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Robust Sliding Mode-H_∞ Control Approach for a Class of Nonlinear Systems Affected by Unmatched Uncertainties using a Poly-quadratic Lyapunov Function

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Abstract: This paper proposes a robust sliding mode-H_∞ control design methodology for a class of nonlinear systems with unmatched parametric uncertainty and external disturbance. The design procedure combines the high robustness of the sliding mode control (SMC) with the H_∞ norm performance. First, based on linear matrix inequalities (LMI) technique and multiple Lyapunov functions approach, the sliding surface design problem is formulated as a H_∞ state-feedback control for a reduced uncertain nonlinear system with polytopic representation. Then, a sliding mode controller that drives the system states to the sliding surface in finite time and maintains a sliding mode is constructed. Finally, a comparative study is done to prove the effectiveness of the results.

Keywords: H_∞ control, poly-quadratic Lyapunov function, sliding mode control, unmatched uncertainties.

1. INTRODUCTION

Uncertainties in mathematical models may arise from externals perturbations, either parameters variation, approximations in the modeling process and unknown dynamics. In order to ensure the systems exhibit good performance in the presence of uncertainties, robust control has received considerable attention.

The H_∞ control is one such strategy that has been extensively applied, in the literature [1–3], for uncertain systems to compact with problems of robust stabilization and disturbance rejection. It provides explicit performance index in the sense of $\mathcal{L}_2$ gain such that the H_∞ norm from the exogenous disturbance input to the controlled output is minimized or guaranteed to be less than or equal to a prescribed value. This theory has also shown the capability of dealing with the model parameter variations and nonlinearities.

In the other hand, sliding mode control (SMC) is well-known as an efficient robust control method and it is widely used due to various advantages this offers which include its height robustness when dynamic plants operating under uncertainty conditions [4]. Generally, the conventional SMC consists of two steps called sliding step and reaching step. Firstly, design of a sliding surface such that the system possesses the desired performance when it is restricted to the surface. Secondly, synthesise a control law which induces a sliding motion on the sliding surface in finite time. For a good survey on the SMC approach, we refer readers to the work of Pisano [5] and references therein. It is important to note that, in the sliding mode, the dynamics behaviour of the system is totally invariant with respect to a subset of uncertainties satisfying the so-called matching condition [6–8]. However, this class of uncertainties has no effect on the system dynamics, as it acts only within channels implicit in the control input. Unlike the matched case, any unmatched (mismatched) uncertainty always affects directly the dynamics even if the plant is in the sliding mode [3, 9–11].

To overcome this difficulty, much effort has been made and many criteria for performance and robustness have been used, over the last decade, to design the sliding surface for systems with mismatched uncertainties. For example, in [12], to minimize the effects of unmatched disturbance, the invariant ellipsoid method is investigated. In [13], the switching surface design problem is formulated in terms of LMIs as a static-output feedback problem with non-matched uncertainties. SMC approach via a nonlinear disturbance observer is used by Yang [14]. References [15, 16] designed a sliding surface minimizing an H_2 performance. To select the switching manifold, a mixed H_2/H_∞ optimization approach is presented in [17, 18].

Among several robust control schemes, SMC and H_∞ are widely accepted as the powerful control methods and the popular convenient strategies for solving the robust control problems. Thus, it is interesting to apply the com-

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bined sliding mode and H_∞ control for uncertain systems. In recent years, some studies emerge in this research field. A method that congregates H_∞ and integral sliding mode control (ISMC) is proposed by Castanos [19, 20]. He examined a matched perturbation and only an unmatched ones. In [21], a dynamic output feedback H_∞ sliding mode controller design problem is devoted. The system has the mismatched uncertainty and external disturbance, once it is in the sliding mode. But, no linearities is considered. In [22], the problem of designing an output-dependent ISMC for systems with mismatched parameter uncertainties along with disturbances and matched nonlinear perturbations is addressed. To design the sliding surface, H_∞ control is used by [23]. The nonlinearity term is also matched.

It easy is to see that, in all previous references as in the majority of literature papers (especially for H_∞-SMC), robust SMC treating the worst case of the simultaneous presence of unmatched parameter uncertainties, unmatched external disturbance and unmatched nonlinearities, is not developed. Due to mathematical complexity, most of the works are, even, based on the restrictive matching condition. Unfortunately, in reality, many kinds of model parameters uncertainties, disturbances and nonlinearities do not satisfy this condition and the situation when they are all mismatched covers several practical uncertain systems [24]. This situation, if not correctly handled, would, absolutely, causes great degradation of the system performances. However, there have been, so far, few results concerning the sliding mode control of this particular type of unmatched systems. As for instance, the recent work of Zhang [24] were a robust SMC-H_∞ for an offshore steel jacket platform is proposed. A comparative study with this work will show the effectiveness and the superiority of our method. There are, so, a lot of space to be improved on sliding mode control for such systems. To be specific, applying H_∞-SMC for systems with parameter uncertainties, external disturbance and nonlinearities together unmatched is, then, a challenging topic. Which motivate us to carry out the present study.

Furthermore, it is worth pointing out that Reference [24] and almost all other papers on SMC required to use the classical quadratic Lyapunov concept for the sliding surface design stage. But, it is well known that utilizing a single Lyapunov matrix (P) to check the stability over the whole uncertain domain (Δ), although appealing from a computational point of view, leads to rather conservatism. To overcame this drawback, many investigations used a parameter dependent Lyapunov functions (PDLF $P(\Delta)$) to assess robust stability and to compute guaranteed performance indices (see, e.g., [25, 26]). The basic idea behind this effective solution is to separate the products of Lyapunov matrices and system matrices ($PA+A^TP$) in the given LMIs by inserting auxiliary slack variables that add some degree of freedom (originally introduced for LTI systems). The problem of adapting the attractable PDLF scheme with other types of systems remains a difficult theme in the field of control. In this sense, although the SMC problem has been widely investigated for uncertain systems, very little results have been available for this interesting research theme, not to mention the case when total unmatched uncertainties are also involved. It is, therefore, the purpose of this study to shorten such a gap.

Motivated by the above analysis, we discuss, in this paper, the problem of sliding mode-H_∞ control for a class of systems subjected to parametric uncertainty, external disturbance and state-nonlinearity function simultaneously unmatched. Additionally, a poly-quadratic Lyapunov function ($P(\lambda)=\sum_{j=1}^{r}\lambda_j P_j$) is introduced to deal with the polytopic type uncertainty. To the best of the authors knowledge this topic is never considered in the research work.

The major contributions of the current study with respect to the related literature can be summarized as follows:

- Combining SMC with H_∞ to attain better performance proprieties.
- Considering the parametric uncertainty, the external disturbance and the state-nonlinearity function to be together unmatched.
- Reducing the conservatism by exploiting a multiple Lyapunov function.
- Developing novel results in terms of LMIs to guarantee both robust asymptotic stability and H_∞ disturbance attenuation performance, in the sliding mode.
- Proposing a robust sliding mode controller that assures the occurrence of the sliding mode in finite time in spite of mismatched uncertainties.

The paper is organized in the following way: Section 2 describes the problem to be considered in this paper. In Section 3, we develop the sliding surface design methodology. Section 4 gives the SMC synthesis method. In Section 5, we show the simulation results and Section 6 concludes the paper.

Notation: The notation used throughout the paper is fairly standard. $\|\cdot\|$ represents the Euclidean norm of a vector or its induced matrix norm. $\Re^n$ denotes the real n-dimensional space. $\Re^{m\times n}$ is the real $(m\times n)$ matrix space. $\mathcal{L}_2[0,\infty)$ denotes the space of square-integrable vector functions over $[0,\infty)$. I and 0 signify the identity matrix and the zero matrix with appropriate dimensions. $(\cdot)^T$ and $(\cdot)^{-1}$ indicate transpose and matrix inverse. "$*$" denotes the symmetric elements of a symmetric matrix. Matrices, if their dimensions are not explicitly specified, are assumed to be compatible for algebraic operations.

2. PROBLEM STATEMENTS

Consider a class of uncertain nonlinear systems described by

$$\dot{x} = (A_0 + \Delta A)x + Bu + Ew + Ff(t,x), \tag{1}$$

$$z = Cx + Dw, \tag{2}$$

where $x \in \Re^n$ is the state, $u \in \Re^m$ is the control input, $z \in \Re^z$ is the regulated output, $A_0 \in \Re^{n\times n}$ is the nominal dynamic matrix; $\Delta A \in \Re^{n\times n}, w \in \mathcal{L}_2[0,\infty)$ and $f(t,x)$ represent, respectively, the parametric uncertainty, the exogenous disturbance signal and the nonlinearity of the system. All matrices are real with appropriate dimensions. The following are assumed to be valid

A1. The pair (A_0, B) is stabilizable.

A2. The input matrix B has full rank m, $m < n$.

A3. The state x is available.

A4. There exist known nonnegative scalar w_0 such that:

$$\|w(t)\| \leq w_0. \tag{3}$$

A5. The state-related nonlinear function $f(t,x)$ is norm-bounded as:

$$\|f(t,x)\| \leq \alpha \|x\|, \tag{4}$$

where $\alpha > 0$ is a known constant.

A6. The parametric uncertainty ΔA is defined in the following affine form as given by Xia [27,28],

$$\Delta A = \sum_{i=1}^{q} \delta_i A_i, \tag{5}$$

where A_i are known constant matrix and δ_i are unknown scaling parameters whose value varies in their ranges $\delta_i \in [-1,1]$ and q is the number of uncertain parameters.

According to assumption A2, there exists, always, a non-singular matrix $T \in \Re^{n\times n}$ [29] and an associated state vector y as

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} x = Tx, \tag{6}$$

where $y_1 \in \Re^{n-m}$, $y_2 \in \Re^m$, $T_1 \in \Re^{(n-m)\times n}$, $T_2 \in \Re^{m\times n}$. Then, the system (1)-(2) can be transformed to the regular form

$$\dot{y} = (\bar{A}_0 + \Delta\bar{A})y + \bar{B}u + \bar{E}w + \bar{F}\bar{f}(t,y), \tag{7}$$

$$z = \bar{C}y + Dw, \tag{8}$$

with

$$\bar{A}_0 = TA_0T^{-1} = \begin{bmatrix} \bar{A}_{01} & \bar{A}_{02} \\ \bar{A}_{03} & \bar{A}_{04} \end{bmatrix},$$

$$\Delta\bar{A} = T\Delta AT^{-1} = \begin{bmatrix} \Delta\bar{A}_1 & \Delta\bar{A}_2 \\ \Delta\bar{A}_3 & \Delta\bar{A}_4 \end{bmatrix} = \sum_{i=1}^{q} \delta_i \times \begin{bmatrix} \bar{A}_{i1} & \bar{A}_{i2} \\ \bar{A}_{i3} & \bar{A}_{i4} \end{bmatrix},$$

$$\bar{f}(t,y) = \begin{bmatrix} \bar{f}_1(t,y_1) = f(t,T_1^T y_1) \\ \bar{f}_2(t,y_2) = f(t,T_2^T y_2) \end{bmatrix},$$

$$\bar{B} = TB = \begin{bmatrix} 0 \\ \bar{B}_2 \end{bmatrix}, \quad \bar{E} = TE = \begin{bmatrix} \bar{E}_1 \\ \bar{E}_2 \end{bmatrix},$$

$$\bar{C} = CT^T = \begin{bmatrix} \bar{C}_1 & \bar{C}_2 \end{bmatrix}, \quad \bar{F} = TF = \begin{bmatrix} \bar{F}_1 \\ \bar{F}_2 \end{bmatrix},$$

and $\bar{B}_2 \in \Re^{m\times m}$ is non-singular. Define the switching variable as

$$\sigma(t) = ST^{-1}Tx = \bar{S}y = \begin{bmatrix} \bar{S}_1 & I_m \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{9}$$

with $\bar{S}_1 \in \Re^{m\times(n-m)}$. Relying on the sliding condition $(\sigma(t) = 0)$, the sliding motion can be described by the following dynamic equation

$$\dot{y}_1 = (\bar{A}_{01} + \Delta\bar{A}_1)y_1 + (\bar{A}_{02} + \Delta\bar{A}_2)y_2 + \bar{E}_1 w + \bar{F}_1\bar{f}_1(t,y_1), \tag{10}$$

$$z = \bar{C}_1 y_1 + \bar{C}_2 y_2 + Dw, \tag{11}$$

$$y_2 = -\bar{S}_1 y_1. \tag{12}$$

This paper aims at designing a:

1- Sliding surface $\sigma(t) = 0$ such that the sliding mode (10)-(12) is robustly asymptotically stable and fulfills also the H_∞ disturbance attenuation performance.

2- Sliding mode controller which directs the state trajectories of the system (1)-(2) onto the desired sliding surface in finite time and maintains them, for all subsequent time, in this surface, regardless of unmatched parametric uncertainty, disturbances and nonlinearities.

3. SLIDING SURFACE DESIGN

As the polytopic representation is convex, it can be considered the most general ways to preserve the structure information of uncertainty matrices when its model depends affinely on uncertain parameters. Due to this fact, the sliding surface design method developed in this section starts from a polytopic description of the unmatched parameter uncertainty associated with the system matrix.

Since, $\Delta\bar{A}_1$ and $\Delta\bar{A}_2$ are affine in δ_i, the dynamic equation of the sliding mode (10)-(12) can be written as

$$\dot{v} = \Phi(\lambda)v + \Psi(\lambda)u_v + \bar{E}_1 w + \bar{F}_1\bar{f}_1(t,v), \tag{13}$$

$$z = \bar{C}_1 v + \bar{C}_2 u_v + Dw, \tag{14}$$

$$u_v = -\bar{S}_1 v = Kv, \tag{15}$$

where $v = y_1$ is the state of the polytopic system in the sliding mode and $u_v = y_2$ is considered as a fictitious control input for the above system.

According to assumption A5, the state-nonlinearity function $\bar{f}_1(t,v)$ is bounded as

$$\left\|\bar{f}_1(t,v)\right\| \le \alpha \left\|T_1^T v\right\|. \tag{16}$$

Matrices $\Phi(\lambda) = \bar{A}_{01} + \Delta\bar{A}_1$ and $\Psi(\lambda) = \bar{A}_{02} + \Delta\bar{A}_2$ belong to a polytope-type set Ω with known vertices. This set is given by

$$\begin{aligned}\Omega = \{ &< \Phi(\lambda), \Psi(\lambda) > = \sum_{j=1}^{r} \lambda_j < \Phi_j, \Psi_j >; \\ &\sum_{j=1}^{r} \lambda_j = 1 \lambda_j \ge 0; r = 2^q \},\end{aligned} \tag{17}$$

such that, for each vertex $\Theta_j = < \Phi_j, \Psi_j >$, the polytopic coefficient λ_j is expressed as follows [30];

$$\left.\begin{array}{l} \bar{\lambda}_i = \dfrac{1-\delta_i}{2} \quad ; \quad i = 1,\dots,q \\ \tilde{\lambda}_i = \begin{cases} \bar{\lambda}_i & \text{if } -1 \text{ is a coordinate of } \Theta_j \\ 1-\bar{\lambda}_i & \text{if } 1 \text{ is a coordinate of } \Theta_j \end{cases} \end{array}\right\} \tag{18}$$

$$\Rightarrow \lambda_j = \prod_{i=1}^{q} \tilde{\lambda}_i.$$

Also, the pairs $(\Phi_j, \Psi_j), \forall j \in I(1,r)$, are stabilizable for all admissible uncertainties in the parameter box.

Remark 1: As demonstrated in [31], the polytopic-type uncertainty describes physical parameter uncertainties more precisely than the norm-bounded uncertainty [24] and eliminates the conservatism usually caused by the latter.

The closed-loop system, in the sliding mode, is given, then, by

$$\dot{v} = (\Phi(\lambda) + \Psi(\lambda)K)v + \bar{E}_1 w + \bar{F}_1 \bar{f}_1(t,v), \tag{19}$$

$$z = \left(\bar{C}_1 + \bar{C}_2 K\right) v + Dw. \tag{20}$$

Note that the matrix D is assumed to satisfy the following condition

$$D^T D < \gamma^2 I. \tag{21}$$

It is clear that, the sliding mode dynamics will be determined by the choice of the state-feedback gain matrix K. It should be selected such that the sliding motion (19)-(20) is robustly asymptotically stable and satisfies $\|z\| < \gamma\|w\|$. In other meaning, for a prescribed scalar $\gamma > 0$

$$J(w) = \int_0^\infty \left(z^T z - \gamma^2 w^T w\right) dt < 0 \tag{22}$$

under zero-initial condition for all nonzero $w \in \mathcal{L}_2[0,\infty)$. Firstly, we will analyze the robust asymptotic stability ($w = 0$) and the H_∞ norm performance of the sliding mode dynamics (19)-(20), by giving the following theorem.

Theorem 1: Let $\mu = \alpha^{-2}$. Suppose that a scalar $\gamma > 0$ is given. Under condition (21), if there exist a scalar $\varepsilon_2 > 0$, any appropriately matrices G_1 and G_2 and a parameter-dependent matrix $P(\lambda) > 0$ such that

$$\Gamma(\lambda) = \begin{bmatrix} \Gamma_1 & \Gamma_2 & -G_1\bar{F}_1 & \Gamma_3 \\ * & G_2 + G_2^T & -G_2\bar{F}_1 & -G_2\bar{E}_1 \\ * & * & -\varepsilon_2^{-1} & 0 \\ * & * & * & -\gamma^2 + D^T D \end{bmatrix} < 0, \tag{23}$$

where $\Gamma_1 = -G_1[\Phi(\lambda) + \Psi(\lambda)K] - [\Phi(\lambda) + \Psi(\lambda)K]^T G_1^T + [\bar{C}_1 + \bar{C}_2 K]^T [\bar{C}_1 + \bar{C}_2 K] + \varepsilon_2^{-1}\mu^{-1} T_1 T_1^T, \Gamma_2 = P(\lambda) + G_1 - [\Phi(\lambda) + \Psi(\lambda)K]^T G_2^T, \Gamma_3 = -G_1\bar{E}_1 + [\bar{C}_1 + \bar{C}_2 K]^T D$. Then, the sliding mode (19)-(20) is robustly asymptotically stable with an H_∞ disturbance attenuation level bound γ.

Proof: Suppose that (23) holds. By applying Schur complement, we have

$$\begin{bmatrix} \bar{\Gamma}_1 & \Gamma_2 & -G_1\bar{F}_1 & -G_1\bar{E}_1 & [\bar{C}_1 + \bar{C}_2 K]^T \\ * & G_2 + G_2^T & -G_2\bar{F}_1 & -G_2\bar{E}_1 & 0 \\ * & * & -\varepsilon_2^{-1} & 0 & 0 \\ * & * & * & -\gamma^2 & D^T \\ * & * & * & * & -I \end{bmatrix} < 0, \tag{24}$$

where $\bar{\Gamma}_1 = -G_1[\Phi(\lambda) + \Psi(\lambda)K] - [\Phi(\lambda) + \Psi(\lambda)K]^T G_1^T + \varepsilon_2^{-1}\mu^{-1} T_1 T_1^T$. Also, we can obtain

$$\bar{\Gamma}(\lambda) = \begin{bmatrix} \bar{\Gamma}_1 & \Gamma_2 & -G_1\bar{F}_1 \\ * & G_2 + G_2^T & -G_2\bar{F}_1 \\ * & * & -\varepsilon_2^{-1} \end{bmatrix} < 0. \tag{25}$$

In other hand, let us define a poly-quadratic Lyapunov function

$$V(v) = v^T P(\lambda) v = v^T \sum_{j=1}^{r} \lambda_j P_j v, \tag{26}$$

where $P_j = P_j^T \in \Re^{(n-m)\times(n-m)} > 0, j = 1,\dots,r$ need to be determined. From (16) and (19), we can obtain

$$\mu^{-1} v^T T_1 T_1^T v - \bar{f}_1^T(t,v)\bar{f}_1(t,v) \ge 0, \tag{27}$$

$$\begin{aligned}&2\left[v^T G_1 + \dot{v}^T G_2\right]\left[\dot{v} - (\Phi(\lambda) + \Psi(\lambda)K)v \right.\\ &\quad \left. - \bar{E}_1 w - \bar{F}_1 \bar{f}_1(t,v)\right] = 0,\end{aligned} \tag{28}$$

where G_1 and G_2 are any appropriately matrices.
We obtain, then

$$\begin{aligned}\Sigma = &\left[\mu^{-1} v^T T_1 T_1^T v - \bar{f}_1^T(t,v)\bar{f}_1(t,v)\right] \\ &\times \varepsilon_2^{-1} + 2\left[v^T G_1 + \dot{v}^T G_2\right]\left[\dot{v} - (\Phi(\lambda)\right. \\ &\left. + \Psi(\lambda)K)v - \bar{E}_1 w - \bar{F}_1 \bar{f}_1(t,v)\right] \ge 0.\end{aligned} \tag{29}$$

Calculating the time derivative of $V(v)$ along the solution of (19) and adding Σ to it gives

$$\dot{V}(v) + \Sigma = v^T P(\lambda)\dot{v} + \dot{v}^T P(\lambda) v + \Sigma. \tag{30}$$

If $w = 0$. After some manipulation, it follows that

$$\dot{V}(v) + \Sigma = \bar{\vartheta}^T \bar{\Gamma}(\lambda) \bar{\vartheta}_{|w=0}, \tag{31}$$

where $\bar{\Gamma}(\lambda)$ is defined in (25) and $\bar{\vartheta} = \begin{bmatrix} v^T & \dot{v}^T & \bar{f}_1^T \end{bmatrix}^T$. As $\bar{\Gamma}(\lambda) < 0$, it follows that for all $\bar{\vartheta} \neq 0$, $\dot{V}(v) + \Sigma < 0$. Thus, $\dot{V}(v) < 0$. So, the sliding mode (19)-(20) is asymptotically stable. To establish the H_∞ norm performance for the sliding mode (19)-(20), we take

$$\dot{V}(v) + \Sigma + z^T z - \gamma^2 w^T w = \vartheta^T \Gamma(\lambda) \vartheta, \tag{32}$$

where $\vartheta = \begin{bmatrix} v^T & \dot{v}^T & \bar{f}_1^T & w^T \end{bmatrix}^T$ and $\Gamma(\lambda)$ is defined in (23). For any $\vartheta \neq 0$, if $\Gamma(\lambda) < 0$, then

$$\dot{V}(v) + z^T z - \gamma^2 w^T w < 0. \tag{33}$$

Integrating (33) from 0 to ∞, it follows that

$$V(v(\infty)) - V(v(0)) + \int_0^\infty \left(z^T z - \gamma^2 w^T w\right) dt < 0. \tag{34}$$

For all nonzero $w \in \mathcal{L}_2[0, \infty)$, the initial condition $v(0) = 0$ implies that $J(w) < 0$. This is the end of proof. □

Now, we are ready to determine the H_∞ state-feedback gain.

Theorem 2: Under condition (21), for given scalar $\varepsilon_1 > 0$, if there exist constants $\gamma > 0$, $\varepsilon_2 > 0$, symmetric definite matrices $Y_j = Y_j^T \in \Re^{(n-m)\times(n-m)} > 0$ and any appropriately matrices N and M satisfying, for $j = 1, \ldots, r$:

$$\Lambda_j = \begin{bmatrix} \Lambda_{1j}(N,M) & \Lambda_{2j}(N,M) & -\varepsilon_2 \bar{F}_1 & -\bar{E}_1 & \Lambda_3(N,M) & NT_1 \\ * & \varepsilon_1(N+N^T) & -\varepsilon_1\varepsilon_2 \bar{F}_1 & -\varepsilon_1 \bar{E}_1 & 0 & 0 \\ * & * & -\varepsilon_2 I & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & D^T & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -\varepsilon_2 \mu I \end{bmatrix} < 0, \tag{35}$$

where

$$\Lambda_{1j}(N,M) = -\Phi_j N^T - N\Phi_j^T - \Psi_j M - M^T \Psi_j^T,$$
$$\Lambda_{2j}(N,M) = Y_j + N^T - \varepsilon_1 N \Phi_j^T - \varepsilon_1 M^T \Psi_j^T,$$
$$\Lambda_3(N,M) = N\bar{C}_1^T + M^T \bar{C}_2^T.$$

then, the state feedback control $u_v = Kv = MN^{-T}v$ guarantees the asymptotic robust stability with an H_∞ disturbance attenuation level bound γ, for the sliding motion (19)-(20).

Proof: The proof is based on Theorem 1. Set $G_1 = G, G_2 = \varepsilon_1 G$. Since $G_2 + G_2^T$ in $\Gamma(\lambda)$ is negative definite, evidently G is nonsingular. Then, pre- and post-multiplying the matrix $\Gamma(\lambda)$, respectively, by $diag\left\{G^{-1}, G^{-1}, \varepsilon_2 I, I\right\}$ and $diag\left\{G^{-T}, G^{-T}, \varepsilon_2 I, I\right\}$, yields

$$\begin{bmatrix} \tilde{\Gamma}_1 & \tilde{\Gamma}_2 & -\varepsilon_2 \bar{F}_1 & \tilde{\Gamma}_3 \\ * & \varepsilon_1(G^{-1}+G^{-T}) & -\varepsilon_1\varepsilon_2 \bar{F}_1 & -\varepsilon_1 \bar{E}_1 \\ * & * & -\varepsilon_2 I & 0 \\ * & * & * & -\gamma^2 + D^T D \end{bmatrix} < 0, \tag{36}$$

where

$$\begin{aligned} \tilde{\Gamma}_1 = &-[\Phi(\lambda) + \Psi(\lambda)K]G^{-T} - G^{-1}[\Phi(\lambda) + \Psi(\lambda)K]^T \\ &+ G^{-1}\left[\bar{C}_1 + \bar{C}_2 K\right]^T \left[\bar{C}_1 + \bar{C}_2 K\right] G^{-T} \\ &+ \varepsilon_2^{-1}\mu^{-1} G^{-1} T_1 T_1^T G^{-T}, \\ \tilde{\Gamma}_2 = &\, G^{-1} P(\lambda) G^{-T} + G^{-T} - \varepsilon_1 G^{-1}[\Phi(\lambda) + \Psi(\lambda)K]^T, \\ \tilde{\Gamma}_3 = &-\bar{E}_1 + G^{-1}\left[\bar{C}_1 + \bar{C}_2 K\right]^T D. \end{aligned}$$

Let $N = G^{-1}, K = MN^{-T}$ and $Y(\lambda) = NP(\lambda)N^T$. Applying the Schur complement, gives

$$\Lambda(\lambda) = \begin{bmatrix} \Lambda_1(N,M) & \Lambda_2(N,M) & -\varepsilon_2 \bar{F}_1 & -\bar{E}_1 & \Lambda_3(N,M) & NT_1 \\ * & \varepsilon_1(N+N^T) & -\varepsilon_1\varepsilon_2 \bar{F}_1 & -\varepsilon_1 \bar{E}_1 & 0 & 0 \\ * & * & -\varepsilon_2 I & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & D^T & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -\varepsilon_2 \mu I \end{bmatrix} < 0, \tag{37}$$

where

$$\begin{aligned} \Lambda_1(N,M) = &-\Phi(\lambda) N^T - N\Phi^T(\lambda) - \Psi(\lambda) M \\ &- M^T \Psi^T(\lambda), \Lambda_2(N,M) \\ = &\, Y(\lambda) + N^T - \varepsilon_1 N \Phi^T(\lambda) - \varepsilon_1 M^T \Psi^T(\lambda). \end{aligned}$$

So, by the convexity condition $\Lambda(\lambda) = \sum_{j=1}^{r} \lambda_j \Lambda_j$, it is clear that $J(w) < 0$ for all nonzero $w \in \mathcal{L}_2[0, \infty)$ if LMIs (35) hold. This is the end of proof. □

Remark 2: Note that for the given tuning scalar ε_1, LMIs (35) are linear with respect to Y_j, N, M, ε_2, and therefore can be solved by LMI Toolbox. But, the obtained results are sensitive to the scalar ε_1. The problem is then how to find its optimal value. This can be ascertained by performing a 1-dimensional search.

Remark 3: Theorem 2 exhibits novel sufficient conditions to design sliding surface in terms of LMIs. Moreover, it shows a new LMI representation of H_∞ performance criterion for a class of polytopic nonlinear system.

Additionally, it does not any product of the Lyapunov matrices and the system dynamic matrices by introducing two weighting matrices (G_1 and G_2). This feature enables us to employ a multiple Lyapunov function that can bring further flexibility and reduce the restriction and the conservatism imposed, usually, by the use of single Lyapunov function in the analysis and synthesis problems of systems with polytopic-type uncertainties.

4. SLIDING MODE CONTROL SYNTHESIS

Once the sliding problem has been solved that is the matrix $\bar{S}_1$ has been selected, attention must be turned to synthesize a sliding mode controller that ensures the reachability of the sliding surface $\sigma(t)=0$. In this context, we state the following result.

Theorem 3: Consider the uncertain nonlinear system (1)-(2). For a given positive scalar β, the state trajectories can be driven onto the sliding surface in finite time t_σ and remain, there, subsequently, by the control

$$u=-\bar{B}_2^{-1}(\rho_1\|x\|+\rho_2)sign(\sigma), \tag{38}$$

where
$\rho_1=\left\|\bar{S}T(A_0+\Delta A)\right\|+\alpha\left\|\bar{S}TF\right\|, \rho_2=\left\|\bar{S}TE\right\|w_0+\beta.$

Proof: Consider the Lyapunov function

$$V_\sigma(\sigma)=0.5\,\sigma^T(t)\sigma(t). \tag{39}$$

Using the control law (38), the time derivative of $V_\sigma(\sigma)$, along the trajectory of (7), is given by

$$\begin{aligned}\dot{V}_\sigma(\sigma)=&\sigma^T\dot{\sigma}=\sigma^T\bar{S}\dot{y}=\sigma^T\bar{S}T\dot{x}\\ =&\sigma^T\left[\bar{S}T(A_0+\Delta A)x+\bar{S}TEw+\bar{S}TFf(t,x)\right.\\ &\left.-(\rho_1\|x\|+\rho_2)sign(\sigma)\right]\\ \le&((\left\|\bar{S}T(A_0+\Delta A)\right\|+\alpha\left\|\bar{S}TF\right\|)\|x\|\\ &+\left\|\bar{S}TE\right\|w_0)\|\sigma\|-(\rho_1\|x\|+\rho_2)\|\sigma\|,\end{aligned} \tag{40}$$

we get so

$$\dot{V}_\sigma(\sigma)\le-\beta\|\sigma\|=-\sqrt{2}\beta\sqrt{V_\sigma(\sigma)}. \tag{41}$$

The reachability condition $\sigma^T(t)\dot{\sigma}(t)\le 0$ is assured. Integrating both sides from 0 to $t>0$, we have

$$\sqrt{V_\sigma(t)}-\sqrt{V_\sigma(0)}\le-\frac{\beta}{\sqrt{2}}t. \tag{42}$$

In fact, suppose that the system states cannot reach the sliding mode $\sigma=0$ within finite time, then from $\sqrt{V_\sigma(t)}\le\sqrt{V_\sigma(0)}-\frac{\beta}{\sqrt{2}}t, \sqrt{V_\sigma(t)}$ becomes negative with t sufficiently large. This contradicts with $\sqrt{V_\sigma(t)}$ nonnegative. In this way considering t_σ as the time required to heat $\sigma=0$ and noting that $\sigma(t=t_\sigma)=0$, one has

$$t_\sigma\le\frac{\|\sigma(0)\|}{\beta}. \tag{43}$$

So, the proposed sliding mode control (38) brings the system trajectories onto the switching manifold in finite time t_σ and kept them, there, afterwards. This is the end of proof. □

Remark 4: It is clear that the convergence speed of the system states is determined by β. The larger the value of β, the faster the convergence of the system trajectories. However, it will require a very high control input but in reality the input is always limited within a fixed value. Thus, the parameter β cannot be chosen to be too large. In practice, its value can be properly chosen in order to keep the reaching time, t_σ, as short as possible and a compromise has to be made between the response speed and the control input.

Remark 5: The main disadvantage of the SMC is chattering around the switching manifold during the sliding phase. In practical, the effect of chattering can be eliminated by introducing the sigmoid function such that $sign(\sigma)\simeq\dfrac{\sigma}{\|\sigma\|+\eta}$ where η is a small positive constant. But this approximation (frequently used in SMC) leads to obtain a quasi-sliding mode surface (the trajectories of the system reach a small bounded region around the real sliding manifold). As η tends to be zero, the performance of the approximated control law can be made arbitrarily close to that of the original control law. It can be used to trade off the requirement of maintaining ideal robustness performance with that of ensuring a smooth control action.

For comparison purposes, we now consider the case of $\Delta A=0$. In this case, the system is in the form

$$\begin{cases}\dot{x}=A_0x+Bu+Ew+Ff(t,x),\\ z=Cx+Dw,\end{cases} \tag{44}$$

and the dynamic equation of the sliding mode can be written as

$$\begin{cases}\dot{v}=\bar{A}_{01}v+\bar{A}_{02}u_{v0}+\bar{E}_1w+\bar{F}_1\bar{f}_1(t,v),\\ \bar{C}_1v+\bar{C}_2u_{v0}+Dw,\\ u_{v0}=K_{H_\infty SMC}\,v=-\bar{S}_1v.\end{cases} \tag{45}$$

By Theorems 2 and 3, we have the following result for the system (44).

Corollary 1: Consider the system (44). For a given scalar $\beta>0$, suppose the sliding function is defined as (9) and the sliding mode control law is designed as

$$u_0=-\bar{B}_2^{-1}(\rho_{10}\|x\|+\rho_2)sign(\sigma), \tag{46}$$

where $\rho_{10}=\left\|\bar{S}TA_0\right\|+\alpha\left\|\bar{S}TF\right\|$.
Under condition (21), if there exist constants $\gamma>0$, $\varepsilon_2>0$, symmetric definite matrice $Y_0=Y_0^T\in\Re^{(n-m)\times(n-m)}>0$ and any appropriately matrices N and M satisfying, for a selected scalar $\varepsilon_1>0$,

$$\left\{\Lambda_0=\Lambda_j\Big|_{\substack{\Lambda_{1j}(N,M)=\Lambda_{10}(N,M)\\ \Lambda_{2j}(N,M)=\Lambda_{20}(N,M)}}\right\}<0, \tag{47}$$

where

$$\Lambda_{10}(N,M) = -\bar{A}_{01}N^T - N\bar{A}_{01}^T - \bar{A}_{02}M - M^T\bar{A}_{02}^T,$$
$$\Lambda_{20}(N,M) = Y_0 + N^T - \varepsilon_1 N\bar{A}_{01}^T - \varepsilon_1 M^T\bar{A}_{02}^T,$$

then, (i) the state feedback control $u_{v0} = MN^{-T}v$ guarantees the asymptotic robust stability with an H_∞ disturbance attenuation level bound γ, for the sliding motion (45) and (ii) the sliding surface is reachable in finite time and the state trajectory of system (44) stays on it thereafter by the control (46).

5. SIMULATION RESULTS

5.1. Offshore steel jacket platform [24, 32]

In this section we design a H_∞-sliding mode control (H_∞SMC) constructed as (46) for the system (44). To demonstrate the efficiency and the superiority of the proposed design scheme, the performances of the system under H_∞SMC, SMHC [24] and methods cited therein (SMC and HIC) are compared. As in [24], the model of an Offshore steel jaket platform is described by the following matrix values

$$A0 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -3.3235 & -0.0212 & 0.0184 & 0.0030 & -5.3449 & -0.8819 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0.0184 & 0.0030 & -118.1385 & -0.1118 & 5.3465 & 0.8822 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -0.0114 & -0.0019 & 0.0114 & 0.0019 & -3.3051 & 0.5454 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0.03445 & 0 & -0.00344628 & 0 & 0.00213 \end{bmatrix}^T,$$

$$E = \begin{bmatrix} 0 & -0.003445 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.00344628 & 0 & 0 \end{bmatrix}^T,$$

$$F = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}^T.$$

The nonlinear self-excited wave force $f(t,x)$ is computed as in [32]. The external disturbance force acting on the first mode $w_1(t)$ is approximated by a uniformly distributed random signal ranging between -4.6×10^5N and 4.6×10^5N, while the external disturbance force acting on the second mode $w_2(t)$ is approximated by a uniformly distributed random signal ranging between -1.1×10^5N and 1.1×10^5N. Since the eigenvalues of matrix A_0 are near the j-axis (-0.0275 ± 1.8116j, -0.0560 ± 10.8690j and -0.2557 ± 1.8060j), the average oscillation amplitudes of the three floors peak to peak is $1.5164m$. As the performance is bad, the offshore platform is very dangerous to work. The main objective of control is, then, to

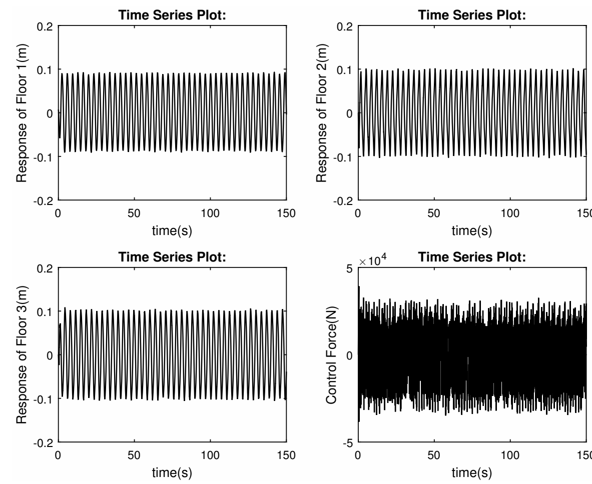

Fig. 1. The responses of the system and the control force.

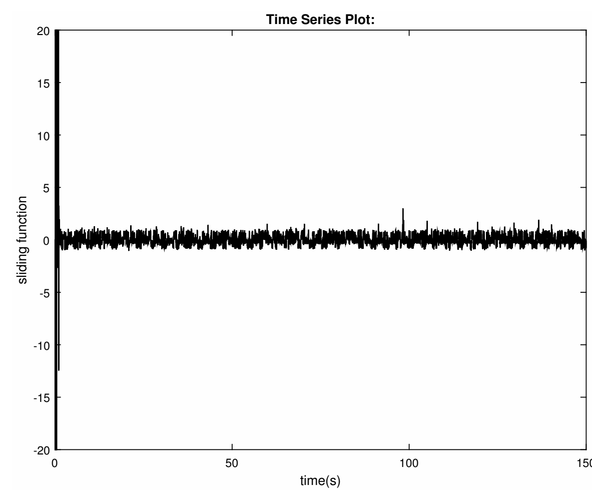

Fig. 2. The evolution of the sliding function $\sigma(t)$.

enhance this performance by reducing the oscillations. Set $\alpha = 0.8$, $\eta = 0.8$ and $\varepsilon_2 = 0.08$. By Corollary 1, we obtain the H_∞-sliding mode gain as

$$K_{H_\infty SMC} = 10^2 \begin{bmatrix} -0.1839, & -0.4835, & 0.1549 & 0.0759, & 0.1239 \end{bmatrix}.$$

The use of the linear sliding function (9) enables us to obtain a ($n-m=5$) reduced order system in the sliding mode (45). In addition, the design strategy in this paper lets the sliding mode gain $K_{H_\infty SMC} = MN^{-T}$ to be independent on lyapunov matrix P which produces, as well recognized, better value results. These advantages are confirmed by the response curves given in Figs. 1 and 2. The responses of the three floors and the required control force when H_∞SMC is applied to the system (44) are presented in Fig. 1. It can be observed that, the peak to peak oscillation amplitudes of the three floors are 0.1886 m, 0.2093 m and 0.2249 m, respectively. The control force peak to peak is about 7.7616×10^4 N. The evolution of the sliding function $\sigma(t)$ is depicted in Fig. 2. The average value of the sliding variable is equal zero. It is clear that the oscillations around the sliding surface are also very less than those under SMHC and SMC.

In Table 1 (by taking into consideration Table 1 [24]), for different values of γ, the oscillation amplitudes of the system and the control force under the (H_∞SMC), SMHC and HIC are compared, where the performances of the system under the SMC and the case of no control are also presented.

From Table 1, Figs. 1-2 and Figs. 2-6 [24], the following can be easily seen

- (H_∞SMC) reduces the oscillation amplitudes of three floors more than those under SMHC, SMC and HIC.
- Compared to others controllers, the required control force under (H_∞SMC) is the smaller.
- The oscillations around the sliding surface are very less than those under SMHC and SMC.

We can conclude that H_∞SMC provides much better performances and the sliding mode-H_∞ control methodology proposed in this study seems to be a good choice for the control design of the Offshore steel jaket platform.

To describe the uncertain dynamic matrix, the norm-bounded uncertainty ($\Delta A = M_1\Delta_1(t)N_1$) has been considered, in [24], by assuming that $\Delta_1(t) = \sin(t)$. This representation has been used to characterize the parameter perturbations. While, we take, in this paper, $\Delta A = \sum_{i=1}^{q} \delta_i A_i; \delta_i \in [-1,1]$. This form directly targets the uncertainty parameter structures ($\theta_i = \theta_{0i} + \delta_i\Delta(\theta_i)$). It necessities the know of the exact number (q) of uncertain parameters (θ_i) and the ranges of parameter variations ($\underline{\theta}_i = \theta_0 - 0.5\Delta(\theta_i) \le \theta_i \le \bar{\theta}_i = \theta_0 + 0.5\Delta(\theta_i)$). However, these informations do not exist in [24]. So, The comparison can't be done in the case of $\Delta A \neq 0$ because there are not the same hypothesis conditions. But, it is important to highlight that the polytopic-type uncertainty, as demonstrated in [31], describes physical parameter uncertainties more precisely than the norm-bounded uncertainty [24] and eliminates the conservatism usually caused by the latter. To illustrate the validity of our method in the case of $\Delta A \neq 0$ a numerical example is given below.

5.2. Numerical example

Consider the uncertain nonlinear system (1)-(2) with

$$A_0 = \begin{bmatrix} -0.7113 & -1.4359 & \theta_0 \\ -2.0813 & -9.5283 & 4.4529 \\ 0.2690 & 0.3648 & -0.1169 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} 0 & 0 & \Delta\theta \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1.5363 \\ 2.0831 \\ -0.6675 \end{bmatrix},$$

Table 1. Control forces and oscillation amplitudes of three floors of the system under different controllers.

γ	Control	Floor 1 (m)	Floor 2 (m)	Floor 3 (m)	u(10^5 N)
/	u=0	1.4159	1.5270	1.6061	/
	SMC	0.2333	0.2537	0.2688	1.8401
0.2	H_∞SMC	0.1886	0.2093	0.2249	0.7762
	SMHC	0.1998	0.2177	0.2317	1.5449
	HIC	0.2040	0.2220	0.2358	2.6952
0.3	H_∞SMC	0.1906	0.2111	0.2255	0.7786
	SMHC	0.2027	0.2208	0.2352	1.5564
	HIC	0.2046	0.2226	0.2364	2.7102
0.4	H_∞SMC	0.1913	0.2119	0.2271	0.8007
	SMHC	0.2026	0.2204	0.2343	1.5137
	HIC	0.2036	0.2216	0.2354	2.0259
0.5	H_∞SMC	0.1924	0.2135	0.2280	0.8021
	SMHC	0.2043	0.2226	0.2371	1.6039
	HIC	0.2041	0.2221	0.2357	2.0242
0.7	H_∞SMC	0.1938	0.2146	0.2286	0.8024
	SMHC	0.2041	0.2221	0.2361	1.5480
	HIC	0.2040	0.2220	0.2357	2.0458

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E = \begin{bmatrix} 0.4 \\ -0.2 \\ 0.2 \end{bmatrix}, \quad F = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},$$

$$D = 0,$$

such that $\theta = \theta_0 + \delta_1\Delta\theta$ is the uncertain parameter when $\theta_0 = -10.0778, \Delta\theta = 2$ and $-1 \le \delta_1 \le 1$. The non linear term is $f(t,x) = 0.9\,\|x(t)\|$. Thus, we take $\alpha = 1$. The transformation matrix is calculated as

$$T = \left[\begin{array}{ccc} -0.7793 & 0.6143 & 0.1236 \\ 0.2497 & 0.1236 & 0.9604 \\ \hline -0.5747 & -0.7793 & 0.2497 \end{array}\right].$$

The reduced system (7)-(8) is described, then, by

$$\bar{A}_0 = \left[\begin{array}{cc|c} -1.0356 & 9.4040 & 6.6925 \\ -0.8478 & -2.1869 & 0.5064 \\ \hline 3.7731 & 3.7568 & -7.1340 \end{array}\right],$$

$$\bar{A}_1 = \left[\begin{array}{cc|c} -0.1926 & -1.4969 & -0.3892 \\ 0.0617 & 0.4797 & 0.1247 \\ \hline -0.1421 & -1.1040 & -0.2870 \end{array}\right],$$

$$\bar{C} = \left[\begin{array}{cc|c} -0.7793 & 0.2497 & -0.5747 \\ 0.6143 & 0.1236 & -0.7793 \\ 0.1236 & 0.9604 & 0.2497 \end{array}\right],$$

$$\bar{B} = \left[\begin{array}{c} 0 \\ 0 \\ \hline -2.673 \end{array}\right], \quad \bar{E} = \left[\begin{array}{c} -0.4099 \\ 0.2673 \\ \hline -0.0241 \end{array}\right],$$

$$\bar{F} = \left[\begin{array}{c} -0.0414 \\ \hline 1.3337 \\ \hline -1.1043 \end{array}\right], \quad \bar{f}(t,y) = \left[\begin{array}{c} 0.9\left\|T_1^T y_1\right\| \\ \hline 0.9\left\|T_2^T y_2\right\| \end{array}\right].$$

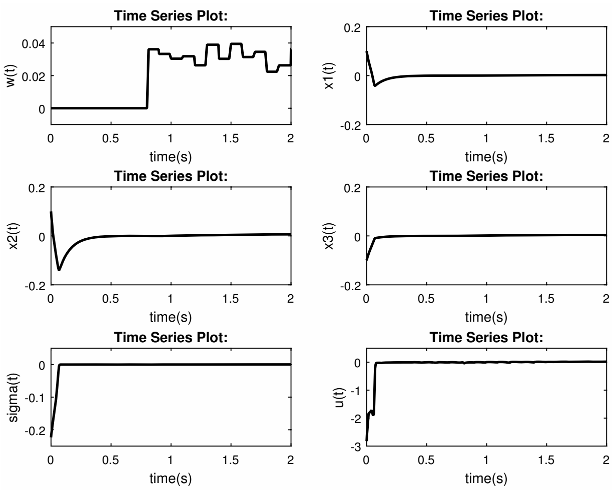

Fig. 3. The responses of $w(t)$, $x(t)$, $\sigma(t)$ and $u(t)$, respectively.

Therefore, the vertices $\{(\Phi_1,\Psi_1);(\Phi_2,\Psi_2)\}$ of the polytope are

$$\Phi_1 = \bar{A}_{01} + \bar{A}_{11} = \begin{bmatrix} -1.2282 & 7.9071 \\ -0.7860 & -1.7072 \end{bmatrix},$$

$$\Psi_1 = \bar{A}_{02} + \bar{A}_{12} = \begin{bmatrix} 6.3033 \\ 0.6311 \end{bmatrix},$$

$$\Phi_2 = \bar{A}_{01} - \bar{A}_{11} = \begin{bmatrix} -0.8430 & 10.9008 \\ -0.9095 & -2.6665 \end{bmatrix},$$

$$\Psi_2 = \bar{A}_{02} - \bar{A}_{12} = \begin{bmatrix} 7.0817 \\ 0.3817 \end{bmatrix}.$$

Taking $\varepsilon_1 = 0.01$ and solving the LMIs (35) in Theorem 2, give the following values

$$\gamma = 0.3255,\ \bar{S}_1 = \begin{bmatrix} 2.5348 & -0.1752 \end{bmatrix}.$$

Hence, the sliding variable (9) and the SMC law (38) are designed as

$$\sigma(t) = \begin{bmatrix} -2.5939 & 0.7563 & 0.3947 \end{bmatrix} x(t),$$

$$u(t) = (12.5978\,\|x(t)\| + 0.769)\frac{\sigma(t)}{\|\sigma(t)\| + 0.01}.$$

The mismatched disturbance signal $w(t)$ is injected, after a time of 0.8 s as shown in Fig. 3. The simulation results are achieved by taking $\delta_1 = -0.5$ and $\beta = 2$ and $x_0 = \begin{bmatrix} 0.1 & 0.1 & -0.1 \end{bmatrix}^T$. Fig 3 shows the evolution responses of the system state $x(t)$, the sliding variable $\sigma(t)$ and the control signal $u(t)$. It is easy to see that the system is stable after a short finite time. The states reach the sliding surface and stay within a small bounded vicinity around this surface, regardless of non-matched uncertainties. It is clear that the controller provides good robustness and performances qualities for the uncertain system. Therefore, the proposed method is an efficient tool to control the considered class of uncertain systems.

6. CONCLUSION

This paper has concerned with the sliding mode-H_∞ control problem for a class of nonlinear systems affected by mismatched uncertainties. Novel LMI conditions that guarantee the robust asymptotic stability with H_∞ disturbance attenuation for the sliding mode dynamics have been developed. A poly-quadratic Lyapunov function has been employed to deal with the polytopic uncertainty witch enable us to reduce the restriction and the conservatism imposed, usually, by the use of single Lyapunov function. The control law is designed to force system trajectories toward the sliding manifold in finite time and maintain them on the manifolds after that in spite of unmatched uncertainties. A comparative study and a numerical example have demonstrated the efficacy of the proposed design methodology.

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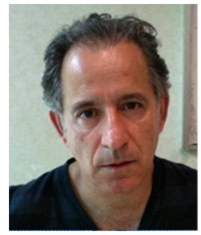

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