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Modeling and Simulation of Quadrotor UAV with Tilting Rotors

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Abstract: Quadrotors have recently been drawing greater research and commercial attention to the point that they have become one of the most popular types of unmanned aerial vehicles. Their applications vary from entertainment to transportation, commercial and even military applications. In this paper, a novel quadrotor design is proposed. The design decouples all motions by allowing each rotor to tilt in two directions about the quadrotor fixed frame. This modification improves the stability and safety of the quadrotor and gives it more manoeuvrability and robustness. The mathematical model of the proposed system is carried out using Newton-Euler technique. Several flight scenarios are also simulated under a simple PID controller to illustrate the superiority over conventional quadrotor designs.

Keywords: Degrees of freedom, fault tolerance, quadrotor, tilting rotor, UAV, underactuated.

1. INTRODUCTION

Quadrotors have recently become a focus of research in Unmanned Aerial Vehicle (UAV) and flying robots applications. Quadrotor Air Vehicles (QRAV) may be employed in a wide range of commercial and military applications. Such applications may include: heavy transportation, construction of bridges and buildings, assembly of large pieces in factories, and rescue operations after natural disasters where roads and bridges are no longer usable. For military applications, QRAV may perform vertical takeoff and landing (VTOL) and can be used in manned operations for effective transport and for military deployment operations in hostile environments where VTOL is a requirement. Additionally, QRAV can have manoeuvrability that may be superior to helicopters, such as the APACHE helicopter.

In [1], one of the first tilt-wing VTOL aircrafts was designed and tested to explore the feasibility of transition from hover to forward flight. Various problems that are related to the performance and control characteristics were discussed. However, at the time, the paper concluded that it was very early to determine control requirement due to lack of flights data. [2] presents a general control approach of autonomously flying VTOL robots that takes advantage of the similarity in motion description in different VTOL robots. This control scheme is based on linearization using inversion of the model blocks. It was shown that this general scheme with inversion of the model blocks works even if the non-linear parameters are unknown.

Specifically, quadrotors are one type of VTOL that has been under the focus of extensive research due to their simple design and high agility. Conventional quadrotor is typically underactuated. It is composed of four fixed rotors which provide four input variables and has six degrees of freedom (DOF), 3 position and 3 orientations. The underactuated nature of typical quadrotors forces two translational motions to be coupled with two rotational orientations, i.e., the x and y translation motions are coupled with the two rotational angles pitch and roll respectively. This coupling reduces the manoeuvrability and agility and severely limits tracking capabilities. For example, to move forward or sideways, roll or pitch angles is compromised and the UAV has to tilt. The UAV cannot go through tight openings and can't hover while having a tilted orientation. While these limitations are not of big impact on ordinary missions, critical missions demand much higher manoeuvrability.

Conventional quadrotor modeling and control were extensively covered in the literature. In [3], the author describes an efficient and robust quadrotor for both indoor and outdoor. The paper presented an improvement to overcome uncertainty in position control and instability during fast maneuvers caused by low frequency in the control.

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[4] presents development and accurate modeling for a quadrotor UAV. The developed system was equipped with necessary devices and sensors. Rigorous dynamic model and robust flight control is developed. The controller presented is a disturbance observer based controller.

In [5], a fully autonomous quadrotor was presented for indoor applications. While navigation of outdoors quadrotor depends mainly on Global Positioning System (GPS), indoor quadrotors can not rely on that type of systems. Hence, the paper presented a navigation systems to enable small quadrotors to operate autonomously in closed environments. The approach was an extension and adaptation of techniques successfully implemented in ground robots.

[6] presented a customized design for control system validation. The paper presented an L1 mathematical modeling that defines the 6 degrees of freedom. The model is described in details and the complexity of L1 was claimed to aim at describing the key features of the flight. In [7], a relatively large quadrotor was presented. The UAV was designed to weigh 4 kg and have a payload of 1 kg. This improvement aimed to exploit the advantages of quadrotor maneuverability in more applications where carrying objects is desirable.

Many breakthroughs emerged by researchers trying to overcome the actuation difficulties in UAV's. Tilt wing mechanism was proposed in [8] and tilt rotor actuation in [9, 10]. In [8], a hybrid system of an aerial vehicle was presented that has a tilt-wing mechanism. The vehicle is capable of vertical takeoff/landing like a helicopter as well as flying horizontally like an airplane. This is done by mounting four rotors on four rotating wings. [9] presented a mini tilt-rotor UAV with two rotors. Modeling of the system was discussed and the dynamics of the 6 DOF's were split into three subdynamics to simplify the control task. The system was equipped with extra mass to introduce a pendular damped effect. In [10], a proposed system with two rotors was presented. The two rotors are allowed to tilt laterally and longitudinally to control the thrust direction. A prototype was implemented and tested and showed promising results in terms of hovering and pitch stability.

In order to maintain a zero net yaw moment, [11] proposed slightly slanted two opposite propellers with a small angle. It was shown that the four main movements : roll, pitch, yaw and heave can be completely separated using this design. In [12], a novel quadrotor design was presented. The four rotors were allowed to rotate about their axes w.r.t the main rotor body. This adds four extra inputs to have a total of eight inputs to the quadrotor. The design provides full actuation to the quadrotor position/orientation with two extra inputs. Fault tolerance has been one of the main concerns in the area of flying vehicles. [13] discusses fault tolerance in system design. Many researchers targeted fault tolerance in quadrotors from the control point of view. In [14], the case of a quadrotor with one faulty rotor is investigated. A double control loop architecture was proposed to assure trajectory tracking on translational motions as well as roll and pitch rotations. The method was claimed to achieve the desired control with acceptable behavior. However, yaw movement is compromised and the quadrotor keeps rotating around its z-axis.

A sliding mode approach to control quadrotor UAV in case of external disturbance and actuator fault was used in [15]. The method was proven to distinguish between disturbances and faults and the simulation verified the effectiveness of the method. However, the same challenge of yaw angle occurs. Yaw motion tends to go out of control. Considerable research targeting fault tolerance suggest the use of actuator redundancy. In [16], an integral sliding control was used to handle total actuator failures directly without changing the baseline controller. The controller takes advantage of the present redundancy of actuators without the need for fault detection and isolation. Similarly, [17] presented a fault tolerant controller that uses Linear Parameter Varying (LPV) sliding mode technique to exploit redundancy without the need for fault detection and isolation.

In this paper, a novel quadrotor design is introduced that has advantages for both manned and unmanned applications. Each rotor is allowed to tilt around two axes w.r.t fixed body frame. The total number of inputs is increased to twelve. With this design, each of the twelve states (outputs) (6 positions/orientations - 6 transitional/rotational speeds) can be controlled independently and freely. In addition, the system can perform all the desired control objectives with half of its actuators faulty.

On the other hand, this addition of actuator increases the system complexity and have undesired aerodynamic effects. However, added complexity can be justified by the advantages of this design and the controller is expected to compensate for those effects. More advantages and distinguished capabilities for critical missions are discussed later in this paper. A preliminary version of this paper has been presented and published in [18]. This paper is an extended version with more details.

This paper is organized as follows: Section 2 presents the dynamic model of quadrotor with two DOF tilting propellers. Section 3 analyzes the model and studies the advantages over conventional designs. After that, Section 4 presents the controller design and simulation tests. Finally, the paper is concluded in Section 5.

2. SYSTEM MODEL

2.1. Frames and rotation matrices

The quadrotor can be considered as five rigid bodies connected together and are in relative motion around themselves. Those five bodies are the quadrotor body itself $\boldsymbol{B}$, and four propellers $\boldsymbol{P}_i$ attached to the body.

Let $\mathcal{F}_E : \{O_E; \quad X_E, \quad Y_E, \quad Z_E\}$ be a world inertial

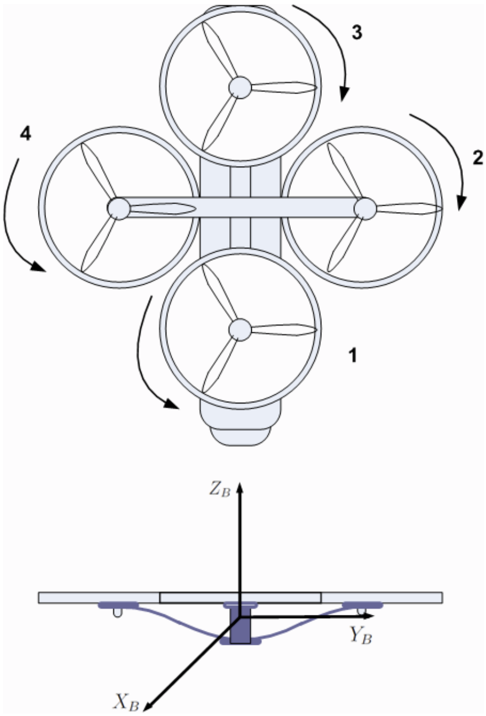

Fig. 1. Rotors positions.

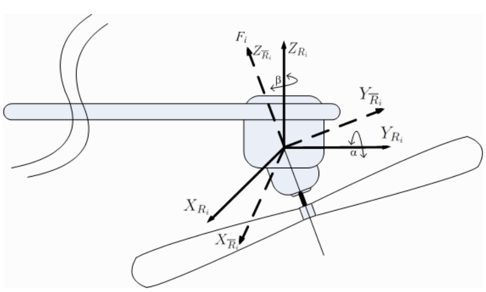

Fig. 2. Tilt angles of the rotor w.r.t fixed body frames.

frame and $\mathcal{F}_B : \{O_B; \quad X_B, \quad Y_B, \quad Z_B\}$ be the quadrotor body frame attached to its center of gravity Fig.1. In addition, the rotors-fixed frames are taken to be parallel to each other and parallel to the quadrotor body frame and are given by $\mathcal{F}_{P_i} : \{O_{P_i}; \quad X_{P_i}, \quad Y_{P_i}, \quad Z_{P_i}\}, i = 1,...,4$.

The orientation of each of the rotors is controlled by two rotations with respect to the rotor-fixed frame; α_i, a rotation about Y_{P_i}, and β_i , about Z_{P_i}. As shown in Fig.2, this rotation creates a second rotating frame for the rotors, $\mathcal{F}_{\overline{P}_i} : \{O_{\overline{P}_i}; \quad X_{\overline{P}_i}, \quad Y_{\overline{P}_i}, \quad Z_{\overline{P}_i}\}, i = 1,...,4$.

When the rotors are aligned along Z_{P_i}, rotor 1 and rotor 2 are assumed to rotate counter-clock-wise CCW, while rotor 3 and rotor 4 rotate clock-wise CW. The forward direction is taken arbitrary to be along X_B

Let $R^{P_i}_{\overline{P}_i}$ be the rotational matrix from the rotors-rotating frame $O_{\overline{P}_i}$ to the rotors-fixed frame O_{P_i}. Since the rotors-fixed frames O_{P_i} are parallel to the body-fixed frame O_B at the center of gravity, then

$$R^{P_i}_{\overline{P}_i} = R^{B}_{\overline{P}_i} = \begin{bmatrix} 0 & 0 & C\beta_i S\alpha_i \\ 0 & 0 & S\beta_i S\alpha_i \\ 0 & 0 & C\alpha_i \end{bmatrix}, \tag{1}$$

where $C(.)$ and $S(.)$ denote $cos(.)$ and $sin(.)$ respectively.

A full rotation of the quadrotor body with respect to the inertial frame can be described by three consecutive rotations about the three body axes, roll rotation Φ about the body x-axis, pitch rotation Θ about the body y-axis and yaw rotation Ψ about the body z-axis. Then R^E_B is the body transformation matrix with respect to the earth inertial frame, and is given by

$$\begin{aligned} R^E_B &= R_\Psi \bullet R_\Theta \bullet R_\Phi \\ &= \begin{bmatrix} C\Psi & -S\Psi & 0 \\ S\Psi & C\Psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\Theta & 0 & S\Theta \\ 0 & 1 & 0 \\ -S\Theta & 0 & C\Theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\Phi & -S\phi \\ 0 & S\Phi & C\phi \end{bmatrix} \\ &= \begin{bmatrix} C\Psi C\Theta & -S\Psi C\Phi + C\Psi S\Theta S\Phi & S\Psi s\Phi + C\Psi S\Theta C\Phi \\ S\Psi C\Theta & C\Psi C\Phi + s\Psi S\Theta S\Phi & -C\Psi S\Phi + S\Psi S\Theta C\Phi \\ -S\Theta & C\Theta S\Phi & C\Theta C\Phi \end{bmatrix}. \end{aligned} \tag{2}$$

The relationship between the body-fixed angular velocity vector $[p \quad q \quad r]^T$ and Euler-Angles rates $[\dot{\Phi} \quad \dot{\Theta} \quad \dot{\Psi}]^T$ is given by

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & -S\Theta \\ 0 & C\Phi & S\Phi C\Theta \\ 0 & -S\Phi & C\Phi C\Theta \end{bmatrix} \begin{bmatrix} \dot{\Phi} \\ \dot{\Theta} \\ \dot{\Psi} \end{bmatrix}. \tag{3}$$

2.2. Quadrotor dynamics

Assume that the rotational speed of the rotor i is given by ω_i. Then we can say that the lifting thrust is given $b\omega_i^2$ and the drag moment is given by $d\omega_i^2$, where b and d are the thrust and drag moment constants respectively.

The thrust components of the i^{th} rotor at the body C.G. are then given by

$$F_i = \begin{bmatrix} 0 & 0 & C\beta_i S\alpha_i \\ 0 & 0 & S\beta_i S\alpha_i \\ 0 & 0 & C\alpha_i \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ b\omega_i^2 \end{bmatrix}. \tag{4}$$

Similarly, the moments of a titled rotor consist of two parts, the drag moment, and the moments generated by the thrust components. These two components can be expressed as

$$M_i = \begin{bmatrix} 0 & 0 & C\beta_i S\alpha_i \\ 0 & 0 & S\beta_i S\alpha_i \\ 0 & 0 & C\alpha_i \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ d\omega_i^2 \delta(i) \end{bmatrix} + r_i \times F_i, \tag{5}$$

where $\delta = [1, 1, -1, -1]$ and r_i is the vector from center of gravity to the reference point of the rotors, i.e.,

$$r_1 = [l, 0, -h]^T, \qquad r_2 = [0, l, -h]^T,$$
$$r_3 = [-l, 0, -h]^T, \qquad r_4 = [0, -l, -h]^T,$$

h and l represent the vertical and horizontal displacements from the center of gravity to the rotors respectively.

The quadrotor position vector η and body angular velocities vector Ω are given by

$$\eta = [x \quad y \quad z]^T,$$
$$\Omega = [p \quad q \quad r]^T.$$

The summation of forces acting on the quadrotor body is then given by the dynamic equation [19]:

$$m\ddot{\eta} = mg_z - K\dot{\eta} + R_B^E \sum_{i=1}^{4} F_i, \quad (6)$$

where m is the mass of the quadrotor, and

$$g_z = [0 \quad 0 \quad -g]^T.$$

K is the matrix of drag constants, and is given by

$$K = \begin{bmatrix} K_1 & 0 & 0 \\ 0 & K_2 & 0 \\ 0 & 0 & K_3 \end{bmatrix}.$$

The aerodynamic drag can be modeled as $K\dot{\eta}^2$. However, it's often ignored in the literature for control purpose. In this development, a first order approximation of air friction and drag $K\dot{\eta}$ is used to simplify the problem.

The rotation dynamic equation is then given by:

$$I\dot{\Omega} = -(\Omega \times I\Omega) - M_G + \sum_{i=1}^{4} M_i, \quad (7)$$

where I is the inertia matrix of the quadrotor, and is given by

$$I = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix}.$$

M_G is the gyroscopic forces, and is given by

$$M_G = \sum_{i=1}^{4} I_R(\Omega \times \overline{\omega}_i)\delta(i). \quad (8)$$

I_R is the rotor moment of inertia. And

$$\overline{\omega}_i = \begin{bmatrix} 0 & 0 & C\beta_i S\alpha_i \\ 0 & 0 & S\beta_i S\alpha_i \\ 0 & 0 & C\alpha_i \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \omega_i \end{bmatrix}. \quad (9)$$

The equations of motion can be put in the standard form

$$\dot{X} = f(X,U),$$

where

$$X = [\dot{\eta} \quad \Omega]^T,$$
$$U = [\omega_1 \alpha_1 \beta_1 \omega_2 \alpha_2 \beta_2 \omega_3 \alpha_3 \beta_3 \omega_4 \alpha_4 \beta_4]^T,$$
$$\begin{bmatrix} \ddot{\eta} \\ \dot{\Omega} \end{bmatrix} = \begin{bmatrix} g_z - \frac{K}{m}\dot{\eta} + \frac{R_B^E}{m}\sum_{i=1}^{4} F_i \\ -(I^{-1}\Omega \times \Omega) - I^{-1}M_G + I^{-1}\sum_{i=1}^{4} M_i \end{bmatrix}, \quad (10)$$

where F_i and M_i are related to the elements of the input vector U through equations 4 and 5 respectively.

3. ANALYSIS AND ADVANTAGES

This proposed design can offer many advantages over all the existing designs in the literature. Some of these advantages are tested in this paper while other are left for future work. To start with, all the positions and orientations can be controlled completely independantly This means that the quadrotor can move on a certain trajectory while maintaining specified speeds and orientations which gives this design superior manoeuvrability. The free inputs can further be used to achieve additional tasks such as overcoming gust disturbances or even as brakes. On the other hand, while the additional inputs may be of great use during critical missions, they can be turned completely off when not needed to save power and reduce control complexity. In fact, the quadrotor is still fully actuated and the motions are completely decoupled using only any two opposite rotors. Failure of any of the rotors would not compromise the safety of the flight or behavior. Furthermore, if the rotors are allowed to rotate freely in a hemisphere, i.e., α is allowed to reach proper angles; and motors are strong enough, the quadrotor could land safely with only one rotor functioning.

A necessary and sufficient condition for the quadrotor's motions to be decoupled and completely independent with only two opposite rotors running, is that equations 4 and 5 of forces and moments are independent with only two opposite rotors running. And since the relation between the actual inputs and forces and moments is nonlinear, and it's not convenient to check independence in nonlinear equations, a change of variables is introduced. Let

$$A = C\beta_1 S\alpha_1 \omega_1^2,$$
$$B = C\beta_3 S\alpha_3 \omega_3^2,$$
$$C = S\beta_1 S\alpha_1 \omega_1^2,$$
$$D = S\beta_3 S\alpha_3 \omega_3^2,$$
$$E = C\alpha_1 \omega_,^2$$
$$F = C\alpha_3 \omega_3^2.$$

Those variables (A through F) can be manipulated freely and independently. That's to say, one can find proper values of β, α and ω that produce any arbitrary values of the variables A through F. To prove that, take the equations of A, C and E where they share the same input variables α_1, ω_1 and β. For any arbitrary values of A and C, the ratio $\frac{A}{C} = \frac{C\beta_1}{S\beta_1}$ determines the value of the variable β_1. The other two variables are determined by the values of either A or C and the value of E (the ratio $\frac{A}{C}$ is already determined hence fixing one variable will automatically fix the other). Now, rewriting the equation of forces and moments 4 and 5 with the new variables and with only rotors 1 and 3 running:

$$\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = b \begin{bmatrix} A+B \\ C+D \\ E+F \end{bmatrix}, \tag{11}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} d(A-B) \quad +bh(C+D) \\ d(C-D) \quad -bh(A+B) \quad +bl(F-E) \\ d(E-F) \quad +bl(C-D) \end{bmatrix}. \tag{12}$$

Combining the two equations

$$\begin{bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} b & b & 0 & 0 & 0 & 0 \\ 0 & 0 & b & b & 0 & 0 \\ 0 & 0 & 0 & 0 & b & b \\ d & -d & bh & bh & 0 & 0 \\ -bh & -bh & d & -d-bh & -bl & bl \\ 0 & 0 & bl & -bl & d & -d \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \\ E \\ F \end{bmatrix}. \tag{13}$$

This matrix is full rank. Which means that the forces and moments are independent and there's always a combination of variables A through F that produce any arbitrary values of forces and moments with only rotors 1 and 3 running. The same proof can be carried out for rotors 2 and 4 as well.

Considering the aforementioned advantages, this design can serve for many critical applications. The fact that the motions are completely decoupled and that the quadrotor doesn't need to pitch to go forward nor to roll for lateral motions; this fact makes the quadrotor very suitable for sensitive payload as it provides a very smooth ride. Surveillance and monitoring could be improved as the quadrotor can fly at precise attitudes with precise speeds and orientations. This can be very suitable for military applications.

4. SIMULATION TESTS

In the following discussion, several maneuvers are simulated to test the decoupling of motions and fault tolerance. The rotational friction is assumed to be zero during all simulations. It's important to point out that the objective here is not to design a high performance controller but instead, a simple controller that will be able to highlight the advantages mentioned in this chapter. To achieve this objective with lowest complexity, trivial pairings are done between inputs and outputs. The sum of the four propellers' speeds ω_i's is paired with the quadrotor elevation z. This sum is given the notion ω_z.

In addition, the angles α_2 and α_4 of rotors 2 and 4 respectively are paired with the velocity along x-axis. The two angles are set to be equal and are given the notion α_x. While the angles α_1 and α_3 of rotors 1 and 3 respectively are paired with the velocity along y-axis. Similarly, these two angles are set to be equal and are given the notion α_y.

The rest of the inputs and states will be ignored at this stage. The control scheme used is a simple PID controller with partial state feedback.

Let e_z, $e_{\dot{x}}$ and $e_{\dot{y}}$ be the error in elevation, error in velocity along the x-axis and error in velocity along the y-axis respectively. And let

$$U_C = [\omega_z \quad \alpha_x \quad \alpha_y]^T,$$
$$e = [e_z \quad e_{\dot{x}} \quad e_{\dot{y}}]^T,$$
$$K_P = \begin{bmatrix} K_P1 & 0 & 0 \\ 0 & K_P2 & 0 \\ 0 & 0 & K_P3 \end{bmatrix},$$
$$K_I = \begin{bmatrix} K_I1 & 0 & 0 \\ 0 & K_I2 & 0 \\ 0 & 0 & K_I3 \end{bmatrix},$$
$$K_D = \begin{bmatrix} K_D1 & 0 & 0 \\ 0 & K_D2 & 0 \\ 0 & 0 & K_D3 \end{bmatrix}.$$

Then, the PID controller is defined as follows:

$$U_C = K_P e + K_I \int e\,\mathrm{d}t + K_D \frac{de}{dt}. \tag{14}$$

A separate PID controller is used to control the roll angle to avoid coupling. The roll angle is paired with the difference $U_\Phi = \omega_4 - \omega_2$ as the torque is proportional to this difference. Then, roll controller is given by

$$U_\Phi = K_{P\Phi} e_\Phi + K_{I\Phi} \int e_\Phi\,\mathrm{d}t + K_{D\Phi} \frac{de_\Phi}{dt},$$

where $K_{P\Phi}$, $K_{I\Phi}$ and $K_{D\Phi}$ are the PID gains. The system's parameters used in the simulations are listed in Table 1.

Table 1. Values of medel and controller parameters *These values were taken from [19].

Parameter	Value	Unit
g	9.8	m/s^2
b^*	2.92×10^{-6}	$kg.m$
d^*	1.12×10^{-7}	$kg.m^2$
m^*	0.5	kg
l^*	0.2	m
h	0	m
ω_{max}	$(10000)\frac{2\pi}{60}$	rpm
I_x^*	4.85×10^{-3}	$kg.m^2$
I_y^*	4.85×10^{-3}	$kg.m^2$
I_z^*	4.81×10^{-3}	$kg.m^2$

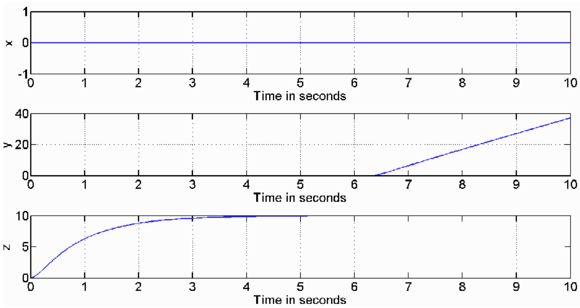

Fig. 3. Flight 1: x, y and z positions in (m).

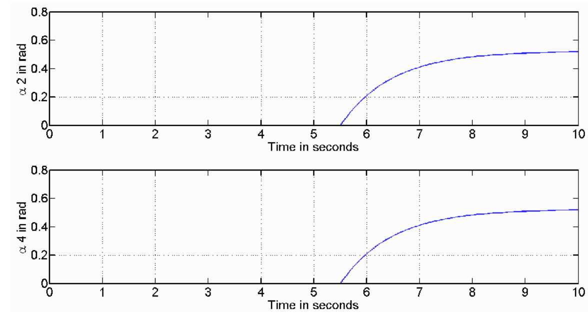

Fig. 6. Flight 1: Rotors 2 and 4 α rotation angle; $\beta = \Pi/2$.

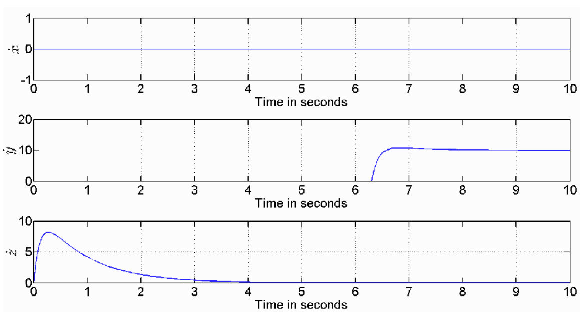

Fig. 4. Flight 1: velocities $\dot{x}$, $\dot{y}$ and $\dot{z}$ in (m/s).

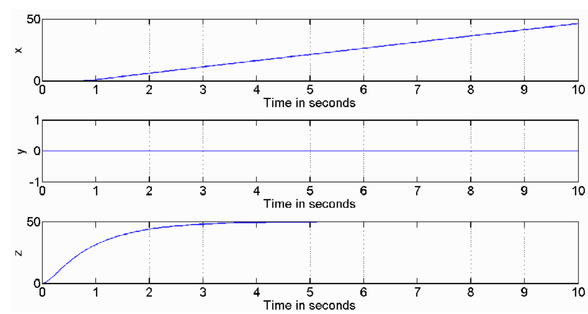

Fig. 7. Flight 2: x, y and z positions in (m).

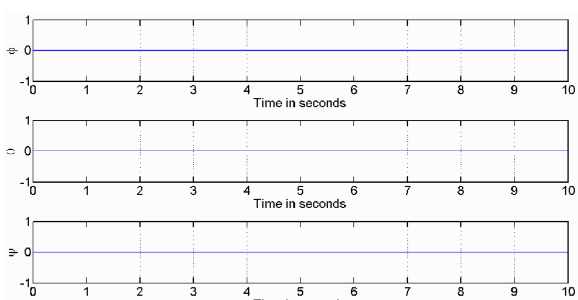

Fig. 5. Flight 1: The three orientation angles Φ, Θ and Ψ in (rad).

4.1. Decoupling

This subsection presents three flight simulations to demonstrate decoupling of motions. The objective of these three tests is to observe how the quadrotor can follow arbitrary trajectories with specific motions without compromising other motions and/or orientations.

In the first flight, the quadrotor elevates up to 10 m and then moves laterally with a velocity of 10 m/s, which is shown in Figs. 3-6. Fig. 3 shows the x, y and z positions where it's evident that the quad achieved the vertical position of 10 m. In Fig. 4, it can be seen that the lateral velocity $\dot{y}$ is achieved while all the orientation angles shown in Fig. 5 are kept fixed at zero. Fig. 6 shows the input angle α in rotors 2 and 4 that are manipulated to achieve this mission.

In the second flight, the quadrotor elevates to 50 meters, and while it's still elevating it's commanded to forward with a speed of 5 m/s . Similar to the figures in the first flight test, Figs. 7 - 9 show that the desired positions, velocities and orientation angles are achieved independently.

In the third flight test, the quadrotor is commanded to hover, then thrust move laterally with a constant speed of 10 m/s, which is the same as flight 1, except that this time the quad has to maintain a constant roll angle of $\pi/6$. The results of this flight are shown in Figs. 10 - 12. This is a very strong test as the results in the figures show that not only the quadrotor can follow certain translational trajectories while keeping orientation angles undisturbed, but also it can achieve desired orientation angles completely independently of its translational path.

Three important observations can be made from the simulation results. Firstly, It's evident that horizontal motion didn't compromise the vertical position of the quadrotor. The second observation is that in both flight 1 and 2, the three orientation angles, Φ, Θ and Ψ remained unchanged. This is a very important result as it shows the smoothness of the flight and the absence of perturbation. Finally, flight 3 shows that control objectives were carried out simultaneously and independently while maintaining other motions undisturbed.

4.2. Fault tolerance

During the fourth flight, two rotors (rotors 1 and 3) are assumed to be faulty (or turned off). The control scheme

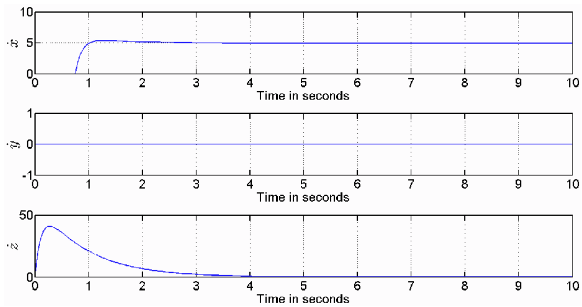

Fig. 8. Flight 2: velocities $\dot{x}$, $\dot{y}$ and $\dot{z}$ in (m/s).

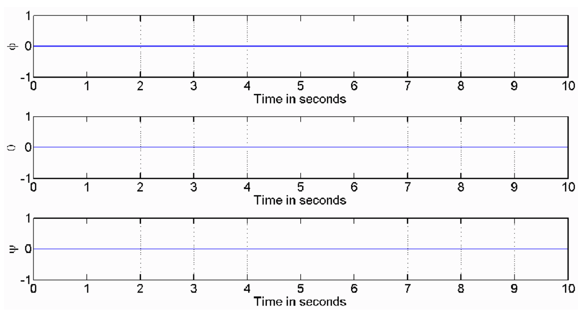

Fig. 9. Flight 2: The three orientation angles Φ, Θ and Ψ in (rad).

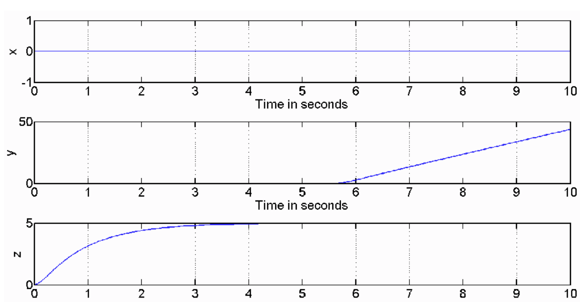

Fig. 10. Flight 3: x, y and z positions in (m).

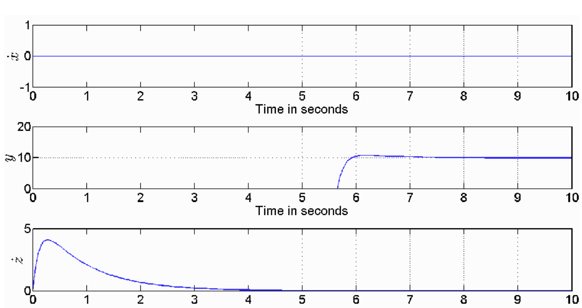

Fig. 11. Flight 3: velocities $\dot{x}$, $\dot{y}$ and $\dot{z}$ in (m/s).

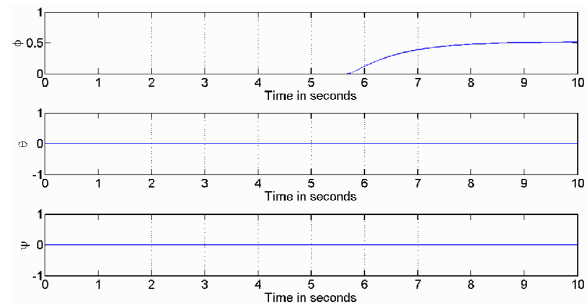

Fig. 12. Flight 3: The three orientation angles Φ, Θ and Ψ in (rad).

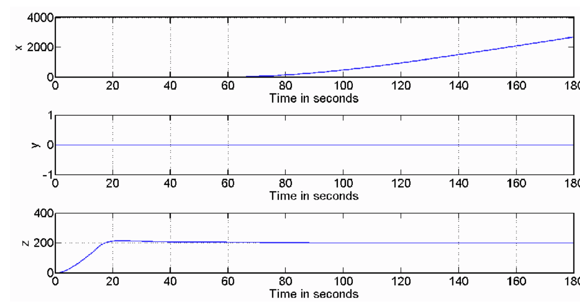

Fig. 13. Flight 4: x, y and z positions in (m).

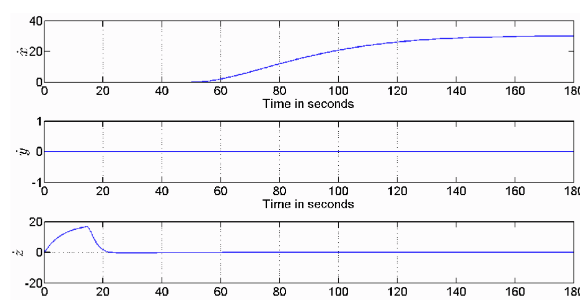

Fig. 14. Flight 4: velocities $\dot{x}$, $\dot{y}$ and $\dot{z}$ in (m/s).

has to be modified because of this actuator loss. The modification here is simple, instead of manipulating α_y to control the velocity along the y-axis, α_x is used as input with β_2 and β_4 are set to $\pi/2$.

With only two rotors remaining (rotor 2 and 4), The quadrotor is commanded to perform a similar flight to the first from the previous section. It's set to elevates up to $200m$ and then thrust forward with a fixed velocity of $30m/s$. Figs. 13 - 17 show the response. The figures show the positions, velocities and orientation angles respectively. Again, it's evident that the control objectives are still achieved independently of each other with only two rotors running. Fig. 16 shows α angles in rotors 2 and 4 while Fig. 17 shows the speeds of the four rotors where it's clear that only rotor 2 and 4 are running. Note that the figure is zoomed in for rotor 2 to show the change in the propeller speed during the maneuver.

The simulation shows clearly that only two rotors are enough to control and decouple the outputs of the system.

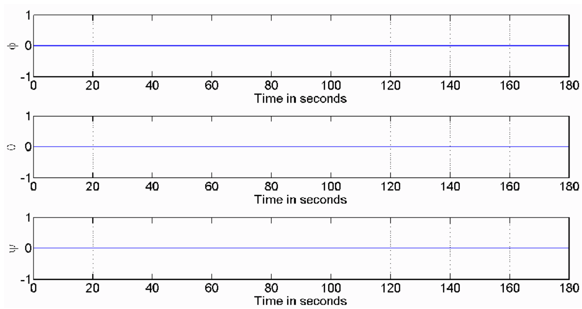

Fig. 15. Flight 4: The three orientation angles Φ, Θ and Ψ in (rad).

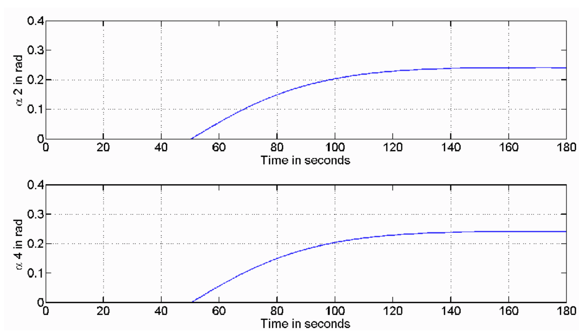

Fig. 16. Flight 4: Rotors 2 and 4 α rotation angle; $\beta = 0$.

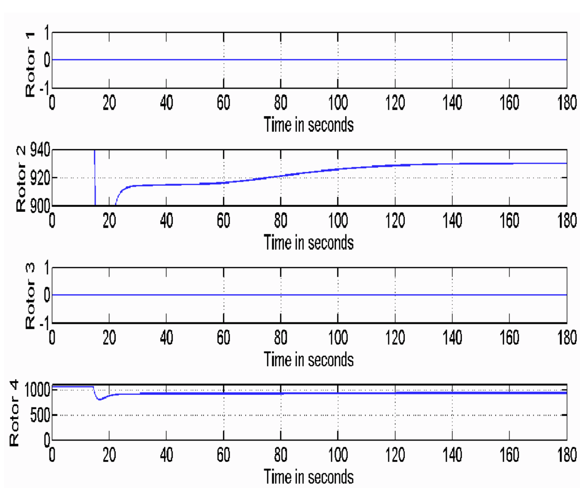

Fig. 17. Flight 4: the four rotors speeds in (rpm).

In fact, with only two rotors running, six inputs are available which means that the system is fully actuated and suitable for conventional missions.

5. CONCLUSION

This paper presented a proposed design for quadrotor. The mathematical model is presented based on Newton-Euler formulation. The proposed quadrotor has all of its rotors allowed to tilt with two degrees of freedom w.r.t the fixed body frame. With this addition of 8 inputs, the system is fully actuated and capable of tracking more outputs separately and independently (6 positions/orientations and 6 translational/rotational speeds). This improves maneuverability and agility of quadrotor and enhances fault tolerance capabilities. In fact, the system was proven to be completely operational with only any two opposite rotors running. Several flight simulations were carried out. The simulations demonstrated some of the advantages of this design over the conventional quadrotor such as complete decoupling of motions, energy saving, ability to track arbitrary trajectories for all outputs independently, robustness and tolerance against various failures.

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