissible, if it is regular, impulse free and stochastically stable.

Definition 2 [18]**:** The singular Markovian jump time delay system (1) is said to be regular, impulse-free and stochastically stabilizable via state feedback if there exists a control

$$u(t)=K(r_t)x(t) \tag{4}$$

with $K(i)$, when $r_t=i\in\mathcal{L}$, a constant matrix, such that the closed-loop system is regular, impulse-free and stochastically stable.

For notational simplicity, in the sequel, for each possible $r_t=i$, $i\in\mathcal{L}$, matrix $A(r_t)$ will be denoted by A_i, and $A_k(r_t)$ by A_{ki} and so on.

3. MAIN RESULTS

In this section, we will first investigate the stochastic admissibility of the unforced system (1), sufficient conditions are established to check the regularity, absence of impulse and stochastic stability of the considered system. Then, we will consider the issue of designing a state feedback controller that makes the resulting closed-loop system to be stochastically admissible.

Theorem 1: Given positive scalars d_k and $u_k<1$ $(k=1,\cdots,p)$. The system (1) with $u(t)=0$ is stochastically admissible if there exist matrices $Z_k>0$, $Q_{ki}>0$, $Q_k>0$, $R_k>0$, P_i and N_{ki} $(k=1,\cdots,p)$ such that for all mode

$i \in \mathcal{L}$, the following inequalities hold,

$$\Theta_i = \begin{bmatrix} \Theta_{i11} & \Theta_{i12} & \Theta_{i13} & A_i^T Z & \Theta_{i15} & \Theta_{i16} \\ * & \Theta_{i22} & 0 & \Theta_{i24} & \Theta_{i25} & 0 \\ * & * & \Theta_{i33} & 0 & 0 & 0 \\ * & * & * & -Z & 0 & 0 \\ * & * & * & * & \Theta_{i55} & 0 \\ * & * & * & * & * & \Theta_{i66} \end{bmatrix} < 0, \tag{5}$$

$$\sum_{j=1}^{N} \pi_{ij} Q_{ki} \leq Q_k, \tag{6}$$

with the following constraint

$$E_i^T P_i = P_i^T E_i \geq 0, \tag{7}$$

where

$$\begin{aligned}
\Theta_{i11} &= P_i^T A_i + A_i^T P_i + \sum_{k=1}^{p} (Q_{ki} + d_k Q_k + R_k + N_{ki}^T E_i \\
&\quad + E_i^T N_{ki}) + \sum_{j=1}^{N} \pi_{ij} E_j^T P_j, \\
\Theta_{i12} &= [P_i^T A_{1i} \quad P_i^T A_{2i} \quad \cdots \quad P_i^T A_{pi}], \\
\Theta_{i13} &= [N_{1i}^T E_i \quad N_{2i}^T E_i \quad \cdots \quad N_{pi}^T E_i], \\
\Theta_{i15} &= [A_1^T H \quad A_2^T H \quad \cdots \quad A_N^T H], \\
\Theta_{i16} &= [d_1 N_{1i}^T \quad d_2 N_{2i}^T \quad \cdots \quad d_p N_{pi}^T, \\
\Theta_{i22} &= \mathrm{diag}\{-(1-u_1)R_1, -(1-u_2)R_2, \cdots, \\
&\quad -(1-u_p)R_p\}, \\
\Theta_{i24} &= \begin{bmatrix} A_{1i}^T Z \\ A_{2i}^T Z \\ \vdots \\ A_{pi}^T Z \end{bmatrix}, \\
\Theta_{i25} &= \begin{bmatrix} A_{11}^T H & A_{12}^T H & \cdots & A_{1N}^T H \\ A_{21}^T H & A_{22}^T H & \cdots & A_{2N}^T H \\ \vdots & \vdots & \ddots & \vdots \\ A_{p1}^T H & A_{p2}^T H & \cdots & A_{pN}^T H \end{bmatrix}, \\
\Theta_{i33} &= diag\{-Q_{1i}, -Q_{2i}, \cdots, -Q_{pi}\}, \\
\Theta_{i55} &= diag\{-H, -H, \cdots, -H\}, \\
\Theta_{i66} &= [-d_1 Z_1, -d_2 Z_2, \cdots, -d_p Z_p,], \\
Z &= \sum_{k=1}^{p} d_k Z_k, \\
H &= \sum_{k=1}^{p} \frac{\overline{\pi} d_k^2}{2} Z_k, \\
\overline{\pi} &= max\{-\pi_{11}, -\pi_{22}, \cdots, -\pi_{NN}\}.
\end{aligned}$$

Proof: Firstly, we will show that the unforced system (1) is regular and impulse-free. Since $\mathrm{rank}(E_i) = r_i \leq n$, there exist two nonsingular matrices L_i and G_i such that

$$L_i E_i G_i = \begin{bmatrix} I_{r_i} & 0 \\ 0 & 0 \end{bmatrix}. \tag{8}$$

Denote

$$L_i A_i G_i = \begin{bmatrix} A_{i11} & A_{i12} \\ A_{i21} & A_{i22} \end{bmatrix}, L_i^{-T} P_i G_i = \begin{bmatrix} P_{i11} & P_{i12} \\ P_{i21} & P_{i22} \end{bmatrix},$$
$$L_i N_{ki} G_i = \begin{bmatrix} N_{ki11} & N_{ki12} \\ N_{ki21} & N_{ki22} \end{bmatrix}. \tag{9}$$

According to (8) and (9), pre- and post-multiplying (7) by G_i^T and G_i, it can be obtained that

$$P_{i11} = P_{i11}^T \geq 0, \quad P_{i12} = 0. \tag{10}$$

Also, it can be found from $\Theta_{i11} < 0$ that

$$\Psi_i = P_i^T A_i + A_i^T P_i + \sum_{k=1}^{p} (N_{ki}^T E_i + E_i^T N_{ki}) + \pi_{ij} E_i^T P_i < 0. \tag{11}$$

Then, pre- and post-multiplying (11) by G_i^T and G_i, respectively, we get

$$G_i^T \Psi_i G_i = \begin{bmatrix} T_{i11} & T_{i12} \\ * & T_{i22} \end{bmatrix} < 0, \tag{12}$$

where $T_{i22} = P_{i22}^T A_{i22} + A_{i22}^T P_{i22}$. For T_{i11} and T_{i12} are not used in the following discussion, so we omit the real expressions for them here. From (12), we know that

$$P_{i22}^T A_{i22} + A_{i22}^T P_{i22} < 0, \tag{13}$$

which implies A_{i22} is nonsingular and thus the pair (E_i, A_i) is regular and impulse free. Hence, by definition (1), the unforced system (1) is regular and impulse free.

Next, we prove that the unforced system (1) is stochastically stable. To this end, we define the following Lyapunov functional:

$$V(x(t),i,t) = V_1(x(t),i,t) + \sum_{k=1}^{p} \sum_{j=1}^{3} V_{kj}(x(t),i,t), \tag{14}$$

where

$$\begin{aligned}
V_1(x(t),i,t) &= x^T(t) E_i^T P_i x(t), \\
V_{k1}(x(t),i,t) &= \int_{t-d_k(t)}^{t} x^T(s) R_k x(s) \mathrm{d}s \\
&\quad + \int_{t-d_k}^{t} x^T(s) Q_{ki} x(s) \mathrm{d}s \\
&\quad + \int_{-d_k}^{0} \int_{t+\theta}^{t} x^T(s) Q_k x(s) \mathrm{d}s \mathrm{d}\theta, \\
V_{k2}(x(t),i,t) &= \int_{-d_k}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) E_i^T Z_k E_i \dot{x}(s) \mathrm{d}s \mathrm{d}\theta, \\
V_{k3}&(x(t),i,t) \\
&= \overline{\pi} \sum_{j=1}^{N} \int_{-d_k}^{0} \int_{\theta}^{0} \int_{t+\alpha}^{t} \dot{x}^T(s) E_j^T Z_k E_j \dot{x}(s) \mathrm{d}s \mathrm{d}\alpha \mathrm{d}\theta,
\end{aligned}$$

Let $\mathfrak{L}$ be the weak infinitesimal generator of the random process. Then, for each $i \in \mathcal{L}$ and $t \geq 0$, we have

$$\begin{aligned}\mathfrak{L}V_1(x(t),i,t) &= 2x^T(t)P_i^T E_i\dot{x}(t) + \sum_{j=1}^{N}\pi_{ij}x^T(t)E_i^T P_i x(t)\\ &= 2x^T(t)P_i^T[A_i x(t) + \sum_{k=1}^{p}A_{ki}x(t-d_k(t))]\\ &\quad + \sum_{j=1}^{N}\pi_{ij}x^T(t)E_j^T P_j x(t), \qquad (15)\end{aligned}$$

$$\begin{aligned}\mathfrak{L}V_{k1}(x(t),i,t) \leq\ & x^T(t)(R_k + Q_{ki} + d_k Q_k)x(t)\\ & -(1-u_k)x^T(t-d_k(t))R_k x(t-d_k(t))\\ & -x^T(t-d_k)Q_{ki}x(t-d_k)\\ & +\int_{t-d_k}^{t} x^T(s)\sum_{j=1}^{N}\pi_{ij}Q_{kj}x(s)\mathrm{d}s\\ & +\int_{t-d_k}^{t} x^T(s)Q_k x(s)\mathrm{d}s \qquad (16)\end{aligned}$$

$$\begin{aligned}\mathfrak{L}V_{k2}(x(t),i,t) =\ & d_k\dot{x}^T(t)E_i^T Z_k E_i\dot{x}(t)\\ & -\int_{t-d_k}^{t}\dot{x}^T(s)E_i^T Z_k E_i\dot{x}(s)\mathrm{d}s\\ & +\int_{-d_k}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)\sum_{j=1}^{N}\pi_{ij}E_j^T Z_k E_j\dot{x}(s)\mathrm{d}s,\end{aligned} \qquad (17)$$

$$\begin{aligned}\mathfrak{L}V_{k3}(x(t),i,t) =\ & \frac{\overline{\pi}d_k^2}{2}\sum_{j=1}^{N}\dot{x}^T(t)E_j^T Z_k E_j\dot{x}(s)\\ & -\overline{\pi}\int_{-d_k}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)\sum_{j=1}^{N}E_j^T Z_k E_j\dot{x}(s)\mathrm{d}s\mathrm{d}\theta,\end{aligned} \qquad (18)$$

what's more, from the Newton-Leibniz formula we know

$$\sum_{k=1}^{p}2x^T(t)N_{ki}^T[E_i x(t) - E_i x(t-d_k(t)) - \int_{t-d_k}^{t}E_i\dot{x}(s)\mathrm{d}s] = 0. \qquad (19)$$

Using (5), (6) and (15)-(19), we can get

$$\begin{aligned}\mathfrak{L}V(x(t),i,t) \leq\ & \mathfrak{L}V(x(t),i,t)\\ & -\sum_{k=1}^{p}\int_{-d_k}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)\sum_{j=1}^{N}\pi_{ij}E_j^T Z_k E_j\dot{x}(s)\mathrm{d}s\\ & +\sum_{k=1}^{p}\overline{\pi}\int_{-d_k}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)\sum_{j=1}^{N}E_j^T Z_k E_j\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\\ \leq\ & \zeta^T(t)\overline{\Theta}_i\zeta(t) - \sum_{k=1}^{p}\int_{-d_k}^{0}[N_{ki}x(t) + Z_k E_i\dot{x}(s)]^T\\ & Z_k^{-1}[N_{ki}x(t) + Z_k E_i\dot{x}(s)]\mathrm{d}s\\ \leq\ & -\tau_i x^T(t)x(t),\end{aligned}$$

where

$$\begin{aligned}&\tau_i = -\lambda_{max}(\overline{\Theta}_i),\\ &\zeta^T(t) = [x^T(t) \quad \omega_1^T(t) \quad \omega_2^T(t)],\\ &\omega_1^T(t) = [x^T(t-d_1(t)) \quad x^T(t-d_2(t)) \quad \cdots\\ &\qquad x^T(t-d_p(t))],\\ &\omega_2^T(t) = [x^T(t-d_1) \quad x^T(t-d_2) \quad \cdots \quad x^T(t-d_p)].\end{aligned}$$

Here

$$\begin{aligned}\overline{\Theta}_i =\ & \begin{bmatrix}\widetilde{\Theta}_i & \Theta_{i12} & \Theta_{i13}\\ * & \Theta_{i22} & 0\\ * & * & \Theta_{i33}\end{bmatrix}\\ & + \begin{bmatrix}A_i^T\\ A_{1i}^T\\ \vdots\\ A_{pi}^T\\ 0\end{bmatrix}\left(\sum_{k=1}^{p}d_k Z_k\right)\begin{bmatrix}A_i^T\\ A_{1i}^T\\ \vdots\\ A_{pi}^T\\ 0\end{bmatrix}^T\\ & + \sum_{j=1}^{N}\begin{bmatrix}A_j^T\\ A_{1j}^T\\ \vdots\\ A_{pj}^T\\ 0\end{bmatrix}\left(\sum_{k=1}^{p}\frac{\overline{\pi}d_k^2}{2}Z_k\right)\begin{bmatrix}A_j^T\\ A_{1j}^T\\ \vdots\\ A_{pj}^T\\ 0\end{bmatrix}^T,\end{aligned} \qquad (20)$$

with

$$\begin{aligned}\widetilde{\Theta}_i =\ & P_i^T A_i + A_i^T P_i + \sum_{k=1}^{p}(Q_{ki} + d_k Q_k + R_k + N_{ki}^T E_i\\ & + E_i^T N_{ki} + d_k N_{ki}^T Z_k^{-1} N_{ki}) + \sum_{j=1}^{N}\pi_{ij}E_j^T P_j.\end{aligned}$$

Noting (5), by using schur complement, (5) is equivalent to $\overline{\Theta}_i < 0$.

Then by the Dynkin's formula, we know for any $t \geq 0$

$$\begin{aligned}&\mathbf{E}\{V((x(t),i,t)) - V(x(0),r_0,0)\}\\ &\quad = \mathbf{E}\left\{\int_0^t \mathfrak{L}V(x(s),i,t)\mathrm{d}s\right\}\\ &\quad \leq -\tau\mathbf{E}\left\{\int_0^t \|x(s)\|^2\mathrm{d}s\right\},\end{aligned} \qquad (21)$$

where

$$\tau = min\{\tau_1, \tau_2, \cdots, \tau_N\}.$$

Thus, we have

$$\lim_{T\to\infty}\mathbf{E}\left\{\int_0^T \|x(s)\|^2\mathrm{d}s\right\} \leq \frac{1}{\tau}V(x(0),r_0,0). \qquad (22)$$

Therefore, by Definition 1, the unforced system (1) is stochastically stable. This completes the proof.

Remark 1: The above result gives the solution to the problem of stochastic admissibility of the unforced system (1). As we can see, the singular matrix studied is mode-dependent, so what if the singular matrix is mode-independent (that is $E_i = E$). We also have a conclusion for this case.

The special system considered as follows:

$$\begin{cases} E\dot{x}(t) = A(r_t)x(t) + \sum_{k=1}^{p} A_k(r_t)x(t-d_k(t)) \\ x(t) = \varphi(t), \quad t \in [-d, 0]. \end{cases} \tag{23}$$

The following theorem gives a new solution to the stochastic admissibility of the system (23).

Theorem 2: Given positive scalars d_k and $u_k < 1$ ($k = 1, \cdots, p$). The system (23) is stochastically admissible if there exist matrices $Z_k > 0$, $Q_{ki} > 0$, $Q_k > 0$, $R_k > 0$, P_i and N_{ki} ($k = 1, \cdots, p$) such that for all mode $i \in \mathcal{L}$, the following inequalities hold,

$$\Theta_i = \begin{bmatrix} \widetilde{\Theta}_{i11} & \Theta_{i12} & \widetilde{\Theta}_{i13} & A_i^T Z & \widetilde{\Theta}_{i15} \\ * & \Theta_{i22} & 0 & \Theta_{i24} & 0 \\ * & * & \Theta_{i33} & 0 & 0 \\ * & * & * & -Z & 0 \\ * & * & * & * & \widetilde{\Theta}_{i55} \end{bmatrix} < 0, \tag{24}$$

$$\sum_{j=1}^{N} \pi_{ij} Q_{ki} \leq Q_k, \tag{25}$$

with the following constraint

$$E^T P_i = P_i^T E \geq 0, \tag{26}$$

where

$$\begin{aligned} \widetilde{\Theta}_{i11} &= P_i^T A_i + A_i^T P_i + \sum_{k=1}^{p} (Q_{ki} + d_k Q_k + R_k + N_{ki}^T E \\ &\quad + E^T N_{ki}) + \sum_{j=1}^{N} \pi_{ij} E^T P_j, \\ \widetilde{\Theta}_{i13} &= [N_{1i}^T E \quad N_{2i}^T E \quad \cdots \quad N_{pi}^T E], \quad \widetilde{\Theta}_{i15} = \Theta_{i16}, \\ \widetilde{\Theta}_{i55} &= \Theta_{i66}, \end{aligned}$$

here, Θ_{i12}, Θ_{i16}, Θ_{i22}, Θ_{i24}, Θ_{i33}, Θ_{i66} and Z are defined as in Theorem 1.

Proof: Choose the following Lyapunov functional candidate for system (23):

$$V(x(t), i, t) = V_1(x(t), i, t) + \sum_{k=1}^{p} \sum_{j=1}^{2} V_{kj}(x(t), i, t), \tag{27}$$

where

$$\begin{aligned} V_1(x(t), i, t) &= x^T(t) E_i^T P_i x(t), \\ V_{k1}(x(t), i, t) &= \int_{t-d_k(t)}^{t} x^T(s) R_k x(s) \mathrm{d}s \\ &\quad + \int_{t-d_k}^{t} x^T(s) Q_{ki} x(s) \mathrm{d}s \\ &\quad + \int_{-d_k}^{0} \int_{t+\theta}^{t} x^T(s) Q_k x(s) \mathrm{d}s \mathrm{d}\theta, \\ V_{k2}(x(t), i, t) &= \int_{-d_k}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) E^T Z_k E \dot{x}(s) \mathrm{d}s \mathrm{d}\theta. \end{aligned}$$

Taking a similar method to the proof of Theorem 1, we can obtain

$$\mathfrak{L}V(x(t), i, t) \leq -\tau_i' x^T(t) x(t).$$

Then we have

$$\lim_{T\to\infty} \mathbf{E}\left\{ \int_0^T \|x(s)\|^2 \mathrm{d}s \right\} \leq \frac{1}{\tau'} V(x(0), r_0, 0). \tag{28}$$

This completes the proof.

Remark 2: The above results consider the stochastic admissibility of the unforced system (1) and (23). Furthermore, similar to [8, 14, 22, 24], we could study the mean-square exponential stability of the considered system that there exist scalars $\alpha > 0$ and $\beta > 0$ such that

$$\mathbf{E}\{\|x(t)\|^2\} \leq \alpha e^{-\beta t} \sup_{-d \leq s \leq 0} \mathbf{E}\{\|\varphi(t)\|^2\}.$$

Thus, the mean-square exponential admissibility problem of the considered system is given. The detailed proof is omitted here.

Next, we are ready to investigate the problem of state feedback control for the system (1). For convenience, the singular matrix considered is mode-independent and we give the result based on Theorem 2. By the controller (4), the resulting closed-loop system can be described as:

$$\begin{cases} E\dot{x}(t) = (A(r_t) + B(r_t)K(r_t))x(t) \\ \qquad\qquad + \sum_{k=1}^{p} A_k(r_t)x(t-d_k(t)) \\ x(t) = \varphi(t), \quad t \in [-d, 0]. \end{cases} \tag{29}$$

Before giving the result, the following lemma is recalled.

Lemma 1 [25]**:** Let X_i be symmetric such that $E_L^T X_i E_L > 0$ and Y_i is nonsingular. Then, $X_i E + U Y_i W^T$ is nonsingular and its inverse is expressed as

$$(X_i E + U Y_i W^T)^{-1} = \mathcal{X}_i E^T + W \mathcal{Y}_i U, \tag{30}$$

where $\mathcal{X}_i$ is a symmetric matrix and $\mathcal{Y}_i$ is a nonsingular matrix with

$$E_R^T \mathcal{X}_i E_R = (E_L^T X_i E_L)^{-1}, \quad \mathcal{Y}_i = (W^T W)^{-1} Y_i^{-1} (U U^T)^{-1},$$

$U \in \mathbb{R}^{(n-r)\times n}$ and $W \in \mathbb{R}^{n\times(n-r)}$ are any matrices with full rank satisfy $UE = 0$ and $EW = 0$; E is decomposed as $E = E_L E_R^T$, here $E_L \in \mathbb{R}^{n\times r}$, $E_R \in \mathbb{R}^{n\times r}$ are of full column rank.

Theorem 3: Given positive scalars d_k, $u_k < 1$, a_i and b_{ki} $(k = 1, \cdots, p)$. The system (29) is stochastically admissible if there exist nonsingular matrices $\mathcal{Y}_i$, matrices $\mathcal{X}_i > 0$, $\mathcal{Q}_{ki} > 0$, $\mathcal{R}_k > 0$, $\mathcal{Z}_k > 0$ $(k = 1, \cdots, p)$, $\mathcal{K}_i \in \mathbb{R}^{m\times n}$ and $\mathcal{W}_i \in \mathbb{R}^{m\times(n-r)}$ such that for all mode $i \in \mathcal{L}$, the following inequalities hold,

$$\Phi_i = \begin{bmatrix} \Phi_{i11} & \Phi_{i12} & \Phi_{i13} & \Phi_{i14} & \tilde{\Phi}_{i1} \\ * & \Phi_{i22} & 0 & \Phi_{i24} & 0 \\ * & * & \Phi_{i33} & 0 & 0 \\ * & * & * & \Phi_{i44} & 0 \\ * & * & * & * & \tilde{\Phi}_{i2} \end{bmatrix} < 0, \quad (31)$$

$$\sum_{j=1}^{N} \pi_{ij} a_j \le 1, \quad (32)$$

where $\mathcal{P}_i = \mathcal{X}_i E^T + W\mathcal{Y}_i U$ and

$$\begin{aligned}
\tilde{\Phi}_{i1} &= [\Phi_{i15} \quad \Phi_{i16} \quad \Phi_{i17}],\\
\tilde{\Phi}_{i2} &= diag\{\Phi_{i55}, \Phi_{i66}, \Phi_{i77}\},\\
\Phi_{i11} &= A_i\mathcal{P}_i + B_i(\mathcal{K}_iE + \mathcal{W}_iU) + (\mathcal{K}_iE + \mathcal{W}_iU)^T B_i^T\\
&\quad + \sum_{k=1}^{p}\{(1 + a_i^{-1}d_k)\mathcal{Q}_{ki} + b_{ki}(E\mathcal{P}_i + \mathcal{P}_i^T E^T)\}\\
&\quad + \mathcal{P}_i^T A_i^T + \pi_{ii}(E\mathcal{P}_i + E^T\mathcal{P}_i^T - E\mathcal{X}_iE^T),\\
\Phi_{i12} &= [A_{1i}\mathcal{P}_i \quad A_{2i}\mathcal{P}_i \quad \cdots \quad A_{pi}\mathcal{P}_i],\\
\Phi_{i13} &= [b_{1i}E\mathcal{P}_i \quad b_{2i}E\mathcal{P}_i \quad \cdots b_{pi}E\mathcal{P}_i],\\
\Phi_{i14} &= [\mathcal{P}_i^T A_i^T + (\mathcal{K}_iE + \mathcal{W}_iU)^T B_i^T]\\
&\quad \times [d_1I \quad d_2I \quad \cdots \quad d_pI],\\
\Phi_{i15} &= [b_{1i}d_1\mathcal{P}_i \quad b_{2i}d_2\mathcal{P}_i \quad \cdots \quad b_{pi}d_p\mathcal{P}_i],\\
\Phi_{i16} &= \overbrace{[P_i \quad \cdots \quad P_i]}^{1\times p},\\
\Phi_{i17} &= [\pi_{i1}\mathcal{P}_i^T E_R \quad \pi_{i2}\mathcal{P}_i^T E_R \quad \cdots \quad \pi_{i(i-1)}\mathcal{P}_i^T E_R\\
&\quad \pi_{i(i+1)}\mathcal{P}_i^T E_R \quad \cdots \quad \pi_{iN}\mathcal{P}_i^T E_R],\\
\Phi_{i22} &= diag\{-(1-u_1)(\mathcal{P}_i + \mathcal{P}_i^T - \mathcal{R}_1),)\\
&\quad - (1-u_2(\mathcal{P}_i + \mathcal{P}_i^T - \mathcal{R}_2), \cdots,\\
&\quad - (1-u_p)(\mathcal{P}_i + \mathcal{P}_i^T - \mathcal{R}_p)\},\\
\Phi_{i24} &= \begin{bmatrix} d_1\mathcal{P}_i^T A_{1i}^T & d_2\mathcal{P}_i^T A_{1i}^T & \cdots & d_p\mathcal{P}_i^T A_{1i}^T \\ d_1\mathcal{P}_i^T A_{2i}^T & d_2\mathcal{P}_i^T A_{2i}^T & \cdots & d_p\mathcal{P}_i^T A_{2i}^T \\ \vdots & \vdots & \ddots & \vdots \\ d_1\mathcal{P}_i^T A_{pi}^T & d_2\mathcal{P}_i^T A_{pi}^T & \cdots & d_p\mathcal{P}_i^T A_{pi}^T \end{bmatrix},\\
\Phi_{i33} &= diag\{-\mathcal{Q}_{1i}, -\mathcal{Q}_{2i}, \cdots, -\mathcal{Q}_{pi}\},\\
\Phi_{i44} &= diag\{-d_1\mathcal{Z}_1, -d_2\mathcal{Z}_2, \cdots, -d_p\mathcal{Z}_p\},\\
\Phi_{i55} &= diag\{-d_1(\mathcal{P}_i + \mathcal{P}_i^T - \mathcal{Z}_1), -d_2(\mathcal{P}_i + \mathcal{P}_i^T\\
&\quad - \mathcal{Z}_2), \cdots, -d_p(\mathcal{P}_i + \mathcal{P}_i^T - \mathcal{Z}_p)\},\\
\Phi_{i66} &= diag\{-\mathcal{R}_1, -\mathcal{R}_2, \cdots, -\mathcal{R}_p,\},\\
\Phi_{i77} &= diag\{-\pi_{i1}E_R^T\mathcal{X}_1E_R, \cdots, -\pi_{i(i-1)}E_R^T\mathcal{X}_{(i-1)}E_R,\\
&\quad - \pi_{i(i+1)}E_R^T\mathcal{X}(i+1)E_R, \cdots, -\pi_{iN}E_R^T\mathcal{X}_NE_R,\}.
\end{aligned}$$

Moreover, the parameter K_i is given by

$$K_i = (\mathcal{K}_iE^T + \mathcal{W}_iU)(\mathcal{X}_iE^T + W\mathcal{Y}_iU)^{-1}. \quad (33)$$

Proof: Let $P_i \triangleq X_iE + UY_iW^T$, according to Lemma 1, P_i is nonsingular, so we defined $\mathcal{P}_i \triangleq P_i^{-1} = \mathcal{X}_iE^T + W\mathcal{Y}_iU$. In the proof of Theorem 2, we know LMI (24) has an equivalent form as follows:

$$\begin{bmatrix} \widetilde{\Theta}_{i11} & \Theta_{i12} & \widetilde{\Theta}_{i13} & \widetilde{\Theta}_{i14} & \widetilde{\Theta}_{i15} \\ * & \Theta_{i22} & 0 & \widetilde{\Theta}_{i24} & 0 \\ * & * & \Theta_{i33} & 0 & 0 \\ * & * & * & \widetilde{\Theta}_{i44} & 0 \\ * & * & * & * & \widetilde{\Theta}_{i55} \end{bmatrix} < 0, \quad (34)$$

where

$$\begin{aligned}
\widetilde{\Theta}_{i14} &= [d_1A_i^TZ_1 \quad d_2A_i^TZ_p \quad \cdots \quad d_pA_i^TZ_p],\\
\widetilde{\Theta}_{i24} &= \begin{bmatrix} d_1A_{1i}^TZ_1 & d_2A_{1i}^TZ_2 & \cdots & d_pA_{1i}^TZ_p \\ d_1A_{2i}^TZ_1 & d_2A_{2i}^TZ_2 & \cdots & d_pA_{2i}^TZ_p \\ \vdots & \vdots & \ddots & \vdots \\ d_1A_{pi}^TZ_1 & d_2A_{pi}^TZ_2 & \cdots & d_pA_{pi}^TZ_p \end{bmatrix},\\
\widetilde{\Theta}_{i44} &= [-d_1Z_1, -d_2Z_2, \cdots, -d_pZ_p,].
\end{aligned}$$

Now, denoting $\mathcal{P}_i^TQ_{ki}\mathcal{P}_i = \mathcal{Q}_{ki}$, $R_k^{-1} = \mathcal{R}_k$, $Z_k^{-1} = \mathcal{Z}_k$, and in Theorem 2, letting $Q_{ki} = a_iQ_k$, $N_{ki} = b_{ki}P_i$, here, a_i and b_{ki} are tuning scalars, $(k = 1, 2, \cdots, p)$.

Pre- and post-multiplying (34) by

$$diag\{\mathcal{P}_i^T, \overbrace{\mathcal{P}_i^T \quad \cdots \quad \mathcal{P}_i^T}^{p}, \overbrace{\mathcal{P}_i^T \quad \cdots \quad \mathcal{P}_i^T}^{p}, \mathcal{Z}_1^T, \cdots, \mathcal{Z}_p^T, \mathcal{Z}_1^T, \cdots, \mathcal{Z}_p^T\}$$

and its transpose respectively, we have

$$\begin{bmatrix} \widetilde{\Phi}_{i11} & \Phi_{i12} & \Phi_{i13} & \widetilde{\Phi}_{i14} & \Phi_{i15} \\ * & \widetilde{\Phi}_{i22} & 0 & \Phi_{i24} & 0 \\ * & * & \Phi_{i33} & 0 & 0 \\ * & * & * & \Phi_{i44} & 0 \\ * & * & * & * & \widetilde{\Phi}_{i55} \end{bmatrix} < 0, \quad (35)$$

where

$$\begin{aligned}
\widetilde{\Phi}_{i11} &= \overline{A}_i\mathcal{P}_i + \mathcal{P}_i^T\overline{A}_i^T + \sum_{k=1}^{p}\{(1 + a_i^{-1}d_k)\mathcal{Q}_{ki}\\
&\quad + b_{ki}(E\mathcal{P}_i + \mathcal{P}_i^TE^T) + \mathcal{P}_i^TR_k\mathcal{P}_i\}\\
&\quad + \sum_{j=1}^{N}\pi_{ij}\mathcal{P}_i^TE^TX_jE\mathcal{P}_i,\\
\widetilde{\Phi}_{i14} &= [d_1\mathcal{P}_i^T\overline{A}_i^T \quad d_2\mathcal{P}_i^T\overline{A}_i^T \quad \cdots \quad d_p\mathcal{P}_i^T\overline{A}_i^T],\\
\widetilde{\Phi}_{i22} &= diag\{-(1-u_1)\mathcal{P}_i^TR_1\mathcal{P}_i, -(1-u_2)\mathcal{P}_i^TR_2\mathcal{P}_i,\\
&\quad \cdots, -(1-u_p)\mathcal{P}_i^TR_p\mathcal{P}_i,\},\\
\widetilde{\Phi}_{i55} &= diag\{-d_1\mathcal{P}_i^TZ_1\mathcal{P}_i, -d_2\mathcal{P}_i^TZ_2\mathcal{P}_i, \cdots,\\
&\quad -d_p\mathcal{P}_i^TZ_p\mathcal{P}_i,\}, \quad \overline{A}_i = A_i + B_iK_i.
\end{aligned}$$

Letting $\mathcal{K}_i = K_i\mathcal{X}_i$, $\mathcal{W}_i = K_iW\mathcal{Y}_i$ and taking Lemma 1 into consideration, we have

$$\begin{aligned} &A_i\mathcal{P}_i + \mathcal{P}_i^TA_i^T + B_i(\mathcal{K}_iE + \mathcal{W}_iU) + (\mathcal{K}_iE + \mathcal{W}_iU)^TB_i^T \\ &+ \sum_{k=1}^{p}\{(1+a_i^{-1}d_k)\mathcal{Q}_{ki} + b_{ki}(E\mathcal{P}_i + \mathcal{P}_i^TE^T) \\ &+ \mathcal{P}_i^TR_k\mathcal{P}_i\} + \pi_{ii}\mathcal{P}_i^TE_R(E_R\mathcal{X}_iE_R)^{-1}E_R^T\mathcal{P}_i \\ &+ \sum_{j=1,j\neq i}^{N}\pi_{ij}\mathcal{P}_i^TE_R(E_R\mathcal{X}_jE_R)^{-1}E_R^T\mathcal{P}_i < 0. \end{aligned}$$

On the other hand, notice that

$$\begin{aligned} &\mathcal{P}_i^T\mathcal{Z}_k\mathcal{P}_i - \mathcal{P}_i^T - \mathcal{P}_i + \mathcal{Z}_i \\ &\quad = (\mathcal{P}_i - \mathcal{Z}_i)^T\mathcal{Z}_k(\mathcal{P}_i - \mathcal{Z}_i) \\ &\quad \geq 0, \end{aligned}$$

which implies

$$-d_k\mathcal{P}_i^T\mathcal{Z}_k\mathcal{P}_i \leq -d_k(\mathcal{P}_i^T + \mathcal{P}_i - \mathcal{Z}_i).$$

Also

$$-d_k\mathcal{P}_i^T\mathcal{R}_k\mathcal{P}_i \leq -d_k(\mathcal{P}_i^T + \mathcal{P}_i - \mathcal{R}_i),$$

and

$$\begin{aligned} &\pi_{ii}\mathcal{P}_i^TE_R(E_R\mathcal{X}_iE_R)^{-1}E_R^T\mathcal{P}_i \\ &\leq \pi_{ii}(E\mathcal{P}_i + \mathcal{P}_i^TE^T - E\mathcal{X}_iE^T), \end{aligned}$$

which means (31) is equivalent to (35) by Schur complement. This completes the proof.

4. ILLUSTRATIVE EXAMPLES

The following examples are presented to illustrate the proposed results with two operating modes (that is $N = 2$).

Example 1: Consider the singular system (1) with the following parameters:

$$E_1 = \begin{bmatrix} -1 & 2 \\ -2 & 4 \end{bmatrix}, A_1 = \begin{bmatrix} -2.3 & 0.8 \\ -0.6 & -2.5 \end{bmatrix},$$
$$A_{11} = \begin{bmatrix} -0.7 & -1.2 \\ -0.6 & -2.0 \end{bmatrix}, A_{12} = \begin{bmatrix} 1 & -0.8 \\ 1.3 & -1 \end{bmatrix},$$
$$E_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} -1 & -0.8 \\ 0.6 & -2.0 \end{bmatrix},$$
$$A_{21} = \begin{bmatrix} -0.5 & 0.3 \\ 0.6 & -1.5 \end{bmatrix}, A_{22} = \begin{bmatrix} -1 & -0.5 \\ 1 & 0.8 \end{bmatrix},$$

and the following transition probability matrix is considered:

$$\Pi = \begin{bmatrix} -a & a \\ 0.6 & -0.6 \end{bmatrix}.$$

Table 1. Maximum d_2 for different a.

a	0.1	0.3	0.5	0.8	1.0
d_2	1.3782	1.3184	1.2959	1.2806	1.2781

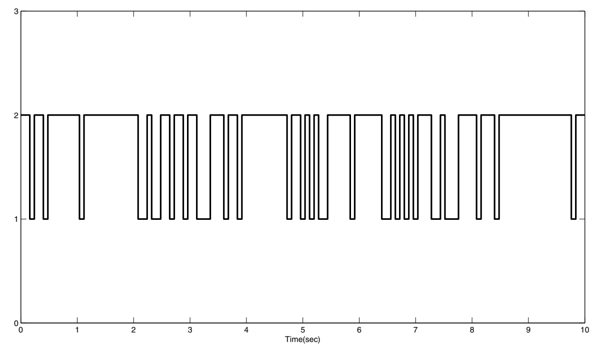

Fig. 1. Jumping modes with 100 random samplings.

Simulation results show that the considered system is unstable for large time-delays. For given $d_1 = 0.5$, $u_1 = u_2 = 0.1$, the maximum allowable d_2 as a changes, the result shows in Table 1.

Example 2: Consider system (29) with the following parameters:

$$E = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}, A_1 = \begin{bmatrix} -15.1 & -3.5 \\ 1 & 1.5 \end{bmatrix},$$
$$A_{11} = \begin{bmatrix} 3.2 & -1.2 \\ 1.2 & 2.0 \end{bmatrix}, A_{12} = \begin{bmatrix} 1 & -3.3 \\ -2.5 & -1 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 5.2 & 0.8 \\ 1.3 & -10 \end{bmatrix}, A_{21} = \begin{bmatrix} -0.7 & -1.2 \\ -0.6 & -2.0 \end{bmatrix},$$
$$A_{22} = \begin{bmatrix} -2.1 & 6.7 \\ -2 & 3.2 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, B_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix},$$

and transition probability matrix choose as:

$$\Pi = \begin{bmatrix} -0.8 & 0.8 \\ 0.6 & -0.6 \end{bmatrix}.$$

In solving LMI (34), we set the tuning scalars $a_i = 0.5$, $b_{ki} = 1$, $E_L = [1 \quad 2]^T$, $E_R = [1 \quad 2]^T$. Fig. 1 and Fig. 2 show the simulation results when $d_1 = 0.5 + 0.5\sin(2t)$ and $d_2 = 1.0 + 0.1\sin t$, the initial condition is assumed to be $\varphi(t) = [-1.5 \quad 2]^T$ for all $t \in [-1.5, 0]$. On the other hand, the parameters are given as:

$$\begin{aligned} K_1 &= [-3.0871 \quad -1.6503], \\ K_2 &= [-6.1004 \quad -1.2209]. \end{aligned}$$

5. CONCLUSIONS

This paper has discussed the problems of stochastic admissibility and stabilization for a class of singu-

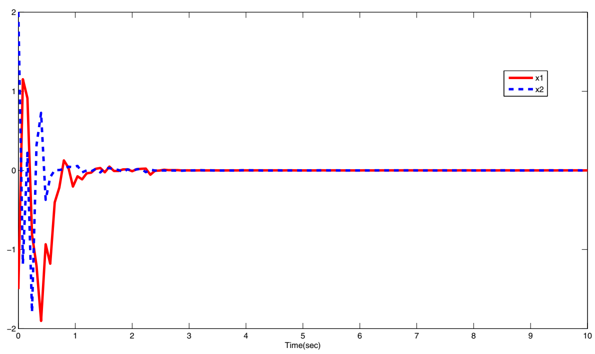

Fig. 2. State response of the closed-loop system.

lar Markovian jump systems with multiple time-varying delays, mode-dependent singular matrix $E(r_t)$ has been considered in the system. Based on Lyapunov functional method, sufficient conditions in terms of strict LMIs have been presented to guarantee the closed-loop system to be stochastically admissible and stochastically stabilizable. Finally, numerical examples have been provided to show the effectiveness of the proposed results.

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