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# Internal Model Control Based PID Tuning Using First-Order Filter

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**Abstract:** The selection of filter plays an important role in internal model control (IMC) based controller design. As per the rule, for a minimum-phase delay free plant, IMC based controller is obtained by augmenting a filter. We suggest the use of first-order filter for controller design, which is then parameterized to a conventional PID controller. The proposed scheme brings filter size reduction, closed-loop bandwidth enhancement, and easy formulation of the PID structure. The proposed scheme is applied to some class of linear and nonlinear processes. Further the hardware testing for velocity control of precision modular servo system which contains DC servomotor is carried out through this scheme. Quantitative comparison of servo, regulatory and optimal attributes of the proposed scheme with other popular IMC-PID control techniques depicts sharp reference tracking, good disturbance rejection and minimum integral error performance.

**Keywords:** Bandwidth, closed-loop response, DC servomotor, disturbance rejection, PID control.

## 1. INTRODUCTION

The synthesis of PID controller from internal model control (IMC) scheme has brought a surge in PID tuning research [1–3]. This is because all the three parameters of PID controller can be obtained from a single parameter called IMC filter time constant $'\lambda'$ and $\lambda > 0$. In recent years, PID tuning based on soft computing techniques (such as genetic algorithm and fuzzy logic concepts [4,5]) and robust optimization (such as $H_\infty$ [6]) are reported. However in comparison to these schemes, IMC scheme is simple, robust, suboptimal and can be easily implemented to linear, nonlinear and time-delayed systems. Therefore, IMC scheme has been intensively studied. In this technique, it is observed that selection and structure of filter plays an important role in determining the PID parameters [7,8]. The filter ensures closed-loop stability, physical realization, robustness to parametric/modeling error, $H_2$ optimality, and robust performance to reference tracking and disturbance rejection [9]. Various high-order filter structures are proposed [10,11] but the choice of filter $F(s)$ for designing the controller is generally a low-pass filter of the form

$$F(s) = (1+\lambda s)^{-n}, \ \ n \in I. \tag{1}$$

While designing IMC controller $Q(s)$, the value of $n$ is considered higher than the order of plant and then $Q(s)$ is parameterized into controller $C(s)$ in a framework of conventional unity feedback control system, using

$$C(s) \triangleq \frac{Q(s)}{1-\bar{P}(s)Q(s)} = \frac{F(s)\bar{P}^{-1}(s)}{1-F(s)}, \tag{2}$$

where $\bar{P}(s)$ is a model of plant and $Q(s) = \bar{P}^{-1}(s)F(s)$. The next step is to reconfigure $C(s)$ into an ideal PID form given by $C(s) = K_P + K_I/s + K_D s$.

Researchers have presented various IMC schemes to obtain PID tuning. Some schemes have complex mathematical manipulations whereas other require additional filter with PID [12–14]. For example, IMC-PID scheme presented by Rivera et al. [1] depicted that converting $Q(s)$ into $C(s)$ yields additional lag-term for some plants of first and second-order and integrating type. Hang et al. showed that IMC-PID fails for lag dominated plant with relative small dead-time [15]. Horn and co-workers framed IMC-PID controller in series with second or high-order filter that leads to high-order controller with complex tuning formulation [16]. Furthermore, some recent PID tuning techniques are evaluated on the concept of percentage overshoot specification, maximum complementary sensitivity function, two-degree-of-freedom structure, $H_\infty$ optimization and frequency dependent uncertainty constraints [17–21]. However, after going through these techniques, we realized that PID can be evolved in another fashion through basic or traditional IMC concept.

In this paper with the aim of reducing the computational effort and modeling error, IMC based PID tuning method is proposed. Here, an attempt has been made to exhibit a simple and optimal IMC-PID methodology on the basis of selection of filter. Normally, to implement the IMC

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scheme, $\bar{P}(s)$ is generally modeled as a first/second-order plus dead time (FOPDT/SOPDT) system. For FOPDT system, IMC scheme yields PI controller when order of filter in (1) is $n = 1$ whereas for SOPDT system, it generates PID augmented with extra lag-term when $n = 2$ [1]. Therefore, an IMC-PID technique is proposed using first-order filter. It is shown that there is no need to use high-order filter for high-order plants. The first-order filter is just sufficient for second-order plant or approximated model instead of considering $n \geq 2$ in (1). We observe the following benefits of the proposed scheme: (i) the PID structure obtained is free from additional lag-term (of the form $\frac{1}{1+\phi s}$, $\phi > 0$), (ii) the bandwidth of the closed-loop system increases, (iii) the formulation and evaluation of tuning parameters are simple, (iv) the size of filter reduces thereby making easy hardware implementation.

The rest of the paper is organized in the following six sections. The controller formulation and its advantages are described in Section 2. Definition of performance indices for optimality testing are explained in Section 3. Simulation results for different linear and nonlinear systems are presented in Section 4 followed by the real-time implementation on PMS system in Section 5. Finally conclusions are drawn in Section 6.

## 2. CONTROL SCHEME AND ITS UTILITY

### 2.1. Proposed PID controller formulation

Given a plant of the generalized form

$$P(s) = \frac{a_x s^x + a_{x-1}s^{x-1} + a_{x-2}s^{x-2} + \ldots + a_0}{b_y s^y + b_{y-1}s^{y-1} + b_{y-2}s^{y-2} + \ldots + b_0}, \ \ x \leq y, \quad (3)$$

let us consider its minimum phase, delay-free approximated second-order model as

$$\bar{P}(s) = K/(as^2 + bs + c), \ \ K, a, b, c > 0. \quad (4)$$

Invert $\bar{P}(s)$ and augment the first-order filter of form (1) with $n = 1$ to obtain IMC based controller as

$$Q(s) = (as^2 + bs + c)/K(1 + \lambda s). \quad (5)$$

Note that $Q(s)$ is improper. Now on manipulating $C(s)$ using (2) gives PID form: $C(s) = K_P + K_I s^{-1} + K_D s$, where

$$K_P = b/K\lambda, \ \ K_I = c/K\lambda, \ \ K_D = a/K\lambda, \quad (6)$$

and $\lambda$ is the single tuning parameter. The value of $\lambda$ can be selected offline or online. In this control scheme, the order of filter instead of plant is reduced. Note that if $\bar{P}(s)$ contains delay-term ($e^{-\theta s}$) then we factorize the plant into two parts as $\bar{P}(s) = \bar{P}_-(s)\bar{P}_+(s)$ where $\bar{P}_+(s)$ contains the delay term and $\bar{P}_-(s)$ contains the rest portion of $\bar{P}(s)$. Now using $\bar{P}_-(s)$ only, we can obtain the PID controller. Note that the proposed scheme effectively compensates the time delay because the controller includes the characteristics of the plant and therefore it does not wait for the plant output that originates after the time delay [22].

### 2.2. Utility of the proposed scheme

We observe that the proposed scheme prevents plant order reduction to avoid modeling error, which improves the robustness of the control system. However, the control practitioners use FOPDT model of high-order process for controller synthesis and analysis. The delay term in FOPDT model inherits the property of instability. One can develop a FOPDT model using methods like maximum slope method, two-point method for estimating time constant, time constant (63.2% calculation) based approach, etc [22]. Generally, the modeling is done in such a way that only steady-state response of model matches with the original plant. This is illustrated in theorem 1 but the information about transient-state response and system properties (like peak overshoot, relative stability, etc) cannot be retained in such type of system identification. Moreover, estimating the plant to FOPDT brings variation in phase response at high frequency zone of operation because the phase of modeling error is high. This is also explained through the corollary 1.

**Theorem 1:** For the reduced model given by (7) of the system described by (4)

$$\tilde{P}(s) = \frac{\alpha}{\beta s + 1} e^{-\gamma s}, \ \ \alpha, \beta, \gamma \geq 0, \quad (7)$$

where for steady-state tracking $\lim_{s\to 0} \bar{P}(s) = \lim_{s\to 0} \tilde{P}(s)$ gives

$$K/c = \alpha, \quad (8)$$

the modeling error $\Delta$ defined by

$$\Delta(s) \equiv \bar{P}(s) - \tilde{P}(s) \quad (9)$$

is zero at steady state for unit step and impulse type inputs.

**Proof:** On substituting (4) and (7) in (9), we get

$$\Delta(s) = \frac{-\alpha a e^{-\gamma s}s^2 + (K\beta - \alpha b e^{-\gamma s})s + (K - \alpha c e^{-\gamma s})}{a\beta s^3 + (b\beta + a)s^2 + (c\tau + b)s + c}. \quad (10)$$

Let $\Delta_\delta(t)$ and $\Delta_u(t)$ be the time domain error response for the unit impulse and step inputs, respectively. After applying final value theorem of Laplace transformation to (10), we get steady state error as

$$\lim_{t\to\infty} \Delta_\delta(t) = \lim_{s\to 0} s\Delta(s) = 0, \quad (11)$$

and

$$\lim_{t\to\infty} \Delta_u(t) = \lim_{s\to 0} \Delta(s) = 0. \quad (12)$$

Therefore from (11) and (12), it is clear that modeling errors are zero at steady state while in transient state, errors are unknown. □

**Corollary 1:** For modeling error $\Delta$ described in (10), the magnitude is zero and phase is finite at high frequency.

**Proof:** On putting $s = j\omega$ and evaluating $\Delta(j\omega)$ from (10), we get

$$\Delta(j\omega) = \frac{M + jN}{Q + jR}, \tag{13}$$

where

$$\begin{aligned} M &= a\alpha\omega^2\cos(\gamma\omega) - b\alpha\omega\sin(\gamma\omega) + K(1-\cos(\gamma\omega)),\\ N &= a\alpha\omega^2\sin(\gamma\omega) + K\beta\omega - b\alpha\omega\cos(\gamma\omega) + K\sin(\gamma\omega),\\ Q &= c - \omega^2(a + b\beta),\ \ R = (c\tau + b)\omega - a\beta\omega^3. \end{aligned} \tag{14}$$

After calculating magnitude and phase at high frequency, we get

$$\lim_{\omega\to\infty} |\Delta(j\omega)| = 0, \tag{15}$$

and

$$\lim_{\omega\to\infty} (\angle\Delta(j\omega)) \neq 0. \tag{16}$$

This is a required proof. □

The other advantage of proposed scheme is that if we select the filter of second-order ($n = 2$) in (1) then $Q(s)$ in (5) becomes semi-proper but the corresponding feedback controller $C(s)$ leads to PID controller augmented with extra lag-term of the form $(1/(\phi + s))$. This lag-term reduces the speed of response and increases the settling time of the overall control system. To avoid this situation, the plant is approximated to FOPDT model but reducing the original plant into FOPDT may create plant-model mismatch, which should be avoided. The plant-model mismatch does not create any instability at low frequency in presence of disturbance but deteriorates the performance at high frequency. Some characteristics of actual system also vanish on reducing the plant dynamics. Therefore, it is appropriate to adopt controller reduction scheme (i.e., selecting the low-order filter in case of IMC here) rather than diminishing the system properties through model order reduction algorithm.

**Theorem 2:** Suppose $P(s)$ as expressed in (3) be the plant and $C(s)$ be a controller parameterized from IMC based controller, then the closed-loop transfer function $T(s)$ of the control system is the filter $F(s)$ used in IMC design.

**Proof:** We select the filter $F(s) = (1 + \lambda s)^{-n};\ \ n \geq y - x$ to obtain IMC based controller $Q(s) = P^{-1}(s)F(s)$. Now, we convert $Q(s)$ into $C(s)$ using (2) and substitute $C(s)$ in $T(s) = P(s)C(s)(1 + P(s)C(s))^{-1}$ to obtain the overall closed-loop transfer function of the control system which yields $T(s) = F(s)$. □

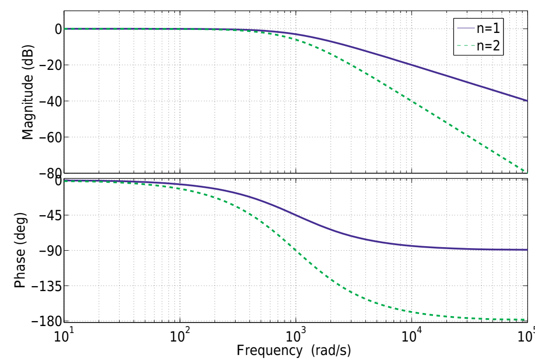



Fig. 1. Frequency response of filter $F(s)$ ($\lambda = 0.001$).

Thus Theorem 2 indicates that the behavior of the closed-loop response of the control system totally depend upon the filter opted. If we see the Bode plots of filters in Fig. 1, we can say that the bandwidth of the first-order filter is more than that of the second-order filter. Since the closed-loop transfer function is equal to filter transfer function, therefore bandwidth of the closed-loop system increases when a low-order filter is used. The higher the order of filter, lower the bandwidth. This can easily be proved from sensitivity function $\sum(s)$ of the closed-loop system given by

$$\sum(s) \triangleq (1 + \bar{P}(s)C(s))^{-1} = 1 - \bar{P}(s)Q(s). \tag{17}$$

For the second-order systems (4), $\sum(s)$ can be obtained as

$$\sum(s) = \left\langle \begin{array}{l} \lambda s(1+\lambda s)^{-1}\ \ \forall n = 1,\\ (2\lambda s + \lambda^2 s^2)(1+\lambda s)^{-2}\ \ \forall n = 2. \end{array}\right. \tag{18}$$

To calculate the bandwidth $\omega_b$, set $|\sum(j\omega)|_{\omega=\omega_b} = 0.707$, which yield

$$\omega_b = \left\langle \begin{array}{l} 1/\lambda\ \ \forall n = 1,\\ 0.402837/\lambda\ \ \forall n = 2. \end{array}\right. \tag{19}$$

From (19), it is clear that the bandwidth of the closed-loop system using first-order filter is 2.48 times greater than that obtained using second-order filter. The increment in bandwidth results in improved disturbance attenuation and sharp reference tracking. Furthermore, (19) also suggests that $\lambda$ should be selected as small as possible. From the above discussion, we can infer that filter-order reduction is a better choice than opting the plant order reduction. Therefore, we augment the first-order filter with the actual plant to design PID controller.

In the proposed design scheme, $\lambda$ is the only variable which needs to be selected properly. Larger the $\lambda$, slower the response and less sensitive to model mismatches, while smaller the $\lambda$, faster the closed-loop response but the controller action may be aggressive and produce tighter response. Thus, a trade-off is required for handling robustness/performance and servo/regulator [1]. However, it is, in fact, very easy to get a good insightful feel for the influence of $\lambda$ on stability and performance if we set its value equal to maximum time constant and then gradually reduce its value. Therefore, after extensive simulation studies, we found that $\lambda$ should be less than the maximum time constant of the plant. This gives a reasonably fast response and good robustness margin. Hence, manual tuning is a viable option, and in many cases it is the preferred choice.

## 3. PERFORMANCE EVALUATION

To test the optimal performance of the control scheme, we select the performance indices (cost functions) in the form of integral error criterion. They are integrated error (*IE*), integral of the squared error (*ISE*), integral of the absolute error (*IAE*), and integral of the time weighted absolute error (*ITAE*) defined, respectively, by

$$\begin{aligned} IE &= \int_0^\infty e(t)dt;\ ISE = \int_0^\infty e(t)^2 dt, \\ IAE &= \int_0^\infty |e(t)|\,dt;\ ITAE = \int_0^\infty t\,|e(t)|\,dt, \end{aligned} \tag{20}$$

where $e(t)$ is the error signal, i.e., the difference between the set-point (desired) input and the actual output. The *IE* index simply accumulates the net error and describes the performance of monotonic response. The *ISE* index denotes indirectly several characteristics like settling time, overshoot, speed of response, and all other important features of the transient response [23]. The *IAE* index is a measure of disturbance rejection for integral controller [24]. The *ITAE* index accounts for long duration error. Here, our objective is to minimize these cost functions to prove the optimality of the proposed scheme.

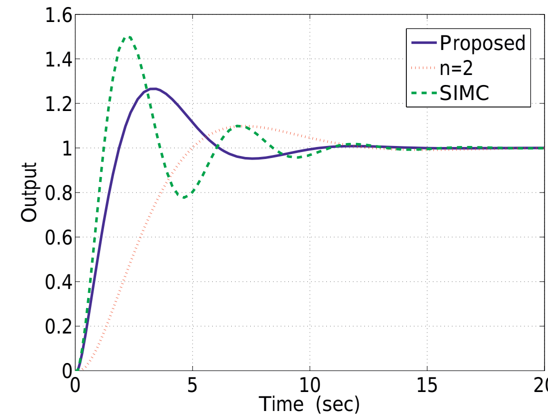
## 4. ILLUSTRATIVE EXAMPLES

In this section, we illustrate our proposed scheme for some linear and nonlinear class of systems.

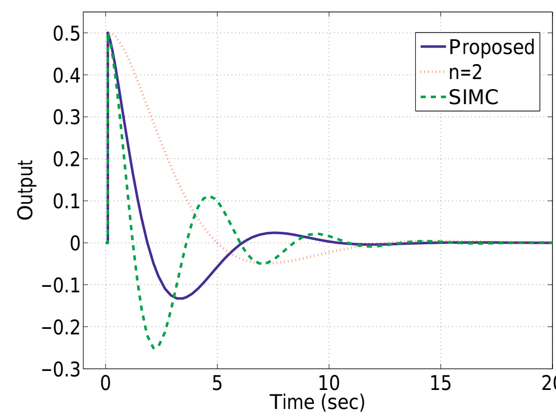
**Example 1:** Consider a repeated pole second-order system

$$\bar{P}(s) = \frac{1}{(s+1)^2}. \tag{21}$$

Using the proposed scheme, the PID parameters for (21) are: $K_P = K_I = K_D = 1/\lambda$. The PID parameters for this system when second-order filter, i.e., $n = 2$ provide the same value of each term but with an extra lag-term of the form $(1+\phi s)^{-1}$ where $\phi = 2/\lambda$. We also stablize (21) using SIMC approach [7] in which we get $\tau_I = \min(1, 4\lambda)$, $K_P = \tau_I/\lambda$, $\tau_D = 1$. During computer simulation for $\lambda = 0.1$, the output response for unit step input is shown in Fig. 2(a) wherein the output using proposed technique




(a)




(b)

Fig. 2. Output for step-type (a) set-point and (b) disturbance of Example 1.

quickly reaches the steady state with less overshoot and oscillations in comparison to SIMC. And, the response using second-order filter ($n = 2$) gives less overshoot but is sluggish in nature due to lag-term. Now to observe the disturbance attenuation property, a step disturbance of amplitude of 0.5 is applied at $t = 0$. In Fig. 2(b), the proposed technique gives faster attenuation than that obtained from second-order filter, whereas SIMC gives oscillatory response during rejection. To test the optimality, the different performance indices are calculated in Table 1 which state that for reference tracking, the proposed technique produces least error among the other applied methods. However for SIMC, the *IE* index is the least due to oscillatory nature of the response.

**Example 2:** An integrating plus first-order plus dead-time system

$$\bar{P}(s) = \frac{1}{s(s+1)}e^{-s} \tag{22}$$

is taken from Kaya [25]. This example gives naturally the

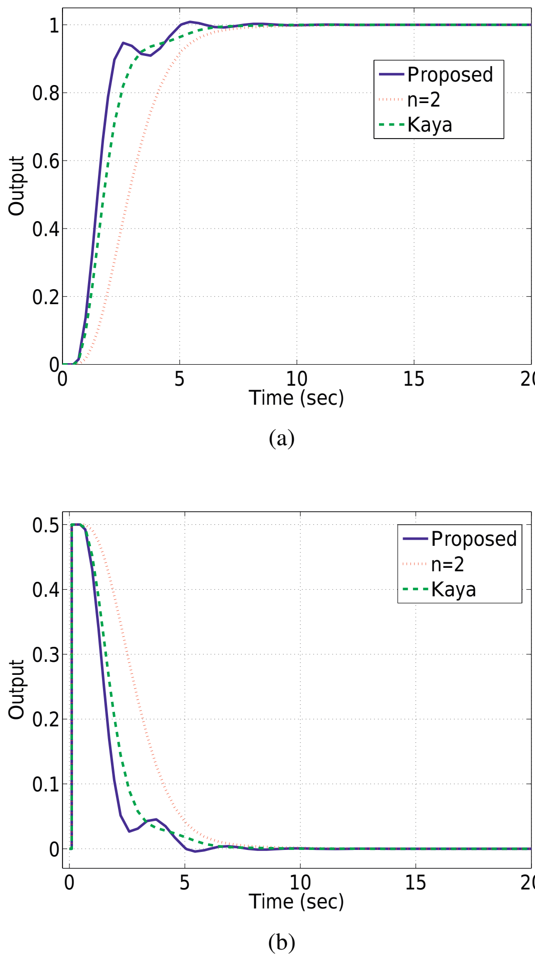



Fig. 3. Output for step-type (a) set-point and (b) disturbance of Example 2.

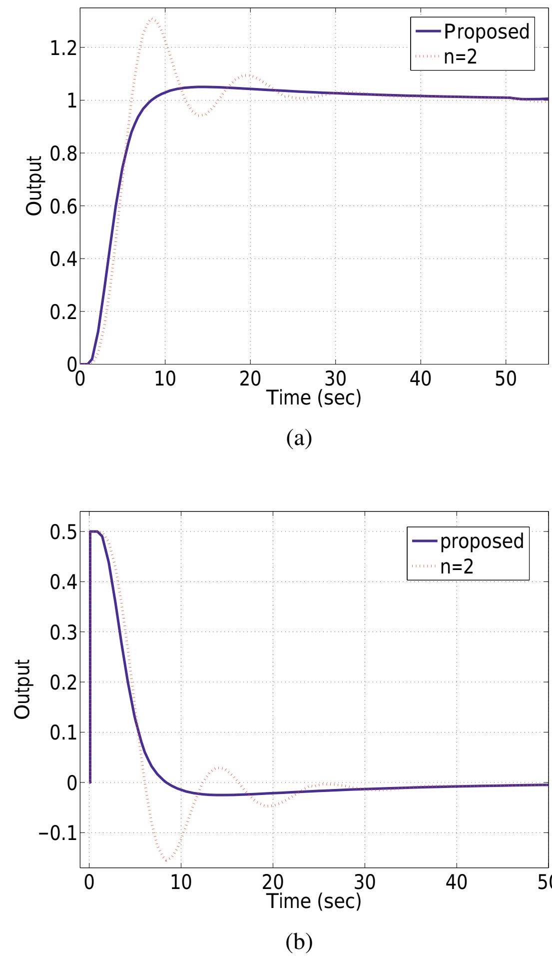



Fig. 4. Output for step-type (a) set-point and (b) disturbance for Example 3.

PD form of controller as integrator is already present in the plant model and if we compare the model with (4), we get $c = 0$. The PD form from proposed method produce $K_P = K_D = 1/\lambda$, and $\lambda = 0.714$. The PD parameters used by Kaya are: $K_P = 1.047$ and $\tau_D = 1$. Along with PID, the second-order filter gives lag-term where $\phi = 2.8011$. The set-point tracking and disturbance responses of the proposed scheme as depicted in Fig. 3 are the fastest and optimal among all (See Table 1, Example 2).

**Example 3:** Consider a lag dominated plant with delay

$$\bar{P}(s) = \frac{1}{(20s+1)(2s+1)}e^{-s}, \tag{23}$$

where the time constant $\tau = 20$ is responsible for generating lag in the response apart from the delay. In order to stabilize this plant through proposed technique, we set $K_P = 22/\lambda$, $K_I = 1/\lambda$, $K_D = 40/\lambda$ and $\lambda = 2.5$. Using second-order filter, the extra term augmented along with PID is $1/(s+0.8)$. The responses of both methods are compared for step reference and disturbance in Fig. 4 where the output of PID with lag term depicts overshoot but the proposed technique shows smooth reference and the response is also optimal (See Table 1, Example 3).

**Example 4:** Consider a non-minimum phase system with time delay from [26]

$$\bar{P}(s) = \frac{(-0.5s+1)}{(s+1)(2s+1)}e^{-s}. \tag{24}$$

Using the proposed technique, the PID parameters are: $K_P = 3/(\lambda+0.5)$, $K_I = 1/(\lambda+0.5)$, $K_D = 2/(\lambda+0.5)$ where we set $\lambda = 3$. Using the second-order filter, the PID controller obtained is

$$C(s) = \frac{1}{\lambda^2}\left(3 + \frac{1}{s} + 2s\right)\left(\frac{1}{s+\phi}\right), \tag{25}$$

where $\phi = (0.5+2\lambda)/\lambda^2$ and $\lambda = 2$. The parameters of Wang's controller [26] are: $K_P = 1.1194$, $K_I = 0.369$, and $K_D = 0.9765$. The closed-loop responses are presented in Fig. 5(a) for step input, where the proposed scheme

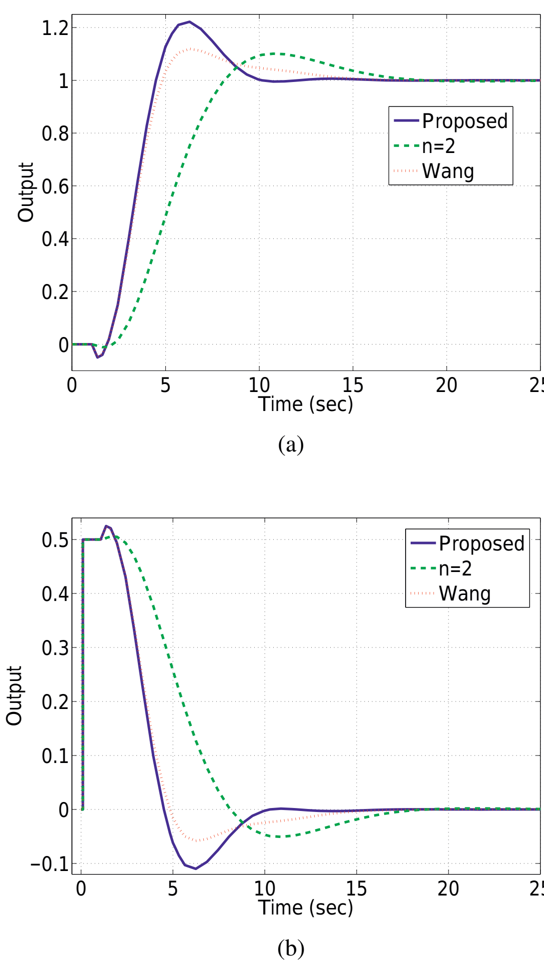



Fig. 5. Output for step-type (a) set-point and (b) disturbance for Example 4.

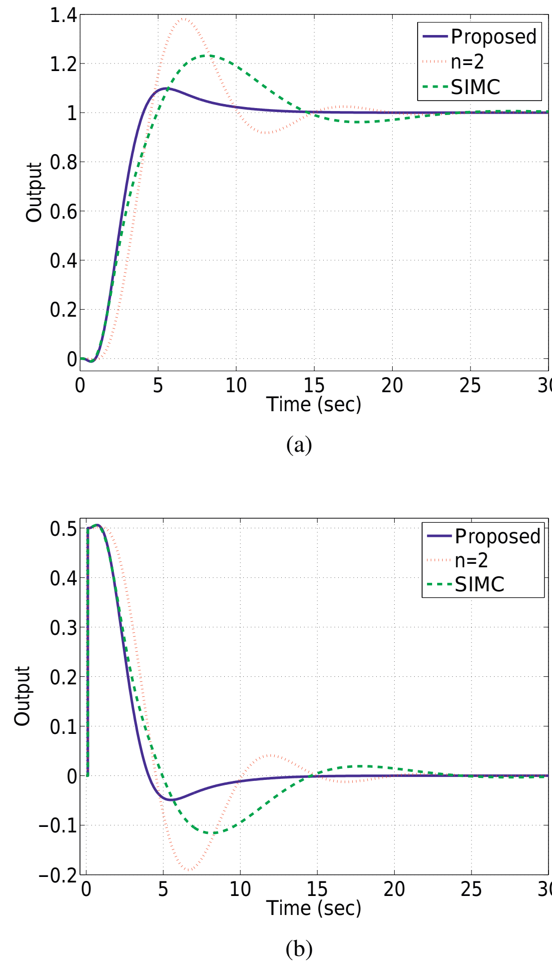



Fig. 6. Output for step-type (a) set-point and (b) disturbance for Example 5.

produces a slight increment in overshoot in comparison to other compared methods however the speed of response is fast and settle quicker to the desired value in comparison to all other methods. Similarly, from Fig. 5(b), disturbance rejection is faster in comparison to other methods. Due to slight increase in overshoot, the *ISE* and *IAE* are slightly increased in comparison to Wang's method whereas as usual it is lesser than that using second-order filter (See Table 1, Example 4).

**Example 5:** Consider a high-order system from [7]

$$P(s) = \frac{(-0.3s+1)(0.08+1)}{(2s+1)(s+1)(0.4s+1)(0.2s+1)(0.05s+1)^3}, \tag{26}$$

and its second-order approximated model

$$\bar{P}(s) = \frac{1}{(2s+1)(1.2s+1)} e^{-0.77s}, \tag{27}$$

where mismatching between plant and its model is present. The PID parameters for this model using proposed technique are $K_P = 3.2/\lambda$, $K_I = 1/\lambda$, $K_D = 2.4/\lambda$ and $\lambda = 2$. Using the second-order filter, the extra lag term obtained is $1/(s+1)$. The SIMC approach gives $K_P = 1.3$, $\tau_I = 2$, $\tau_D = 1.2$. For both reference tracking and disturbance attenuation, the closed-loop performances are shown in Fig. 6 wherein the response is fastest and smoothest with least oscillation using the proposed technique in comparison to SIMC and PID with lag term. The proposed scheme is also optimal in comparison to other techniques (See Table 1, Example 5).

**Example 6:** Consider an undamped oscillator

$$\bar{P}(s) = \frac{1}{s^2+4} \tag{28}$$

for which the controller parameter using proposed method gives $K_I = 4/\lambda$, $K_D = 1/\lambda$ where $\lambda = 15$. Since the system is oscillatory in nature, therefore, $\lambda$ should be large enough to prevent fluctuations to make the system stable. For the same system, the second-order filter gives lag-term

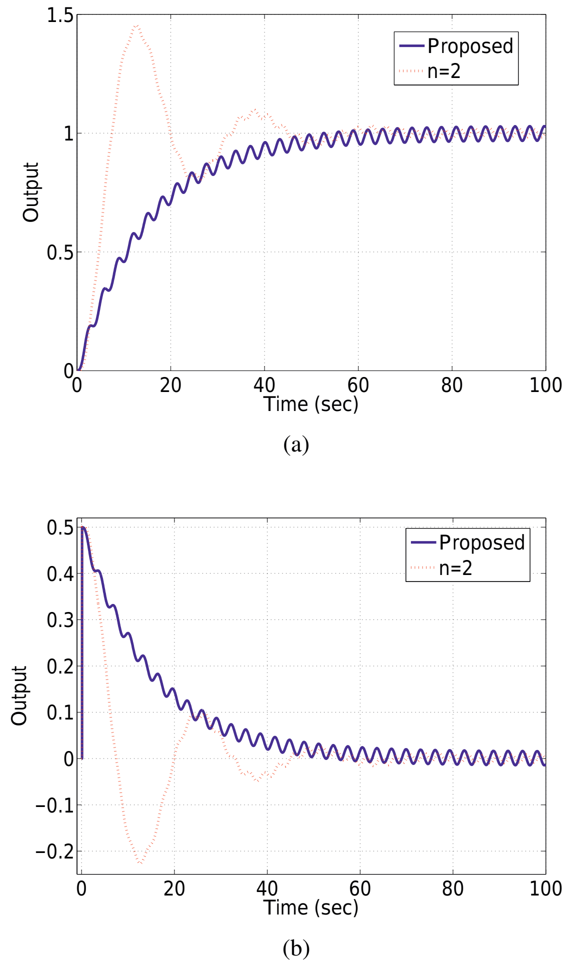



Fig. 7. Output for step-type (a) set-point and (b) disturbance for Example 6.

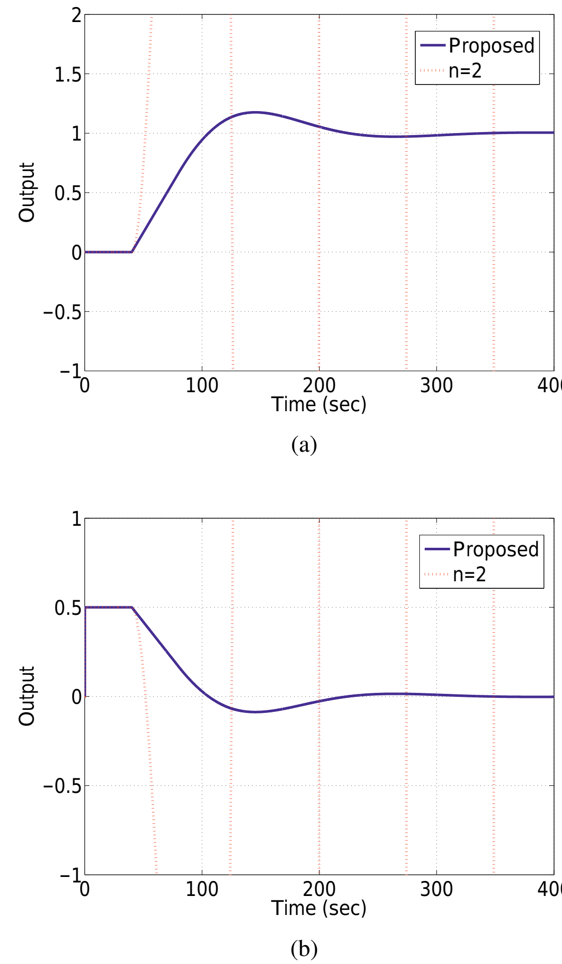



Fig. 8. Output for step-type (a) set-point and (b) disturbance for Example 7.

with $\phi = 0.133$. The step and disturbance rejection performances are shown in Fig. 7 where due to oscillatory nature of the original system (28), the steady state output is oscillatory but is within a 2% of the steady state value whereas huge overshoot is found when second-order filter is used. It is interesting to note that the performance is optimal in case of second-order filter due to presence of overshoot and undershoot (See Table 1, Example 6).

**Example 7:** Consider a slow process with large delay

$$\bar{P}(s) = \frac{0.001667}{(0.05s+1)(0.33s+1)}e^{-40s}. \tag{29}$$

The proposed method gives PID parameters as $K_P = 50/\lambda$, $K_I = 600/\lambda$, $K_D = 1/\lambda$ and $\lambda = 60$. Now, applying the second-order filter, the conventional IMC technique gives an extra lag-term having $\phi = 0.033$. The closed-loop responses in Fig. 8 show that the proposed method yields stable response whereas the lag term creates instability if the higher-order filter is used.

**Example 8:** In continuation to the linear systems, we extend our proposed work to the class of nonlinear system so-called separable system, which comprises of a linear part defined by its transfer function, and a nonlinear part defined by a time-independent relationship $\Gamma$ between its input $u$ and output $u_\Gamma$ as shown in Fig. 9 [27–29].

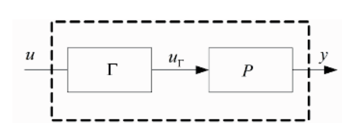



Fig. 9. Nonlinear system with separable nonlinearity.

Consider a second-order integrating type system

$$P(s) = \frac{4}{s(0.5s+1)}, \tag{30}$$

having input saturation nonlinearities defined as

$$\Gamma : \begin{cases} u_\Gamma = u, & |u| \le \delta, \\ u_\Gamma = \delta \operatorname{sgn}(u), & |u| > \delta, \end{cases} \tag{31}$$

where $\delta = 0.05$. For such system, only the linear part

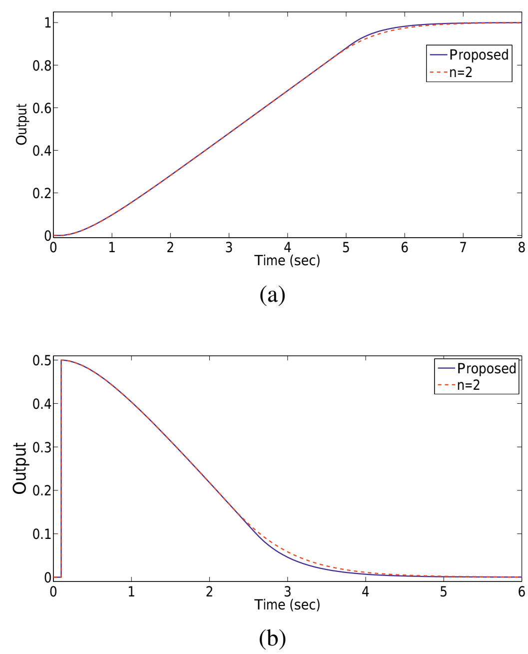



Fig. 10. Output for step-type (a) set-point and (b) disturbance for Example 8.

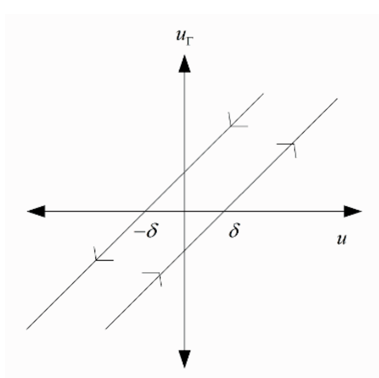



Fig. 11. Input-output plot for backlash nonlinearity.

is utilized for the proposed PID scheme, and we get PD controller as

$$C(s) = \frac{1}{\lambda}\left(\frac{1}{4} + 0.125s\right), \tag{32}$$

where $\lambda = 0.16$. The responses to a unit step input and disturbance rejection of the proposed and PID with $\phi = 2/\lambda$ are shown in Fig. 10 wherein the PID with lag term yields sluggish response in comparison to the proposed one. The performance indices of the proposed scheme are also less (See Table 1, Example 8).

**Example 9:** Consider a force-actuated mass-damper-spring system

$$P(s) = \frac{1}{ms^2 + bs + k}, \tag{33}$$

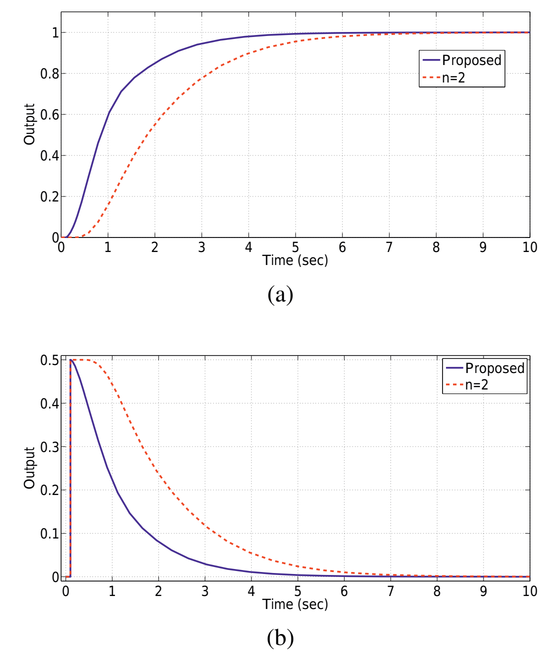



Fig. 12. Output for step-type (a) set-point and (b) disturbance for Example 9.

where $m = 1$, $k = 4$ and $b = 16$. The friction force is represented by the backlash model as shown in Fig. 11, where deadzone is $\delta = 1.5$. We design PID controller in a similar fashion as mentioned in Example 8 by considering only the linear part. The obtained PID controller is

$$C(s) = \frac{1}{\lambda}\left(4 + \frac{16}{s} + s\right), \tag{34}$$

where $\lambda = 1$. The closed-loop responses of the proposed scheme and PID with $\phi = 2$ are shown in Fig. 12 where we can observe the faster tracking of the proposed scheme in comparison to that of PID with lag-term. Similarly, the disturbance rejection is fast for the proposed method. The proposed controller also shows the optimal performance (See Table 1, Example 9).

## 5. EXPERIMENTAL RESULTS

To verify the advantages of the suggested technique, the precision modular servo (PMS) system consisting of DC servomotor from Feedback Instruments Ltd., U.K., is considered (See Fig. 13) [30]. The transfer function for the velocity control of PMS is

$$\bar{P}(s) = \frac{1.362 \times 10^8}{s^2 + 1000s + 8.476 \times 10^4}. \tag{35}$$

Using the procedure defined in section 2.1, the PID parameters with filter of order $n = 1$ and 2, are calculated. To

Table 1. Performance comparison for simulation examples.

| Examples | Method | Reference tracking | | | | Disturbance rejection | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | *ISE* | *IAE* | *ITAE* | *IE* | *ISE* | *IAE* | *ITAE* | *IE* |
| Example 1 | **Proposed** | **0.739** | **1.69** | **4.323** | **0.3004** | **0.1850** | **0.8448** | **2.162** | **-0.1502** |
| | $n=2,\phi=20$ | 1.727 | 2.841 | 7.715 | 2.003 | 0.4319 | 1.421 | 3.857 | -1.001 |
| | SIMC [7] | 0.8221 | 1.974 | 5.929 | 0.1004 | 0.2055 | 0.9868 | 2.965 | -0.05021 |
| Example 2 | **Proposed** | **1.172** | **1.606** | **1.976** | **1.591** | **0.293** | **0.804** | **0.9882** | **-0.7954** |
| | $n=2,\phi=2.8011$ | 2.199 | 2.951 | 5.639 | 2.951 | 0.5498 | 1.475 | 2.82 | -1.475 |
| | Kaya [25] | 1.369 | 1.906 | 2.633 | 1.906 | 0.3422 | 0.9531 | 1.316 | -0.9531 |
| Example 3 | **Proposed** | **2.988** | **5.199** | **49.35** | **2.521** | **0.747** | **2.599** | **24.67** | **-1.26** |
| | $n=2,\phi=0.8$ | 3.799 | 6.628 | 68.2 | 2.058 | 0.9496 | 3.314 | 34.1 | -1.029 |
| Example 4 | **Proposed** | **2.924** | **3.872** | **10.34** | **2.5** | **0.7299** | **1.93** | **5.126** | **-1.25** |
| | $n=2,\phi=1.125$ | 4.229 | 5.6 | 21.22 | 4.501 | 1.057 | 2.801 | 10.63 | -2.25 |
| | Wang *et al*. [26] | 2.894 | 3.819 | 10.61 | 2.71 | 0.723 | 1.911 | 5.341 | -1.355 |
| Example 5 | **Proposed** | **2.002** | **2.82** | **6.418** | **2** | **0.5006** | **1.41** | **3.209** | **-1** |
| | $n=2,\phi=1$ | 3.044 | 4.74 | 19.35 | 1.999 | 0.761 | 2.37 | 9.672 | -0.9994 |
| | SIMC [7] | 2.324 | 4.238 | 21.01 | 1.542 | 0.581 | 2.119 | 10.5 | -0.7708 |
| Example 6 | **Proposed** | **7.563** | **15.56** | **271.6** | **14.97** | **1.891** | **7.779** | **135.8** | **7.486** |
| | $n=2,\phi=0.133$ | 4.882 | 10.79 | 174.8 | 2.005 | 1.22 | 5.397 | 87.39 | 1.002 |
| Example 8 | **Proposed** | **2.122** | **3.002** | **5.98** | **3.002** | **0.3134** | **0.8771** | **1.054** | **0.8771** |
| | $n=2,\phi=12.5$ | 2.123 | 3.019 | 6.079 | 3.019 | 0.3147 | 0.8938 | 1.111 | 0.8938 |
| Example 9 | **Proposed** | **0.5826** | **1.048** | **1.137** | **1.046** | **0.1517** | **0.5474** | **0.6077** | **0.5463** |
| | $n=2,\phi=2$ | 1.383 | 2.094 | 3.32 | 2.094 | 0.3713 | 1.094 | 1.757 | 1.094 |

Table 2. PID parameters and performance comparison for PMS system.

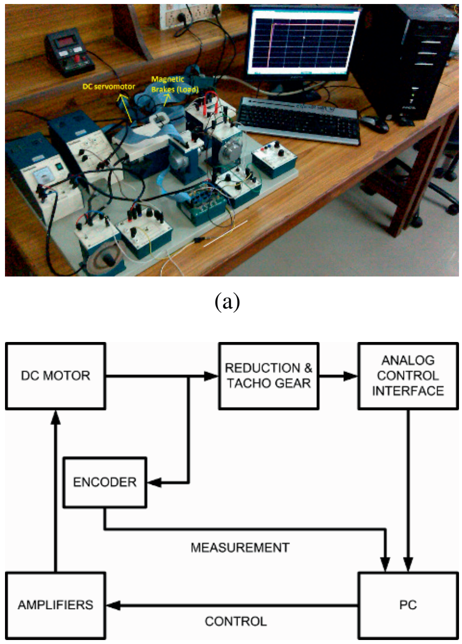
| Method | $K_P$ | $K_I$ | $K_P$ | *ISE* | *IAE* | *ITAE* | *IE* |
|---|---|---|---|---|---|---|---|
| **Proposed method** ($n=1$) | **0.0073** | **0.6223** | **7.3421** x$10^{-6}$ | **1.832** x$10^{4}$ | **60.59** | **165** | **1.232** |
| $n=2$ ($\phi=2000$) | 0.073 | 6.223 | 7.3421 x$10^{-5}$ | 4.138 x$10^{4}$ | 122.4 | 278.6 | 23.91 |
| Hang *et al*. [15] | 6.6564 x$10^{-3}$ | 1.502 x$10^{5}$ | 17.1877 | 1.634 x$10^{4}$ | 3375 | 1.85 x$10^{4}$ | 10.69 |
| Alcántara *et al*. [14] | 0.0213 | 7.559 x$10^{4}$ | 8.5425 | 1.634 x$10^{6}$ | 3409 | 1.85 x$10^{4}$ | 16.83 |

evaluate the efficiency of proposed controller, two more PID controllers are designed using the method suggested by Hang *et al*. [15] and Alcántara *et al*. [14]. Table 2 depicts the calculated tuning parameters along with different integral error performance indices for reference tracking. Note that for all methods, $\lambda = 0.001$ is considered.

The time domain step responses are depicted in Fig. 14(a), which states that the speed of response and settling time for the proposed controller is better than the PID with lag term whereas the other two methods, i.e., Hang *et al*. [15] and Alcántara *et al*. [14] keep on oscillating about the reference speed of 1000 rpm. In the experimental results, the data points on x-axis in Fig. 14 have been divided by 100 to convert the axis scale into time-scale of "seconds" unit. To observe the disturbance rejection performance, load is applied through magnetic brakes at 8 sec as shown in Fig. 14(b) and removed at 11 sec during operation and it is observed that the proposed scheme yield good disturbance rejection performance among all.

## 6. CONCLUSION AND FUTURE SCOPE

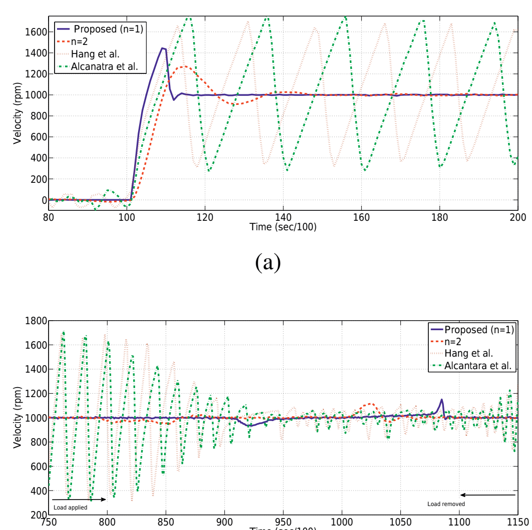
In this paper, IMC based PID tuning scheme is suggested for the stable linear and separable nonlinear systems after employing first-order filter in place of second or higher-order filter. The proposed technique is further implemented on the real-time PMS system to show its effectiveness in bringing good servo, regulator, and optimal performances. Through the simulation and hardware re-




(a)

(b)

Fig. 13. (a) Pictorial and (b) block diagram of PMS set-up.




(a)

(b)

Fig. 14. Output for step-type (a) set-point and (b) disturbance for PMS system.

sults, it is shown that the optimality of the controller performance depends upon the choice of filter selected. The proposed technique can be useful in industrial applications like power electronics, electrical drives, robotics, process control, aerospace engineering, etc.

We have covered varieties of second- and higher-order processes. The presented examples are stable in nature but the design scheme for unstable systems is yet to explore. The proposed scheme is further applied for a class of nonlinear systems so called separable systems but the control of pure nonlinear system is not described. Apart from this, we have opted manual $\lambda$ tuning, thus a mathematical formulation of tuning law is also required. These research gaps pave the path for the control researchers to proceed further in this area.

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