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Delay-dependent H_∞ Control for a Class of Uncertain Time-delay Singular Markovian Jump Systems via Hybrid Impulsive Control

Hui Lv, Qingling Zhang*, and Junchao Ren

Abstract: This paper deals with the problem of robust normalization and delay-dependent H_∞ control for a class of singular Markovian jump systems with norm-bounded parameter uncertainties and time delay. A new impulsive and proportional-derivative control strategy with memory is presented, which results in a novel class of hybrid impulsive systems. Sufficient conditions are developed to guarantee that the resultant closed-loop system is not only robust normal and stochastically stable, but also satisfies a prescribed H_∞ performance level for all delays no larger than a given upper bound. In addition, the explicit expression of the desired impulsive control gains is also given together with the design approach. Finally, two numerical examples are provided to illustrate the effectiveness of the proposed methods.

Keywords: Delay-dependent, H_∞ control, impulsive control, proportional-derivative control with memory, robust normalization, uncertain singular Markovian jump systems.

1. INTRODUCTION

Recently, more attention has been paid to the study of singular Markovian jump linear systems (SMJSs) with time delay, in which the mode operation process is a continuous Markov Chain taking values in a finite set [1–10]. Time delays are frequently encountered in a variety of engineering systems and a relatively small time delay may destroy the systems. The results on such systems can be classified into two types: delay-independent [11, 12] and delay dependent [1, 13–16]. It has been shown that the delay-dependent results are less conservative than the delay independent ones especially when time delays are small [15]. On the other hand, impulsive control is an effective way to stabilize a complicated system by using simple control impulses and have been investigated for various types of systems, such as singular systems, Markovian jump systems and time-delay systems [17,18].

In this paper, the problem of delay-dependent H_∞ control is studied for a class of time-delay SMJSs with norm-bounded parameter uncertainties in both derivative and system matrices. To the best of our knowledge, there are few results available in the literature for this problem, which motivates our current research.

In our approach, an impulsive and proportional derivative memory state feedback controller (IPDMSFC) is proposed to solve this problem. The derivative part of the hybrid controller is to normalize the uncertain SMJSs, whereas the impulsive part is to guarantee that the value of the Lyapunov-Krasovskii functional does not increase at each switching time instant. By adopting appropriate congruence transformations and free-connection weighting matrices, sufficient conditions are provided in terms of feasible matrix inequalities such that the resultant closed-loop system is not only robust normal and stochastically stable, but also satisfies a prescribed H_∞ performance level for all delays no larger than a given upper bound. The gain matrices of the impulsive control part are parameter variables, which can be solved together with the design approach. This is different from the results of [17, 18], in which the gain of the impulsive control is given as a constant matrix in advance. Our design idea can thus provide more design freedom than those in the existing literature. Finally, two numerical examples demonstrate the effectiveness of the presented methods.

Notations: $\mathbb{R}^n$ is the n-dimensional Euclidean space and $\mathbb{R}^{m\times n}$ is the set of all $m\times n$ real matrices. $\|.\|$ refers to the Euclidian norm for a vector. $\mathcal{L}_2[0,\infty)$ stands for the space of square integrable functions on $[0,\infty)$. $C_{n,d} = C([-d,0],\mathbb{R}^n)$ denotes the Banach space of continuous vector functions mapping the interval $[-d,0]$ into $\mathbb{R}^n$ with norm $\|\phi(t)\|_d = sup_{-d\le s\le 0}\|\phi(s)\|$. $\mathbb{E}[.]$ denotes the expectation operator with respect to some probability measure $\mathcal{P}$. '$*$' denotes the term that is induced by symmetry. I

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Hui Lv and Junchao Ren are with the Institute of Systems Science, Northeastern University, Shenyang 110819, China (e-mails: lh8481@tom.com, junchao_r@sohu.com). Qingling Zhang is with the Institute of Systems Science, Key Laboratory of Integrated Automation of Process Industry, Ministry of Education, Northeastern University, Shenyang 110819, China (e-mail: qlzhang@mail.neu.edu.cn).
* Corresponding author.

denotes the identity matrix with appropriate dimension.

2. PROBLEM FORMULATION

Consider the following uncertain singular time-delay systems with Markovian jump parameters

$$\begin{aligned}
&(E(r(t))+\Delta E(r(t)))\dot{x}(t)\\
&\quad=(A(r(t))+\Delta A(r(t)))x(t)\\
&\qquad+(A_d(r(t))+\Delta A_d(r(t)))x(t-d)\\
&\qquad+(B(r(t))+\Delta B(r(t)))u(t)+B_\omega(r(t))\omega(t),\\
&z(t)=C(r(t))x(t)+C_d(r(t))x(t-d)+D(r(t))\omega(t),\\
&x(t)=\phi(t),\quad t\in[-\bar{d},0],
\end{aligned}\tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state vector, $u(t)\in\mathbb{R}^m$ is the control input, $\omega(t)\in\mathbb{R}^q$ is the disturbance input that belongs to $\mathcal{L}_2[0,\infty)$, and $z(t)\in\mathbb{R}^l$ is the controlled output vector. d is the constant time delay of the state in the system which satisfies $0\le d\le\bar{d}$, $\phi(t)\in C_{n,\bar{d}}$ is a compatible vector valued initial function. Matrix $E(r(t))\in\mathbb{R}^{n\times n}$ may be singular, and it is assumed that $rankE(r(t))=n_{r(t)}\le n$. $A(r(t)),A_d(r(t)),B(r(t)),B_\omega(r(t)),C(r(t)),C_d(r(t))$ and $D(r(t))$ are known matrices with compatible dimensions. $\Delta E(r(t))$, $\Delta A(r(t))$, $\Delta A_d(r(t))$ and $\Delta B(r(t))$ are unknown matrices denoting the uncertainties of the system. The mode $\{r(t),t\ge0\}$ (we also denote as $\{r_t,t\ge0\}$) is a right-continuous-time Markov process taking values in a finite state space $\mathcal{S}=\{1,2,\dots,N\}$ with transition probabilities

$$Pr[r(t+\Delta)=j|r(t)=i]=\begin{cases}\pi_{ij}\Delta+o(\Delta) & i\ne j,\\ 1+\pi_{ii}\Delta+o(\Delta) & i=j,\end{cases}\tag{2}$$

where $\Delta>0$, $\lim_{\Delta\to0}o(\Delta)/\Delta=0$ and $\pi_{ij}\ge0$, $i,j\in\mathcal{S}$, $i\ne j$, is the transition rate from the mode i at time t to the mode j at time $t+\Delta$ and $\pi_{ii}=-\sum_{j=1,j\ne i}^N\pi_{ij}$. For simplicity, for each possible value $r(t)=i\in\mathcal{S}$, a matrix $A(r(t))$ is denoted as A_i.

In this paper, for any value $r(t)=i\in\mathcal{S}$, the above uncertainties are assumed as

$$\begin{bmatrix}\Delta E_i & \Delta A_i & \Delta A_{di} & \Delta B_i\end{bmatrix}=M_iF(t)\begin{bmatrix}N_{ei} & N_{ai} & N_{di} & N_{bi}\end{bmatrix},\tag{3}$$

where M_i, N_{ei}, N_{ai}, N_{di} and N_{bi} are known real constant matrices of appropriate dimensions, and the uncertain matrix $F(t)$ satisfies $F^{\mathrm{T}}(t)F(t)\le I$.

The objective of this paper is to design an impulsive and proportional-derivative memory state feedback controller (IPDMSFC) for system (1) in the form of

$$\begin{aligned}
&u(t)=u_1(t)+u_2(t),\\
&u_1(t)=K_a(r(t))x(t)+K_d(r(t))x(t-d)-K_e(r(t))\dot{x}(t),\\
&u_2(t)=\sum_{k=1}^{\infty}G(r(t_k^+))x(t)\delta(t-t_k),\ k=1,2,\dots,
\end{aligned}\tag{4}$$

where $u_1(t)$ is a mode-dependent proportional-derivative state feedback controller with memory and $u_2(t)$ is an impulsive controller. $K_a(r(t))$, $K_d(r(t))$, $K_e(r(t))$ and $G(r(t_k^+))$ are to be designed gain matrices of appropriate dimensions. $\delta(.)$ is the Dirac impulse function, with discontinuous impulsive instants $t_1<t_2<\cdots<t_k<\cdots$, where $t_1>t_0=0$, $\lim_{k\to\infty}t_k=\infty$, and $x(t_k)=x(t_k^-)=\lim_{h\to0^+}x(t_k-h)$, $x(t_k^+)=\lim_{h\to0^+}x(t_k+h)$, $e_x(t_k)\triangleq x(t_k^+)-x(t_k^-)$.

Suppose that when $t\in(t_k,t_{k+1}]$, $r(t)=i$, that is, the ith subsystem is activated. Substituting (4) into the system (1) leads to

$$\begin{aligned}
E_{ci}[x(t_k+h)-x(t_k)]&=\int_{t_k}^{t_k+h}E_{ci}\dot{x}(s)ds\\
&=\int_{t_k}^{t_k+h}[A_{ci}x(s)+A_{cdi}x(s-d)+B_i+\Delta B_i)u_2(s)\\
&\qquad+B_{\omega i}\omega(s)]ds,
\end{aligned}$$

where

$$\begin{aligned}
E_{ci}&=E_i+\Delta E_i+(B_i+\Delta B_i)k_{ei},\\
A_{ci}&=A_i+\Delta A_i+(B_i+\Delta B_i)k_{ai},\\
A_{cdi}&=A_{di}+\Delta A_{di}+(B_i+\Delta B_i)k_{di},
\end{aligned}$$

when $h\to0^+$, it follows that

$$E_{ci}e_x(t_k)=\lim_{h\to0^+}E_{ci}[x(t_k+h)-x(t_k)]=(B_i+\Delta B_i)G_ix(t_k).$$

With controller (4), system (1) becomes an uncertain singular and impulsive Markovian jump time-delay system in the following form

$$\begin{aligned}
&E_c(r_t)\dot{x}(t)=A_c(r_t)x(t)+A_{cd}(r_t)x(t-d)\\
&\qquad\qquad+B_\omega(r_t)\omega(t),\qquad t\in(t_k,t_{k+1}],\\
&E_c(r_t)e_x(t_k)=(B(r_t)+\Delta B(r_t))G(r_{t_k^+})x(t_k),\qquad t=t_k,\\
&z(t)=C(r_t)x(t)+C_d(r_t)x(t-d)+D(r_t)\omega(t),\\
&x(t)=\phi(t),\quad t\in[-\bar{d},0],
\end{aligned}\tag{5}$$

where

$$\begin{aligned}
E_c(r_t)&=E(r_t)+\Delta E(r_t)+(B(r_t)+\Delta B(r_t))K_e(r_t),\\
A_c(r_t)&=A(r_t)+\Delta A(r_t)+(B(r_t)+\Delta B(r_t))K_a(r_t),\\
A_{cd}(r_t)&=A_d(r_t)+\Delta A_d(r_t)+(B(r_t)+\Delta B(r_t))K_d(r_t).
\end{aligned}\tag{6}$$

Definition 1: The hybrid impulsive Markovian jump time-delay system (5) with $\omega(t)=0$ is said to be robustly stochastically stable, if there exists a scalar $M(x_0,\phi(.))>0$ such that

$$\mathbb{E}\left\{\int_0^\infty\|x(s)\|^2ds|r_0,x(s)=\phi(s),s\in[-\bar{d},0]\right\}\le M(x_0,\phi(.))$$

holds for all admissible uncertainties and $r_0\in\mathcal{S}$.

Definition 2: The hybrid impulsive Markovian jump time-delay system (5) is said to be with robustly stochastically stable with H_∞ performance γ, if the system with $\omega(t)=0$ is robustly stochastically stable and the following condition is satisfied under the zero-initial condition

$$\mathbb{E}\left\{\int_0^\infty z^{\mathrm{T}}(t)z(t)dt\right\} \le \gamma^2 \int_0^\infty \omega^{\mathrm{T}}(t)\omega(t)dt \tag{7}$$

for all admissible uncertainties and any non-zero $\omega(t) \in \mathcal{L}_2[0,\infty)$.

Definition 3: Consider the uncertain time-delay SMJS (1). If there exists a controller (4) and a given disturbance attenuation level $\gamma > 0$ such that for all admissible uncertainties, the derivative matrix E_{ci}, $\forall i \in \mathcal{S}$, in the system (5) is invertible and the system (5) is robustly stochastically stable with H_∞ performance γ, then controller (4) is said to be an robust normalization and H_∞ hybrid impulsive controller (RNHIC) for system (1).

Lemma 1 [19]**:** Suppose a piecewise continuous real square matrices $A(t)$, X and $Q>0$, satisfying:

$$A^{\mathrm{T}}(t)X + X^{\mathrm{T}}A(t) + Q < 0$$

for all t. Then, the following hold:

1. $A(t)$ and X are invertible.
2. $\|A^{-1}(t)\| \le \delta$ for some $\delta > 0$.

Lemma 2 [20]**:** For any constant matrix $X \in \mathbb{R}^{n\times n}$, $X = X^{\mathrm{T}} > 0$, scalar $r>0$, and vector function $\dot{x} : [-r,0] \to R^n$ such that the following integration is well defined, then

$$-r\int_{-r}^{0} \dot{x}^{\mathrm{T}}(t+s)X\dot{x}(t+s)ds \le \begin{bmatrix} x^{\mathrm{T}}(t) & x^{\mathrm{T}}(t-r)\end{bmatrix} \begin{bmatrix} -X & X \\ X & -X \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-r)\end{bmatrix}.$$

Lemma 3 [21]**:** Given a positive definite matrix $P \in \mathbb{R}^{n\times n}$ and a symmetric matrix $Q \in \mathbb{R}^{n\times n}$, then

$$\begin{aligned}\lambda_{min}(P^{-1}Q)x^{\mathrm{T}}(t)Px(t) &\le x^{\mathrm{T}}(t)Qx(t) \\ &\le \lambda_{max}(P^{-1}Q)x^{\mathrm{T}}(t)Px(t)\end{aligned}$$

for all $x(t) \in \mathbb{R}^n$.

Lemma 4 [22]**:** Given a symmetric matrix Z and matrices X and Y of appropriate dimensions, then

$$Z + XF(t)Y + (XF(t)Y)^{\mathrm{T}} < 0$$

for all $F(t)$ satisfying $F^{\mathrm{T}}(t)F(t) \le I$, if and only if there exists a scalar $\varepsilon > 0$ such that

$$Z + \varepsilon XX^{\mathrm{T}} + \varepsilon^{-1}Y^{\mathrm{T}}Y < 0.$$

3. MAIN RESULTS

In this section, a set of sufficient conditions is derived to guarantee that the system (1) is normal and robustly stochastically stable with H_∞ performance γ under IPDMSFC (4).

3.1. Existence conditions of RNHIC

In this part, the existence conditions of RNHIC for system (1) are presented by the following theorem.

Theorem 1: For prescribed scalars $\bar{d} > 0$ and $\gamma > 0$, controller (4) is an RNHIC for system (1) if there exist symmetric positive-definite matrices P_i, Q_i, Q, Z, and matrices T_{1i}, T_{2i} such that the following set of inequalities hold for each $i \in \mathcal{S}$ and $k = 1,2,\ldots$

$$\Omega_i = \begin{bmatrix} \Omega_{11i} & \Omega_{12i} & \Omega_{13i} & T_{1i}^{\mathrm{T}}B_{\omega i} & C_i^{\mathrm{T}} \\ * & \Omega_{22i} & \Omega_{23i} & T_{2i}^{\mathrm{T}}B_{\omega i} & 0 \\ * & * & \Omega_{33i} & 0 & C_{di}^{\mathrm{T}} \\ * & * & * & -\gamma^2 I & D_i^{\mathrm{T}} \\ * & * & * & * & -I \end{bmatrix} < 0, \tag{8}$$

$$\textstyle\sum_{j=1}^{N} \pi_{ij}Q_j < Q, \tag{9}$$

$$0 < \beta_k \le 1, \tag{10}$$

where

$$\begin{aligned}
&\Omega_{11i} = A_{ci}^{\mathrm{T}}T_{1i} + T_{1i}^{\mathrm{T}}A_{ci} + \textstyle\sum_{j=1}^{N}\pi_{ij}P_j + Q_i + \bar{d}Q - Z,\\
&\Omega_{12i} = P_i - T_{1i}^{\mathrm{T}}E_{ci} + A_{ci}^{\mathrm{T}}T_{2i},\ \Omega_{13i} = T_{1i}^{\mathrm{T}}A_{cdi} + Z,\\
&\Omega_{22i} = -T_{2i}^{\mathrm{T}}E_{ci} - E_{ci}^{\mathrm{T}}T_{2i} + \bar{d}^2 Z,\\
&\Omega_{23i} = T_{2i}^{\mathrm{T}}A_{cdi},\ \Omega_{33i} = -Q_i - Z,\\
&\quad \beta_k = \lambda_{max}\{P^{-1}(r_{t_k^-})[I + E_{ci}^{-1}(B_i + \Delta B_i)G_i]^{\mathrm{T}}\\
&\qquad \times P(r_{t_k^+})[I + E_{ci}^{-1}(B_i + \Delta B_i)G_i]\}.
\end{aligned}$$

Proof: Suppose there exist symmetric positive-definite matrices P_i, Q_i, Q, Z, matrices T_{1i}, T_{2i}, and the control law (4) such that (8) holds. According to Lemma 1 and (8), it is obtained that the derivative matrix E_{ci}, $i \in \mathcal{S}$, is invertible and $\|E_{ci}^{-1}\|$ is bounded for all admissible uncertainties.

Next, we will show the robust stochastic stability of the system (5). Define a new process $\{(x_t, r_t), t \ge 0\}$ by $\{x_t = x(t+\theta), -2d \le \theta \le 0\}$, then $\{(x_t, r_t), t \ge d\}$ is a Markov process with initial state $(\phi(.), r_0)$. For $t \ge d$, define a stochastic Lyapunov candidate for system (5) as

$$V(x_t, r_t, t) = \textstyle\sum_{\mu=1}^{4} V_\mu(x_t, r_t, t), \tag{11}$$

where

$$\begin{aligned}
V_1(x_t, r_t, t) &= x^{\mathrm{T}}(t)P(r_t)x(t),\\
V_2(x_t, r_t, t) &= \int_{t-d}^{t} x^{\mathrm{T}}(\alpha)Q(r_t)x(\alpha)d\alpha,\\
V_3(x_t, r_t, t) &= \bar{d}\int_{-d}^{0}\int_{t+\beta}^{t} \dot{x}^{\mathrm{T}}(\alpha)Z\dot{x}(\alpha)d\alpha d\beta,\\
V_4(x_t, r_t, t) &= \int_{-d}^{0}\int_{t+\beta}^{t} x^{\mathrm{T}}(\alpha)Qx(\alpha)d\alpha d\beta.
\end{aligned}$$

Let $r(t) = i$, $t \in (t_k, t_{k+1}]$. The following equation holds for any matrices T_{1i} and T_{2i} of appropriate dimensions

$$2[-x^{\mathrm{T}}(t)T_{1i}^{\mathrm{T}} - \dot{x}^{\mathrm{T}}(t)T_{2i}^{\mathrm{T}}]$$

$$\times\ [E_{ci}\dot{x}(t)-A_{ci}x(t)-A_{cdi}x(t-d)-B_{\omega i}\omega(t)]=0. \tag{12}$$

Let $\mathbb{L}$ be the weak infinitesimal operator of the random process $\{x_t, r_t\}$, then for each $i\in\mathcal{S}$

$$\begin{aligned}
&\mathbb{L}V(x_t,i,t)+z^{\mathrm{T}}(t)z(t)-\gamma^2\omega^{\mathrm{T}}(t)\omega(t)\\
&\leq 2x^{\mathrm{T}}(t)P_i\dot{x}(t)+\sum\nolimits_{j=1}^{N}\pi_{ij}x^{\mathrm{T}}(t)P_jx(t)+x^{\mathrm{T}}(t)Q_ix(t)\\
&\quad -x^{\mathrm{T}}(t-d)Q_ix(t-d)+\int_{t-d}^{t}x^{\mathrm{T}}(\alpha)\sum_{j=1}^{N}\pi_{ij}Q_jx(\alpha)d\alpha\\
&\quad +\bar{d}^2\dot{x}^{\mathrm{T}}(t)Z\dot{x}(t)-\bar{d}\int_{t-d}^{t}\dot{x}^{\mathrm{T}}(\alpha)Z\dot{x}(\alpha)d\alpha\\
&\quad +\bar{d}x^{\mathrm{T}}(t)Qx(t)-\int_{t-d}^{t}x^{\mathrm{T}}(\alpha)Qx(\alpha)d\alpha\\
&\quad +[C_ix(t)+C_{di}x(t-d)+D_i\omega(t)]^{\mathrm{T}}\\
&\qquad \times[C_ix(t)+C_{di}x(t-d)+D_i\omega(t)]-\gamma^2\omega^{\mathrm{T}}(t)\omega(t)\\
&\quad +2[-x^{\mathrm{T}}(t)T_{1i}^{\mathrm{T}}-\dot{x}^{\mathrm{T}}(t)T_{2i}^{\mathrm{T}}]\\
&\qquad \times[E_{ci}\dot{x}(t)-A_{ci}x(t)-A_{cdi}x(t-d)-B_{\omega i}\omega(t)].
\end{aligned}$$

According to Lemma 2 and (9), for each $i\in\mathcal{S}$

$$\begin{aligned}
&\mathbb{L}V(x_t,i,t)+z^{\mathrm{T}}(t)z(t)-\gamma^2\omega^{\mathrm{T}}(t)\omega(t)\\
&\leq\zeta^{\mathrm{T}}(t)\left\{\begin{bmatrix}\Omega_{11i}&\Omega_{12i}&\Omega_{13i}&T_{1i}^{\mathrm{T}}B_{\omega i}\\ *&\Omega_{22i}&\Omega_{23i}&T_{2i}^{\mathrm{T}}B_{\omega i}\\ *&*&\Omega_{33i}&0\\ *&*&*&-\gamma^2I\end{bmatrix}\right.\\
&\qquad\left.+\begin{bmatrix}C_i^{\mathrm{T}}\\0\\C_{di}^{\mathrm{T}}\\D_i^{\mathrm{T}}\end{bmatrix}\begin{bmatrix}C_i&0&C_{di}&D_i\end{bmatrix}\right\}\zeta(t),
\end{aligned}\tag{13}$$

where $\zeta(t)=[\ x^{\mathrm{T}}(t)\quad \dot{x}^{\mathrm{T}}(t)\quad x^{\mathrm{T}}(t-d)\quad \omega^{\mathrm{T}}(t)\ ]^{\mathrm{T}}$. It follows from (8) and (13) that

$$\mathbb{L}V(x_t,i,t)<0 \tag{14}$$

for each $i\in\mathcal{S}$ and all admissible uncertainties when $\omega(t)=0$, then there must exist a scalar $\lambda>0$ such that

$$\mathbb{L}V(x_t,i,t)\leq-\lambda\|x(t)\|^2. \tag{15}$$

Now, consider the impulsive system at time point t_k. It follows from (5), (10) and Lemma 3 that

$$\begin{aligned}
&V(x_{t_k^+},r_{t_k^+},t_k^+)\\
&=x^{\mathrm{T}}(t_k^+)P(r_{t_k^+})x(t_k^+)+\int_{t_k^+-d}^{t_k^+}x^{\mathrm{T}}(\alpha)Q(r_{t_k^+})x(\alpha)d\alpha\\
&\quad +\bar{d}\int_{-d}^{0}\int_{t_k^++\beta}^{t_k^+}\dot{x}^{\mathrm{T}}(\alpha)Z\dot{x}(\alpha)d\alpha d\beta\\
&\quad +\int_{-d}^{0}\int_{t_k^++\beta}^{t_k^+}x^{\mathrm{T}}(\alpha)Qx(\alpha)d\alpha d\beta\\
&\leq\lambda_{\max}\{P^{-1}(r_{t_k^-})[I+E_{ci}^{-1}(B_i+\Delta B_i)G_i]^{\mathrm{T}}P(r_{t_k^+})\\
&\qquad\times[I+E_{ci}^{-1}(i)(B_i+\Delta B_i)G_i]\}x^{\mathrm{T}}(t_k)P(r(t_k^-))x(t_k)\\
&\qquad +V_2(x_{t_k^-},r_{t_k^-},t_k^-)+V_3(x_{t_k^-},r_{t_k^-},t_k^-)+V_4(x_{t_k^-},r_{t_k^-},t_k^-)\\
&=\beta_kV_1(x_{t_k^-},r_{t_k^-},t_k^-)+V_2(x_{t_k^-},r_{t_k^-},t_k^-)\\
&\qquad +V_3(x_{t_k^-},r_{t_k^-},t_k^-)+V_4(x_{t_k^-},r_{t_k^-},t_k^-)\\
&\leq V(x_{t_k^-},r_{t_k^-},t_k^-).
\end{aligned}\tag{16}$$

Suppose $d\in(t_p,t_{p+1}]$, $p\in\{0,1,2,\ldots\}$. Based on the Dynkin's formula, for $t\in(t_k,t_{k+1}]$, $k\geq p+1$,

$$\begin{aligned}
&\mathbb{E}[\int_{d}^{t}\mathbb{L}V(x_t,i,t)dt]\\
&=\mathbb{E}\int_{d^+}^{t_{p+1}}\mathbb{L}V(x_t,i,t)dt+\mathbb{E}\int_{t_{p+1}^+}^{t_{p+2}}\mathbb{L}V(x_t,i,t)dt\\
&\quad +\cdots+\mathbb{E}\int_{t_k^+}^{t}\mathbb{L}V(x_t,i,t)dt\\
&=\mathbb{E}[-V(x_d^+,r_d^+,d^+)+\sum\nolimits_{j=p+1}^{k}(V(x_{t_j^-},r_{t_j^-},t_j^-)\\
&\quad -V(x_{t_j^+},r_{t_j^+},t_j^+))+V(x_t,r_t,t)].
\end{aligned}$$

Therefore, for any $t\geq d$,

$$\mathbb{E}V(x_t,r_t,t)-\mathbb{E}V(x_d,r_d,d)\leq-\lambda\mathbb{E}\int_{d}^{t}\|x(s)\|^2ds.$$

From (14) and (16), it follows that

$$\lim_{t\to\infty}V(x_t,r_t,t)=0,$$

which yields

$$\mathbb{E}\int_{d}^{t}\|x(s)\|^2ds\leq\lambda^{-1}\mathbb{E}V(x_d,r_d,d). \tag{17}$$

For $t\in(t_0,t_1]$, it follows from (5) (when $\omega(t)=0$) that

$$\begin{aligned}
\|x(t)\|&=\|x(0)+\int_{0}^{t}[E_{ci}^{-1}A_{ci}x(\alpha)+E_{ci}^{-1}A_{cdi}x(\alpha-d)]d\alpha\|\\
&\leq\|x(0)\|+k_1\int_{0}^{t}[\|x(\alpha)\|+\|x(\alpha-d)\|]d\alpha,
\end{aligned}$$

where $k_1=\max_{i\in\mathcal{S}}\{\|E_{ci}^{-1}\|\|A_{ci}\|,\|E_{ci}^{-1}\|\|A_{cdi}\|\}>0$. Then for any $0\leq t\leq d$, $t\in(t_0,t_1]$,

$$\|x(t)\|\leq(k_1\bar{d}+1)\|\phi\|_{\bar{d}}+k_1\int_{0}^{t}\|x(\alpha)\|d\alpha,$$

which, by the Gronwall-Bellman Lemma, gives that for any $0\leq t\leq d$, $t\in(t_0,t_1]$,

$$\|x(t)\|\leq(k_1\bar{d}+1)\|\phi\|_{\bar{d}}e^{k_1\bar{d}}$$

and

$$\|x(t_1)\|\leq(k_1\bar{d}+1)\|\phi\|_{\bar{d}}e^{k_1\bar{d}}. \tag{18}$$

Because $0<\beta_k\leq1$ for all $k=1,2,\ldots$, it follows from (16) and (18) that

$$\|x(t_1^+)\|\leq\sqrt{\frac{\lambda_{\max}P(r_{t_1^-})}{\lambda_{\min}P(r_{t_1^+})}}\|x(t_1^-)\|\leq\bar{k}_1\|\phi\|_{\bar{d}},$$

where $\bar{k}_1 = \sqrt{\frac{\lambda_{\max} P(r_{t_1^-})}{\lambda_{\min} P(r_{t_1^+})}}(k_1\bar{d}+1)e^{k_1\bar{d}}$. In general, for any $0 \le t \le d$, $t \in (t_s, t_{s+1}]$, $s \in \{0,1,2,\ldots\}$,

$$\begin{aligned} \|x(t)\| &\le \|x(t_s^+)\| + k_1 \int_{t_s^+}^{t} [\|x(\alpha)\| + \|x(\alpha-d)\|] d\alpha \\ &\le (\bar{k}_s + k_1\bar{d})\|\phi\|_{\bar{d}} + k_1 \int_{t_s^+}^{t} \|x(\alpha)\| d\alpha \\ &\le (\bar{k}_s + k_1\bar{d})\|\phi\|_{\bar{d}} e^{k_1\bar{d}}, \end{aligned}$$

where $\bar{k}_0 = 1$ and $\bar{k}_s = \sqrt{\frac{\lambda_{\max} P(r_{t_s^-})}{\lambda_{\min} P(r_{t_s^+})}}(\bar{k}_{s-1} + k_1\bar{d})e^{k_1\bar{d}}$, $s \ge 1$. Hence, there exists a scalar $\bar{k} > 0$ such that

$$\sup_{0\le\alpha\le d} \|x(\alpha)\|^2 \le \bar{k}\|\phi\|_{\bar{d}}^2. \tag{19}$$

Note that

$$\begin{aligned} \int_{-d}^{0}\int_{t+\beta}^{t} \dot{x}^{\mathrm{T}}(\alpha) Z \dot{x}(\alpha) d\alpha d\beta &\le \bar{d}\int_{t-d}^{t} \dot{x}^{\mathrm{T}}(\alpha) Z \dot{x}(\alpha) d\alpha, \\ \int_{-d}^{0}\int_{t+\beta}^{t} x^{\mathrm{T}}(\alpha) Q x(\alpha) d\alpha d\beta &\le \bar{d}\int_{t-d}^{t} x^{\mathrm{T}}(\alpha) Q x(\alpha) d\alpha, \end{aligned}$$

then there exists a scalar ρ such that

$$V(x_d, r_d, d) \le \rho\|\phi\|_{\bar{d}}^2,$$

which together with (17) and (19) implies there exists a scalar ρ such that

$$\begin{aligned} \mathbb{E}\int_0^t \|x(s)\|^2 ds = \mathbb{E}\left\{\int_0^d \|x(s)\|^2 ds\right\} \\ + \mathbb{E}\left\{\int_d^t \|x(s)\|^2 ds\right\} \le \rho\mathbb{E}\|\phi\|_{\bar{d}}^2. \end{aligned}$$

According to Definition 1, the system (5) is robustly stochastically stable for any constant time delay d satisfying $0 \le d \le \bar{d}$.

In the following, we establish the H_∞ performance of the system (5). For this purpose, consider the following index

$$J_{z\omega}(t) = \mathbb{E}\left\{\int_0^t [z^{\mathrm{T}}(s)z(s) - \gamma^2\omega^{\mathrm{T}}(s)\omega(s)]\right\}.$$

Under zero-initial condition, it is easy to see that

$$J_{z\omega}(t) \le \mathbb{E}\left\{\int_0^t [z^{\mathrm{T}}(s)z(s) - \gamma^2\omega^{\mathrm{T}}(s)\omega(s) + \mathbb{L}V(x_s, i, s)]\right\}$$

Via the Schur Complement, it is obtained from (8) and (13) that for all $t > 0$, $J_{z\omega}(t) < 0$. Therefore, (7) is satisfied for all admissible uncertainties and any non-zero $\omega(t) \in \mathcal{L}_2[0,\infty)$. This completes the proof. □

3.2. Controller design

In the following, we seek a design method of the RNHIC for system (1).

Theorem 2: For prescribed scalars $\bar{d} > 0$ and $\gamma > 0$, controller (4) is an RNHIC for system (1) if there exist symmetric positive-definite matrices X_i, $\bar{Q}_i$, $\bar{Q}$, $\bar{Z}$, $\bar{W}$, $\bar{Y}$, matrices U_{1i}, U_{2i}, S_{1i}, S_{2i}, S_{3i}, L_i and scalars $\delta_{1i} > 0$, $\delta_{2i} > 0$ such that the following conditions hold for all $i, j \in \mathcal{S}$, $i \ne j$

$$\begin{bmatrix} \Theta_{11i} & \Theta_{12i} & \Theta_{13i} & 0 & X_iC_i^{\mathrm{T}} & \Theta_{16i} & \Theta_{17i} & X_i & \bar{d}X_i & \bar{d}U_{1i}^{\mathrm{T}} \\ * & \Theta_{22i} & \Theta_{23i} & B_{\omega i} & 0 & \Theta_{26i} & 0 & 0 & 0 & \bar{d}U_{2i}^{\mathrm{T}} \\ * & * & \Theta_{33i} & 0 & \bar{Q}_iC_{di}^{\mathrm{T}} & \Theta_{36i} & 0 & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & D_i^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\delta_{1i}I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Theta_{77i} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\bar{Q}_i & 0 & 0 \\ * & * & * & * & * & * & * & * & -\bar{d}\bar{Q} & 0 \\ * & * & * & * & * & * & * & * & * & -\bar{Z} \end{bmatrix} < 0, \tag{20}$$

$$\begin{bmatrix} \Phi_{11i} & \Phi_{12i} \\ * & \Phi_{22i} \end{bmatrix} < 0, \tag{21}$$

$$\begin{bmatrix} \Sigma_{11ij} & \Sigma_{12i} & \Sigma_{13i} & 0 \\ * & \Sigma_{22i} & \Sigma_{23i} & U_{2i}^{\mathrm{T}} \\ * & * & -\delta_{2i}I & 0 \\ * & * & * & -X_i \end{bmatrix} \le 0, \tag{22}$$

where

$$\begin{aligned} \Theta_{11i} &= U_{1i} + U_{1i}^{\mathrm{T}} + \pi_{ii}X_i - X_i - X_i^{\mathrm{T}} + \bar{Z}, \\ \Theta_{12i} &= X_iA_i^{\mathrm{T}} - U_{1i}{}^{\mathrm{T}}E_i^{\mathrm{T}} + U_{2i} + S_{1i}^{\mathrm{T}}B_i^{\mathrm{T}}, \\ \Theta_{13i} &= \bar{Q}_i,\ \Theta_{16i} = X_iN_{ai}^{\mathrm{T}} - U_{1i}^{\mathrm{T}}N_{ei}^{\mathrm{T}} + S_{1i}^{\mathrm{T}}N_{bi}^{\mathrm{T}}, \\ \Theta_{17i} &= [\sqrt{\pi_{i1}}X_i\ \ldots\ \sqrt{\pi_{i(i-1)}}X_i\ \sqrt{\pi_{i(i+1)}}X_i\ \ldots\ \sqrt{\pi_{iN}}X_i], \\ \Theta_{22i} &= -E_iU_{2i} - U_{2i}^{\mathrm{T}}E_i^{\mathrm{T}} + B_iS_{2i} + S_{2i}^{\mathrm{T}}B_i^{\mathrm{T}} + \delta_{1i}M_iM_i^{\mathrm{T}} + \bar{W}, \\ \Theta_{23i} &= A_{di}\bar{Q}_i + B_iS_{3i},\ \Theta_{26i} = -U_{2i}^{\mathrm{T}}N_{ei}^{\mathrm{T}} + S_{2i}^{\mathrm{T}}N_{bi}^{\mathrm{T}}, \\ \Theta_{33i} &= -\bar{Q}_i + \bar{Y},,\ \Theta_{36i} = \bar{Q}_i^{\mathrm{T}}N_{di}^{\mathrm{T}} + S_{3i}^{\mathrm{T}}N_{bi}^{\mathrm{T}}, \\ \Theta_{77i} &= -diag\{X_1,\ldots,X_{i-1},X_{i+1},\ldots,X_N\}, \\ \Phi_{11i} &= -X_i - X_i^{\mathrm{T}} + \bar{Q} - \sqrt{-\pi_{ii}}X_i - \sqrt{-\pi_{ii}}X_i^{\mathrm{T}} + \bar{Q}_i, \\ \Phi_{12i} &= [\sqrt{\pi_{i1}}X_i\ \ldots\ \sqrt{\pi_{i(i-1)}}X_i\ \sqrt{\pi_{i(i+1)}}X_i\ \ldots\ \sqrt{\pi_{iN}}X_i], \\ \Phi_{22i} &= -diag\{\bar{Q}_1,\ldots,\bar{Q}_{i-1},\bar{Q}_{i+1},\ldots,\bar{Q}_N\}, \\ \Sigma_{11ij} &= -U_{2i}^{\mathrm{T}} - U_{2i} + X_j,\ \Sigma_{12i} = -U_{2i}^{\mathrm{T}}E_i^{\mathrm{T}} + S_{2i}^{\mathrm{T}}B_i^{\mathrm{T}} - L_i^{\mathrm{T}}B_i^{\mathrm{T}}, \\ \Sigma_{13i} &= -U_{2i}^{\mathrm{T}}N_{ei}^{\mathrm{T}} + S_{2i}^{\mathrm{T}}N_{bi}^{\mathrm{T}} - L_i^{\mathrm{T}}N_{bi}^{\mathrm{T}}, \\ \Sigma_{22i} &= -E_iU_{2i} - U_{2i}^{\mathrm{T}}E_i^{\mathrm{T}} + B_iS_{2i} + S_{2i}^{\mathrm{T}}B_i^{\mathrm{T}} + \delta_{2i}M_iM_i^{\mathrm{T}}, \\ \Sigma_{23i} &= -U_{2i}^{\mathrm{T}}N_{ei}^{\mathrm{T}} + S_{2i}^{\mathrm{T}}N_{bi}^{\mathrm{T}}. \end{aligned}$$

In this case, the gains of RNHIC (4) are given by

$$\begin{aligned} K_{ai} &= (S_{1i} - S_{2i}U_{2i}^{-1}U_{1i})X_i^{-1},\ K_{di} = S_{3i}\bar{Q}_i^{-1}, \\ K_{ei} &= -S_{2i}U_{2i}^{-1},\ G_i = L_iU_{2i}^{-1}. \end{aligned} \tag{23}$$

Proof: From Theorem 1, it is seen that there exists an RNHIC for system (1) if (8), (9) and (10) hold for each $i \in \mathcal{S}$ and $k = 1, 2, \cdots$. Pre- and post-multiplying (8) by matrix $\begin{bmatrix} P_i & 0 & 0 & 0 & 0 \\ T_{1i} & T_{2i} & 0 & 0 & 0 \\ 0 & 0 & Q_i & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}^{-\mathrm{T}}$ and its transpose, respectively, and setting $X_i = P_i^{-1}$, $U_{1i} = -T_{2i}^{-1}T_{1i}P_i^{-1}$, $U_{2i} = T_{2i}^{-1}$, $\bar{Q}_i = Q_i^{-1}$, then (8) becomes

$$\begin{bmatrix} \Pi_{11i} & \Pi_{12i} & \Pi_{13i} & 0 & X_i^{\mathrm{T}}C_i^{\mathrm{T}} \\ * & \Pi_{22i} & \Pi_{23i} & B_{\omega i} & 0 \\ * & * & \Pi_{33i} & 0 & \bar{Q}_i^{\mathrm{T}}C_{di}^{\mathrm{T}} \\ * & * & * & -\gamma^2 I & D_i^{\mathrm{T}} \\ * & * & * & * & -I \end{bmatrix} < 0, \qquad (24)$$

where

$$\begin{aligned} \Pi_{11i} &= U_{1i} + U_{1i}^{\mathrm{T}} + X_i^{\mathrm{T}}\textstyle\sum_{j=1}^{N} \pi_{ij}P_jX_i + X_i^{\mathrm{T}}Q_iX_i \\ &\quad + \bar{d}X_i^{\mathrm{T}}QX_i + \bar{d}^2U_{1i}^{\mathrm{T}}ZU_{1i} - X_i^{\mathrm{T}}ZX_i, \\ \Pi_{12i} &= U_{2i} + X_i^{\mathrm{T}}A_{ci}^{\mathrm{T}} - U_{1i}^{\mathrm{T}}E_{ci}^{\mathrm{T}} + \bar{d}^2U_{1i}^{\mathrm{T}}ZU_{2i},\ \Pi_{13i} = X_i^{\mathrm{T}}Z\bar{Q}_i, \\ \Pi_{22i} &= -E_{ci}U_{2i} - U_{2i}^{\mathrm{T}}E_{ci}^{\mathrm{T}} + \bar{d}^2U_{2i}^{\mathrm{T}}ZU_{2i}, \\ \Pi_{23i} &= A_{cdi}\bar{Q}_i,\ \Pi_{33i} = -\bar{Q}_i - \bar{Q}_i^{\mathrm{T}}Z\bar{Q}_i. \end{aligned}$$

Note that for any positive matrices $W > 0$ and $Y > 0$,

$$\begin{aligned} &\begin{bmatrix} -X_i^{\mathrm{T}}ZX_i & 0 & X_i^{\mathrm{T}}Z\bar{Q}_i \\ 0 & 0 & 0 \\ * & 0 & -\bar{Q}_i^{\mathrm{T}}Z\bar{Q}_i \end{bmatrix} \\ &= -\begin{bmatrix} X_i^{\mathrm{T}} & 0 & 0 \\ 0 & 0 & 0 \\ -\bar{Q}_i^{\mathrm{T}} & 0 & 0 \end{bmatrix}\begin{bmatrix} Z & 0 & 0 \\ 0 & W & 0 \\ 0 & 0 & Y \end{bmatrix}\begin{bmatrix} X_i & 0 & -\bar{Q}_i \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ &\le -\begin{bmatrix} X_i^{\mathrm{T}} & 0 & 0 \\ 0 & 0 & 0 \\ -\bar{Q}_i^{\mathrm{T}} & 0 & 0 \end{bmatrix} - \begin{bmatrix} X_i & 0 & -\bar{Q}_i \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} \bar{Z} & 0 & 0 \\ 0 & \bar{W} & 0 \\ 0 & 0 & \bar{Y} \end{bmatrix}, \end{aligned} \qquad (25)$$

where $\bar{Z} = Z^{-1}$, $\bar{W} = W^{-1}$ and $\bar{Y} = Y^{-1}$. By (23), it is easy to show that

$$S_{1i} = K_{ai}X_i - K_{ei}U_{1i},\ S_{2i} = -K_{ei}U_{2i},\ S_{3i} = K_{di}\bar{Q}_i. \qquad (26)$$

Taking into account (25) and let $\bar{Q} = Q^{-1}$. Via Lemma 4 and the Schur Complement, conditions (20) implies that (8) holds by substituting (3) and (6) into (24). Pre- and post-multiplying (9) by X_i^{T} and X_i, respectively, then (21) implies that (9) holds basing on the fact that

$$\begin{aligned} &-X_i^{\mathrm{T}}QX_i \le -X_i - X_i^{\mathrm{T}} + \bar{Q}, \\ &\pi_{ii}X_i^{\mathrm{T}}Q_iX_i \le -\sqrt{-\pi_{ii}}X_i - \sqrt{-\pi_{ii}}X_i^{\mathrm{T}} + \bar{Q}_i. \end{aligned}$$

On the other hand, condition (10) in Theorem 1 is equivalent to

$$\begin{bmatrix} P_j & (I + E_{ci}^{-1}(B_i + \Delta B_i)G_i)^{\mathrm{T}} \\ * & P_i^{-1} \end{bmatrix} \ge 0, \qquad (27)$$

where $i, j \in \mathcal{S}, i \neq j$. Pre- and post-multiply (27) by matrix $\begin{bmatrix} U_{2i}^{\mathrm{T}} & 0 \\ 0 & E_{ci} \end{bmatrix}$ and its transpose, respectively,

$$\begin{bmatrix} -U_{2i}^{\mathrm{T}}P_jU_{2i} & -U_{2i}^{\mathrm{T}}E_{ci}^{\mathrm{T}} - U_{2i}^{\mathrm{T}}G_i^{\mathrm{T}}(B_i + \Delta B_i)^{\mathrm{T}} \\ * & -E_{ci}P_i^{-1}E_{ci}^{\mathrm{T}} \end{bmatrix} \le 0. \qquad (28)$$

Setting $L_i = G_iU_{2i}$, via Lemma 4 and the Schur Complement, it is concluded that condition (22) implies (10) holds based on the fact that

$$\begin{aligned} &-U_{2i}^{\mathrm{T}}P_jU_{2i} \le -U_{2i}^{\mathrm{T}} - U_{2i} + X_j, \\ &-E_{ci}P_i^{-1}E_{ci}^{\mathrm{T}} \le -E_{ci}U_{2i} - U_{2i}^{\mathrm{T}}E_{ci}^{\mathrm{T}} + U_{2i}^{\mathrm{T}}P_iU_{2i}. \end{aligned}$$

This completes the proof. □

In (23), if $S_{2i} = 0$, then $K_{ei} = 0$, or vice versa. Thus, in the case of $rank(E(r(t)) + \Delta E(r(t))) = n$, the following result can be obtained directly:

Corollary 1: For prescribed scalars $\bar{d} > 0$ and $\gamma > 0$, under the constraint of $rank(E(r(t)) + \Delta E(r(t))) = n$ (the system dimension), controller (4) is an RNHIC for system (1) if there exist symmetric positive-definite matrices X_i, $\bar{Q}_i$, $\bar{Q}$, $\bar{Z}$, $\bar{W}$, $\bar{Y}$, matrices U_{1i}, U_{2i}, S_{1i}, S_{3i}, L_i and scalars $\delta_{1i} > 0$, $\delta_{2i} > 0$ such that (21) and the following conditions hold for all $i, j \in \mathcal{S}$, $i \neq j$

$$\begin{bmatrix} \Theta_{11i} & \Theta_{12i} & \Theta_{13i} & 0 & X_iC_i^{\mathrm{T}} & \Theta_{16i} & \Theta_{17i} & X_i & \bar{d}X_i & \bar{d}U_{1i}^{\mathrm{T}} \\ * & \hat{\Theta}_{22i} & \Theta_{23i} & B_{\omega i} & 0 & \hat{\Theta}_{26i} & 0 & 0 & 0 & \bar{d}U_{2i}^{\mathrm{T}} \\ * & * & \Theta_{33i} & 0 & \bar{Q}_iC_{di}^{\mathrm{T}} & \Theta_{36i} & 0 & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & D_i^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\delta_{1i}I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Theta_{77i} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\bar{Q}_i & 0 & 0 \\ * & * & * & * & * & * & * & * & -\bar{d}\bar{Q} & 0 \\ * & * & * & * & * & * & * & * & * & -\bar{Z} \end{bmatrix} < 0, \qquad (29)$$

$$\begin{bmatrix} \Sigma_{11ij} & \hat{\Sigma}_{12i} & \hat{\Sigma}_{13i} & 0 \\ * & \hat{\Sigma}_{22i} & \hat{\Sigma}_{23i} & U_{2i}^{\mathrm{T}} \\ * & * & -\delta_{2i}I & 0 \\ * & * & * & -X_i \end{bmatrix} \le 0, \qquad (30)$$

where

$$\begin{aligned} &\hat{\Theta}_{22i} = -E_iU_{2i} - U_{2i}^{\mathrm{T}}E_i^{\mathrm{T}} + \delta_{1i}M_iM_i^{\mathrm{T}} + \bar{W},\ \hat{\Theta}_{26i} = -U_{2i}^{\mathrm{T}}N_{ei}^{\mathrm{T}}, \\ &\hat{\Sigma}_{12i} = -U_{2i}^{\mathrm{T}}E_i^{\mathrm{T}} - L_i^{\mathrm{T}}B_i^{\mathrm{T}},\ \hat{\Sigma}_{13i} = -U_{2i}^{\mathrm{T}}N_{ei}^{\mathrm{T}} - L_i^{\mathrm{T}}N_{bi}^{\mathrm{T}}, \\ &\hat{\Sigma}_{22i} = -E_iU_{2i} - U_{2i}^{\mathrm{T}}E_i^{\mathrm{T}} + \delta_{2i}M_iM_i^{\mathrm{T}},\ \hat{\Sigma}_{23i} = -U_{2i}^{\mathrm{T}}N_{ei}^{\mathrm{T}}, \end{aligned}$$

and the other terms are the same as the ones in Theorem 2. In this case, the gains of RNHIC (4) are given by

$$K_{ai} = S_{1i}X_i^{-1},\ K_{di} = S_{3i}\bar{Q}_i^{-1},\ K_{ei} = 0,\ G_i = Z_iV_{3i}^{-1}.$$

4. ILLUSTRATIVE EXAMPLES

Example 1: Consider a time-delay SMJS described by (1) with two modes, i.e., $\mathcal{S} = \{1, 2\}$. The system parameters are as follows:

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ A_1 = \begin{bmatrix} 0.2 & -0.3 & 1 \\ 0.7 & -1 & -0.5 \\ 0.1 & 0 & 0.4 \end{bmatrix},$$

Table 1. Maximum allowed $\bar{d}$ for different γ.

γ	1.0	1.25	1.5	1.75	2.0
$\bar{d}_{max}$	0.5901	0.6178	0.6375	0.6522	0.6637

Table 2. Minimum allowed γ for different $\bar{d}$.

$\bar{d}$	0.1	0.2	0.3	0.4	0.5
γ_{min}	0.2180	0.2476	0.3006	0.3936	0.5826

$$A_{d1} = \begin{bmatrix} -0.5 & 0.2 & 1 \\ 1.2 & 0.3 & 0.9 \\ -0.3 & 1 & -0.2 \end{bmatrix},\ B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \end{bmatrix},$$

$$B_{\omega 1} = \begin{bmatrix} 1 & 0.3 \\ 0.2 & 2 \\ 0 & -0.5 \end{bmatrix},\ C_1 = \begin{bmatrix} 1 & 0.2 & 0 \\ 0 & 0.1 & 0.5 \end{bmatrix},$$

$$C_{d1} = 0,\ D_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$E_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ A_2 = \begin{bmatrix} 0.1 & -1 & 0 \\ -0.2 & -1 & 0.4 \\ 0 & 0.3 & 0.1 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} 0.1 & 0.1 & 0.5 \\ 0.5 & -0.2 & 1 \\ -0.2 & 0.5 & -0.1 \end{bmatrix},\ B_2 = \begin{bmatrix} 0 & -0.3 \\ -1 & 0 \\ 1 & 1 \end{bmatrix},$$

$$B_{\omega 2} = \begin{bmatrix} 0.1 & -1 \\ 0.5 & 1 \\ 0.1 & 0 \end{bmatrix},\ C_2 = \begin{bmatrix} 0.1 & 0 & 1 \\ -1 & 0 & 0.3 \end{bmatrix},$$

$$C_{d2} = 0,\ D_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}.$$

The norm-bounded uncertainties satisfying (3) are described as

$$\begin{aligned} M_1 &= \begin{bmatrix} 0.3 & 0.4 & 0.3 \end{bmatrix}^{\mathrm{T}},\ N_{e1} = \begin{bmatrix} 0.7 & 0.7 & 0.2 \end{bmatrix}, \\ N_{a1} &= \begin{bmatrix} 0.2 & 0.4 & 0.3 \end{bmatrix},\ N_{d1} = \begin{bmatrix} 0.1 & 0.2 & 0.3 \end{bmatrix}, \\ N_{b1} &= \begin{bmatrix} 0.5 & 0.2 \end{bmatrix},\ M_2 = \begin{bmatrix} 0.3 & 0.4 & 0.3 \end{bmatrix}^{\mathrm{T}}, \\ N_{e2} &= \begin{bmatrix} 0.3 & 0.2 & 0.2 \end{bmatrix},\ N_{a2} = \begin{bmatrix} 0.3 & 0.5 & 0.2 \end{bmatrix}, \\ N_{d2} &= \begin{bmatrix} 0.4 & 0.1 & 0.2 \end{bmatrix},\ N_{b2} = \begin{bmatrix} 0.3 & 0.3 \end{bmatrix}, \end{aligned}$$

and the uncertain matrix is given as $F(t) = \sin(\frac{t+0.1}{2})$. It is easy to see that $rank(E_i + \Delta E_i) \not\equiv 3$ (the system dimension), $i = 1,2$, which means that the original system is not normal. The transition rate matrix is given as

$$\Pi = \begin{bmatrix} -5 & 5 \\ 7 & -7 \end{bmatrix}.$$

By solving the matrix inequalities (20)-(22), we can compute the maximum allowed time-delay $\bar{d}$ for given $\gamma > 0$ and the minimum allowed γ for given $\bar{d} > 0$. Table 1 and Table 2 presents the calculated results, respectively.

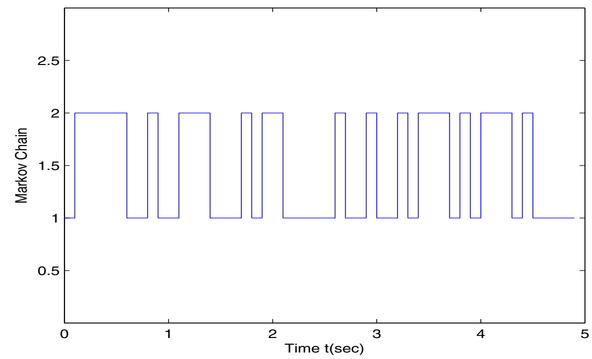

Fig. 1. The Markov process.

When $\bar{d} = 0.3$, $\gamma = 1.2$, an RNHIC for system (1) can be obtained by Theorem 2. The gain matrices of RNHIC are computed as

$$K_{a1} = \begin{bmatrix} -59.4351 & -8.4650 & 35.2804 \\ -97.7032 & -20.2983 & -79.0948 \end{bmatrix},$$

$$K_{d1} = \begin{bmatrix} 0.0742 & -0.0041 & -1.0347 \\ 0.0280 & -0.8505 & -0.8378 \end{bmatrix},$$

$$K_{e1} = \begin{bmatrix} 3.8769 & 0.4590 & -2.9862 \\ 7.2878 & 1.2793 & 4.0004 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} -4.3602 & -0.8825 & 3.0143 \\ -7.2933 & -2.1840 & -3.9715 \end{bmatrix},$$

$$K_{a2} = \begin{bmatrix} -61.2589 & -6.8518 & -27.7043 \\ 90.1553 & 5.1558 & -69.7300 \end{bmatrix},$$

$$K_{d2} = \begin{bmatrix} 0.3893 & -0.4310 & 0.1597 \\ -0.4989 & -0.2997 & -1.0661 \end{bmatrix},$$

$$K_{e2} = \begin{bmatrix} 4.8689 & 1.2858 & 0.9818 \\ -4.0502 & -1.6334 & 4.6704 \end{bmatrix},$$

$$G_2 = \begin{bmatrix} -5.0380 & -0.2194 & -0.7291 \\ 3.4805 & 0.4715 & -5.2657 \end{bmatrix}.$$

For any $t \in [0,\infty)$ and with the designed controller aforementioned, the rank of the derivative matrix of the corresponding closed-loop system is $rank(E_{ci}) = 3$, $i = 1,2$, which implies that the closed-loop system is normalized.

The Markov process is shown in Fig. 1, while the state responses of the open-loop (when $\omega(t) = 0$) and corresponding closed-loop system with $\phi(t) = \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}^{\mathrm{T}}$, $t \in [-0.8, 0]$ are illustrated by Fig. 2 and Fig. 3, respectively. Simulation results show that the closed-loop system is robustly stochastically stabilized by IPDMSFC (4).

Example 2: Consider time-delay SMJS (1) with no uncertainties in system matrices (i.e., $\Delta E_i = \Delta A_i = \Delta A_{di} = \Delta B_i = 0$) and $E_i = I$, $D_i = 0$, $i = 1,2$, whose parameters are described as follows:

$$A_1 = \begin{bmatrix} -3.5 & 0.8 \\ -0.6 & -3.3 \end{bmatrix},\ A_{d1} = \begin{bmatrix} -0.9 & -1.3 \\ -0.7 & -2.1 \end{bmatrix},$$

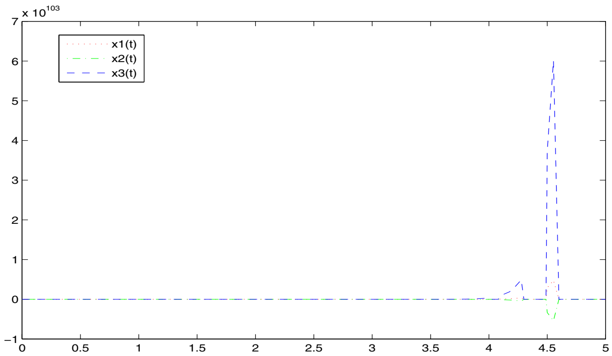

Fig. 2. The state trajectories of the open-loop system.

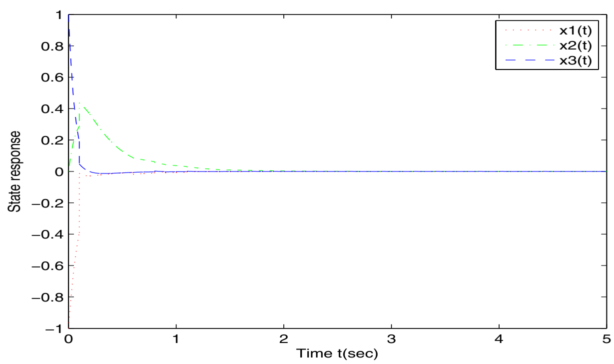

Fig. 3. The state trajectories of the closed-loop system.

$$B_1 = \begin{bmatrix} 0.8 \\ 0.1 \end{bmatrix}, B_{\omega 1} = \begin{bmatrix} 0.5 \\ 0.4 \end{bmatrix}, C_1 = \begin{bmatrix} 0.1 & 0.3 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -2.5 & 0.3 \\ 1.4 & -0.1 \end{bmatrix}, A_{d2} = \begin{bmatrix} -2.8 & 0.5 \\ -0.8 & -1.0 \end{bmatrix},$$
$$B_2 = \begin{bmatrix} 0.1 \\ 0.5 \end{bmatrix}, B_{\omega 2} = \begin{bmatrix} 0.3 \\ 0.2 \end{bmatrix}, C_2 = \begin{bmatrix} 0.2 & 0.15 \end{bmatrix}.$$

The transition rate matrix is given as

$$\Pi = \begin{bmatrix} -0.2 & 0.2 \\ 0.8 & -0.8 \end{bmatrix}.$$

The objective is to design a state feedback controller such that the resulting closed-loop system is robustly stochastically stable (when $\omega(t) = 0$) and has an H_∞ performance. Because $E_i = I$, $i = 1,2$, is non-singular, we can obtain the RNHIC for this system whether there is a derivative part in controller (4) or not (i.e., by Theorem 2 when $K_{ei} \neq 0$ or by Corollary 1 when $K_{ei} = 0$). We compute the minimum attenuation level γ by using Theorem 2 in [1] ($k = 0.5$), Theorem 4 in [16], Corollary 1 and Theorem 2 in this paper, respectively. Table 3 presents the comparison results on minimum allowed γ for various $\bar{d}$ by different methods.

It is clear that the minimal value of γ calculated by our results (whether there is a derivative part in controller (4) or not) are lower than those in [1] and [16]. Moreover, there is no solution to a mode-dependent controller by the method proposed in [16] if $\bar{d} > 0.6589$, where the designed controller is proportional. It is worth pointing out that the minimal value of γ is decreased due to the addition of derivative part in IPDMSFC (4).

Table 3. Comparisons of γ_{min} by different methods.

$\bar{d}$	0.4	0.5	0.6	0.7
[1]	0.0621	0.0740	0.0870	0.1028
[16]	0.0481	0.2443	0.9944	———
Corollary 1	0.0325	0.0394	0.0492	0.0731
Theorem 2	0.0222	0.0283	0.0400	0.0726

5. CONCLUSION

This paper has investigated the problem of robust normalization and H_∞ control for uncertain singular Markovian jump systems with time delay. A new hybrid impulsive controller has been proposed to ensure the normalization, robust stochastic stability and H_∞ performance of the closed-loop system simultaneously. Based on certain matrix conditions, an explicit desired impulsive and proportional-derivative memory state feedback controller has also been given. Illustrative examples have been provided to illustrate the effectiveness of our methods.

REFERENCES

[1] J. R. Wang, H. Wang, A. K. Xue, and R. Q. Lu, "Delay-dependent H_∞ control for singular Markovian jump systems with time delay," *Nonlinear Analysis: Hybrid Systems*, vol. 8, pp. 1-12, 2013. [click]

[2] G. L. Wang, "Robust stabilization of singular Markovian jump systems with uncertain switching," *International Journal of Control, Automation and Systems*, vol. 11, no. 1, pp. 188-193, 2013. [click]

[3] P. Shi and F. B. Li, "A survey on Markovian jump systems: Modeling and design," *International Journal of Control, Automation and Systems*, vol. 13, no. 1, pp. 1-16, 2015.

[4] W. L. Li and Y. M. Jia, "Rao-Blackwellised unscented particle filtering for jump Markov non-linear systems: an H_∞ approach," *IET Signal Processing*, vol. 5, no. 2, pp. 187 -193, 2011. [click]

[5] W. L. Li and Y. M. Jia, "Rao-Blackwellised particle filtering and smoothing for jump Markov non-linear systems with mode observation," *IET Signal Processing*, vol. 7, no. 4, pp. 327-336, 2013. [click]

[6] M. Q. Shen and D. Ye, "Improved fuzzy control design for nonlinear Markovian-jump systems with incomplete transition descriptions," *Fuzzy Sets and Systems*, vol. 217, pp. 80-95, 2013. [click]

[7] J. X. Dong and G. H. Yang, "Robust H_2 control of continuous-time Markov jump linear systems," *Automatica*, vol. 44, no. 5, pp. 1431-1436, 2008. [click]

[8] Z. G. Wu, P. Shi, H. Y. Su, and J. Chu, "Asynchronous $l_2 - l_\infty$ filtering for discrete-time stochastic Markov jump systems with randomly occurred sensor nonlinearities," *Automatica*, vol. 50, no. 1, pp. 180-186, 2014. [click]

[9] T. H. Lee, Q. Ma, S. Xu, and J. H. Park, "Pinning control for cluster synchronisation of complex dynamical networks with semi-Markovian jump topology," *International Journal of Control*, vol. 88, no. 6, pp. 1223-1235, 2015.

[10] T. H. Lee, J. H. Park, S. M. Lee, and O. M. Kwon, "Robust sampled-data control with random missing data scenario," *International Journal of Control*, vol. 87, no.9, pp. 1957-1969, 2014. [click]

[11] Y. Y. Cao and J. Lam, "Robust H_∞ control of uncertain Markovian jump systems with time-delay," *IEEE Trans. on Automatic Control*, vol. 45, no. 1, pp. 77-83, 2000. [click]

[12] Z. Wang, J. Lam, and X. Liu, "Exponential filtering for uncertain Markovian jump time-delay systems with nonlinear disturbances," *IEEE Trans. on Circuits and Systems II: Express Briefs*, vol. 51, no. 5, pp. 262-268, 2004. [click]

[13] N. Chaibi and E. H. Tissir, "Delay dependent robust stability of singular systems with time-varying delay," *International Journal of Control, Automation and Systems*, vol. 10, no. 3, pp. 632-638, 2012. [click]

[14] T. H. Lee, M. J. Park, J. H. Park and O. M. Kwon, "Extended dissipative analysis for neural networks with time-varying delays," *IEEE Trans. on Neural Networks and Learning Systems*, vol. 25, no. 10, pp. 1936-1941, 2014. [click]

[15] Z. G. Wu, P. Shi, H. Y. Su, and J. Chu, "Stochastic synchronization of Markovian jump neural networks with time-varying delay using sampled-data," *IEEE Trans. on Cybernetics*, vol. 43, no. 6, pp. 1796-1806, 2013. [click]

[16] J. Wu, T. Chen, and L. Wang, "Delay-dependent robust stability and H_∞ control for jump linear systems with delays," *Systems & Control Letters*, vol. 55, no. 11, pp. 939-948, 2006. [click]

[17] H. Zhang, Z. H. Guan, and G. Feng, "Reliable dissipative control for stochastic impulsive systems," *Automatica*, vol. 44, no. 4, pp. 1004-1010, 2008. [click]

[18] B. Li, D. Li, and D. Xu, "Stability analysis for impulsive stochastic delay differential equations with Markovian switching," *Journal of the Franklin Institute*, vol. 350, no. 7, pp. 1848-1864, 2013. [click]

[19] D. Yue, J. Lam, and D. W. C. Ho, "Reliable H_∞ control of uncertain descriptor systems with multiple time delays," *IEE Proceedings of Control Theory and Applications*, vol. 150, no. 6, pp. 557-564, 2003. [click]

[20] Q. L. Han, "Absolute stability of time-delay systems with sector-bounded nonlinearity," *Automatica*, vol. 41, no. 12, pp. 2171-2176, 2005. [click]

[21] L. Huang, *Linear Algebra in System and Control Theory*, Science Publish House, Beijing, 1984.

[22] I. R. Petersen, "A stabilization algorithm for a class of uncertain linear systems," *Systems & Control Letters*, vol. 8, no. 4, pp. 351-357, 1987. [click]

Hui Lv received her B.S. and M.S. degrees in applied mathematics from Liaoning University, Shenyang, China, in 2004 and 2009, respectively. She is pursuing a Ph.D. degree in control theory and control engineering from Northeastern University, Shenyang, China. Her research interests include singular Markovian systems and impulsive control.

Qingling Zhang received the B.S. and M.S. degrees from the Mathematics Department and the Ph.D. degree from the Automatic Control Department of Northeastern University, Shenyang, China, in 1982, 1986 and 1995, respectively. Dr. Zhang is now a Professor at Institute of Systems Science, Northeastern University. His research interests include singular systems and stochastic systems.

Junchao Ren received his B.S. degree in applied mathematics from Northeastern University, Shenyang, China, in 2000 and an M.S. degree in control theory and control engineering from Guangdong University of Technology, Guangzhou, in 2003 and a Ph.D. degree in control theory and control engineering from Northeastern University, Shenyang, China, in 2011. Dr. Ren is currently an associate professor with Institute of Systems Science, Northeastern University, Shenyang, China. His research interests include robust control and singular systems.