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Asymptotical Synchronization for Delayed Stochastic Neural Networks with Uncertainty via Adaptive Control

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Abstract: In this paper, the problem of the adaptive synchronization control is considered for neural networks with uncertainty and stochastic noise. Via utilizing stochastic analysis method and linear matrix inequality (LMI) approach, several sufficient conditions to ensure the adaptive synchronization for neural networks are derived. By the adaptive feedback methods, some suitable parameters update laws are found. Finally, a simulation result is provided to substantiate the effectiveness of the proposed approach.

Keywords: Neural networks, stochastic noises, synchronization control, time-delays, uncertainty.

1. INTRODUCTION

During the past years, neural networks have found their important applications in various fields [1,2] such as affine invariant matching, pattern recognition, associative memory and optimization solvers. It has been known that time delays are often encountered in neural networks. And in the neural networks, time delays are often unavoidably encountered due to the finite speeds of signals switching and transmission between neurons, which may cause undesirable dynamic network behaviors such as oscillation and instability. For neural networks with time-delays, various sufficient conditions have been proposed to guarantee the asymptotic or exponential stability in many of recent literatures, (see e.g., [3–9]). What is more, it is very important to analyze the robustness of delayed neural networks due to practical implementation that inevitably has uncertainties resulting from parameter drifting, fluctuation or modeling errors. In the uncertain neural networks, the interval uncertainty and the norm-bounded uncertainty are the most widely considered two types. Unfortunately, adaptive asymptotical synchronization problem for stochastic neural networks with uncertainties has not been fully investigated yet, which is the partly motivation of our research.

Up to now, the synchronization control of the neural networks has drawn much attention due to its potential applications in many fields, such as signal processing, combinatorial optimization, communication, secure communication, etc. (see e.g., [10–14]). And the synchronization control of neural networks is to achieve the accordance of the states of the drive neural networks and the response neural networks in a moment. Moreover, the adaptive synchronization control for neural networks has been extensively investigated over the last decade due to their successful applications in many areas, (see e.g., [15–19]), such as communication, signal processing and combinatorial optimization, etc.

It should be pointed out that, up to now, the problem of delay-dependent adaptive synchronization control for delayed neural networks with uncertainties and stochastic noises has received very little research attention.

Summarizing the above discussions, the focus of this paper is on the delay-dependent adaptive synchronization control for stochastic delayed neural networks without uncertainties and with uncertainties. The main novelty of our contribution lies in three folds: 1) A new delay-dependent adaptive synchronization control for delayed neural net-

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works with uncertainties and stochastic noise is addressed; 2) Using the adaptive feedback control techniques, adaptive feedback controller is designed; 3) The LMIs method of the adaptive synchronization controller is given by employing a new nonnegative function.

2. PROBLEM FORMULATION AND PRELIMINARIES

In this paper, we consider the delayed neural networks with uncertainties and stochastic noises described by

$$dx(t) = \Big[-\bar{C}x(t) + \bar{A}f(x(t)) + \bar{B}g(x(t-\tau_1(t))) + \bar{D}\int_{t-\tau_2}^{t} h(x(s))ds + J\Big]dt, \tag{1}$$

where $x(t) = (x_1(t), x_2(t), \cdots, x_n(t))^T \in \mathbb{R}^n$ is a real n-vector denoting the state variables associated with the neurons, $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \cdots, f_n(x_n(t)))^T \in \mathbb{R}^n$ denotes the activation function of the neurons, $\tau_1(t)$ and $\tau_2(t)$ are the transmission delay satisfying that $0 < \tau_i(t) \leq \bar{\tau}_i$ and $\dot{\tau}_i(t) \leq \hat{\tau}_i < 1$, where $\bar{\tau}_i$, $\hat{\tau}_i$ are constants, and $i = 1$, 2. $J \in \mathbb{R}^n$ is a constant external input vector. And the matrices $\bar{C}$, $\bar{A}$, $\bar{B}$ and $\bar{D}$, respectively, are the self-feedback matrix, the connection weight matrix and the delayed connection weight matrix satisfying

$$\begin{aligned} &\bar{C} = C + \Delta C(t), \bar{A} = A + \Delta A(t),\\ &\bar{B} = B + \Delta B(t), \bar{D} = D + \Delta D(t), \end{aligned} \tag{2}$$

where $C = \mathrm{diag}\{c_1, c_2, \cdots, c_n\}$, A, B and D are known constant matrices with appropriate dimensions. In addition, $\Delta C(t)$, $\Delta A(t)$, $\Delta B(t)$, $\Delta D(t)$ are the parameter uncertainties, which are assumed to be of the form

$$\begin{aligned} &[\Delta C(t), \Delta A(t), \Delta B(t), \Delta D(t)]\\ &\quad = MF(t)[N_C, N_A, N_B, N_D] \end{aligned} \tag{3}$$

where M, N_C, N_A, N_B and N_D are some given constant matrices with appropriate dimensions, and $F(t)$ is an unknown matrix representing the parameter perturbation which satisfies

$$F^T(t)F(t) \leq I. \tag{4}$$

For the drive systems (1), a response system is constructed as follows:

$$\begin{aligned} dy(t) =& [-\bar{C}y(t) + \bar{A}f(y(t)) + \bar{B}g(y(t-\tau_1(t)))\\ &+ \bar{D}\int_{t-\tau_2(t)}^{t} h(y(s))ds + J + u(t)]dt\\ &+ \sigma(t, y(t)-x(t), y(t-\tau_1(t)) - x(t-\tau_1(t)),\\ &\quad y(t-\tau_2(t)) - x(t-\tau_2(t)))d\omega(t), \end{aligned} \tag{5}$$

where $y(t)$ is the state vector of response system (5), $u(t) = (u_1(t), u_2(t), \cdots, u_n(t))^T \in \mathbb{R}^n$is a control input vector with the form of

$$\begin{aligned} u(t) &= k(t)(y(t) - x(t))\\ &= \mathrm{diag}\{k_1(t), k_2(t), \cdots, k_n(t)\}(y(t) - x(t)), \end{aligned} \tag{6}$$

$\omega(t) = (\omega_1(t),\ \omega_2(t),\ \cdots,\ \omega_n(t))^T$ is an n-dimensional Brown moment defined on a complete probability space $(\Omega, \mathcal{F}, P)$ with a natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$ (i.e., $\mathcal{F}_t = \sigma\{\omega(s)\ :\ 0 \leq s \leq t\}$ is a σ-algebra), and $\sigma\ :\ \mathbb{R}_+ \times \mathbb{S} \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times n}$ is the noise intensity matrix and can be regarded as a result from the occurrence of eternal random fluctuation and other probabilistic causes.

Let $e(t) = y(t) - x(t)$. For the purpose of simplicity, we mark $e(t-\tau(t)) = e_\tau(t)$, $f(e(t)) = f(y(t)) - f(x(t))$, $g(e(t)) = g(y(t)) - g(x(t))$ and $h(e(t)) = h(y(t)) - h(x(t))$. From the drive system (1) and the response system (2), the error system can be represented as follows

$$\begin{aligned} de(t) =& [-\bar{C}e(t) + \bar{A}f(e(t)) + \bar{B}g(e_{\tau_1}(t))\\ &+ \bar{D}\int_{t-\tau_2}^{t} h(e(s))ds + u(t)]dt\\ &+ \sigma(t, e(t), e_{\tau_1}(t), e_{\tau_2}(t))d\omega(t). \end{aligned} \tag{7}$$

To obtain the main result, we need the following assumptions, definition and lemmas.

Assumption 1: The activation functions of the neurons $f(x(t))$, $g(x(t))$ and $h(x(t))$ satisfy the Lipschitz condition. There exist diagonal matrices $L_{ij}^- = \mathrm{diag}(l_{i1}^-, l_{i2}^-, \ldots, l_{in}^-)$ and $L_{ij}^+ = \mathrm{diag}(l_{i1}^+, l_{i2}^+, \ldots, l_{in}^+)$, $(i = 1, 2, 3;\ j = 1, 2, \cdots, n)$ satisfying

$$\begin{aligned} L_{1j}^- &\leq \frac{f_j(u) - f_j(v)}{u - v} \leq L_{1j}^+,\\ L_{2j}^- &\leq \frac{g_j(u) - g_j(v)}{u - v} \leq L_{2j}^+,\\ L_{3j}^- &\leq \frac{h_j(u) - h_j(v)}{u - v} \leq L_{3j}^+, \end{aligned}$$

for all $u, v \in \mathbb{R}^n$, $u \neq v$.

Assumption 2: The noise intensity matrix $\sigma(\cdot,\cdot,\cdot,\cdot)$ satisfies the linear growth condition. That is, there exist positive definite matrices R_1, R_2 and R_3 such that

$$\begin{aligned} &\mathrm{trace}[\sigma^T(t, x_1, x_2, x_3)\sigma(t, x_1, x_2, x_3)]\\ &\quad \leq x_1^T R_1 x_1 + x_2^T R_2 x_2 + x_3^T R_3 x_3, \end{aligned}$$

for all x_1, x_2, $x_3 \in \mathbb{R}^n$ and $t \in \mathbb{R}^+$.

Definition 1: Consider an n-dimensional stochastic delayed differential equation (SDDE, for short)

$$dx(t) = \phi(t, x(t), x_\tau(t))dt + \varphi(t, x(t), x_\tau(t))d\omega(t) \tag{8}$$

on $t \in [0, \infty)$ with the initial data given by

$$\{x(\theta) : -\bar{\tau} \leq \theta \leq 0\} = \xi \in L_{\mathcal{L}_0}^p([-\bar{\tau}, 0]; \mathbb{R}^n).$$

If $V \in C^{2,1}(\mathbb{R}_+ \times \mathbb{R}^n; \mathbb{R}_+)$, define an operator $\mathcal{L}$ from $\mathbb{R}_+ \times \mathbb{R}^n$ to $\mathbb{R}$ by

$$\begin{aligned}&\mathcal{L}V(t,x,x_\tau)\\&= V_t(t,x) + V_x(t,x)\phi(t,x,x_\tau)\\&\quad + (1/2)\mathrm{trace}(\varphi^T(t,x,t_\tau)V_{xx}(t,x)\varphi(t,x,x_\tau)),\end{aligned}$$

where $V_t(t,x) = \frac{\partial V(t,x)}{\partial t}$, $V_{xx}(t,x) = \left(\frac{\partial^2 V(t,x)}{\partial x_j \partial x_k}\right)_{n\times n}$, $V_x(t,x) = \left(\frac{\partial V(t,x)}{\partial x_1}, \frac{\partial V(t,x)}{\partial x_2}, \cdots, \frac{\partial V(t,x)}{\partial x_n}\right)$.

Lemma 1 [20]**:** Assume that there are functions $V \in C^{2,1}(\mathbb{R}^+ \times \mathbb{S} \times \mathbb{R}^n; \mathbb{R}^+)$, $\psi \in L^1(\mathbb{R}^+; \mathbb{R}^+)$ and w_1, $w_2 \in C(\mathbb{R}^n; \mathbb{R}^+)$ such that

$$\begin{aligned}&\mathcal{L}V(t,i,x,y) \le \psi(t) - w_1(x) + w_2(y),\\&\forall (t,i,x,y) \in \mathbb{R}^+ \times \mathbb{S} \times \mathbb{R}^n \times \mathbb{R}^n,\end{aligned} \tag{9}$$

$$w_1(0) = w_2(0) = 0, w_1(x) > w_2(x) \ \ \forall x \neq 0,\ y \neq 0, \tag{10}$$

$$\lim_{|x|\to\infty} \inf_{0\le t<\infty, i\in S} V(t,i,x) = \infty. \tag{11}$$

Then the solution of equation (8) is almost surely asymptotically stable.

Lemma 2 [21]**:** For any vectors $a, b \in \mathbb{R}^n$, the inequality $2a^T b \le a^T X a + b^T X^{-1} b$holds, in which X is any matrix with $X > 0$.

Lemma 3 [21]**:** For any positive definite matrix $X \in \mathbb{R}^{n\times n}$, a scalar $\gamma > 0$, vector function $\psi : [0,\gamma] \to \mathbb{R}^n$such that the integration concerned are well defined, then

$$\left(\int_0^\gamma \psi(s)ds\right)^T X \left(\int_0^\gamma \psi(s)ds\right) \le \gamma \int_0^\gamma \psi^T(s) X \psi(s) ds.$$

Lemma 4 [22]**:** Let $\mathcal{U}$, $\mathcal{V}$ and $\mathcal{Z}$ be real matrices of appropriate dimensions with $\mathcal{Z} = \mathcal{Z}^T$, then

$$\mathcal{Z} + \mathcal{U}\mathcal{V}\mathcal{W} + \mathcal{W}^T \mathcal{V}^T \mathcal{U}^T < 0$$

for all $\mathcal{V}^T\mathcal{V} \le I$, if and only if there exists a scalar $\varepsilon > 0$ such that

$$\mathcal{Z} + \varepsilon^{-1}\mathcal{U}\mathcal{U}^T + \varepsilon \mathcal{W}^T \mathcal{W} < 0.$$

3. MAIN RESULTS

In this section, the adaptive synchronization control for the neural networks (1) and (5) is investigated under Assumptions 1-2. Firstly, the adaptive synchronization control of neural networks (1) and (5) will be handled without uncertainties.

Theorem 1: Under Assumptions 1-2, the stochastic neural networks (1) and (5) without uncertainties can be adaptive almost surely asymptotically synchronized, if there exist positive diagonal matrices H_1, H_2, H_3, $P = \mathrm{diag}(p_1, p_2, \ldots, p_n)$ positive definite matrices Q_1, Q_2, Q_3, S, and a positive scalar η such that the following matrix inequalities hold,

$$P \le \eta I, \tag{12}$$

$$\bar{\tau}_2 S \le Q_2, \tag{13}$$

$$\begin{bmatrix} \Pi_{11} & 0 & PA & 0 & PB & PD \\ * & \Pi_{22} & 0 & 0 & 0 & 0 \\ * & * & -H_1 & 0 & 0 & 0 \\ * & * & * & \bar{\tau}_2 Q_2 - H_2 & 0 & 0 \\ * & * & * & * & -H_2 & 0 \\ * & * & * & * & * & -S \end{bmatrix} < 0, \tag{14}$$

where $\Pi_{11} = -2PC + \eta(R_1 + R_3) + Q_1 + L_1H_1L_1 + L_2H_2L_2$, $\Pi_{22} = \eta R_2 - (1-\hat{\tau}_1)Q_1 + L_3H_3L_3$.

And the adaptive feedback controller is designed as

$$u(t) = k(y(t) - x(t)), \tag{15}$$

where the feedback strength $k = \mathrm{diag}(k_1,\ k_2,\ \ldots,\ k_n)$, is chosen as

$$\dot{k}_j = -\nu_j e_j^2(t) \tag{16}$$

with $\nu_j > 0\ (j = 1, 2, \ldots, n)$, an arbitrary positive constant.

Proof: Consider the following Lyapunov-Krasovskii function for system (7) as follows:

$$V(t,e(t)) = \sum_{i=1}^4 V_i(t,e(t)), \tag{17}$$

where

$$\begin{aligned}V_1(t,e(t)) &= e^T(t)Pe(t),\\V_2(t,e(t)) &= \int_{t-\tau_1(t)}^t e^T(s)Q_1 e(s)ds,\\V_3(t,e(t)) &= \int_{-\tau_2(t)}^0 \int_{t+\theta}^t h^T(e(s))Q_2 h(e(s))dsd\theta,\\V_4(t,e(t)) &= \sum_{j=1}^n \frac{p_j}{\nu_j}k_j^2.\end{aligned}$$

Computing $\mathcal{L}V(t,e(t))$ along the trajectory of error system (7), one can get that

$$\begin{aligned}&\mathcal{L}V_1(t,e(t))\\&= 2e^T(t)P[-Ce(t) + Af(e(t)) + Bg(e_{\tau_1}(t))\\&\quad + D\int_{t-\tau_2(t)}^t h(e(s))ds + k(t)e(t)]\\&\quad + \mathrm{trace}[\sigma^T(t,e(t),e_{\tau_1}(t),e_{\tau_2}(t))\\&\qquad \times P\sigma(t,e(t),e_{\tau_1}(t),e_{\tau_2}(t))]\\&= e^T(t)[-2PC]e(t) + e^T(t)[2PA]f(e(t))\\&\quad + e^T(t)[2PB]g(e_{\tau_1}(t)) + e^T(t)[2PD]\int_{t-\tau_2(t)}^t h(e(s))ds\\&\quad + 2\sum_{j=1}^n p_j k_j(t) e_j^2(t)\end{aligned}$$

$$+\mathrm{trace}[\sigma^T(t,e(t),e_{\tau_1}(t),e_{\tau_2}(t)) \times P\sigma(t,e(t),e_{\tau_1}(t),e_{\tau_2}(t))]. \tag{18}$$

From Lemma 2, yields

$$\begin{aligned} &e^T(t)[2PD]\int_{t-\tau_2(t)}^{t} h(e(s))ds \\ &\le e^T(t)[PDS^{-1}D^TP^T]e(t) \\ &\quad + \left[\int_{t-\tau_2(t)}^{t} h(e(s))ds\right]^T S \left[\int_{t-\tau_2(t)}^{t} h(e(s))ds\right]. \end{aligned} \tag{19}$$

Applying the Lemma 3, one gets

$$\begin{aligned} &\left[\int_{t-\tau_2(t)}^{t} h(e(s))ds\right]^T S \left[\int_{t-\tau_2(t)}^{t} h(e(s))ds\right] \\ &\le \tau_2(t)\int_{t-\tau_2(t)}^{t} h^T(e(s))Sh(e(s))ds \\ &\le \int_{t-\tau_2(t)}^{t} h^T(e(s))[\bar{\tau}_2 S]h(e(s))ds. \end{aligned} \tag{20}$$

It follows from Assumption 2 and inequality (12) that

$$\begin{aligned} &\mathrm{trace}[\sigma^T(t,e(t),e_{\tau_1}(t),e_{\tau_2}(t))P\sigma(t,e(t),e_{\tau_1}(t),e_{\tau_2}(t))] \\ &\le \lambda_{max}(P)\mathrm{trace}[\sigma^T(t,e(t),e_{\tau_1}(t),e_{\tau_2}(t)) \\ &\qquad \times \sigma(t,e(t),e_{\tau_1}(t),e_{\tau_2}(t))] \\ &\le \eta[e^T(t)R_1e(t)+e_{\tau_1}^T(t)R_2e_{\tau_1}(t)+e_{\tau_2}^T(t)R_3e_{\tau_2}(t)], \end{aligned} \tag{21}$$

where $\eta = \lambda_{max}(P)$.

By Itô's differential formula, we have

$$\begin{aligned} &\mathcal{L}V_2(t,e(t)) \\ &= e^T(t)Q_1e(t) - (1-\dot{\tau}_1(t))e_{\tau_1}^T(t)Q_1e_{\tau_1}(t) \\ &\le e^T(t)Q_1e(t) - e_{\tau_1}^T(t)[(1-\hat{\tau}_1)Q_1]e_{\tau_1}(t), \end{aligned} \tag{22}$$

$$\begin{aligned} &\mathcal{L}V_3(t,e(t)) \\ &= \tau_2(t)h^T(e(t))Q_2h(e(t)) \\ &\quad - \int_{t-\tau_2(t)}^{t} h^T(e(s))Q_2h(e(s))ds \\ &\le h^T(e(t))[\bar{\tau}_2Q_2]h(e(t)) \\ &\quad - \int_{t-\tau_2(t)}^{t} h^T(e(s))Q_2h(e(s))ds, \end{aligned} \tag{23}$$

$$\mathcal{L}V_4(t,e(t)) = 2\sum_{j=1}^{n}\frac{p_j}{\nu_j}k_j\dot{k}_j = -2\sum_{j=1}^{n}p_jk_je_j^2(t). \tag{24}$$

Furthermore, the condition (13) yields

$$\begin{aligned} &\int_{t-\tau_2(t)}^{t} h^T(e(s))[\bar{\tau}_2S]h(e(s))ds \\ &\qquad - \int_{t-\tau_2(t)}^{t} h^T(e(s))Q_2h(e(s))ds \le 0. \end{aligned} \tag{25}$$

On the other hand, from Assumption 1, it follows that

$$e^T(t)L_1H_1L_1e(t) - f^T(e(t))H_1f(e(t)) \ge 0, \tag{26}$$

$$e^T(t)L_2H_2L_2e(t) - h^T(e(t))H_2h(e(t)) \ge 0, \tag{27}$$

$$e_{\tau_1}^T(t)L_3H_3L_3e_{\tau_1}(t) - g^T(e_{\tau_1}(t))H_3g(e_{\tau_1}(t)) \ge 0, \tag{28}$$

where $L_i = \mathrm{diag}(l_{i1}, l_{i2}, \ldots, l_{in})$, $l_{ij} = \max\{|l_{ij}^-|, |l_{ij}^+|\}$, $(i = 1, 2, 3)$ for $j = 1, 2, \ldots, n$.

According to (18)-(28), one can obtain that

$$\begin{aligned} &\mathcal{L}V(t,e(t)) \\ &= e^T(t)[-2PC+Q_1+\eta(R_1+R_3)+PDS^{-1}D^TP^T \\ &\quad + L_1H_1L_1 + L_2H_2L_2]e(t) + e^T(t)[2PA]f(e(t)) \\ &\quad + e^T(t)[2PB]g(e_{\tau_1}(t)) + e_{\tau_1}^T(t)[\eta R_2 - (1-\hat{\tau}_1)Q_1 \\ &\quad + L_3H_3L_3]e_{\tau_1}(t) + h^T(e(t))[\bar{\tau}_2Q_2 - H_2]h(e(t)) \\ &\quad + f^T(e(t))[-H_1]f(e(t)) + g^T(e_{\tau_1}(t))[-H_3]g(e_{\tau_1}(t)) \\ &\quad - e^T(t)[\eta R_3]e(t) + e_{\tau_2}^T(t)[\eta R_3]e_{\tau_2}(t) \\ &= \Psi^T(t)\Xi\Psi(t) - e^T(t)[\eta R_3]e(t) + e_{\tau_2}^T(t)[\eta R_3]e_{\tau_2}(t), \end{aligned} \tag{29}$$

where

$$\Psi^T(t) = [e^T(t), e_{\tau_1}^T(t), f^T(e(t)), h^T(e(t)), g^T(e_{\tau_1}(t))]^T,$$

$$\Xi = \begin{bmatrix} \Xi_{11} & 0 & PA & 0 & PB \\ * & \Pi_{22} & 0 & 0 & 0 \\ * & * & -H_1 & 0 & 0 \\ * & * & 0 & \bar{\tau}_2Q_2 - H_2 & 0 \\ * & * & * & 0 & -H_3 \end{bmatrix}, \tag{30}$$

$$\Xi_{11} = -2PC + Q_1 + \eta(R_1+R_3) + L_1H_1L_1 + L_2H_2L_2 + PDS^{-1}D^TP^T.$$

In addition, by Schur complement, inequality (14) is equivalent to (30). Therefore, $\Xi < 0$.

Let $\delta = \lambda_{min}(-\Xi)$, clearly, the constant $\delta > 0$. This together with (29) gives

$$\begin{aligned} \mathcal{L}V(t,e(t)) &\le -e^T(t)(\eta R_3 + \delta I)e(t) \\ &\quad + e_{\tau_2}^T(t)(\eta R_3 - \delta I)e_{\tau_2}(t) \\ &= -\omega_1(e(t)) + \omega_2(e_{\tau_2}(t)), \end{aligned} \tag{31}$$

where $\omega_1(e(t)) = e^T(t)(\eta R_3 + \delta I)e(t)$ and $\omega_2(e(t)) = e_{\tau_2}^T(\eta R_3 - \delta I)e_{\tau_2}(t)$.

Let $\psi(t) = 0$, $\omega_1(e) = e^T(t)(\eta R_3 + \delta I)e(t)$ and $\omega_2(e_{\tau_2}) = e_{\tau_2}^T(t)(\eta R_3 - \delta I)e_{\tau_2}(t)$. Then inequality (31) holds such that inequality (9) holds. $\omega_1(0) = 0$ and $\omega_2(0) = 0$ when $e = 0$ and $e_{\tau_2} = 0$, and it can be seen that $\omega_1(e(t)) > \omega_2(e(t))$ for any $e(t) \neq 0$. So (10) holds. Moreover, (11) holds when $|e| \to \infty$ and $|e_{\tau_2}| \to \infty$. By Lemma 1, therefore, the error system (7) is adaptive almost surely asymptotically stable, and hence the response neural networks (5) can be adaptive almost surely asymptotical synchronized with the drive neural networks (1). This completes the proof.

Remark 6: In Theorem 1, the condition of adaptive synchronization control is obtained for neural networks without uncertainty. Next, we will deal with the synchronization control of neural networks (1) and (5) with uncertainty.

Theorem 2: Under Assumption 1-2, the stochastic neural networks (1) and (5) with uncertainty can be adaptive almost surely asymptotically synchronized, if there exist positive diagonal matrices H_1, H_2, H_3, $P = \mathrm{diag}(p_1, p_2, \ldots, p_n)$, positive definite matrices Q_1, Q_2, S, and a positive scalar η such that the following matrix inequalities hold,

$$P \le \eta I, \tag{32}$$

$$\bar{\tau}_2 S \le Q_2, \tag{33}$$

$$\begin{bmatrix} \Theta_{11} & 0 & PA & 0 & PB & PD & PM & \varepsilon N_C^T \\ * & \Theta_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -H_1 & 0 & 0 & 0 & 0 & \varepsilon N_A^T \\ * & * & * & \Theta_{44} & 0 & 0 & 0 & 0 \\ * & * & * & * & -H_3 & 0 & 0 & \varepsilon N_B^T \\ * & * & * & * & * & -S & 0 & \varepsilon N_D^T \\ * & * & * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0, \tag{34}$$

where $\Theta_{11} = -2PC + \eta(R_1 + R_3) + Q_1 + L_1H_1L_1 + L_2H_2L_2$, $\Theta_{22} = \eta R_2 - (1 - \hat{\tau}_1)Q_1 + L_3H_3L_3$, $\Theta_{44} = \bar{\tau}_2 Q_2 - H_2$.

And the adaptive feedback controller is designed as

$$u(t) = k(y(t) - x(t)), \tag{35}$$

where the feedback strength $k = \mathrm{diag}(k_1, k_2, \ldots, k_n)$, is chosen as

$$\dot{k}_j = -\nu_j e_j^2(t) \tag{36}$$

with $\nu_j > 0\,(j = 1, 2, \ldots, n)$, an arbitrary positive constant.

Proof : Replacing C, A, B and D in (14) by $\bar{C}$, $\bar{A}$, $\bar{B}$, $\bar{D}$, respectively. Then, one gets

$$\begin{bmatrix} \bar{\Theta}_{11} & 0 & P\bar{A} & 0 & P\bar{B} & P\bar{D} \\ * & \Theta_{22} & 0 & 0 & 0 & 0 \\ * & * & -H_1 & 0 & 0 & 0 \\ * & * & * & \Theta_{44} & 0 & 0 \\ * & * & * & * & -H_2 & 0 \\ * & * & * & * & * & -S \end{bmatrix} < 0, \tag{37}$$

where $\bar{\Theta}_{11} = -2P\bar{C} + \eta(R_1 + R_3) + Q_1 + L_1H_1L_1 + L_2H_2L_2$.

By Schur complement lemma, (37) is equivalent to

$$\Theta + \Lambda F \Gamma + (\Lambda F \Gamma)^T < 0, \tag{38}$$

$$\Theta = \begin{bmatrix} \Theta_{11} & 0 & PA & 0 & PB & PD \\ * & \Theta_{22} & 0 & 0 & 0 & 0 \\ * & * & -H_1 & 0 & 0 & 0 \\ * & * & * & \Theta_{44} & 0 & 0 \\ * & * & * & * & -H_3 & 0 \\ * & * & * & * & 0 & -S \end{bmatrix} < 0, \tag{39}$$

$$\Lambda = \begin{bmatrix} (PM)^T & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$

$$\Gamma = \begin{bmatrix} N_C & 0 & N_A & 0 & N_B & N_D \end{bmatrix}.$$

According to Lemma 4, then

$$\Theta + \varepsilon \Lambda^T \Lambda + \varepsilon^{-1} \Gamma \Gamma^T < 0, \tag{40}$$

holds if and only if there exists $\varepsilon > 0$.

Again by Schur complement lemma, (40) is equivalent to (34).This completes the proof.

Remark 7: Up to now, the condition of adaptive synchronization control are got for neural networks with uncertainty and without uncertainty. Note that the obtained criteria are dependent on not only the upper bound but also the lower bound of the time-varying delay, hence less conservative than the traditional delay-independent ones. Also, the LMI-based criteria can be checked efficiently via the Matlab LMI Toolbox.

4. ILLUSTRATIVE EXAMPLE

Consider the delayed neural networks (1), the response stochastic delayed neural networks (5) and the error system (7) with the network parameters given as follows:

$$C = \begin{bmatrix} 2.9 & 0 & 0 \\ 0 & 2.8 & 0 \\ 0 & 0 & 2.3 \end{bmatrix}, A = \begin{bmatrix} 0.2 & 0.18 & 0 \\ 0.3 & 0.19 & 0.2 \\ 0.4 & 0.7 & 0.6 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.3 & 0 & 0.5 \\ 0.4 & 0.5 & 0 \\ 0.2 & 0.7 & 0.4 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$N_C = \begin{bmatrix} 1.3 & 0 & 0 \\ 0 & 0.7 & 0 \\ 0 & 0 & 0.8 \end{bmatrix}, N_A = \begin{bmatrix} 1.3 & 0.48 & 0.3 \\ 0.4 & 0.19 & 0.2 \\ 0.1 & 1.12 & 1.6 \end{bmatrix},$$

$$N_B = \begin{bmatrix} 1.3 & 0 & 0.2 \\ 0.5 & 0.8 & 0.4 \\ 0.6 & 0.9 & 0.1 \end{bmatrix}, N_D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$M = \begin{bmatrix} 4 & 0.2 & 0 \\ 0.16 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$f(e(t)) = g(e(t)) = h(e(t)) = \tanh(e(t)),$$

$$\begin{aligned}\sigma(t, e(t), e(t-\tau_1), e(t-\tau_2)) &= [0.3e_1(t-\tau_1) + 0.4e_2(t-\tau_2); \\ &\quad 0.5e_2(t) + 0.2e_3(t-\tau_2); \\ &\quad 0.3e_3(t) + 0.1e_1(t-\tau_2)]^T,\end{aligned}$$

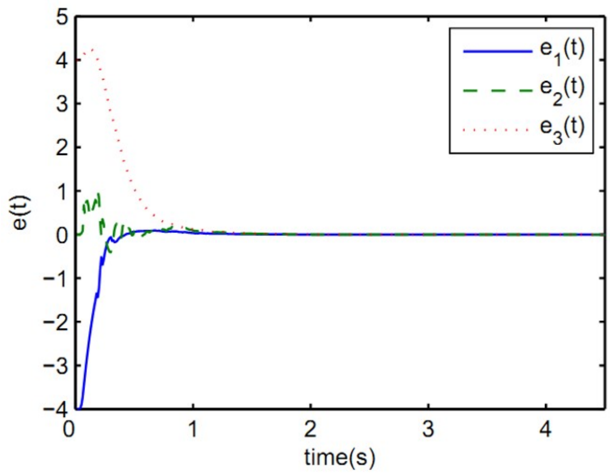

Fig. 1. The dynamic curve of the errors system $e(t)$.

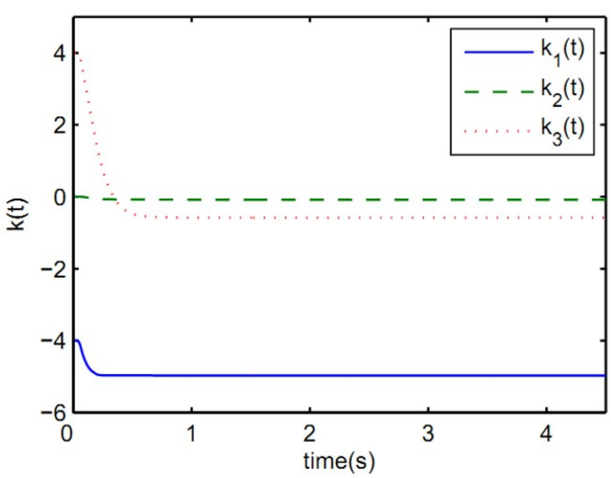

Fig. 2. The dynamic curve of the feedback gain $k(t)$.

$$L_1 = L_2 = L_3 = 1, R_1 = R_2 = R_3 = 0.4I,$$
$$\bar{\tau}_1 = \bar{\tau}_2 = \hat{\tau}_1 = \hat{\tau}_2 = 1.$$

Let the initial data as $e(0) = k(0) = [-4, 0, 4]^T$, we can draw the response curve of $e(t)$ of the errors system, and the dynamic curve of the feedback gain $k(t)$, respectively, as Figs. 1-2. From the simulation figures, one can see that the stochastic delayed neural networks (1) and (5) are adaptive synchronized.

5. CONCLUSION

In this paper, we have dealt with the problem of the adaptive synchronization for neural networks with uncertainties and stochastic noises. By using the adaptive feedback control technique and LMIs method, an adaptive feedback controller has been designed to achieve the synchronization for the stochastic neural networks.

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