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Stability Analysis of Switched Delay Systems with All Subsystems Unstable

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Abstract: The stability problem of switched delay systems with all subsystems unstable is investigated in this paper. A sufficient criterion is firstly proposed to guarantee asymptomatic stability of nonlinear switched delay systems with all subsystems unstable, where the stabilization property of switching behaviors is exploited to compensate the divergence of states. Then by constructing multiple discretized Lyapunov-Krasovskii functionals, stability criteria are developed for linear switched time-invariant and time-varying delay systems with all subsystems unstable. Finally, two examples are provided to illustrate the feasibility, superiority and application of the proposed approach.

Keywords: Asymptomatic stability, multiple discretized Lyapunov-Krasovskii functionals, switched delay systems, unstable subsystems.

1. INTRODUCTION

Switched systems include a series of subsystems described by differential or difference equations and a switching signal governing the switching among these subsystems. And many mechanical systems can be modelled as switched systems, for example, flight control systems [1], electric vehicles [2], chemical processes [3], network control systems [4, 5], etc. Hence, the stability analysis and the controller design for switched systems have become active topics in control theory and application [6–11]. In addition, time delay phenomena actually appear in various practical systems, such as chemical processes, manufacturing systems, network control systems. Since time delays may deteriorate the system performance or even lead to instability, it is significant to analyze the performance of delay systems due to the strong engineering background. Lots of results on time delay systems have been reported in the past decade [12–15].

On the other hand, it is an important issue to design an appropriate switching signal to guarantee system stability. Switching signals are usually divided into two cases: time-dependent switching signals [16–21] and state-dependent switching signals [22, 23]. For the former, the robust reliable control issue is discussed in [17] for uncertain switched nonlinear systems with time delay under asynchronous switching. In [21], the stability and L_2 gain analysis of switched delay systems based on average dwell time method is studied. A dynamic output feedback controller is developed for a class of switched delay systems under asynchronous switching in [20]. For the latter, in [22], the authors have designed the state-dependent switching signal to ensure the stability of linear switched systems with time delay. In [23], the stability problem is discussed for a class of linear switched systems with time-varying delay in the sense of Hurwitz convex combination.

However, it should be pointed out that, for the former, there exists at least one stable subsystem to guarantee the stability of the whole switched delay systems. The basic idea is that stable subsystems are activated for sufficiently long time to compensate the state divergence caused by unstable subsystems. The existing results based on time-dependent switching signals fail to work on switched delay systems with all subsystems unstable. For the latter, state-dependent switching signals can stabilize switched delay systems with all unstable subsystems, but the system states must be fully measurable. So, it is particularly urgent to design time-dependent switching signals for switched delay systems with all subsystems unstable, which motivates the present study.

In this paper, we consider the stability issue of switched delay systems with all subsystems unstable. Firstly, for nonlinear switched delay systems with all unstable subsystems, a sufficient criterion is proposed to ensure asymptotic stability of the given systems. It is worth mentioning that this result can work well when the system states are not measurable because the designed switching signal is time-dependent. Secondly, multiple discretized

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Lyapunov-Krasovskii functionals are constructed to derive stability criteria for linear switched time-invariant and time-varying delay systems with all the subsystems unstable. Finally, two examples are used to illustrate the effectiveness, superiority and application of the proposed method.

2. PROBLEM FORMULATION AND PRELIMINARIES

Let $\mathbb{C}$ denote the space of continuous functions mapping $[-d,0]$, $d>0$, into $\mathbb{R}^n$, that is, $\mathbb{C}:[-d,0]\to\mathbb{R}^n$. For a function $\phi\in\mathbb{C}$, define the norm $\|\cdot\|_{\mathbb{C}}$ by $\|\phi\|_{\mathbb{C}}=\sup_{-d\le\theta\le0}\|\phi(\theta)\|$. The symbol $\mathcal{D}_{\tau_k}$ stands for a set of all switching signals with dwell time $\tau_k=t_{k+1}-t_k, k=0,1,2,\cdots$.

In this paper, consider the following nonlinear switched delay system described by

$$\Sigma_{(1)}:\dot{x}(t)=f_{\sigma(t)}(t,x_t),\ \ x_{t_0}=\varphi(\theta),\quad \theta\in[-d,0],\ (1)$$

where $x(t)\in\mathbb{R}^n$ is the state vector, $f_{\sigma(t)}:\mathbb{R}_{\ge0}\times\mathbb{C}\to\mathbb{R}^n$. $\sigma(t):\mathbb{R}_{\ge0}\to\mathcal{M}=\{1,2,\cdots,m\}$, $m\in\mathbb{Z}^+$, is the switching signal which is assumed to be a piecewise constant function and continuous from the right. m is the number of subsystems. $x_t=x_t(\theta)=x(t+\theta)$, $\theta\in[-d,0]$, with d being a positive constant. $x_t\in\mathbb{C}$, and $\|x_t\|_{\mathbb{C}}=\sup_{-d\le\theta\le0}\|x_t(\theta)\|$. $\varphi(\theta)$ is a continuously differential initial function on $[-d,0]$. $\sigma(t)$ can be given by $\Sigma=\{x_{t_0};(\sigma(t_0),t_0),(\sigma(t_1),t_1),\cdots,(\sigma(t_k),t_k),\cdots,\sigma(t_k)\in\mathcal{M}, k=0,1,2,\cdots\}$, where t_0 is the initial time, x_{t_0} is the initial state function, and t_k, $k=0,1,2,\cdots$, are switching instants. $(\sigma(t_k),t_k)$ means that $\sigma(t_k)$-th subsystem is activated on the interval $[t_k,t_{k+1})$.

In this paper, assume that

A1. All the subsystems are unstable.

A2. $f_i(t,0)\equiv0$, for all t, and $i\in\mathcal{M}$.

A3. For each x_{t_0}, there exists a unique trajectory $x(t;x_{t_0})$ for (1).

A4. No state jump occurs at switching instants.

Definition 1: The switched delay system $\Sigma_{(1)}$ is said to be uniformly stable (US) with respect to $\sigma(t)$ if for any $t_0\in\mathbb{R}_{\ge0}$ and any $\varepsilon>0$, there exists a $\delta=\delta(\varepsilon)>0$ such that $\|x_{t_0}\|_{\mathbb{C}}<\delta$ implies $\|x(t)\|<\varepsilon$ for $t\ge t_0$. Furthermore, if system $\Sigma_{(1)}$ is US and satisfies $\lim\limits_{t\to\infty}x(t)=0$, then system $\Sigma_{(1)}$ is uniformly asymptotically stable (UAS) with respect to $\sigma(t)$.

3. MAIN RESULTS

3.1. Stability analysis of the nonlinear switched delay system

In [24], sufficient criterion has been proposed to guarantee the globally uniformly asymptotical stability of switched systems with all unstable subsystems. Here, we will generalize the result in [24] to the switched delay system (1).

Theorem 1: Consider the nonlinear switched delay system (1), with $d>0$. If there exist continuous differentiable functionals $V_i(t,x_t):\mathbb{R}_{\ge0}\times\mathbb{C}\to\mathbb{R}_{\ge0}$, $i=1,2,\cdots,m$, functions $\alpha_1,\alpha_2\in\mathcal{K}_\infty$, and constants $\eta>0$, $0<\mu<1$, such that for any $i\in\mathcal{M}$, the following inequalities hold

$$\alpha_1(\|x\|)\le V_i(t,x_t)\le\alpha_2(\|x_t\|_{\mathbb{C}}),\tag{2}$$

$$\dot{V}_i(t,x_t)\le\eta V_i(t,x_t),\tag{3}$$

$$V_j(t_k^+,x_{t_k^+})\le\mu V_i(t_k^-,x_{t_k^-}),\quad i\ne j,\tag{4}$$

$$\ln\mu+\eta\tau_k<0,\tag{5}$$

then the nonlinear switched delay system (1) is UAS with respect to the switching signal $\sigma(t)\in\mathcal{D}_{\tau_k}$.

Proof: For system (1), define a multiple Lyapunov functional $V_{\sigma(t)}(t,x_t),\sigma(t)\in\mathcal{M}$. When $t\in[t_k,t_{k+1})$, from (3), it is easy to see that

$$V_{\sigma(t)}(t,x_t)=V_{\sigma(t_k)}(t,x_t)\le e^{\eta(t-t_k)}V_{\sigma(t_k)}(t_k,x_{t_k}).\tag{6}$$

From (4) and (6), we derive

$$\begin{aligned}V_{\sigma(t_{k+1})}(t_{k+1},x_{t_{k+1}})&\le\mu V_{\sigma(t_{k+1}^-)}(t_{k+1}^-,x_{t_{k+1}^-})\\&=\mu V_{\sigma(t_k)}(t_{k+1}^-,x_{t_{k+1}^-})\\&\le\mu e^{\eta(t_{k+1}-t_k)}V_{\sigma(t_k)}(t_k,x_{t_k})\\&=\rho V_{\sigma(t_k)}(t_k,x_{t_k}),\end{aligned}\tag{7}$$

where $\rho=\mu e^{\eta(t_{k+1}-t_k)}$. From (5) and $0<\mu<1$, one gets $0<\rho<1$. Thus, we obtain $V_{\sigma(t_{k+1})}(t_{k+1},x_{t_{k+1}})<V_{\sigma(t_k)}(t_k,x_{t_k})$, which implies that $V_{\sigma(t_k)}(t_k,x_{t_k})$ is strictly decreasing as $k\to\infty$. $\tau_{\max}=\sup_{k=0,1,2,\dots}\tau_k$ exists by (5). For any $\varepsilon>0$, there exists a $\delta=\delta(\varepsilon)=\alpha_2^{-1}(e^{-\eta\tau_{\max}}\alpha_1(\varepsilon))$ such that $\|x_{t_0}\|_{\mathbb{C}}<\delta$ implies $V_{\sigma(t_0)}(t_0,x_{t_0})<\alpha_2(\|x_{t_0}\|_{\mathbb{C}})<e^{-\eta\tau_{\max}}\alpha_1(\varepsilon)$. Since $V_{\sigma(t_k)}(t_k,x_{t_k})$ is strictly decreasing as $k\to\infty$, we have $V_{\sigma(t_k)}(t_k,x_{t_k})<e^{-\eta\tau_{\max}}\alpha_1(\varepsilon)$. This, together with (6), yields $V_{\sigma(t)}(t,x_t)\le e^{\eta(t-t_k)}V_{\sigma(t_k)}(t_k,x_{t_k})<e^{\eta\tau_{\max}}V_{\sigma(t_k)}(t_k,x_{t_k})<\alpha_1(\varepsilon)$. From (2), we have $\|x(t)\|<\varepsilon$. Therefore, we can conclude that system (1) is US.

Since $V_{\sigma(t_k)}(t_k,x_{t_k})$ is strictly decreasing as $k\to\infty$, and bounded from below by zero, one has $V_{\sigma(t_k)}(t_k,x_{t_k})\to l\ge0$, as $k\to\infty$. Furthermore, we obtain

$$\begin{aligned}0=l-l&=\lim_{k\to\infty}V_{\sigma(t_{k+1})}(t_{k+1},x_{t_{k+1}})-\lim_{k\to\infty}V_{\sigma(t_k)}(t_k,x_{t_k})\\&=\lim_{k\to\infty}(V_{\sigma(t_{k+1})}(t_{k+1},x_{t_{k+1}})-V_{\sigma(t_k)}(t_k,x_{t_k}))\\&\le-(1-\rho)\lim_{k\to\infty}V_{\sigma(t_k)}(t_k,x_{t_k})=-(1-\rho)l.\end{aligned}\tag{8}$$

Equation (8) and $0<\rho<1$ imply $l=0$. Thus, $V_{\sigma(t_k)}(t_k,x_{t_k})\to0$ holds as $k\to\infty$. Choose $\tilde{\varepsilon}=e^{-\eta\tau_{\max}}\alpha_1(\varepsilon)$. There exists $K>0,(K\in\mathbb{Z}^+)$ such that $V_{\sigma(t_k)}(t_k,x_{t_k})<\tilde{\varepsilon},k\ge K$. When $t\in[t_k,t_{k+1}),k=K,K+1,\cdots$, we have $V_{\sigma(t)}(t,x_t)\le e^{\eta\tau_{\max}}V_{\sigma(t_k)}(t_k,x_{t_k})<\alpha_1(\varepsilon)$. Choose $T=t_K$. When $t>T$, it is easy to derive $V_{\sigma(t)}(t,x_t)$

$< \alpha_1(\varepsilon)$. From (2), we can see that $\|x(t)\| < \varepsilon$. Furthermore, $\lim\limits_{t\to\infty} x(t) = 0$ holds. Therefore, nonlinear switched delay system (1) is UAS with respect to switching signal $\sigma(t) \in \mathcal{D}_{\tau_k}$.

Remark 1: The condition that functionals $V_i(t,x_t)$, $i = 1,2,\cdots,m$ are continuously differentiable is restrictive. In fact, Theorem 1 can work with the weaker conditions. Namely, functionals $V_i(t,x_t)$, $i = 1,2,\cdots,m$ are just continuous and satisfy (3) except for finite non-differentiable instants.

Remark 2: For nonlinear switched delay system (1), Theorem 1 provides a general conclusion to judge the stability, which can not only apply to the switched time-invariant delay systems with all subsystems unstable but also to the switched time-varying delay systems with all subsystems unstable.

Considering that the bounded quadratic Lyapunov functional is used widely, we state a restricted version of Theorem 1 in the following corollary.

Corollary 1: Consider the nonlinear switched delay system (1). If there exist bounded quadratic continuous differentiable functionals $V_i(t,x_t): \mathbb{R}_{\geq 0} \times \mathbb{C} \to \mathbb{R}_{\geq 0}$, $i = 1,2,\cdots,m$, and constants $\upsilon > 0$, $\eta > 0$, $0 < \mu < 1$, such that for any $i \in \mathcal{M}$, the following inequalities hold

$$\begin{aligned} &V_i(t,x_t) \geq \upsilon \|x\|^2, \\ &\dot{V}_i(t,x_t) \leq \eta V_i(t,x_t), \\ &V_j(t_k^+, x_{t_k^+}) \leq \mu V_i(t_k^-, x_{t_k^-}), \qquad i \neq j, \\ &\ln\mu + \eta\tau_k < 0, \end{aligned}$$

then the nonlinear switched delay system (1) is UAS with respect to the switching signal $\sigma(t) \in \mathcal{D}_{\tau_k}$.

Proof: Since $V_i(t,x_t)$, $i = 1,2,\cdots,m$ are bounded quadratic, there exists $K > 0$ such that $V_i(t,x_t) \leq K\|x_t\|_{\mathbb{C}}^2$, $i = 1,2,\cdots,m$ hold. By Theorem 1, this corollary holds.

3.2. Stability analysis of linear switched time-invariant delay system with all subsystems unstable

Based on Theorem 1, we further discuss the stability problem of the linear switched time-invariant systems with all subsystems unstable.

Consider the linear switched time-invariant delay system given by

$$\begin{aligned} \Sigma_{(2)}: \dot{x}(t) &= A_{\sigma(t)}x(t) + A_{d\sigma(t)}x(t-d), \\ x(t) &= \varphi(t), \qquad\qquad t \in [-d, 0], \end{aligned} \tag{9}$$

where the definitions of $x(t)$ and $\sigma(t)$ are the same with (1). A_i, A_{di} are known real constant matrices with appropriate dimensions. Define $\tau_{\min} = \inf_{k=0,1,2,\cdots} \tau_k$ and $\tau_{\max} = \sup_{k=0,1,2,\cdots} \tau_k$. The symbol $\mathcal{D}_{[\tau_{\min},\tau_{\max}]}$ stands for a set of all switching signals with dwell time $\tau_k \in [\tau_{\min}, \tau_{\max}], k = 0,1,2,\cdots$.

For $t \in [t_k, t_{k+1})$, assume that i-th subsystem is activated. At this time, $[t_k, t_{k+1}) = [t_k, t_k + \tau_{\min}) \cup [t_k + \tau_{\min}, t_{k+1})$. $[t_k, t_k + \tau_{\min})$ is divided into L segments described as $\mathcal{N}_{k,q} = [t_k + \theta_{k,q}, t_k + \theta_{k,q+1})$ of equal length $h = \tau_{\min}/L$. $\theta_{k,q} = qh = q\tau_{\min}/L$ and $[t_k, t_k + \tau_{\min}) = \bigcup_{q=0}^{L-1} \mathcal{N}_{k,q}$. Define a set $\mathcal{L} = \{0,1,2,\cdots,L-1\}$ and let $P_{i,q} > 0$, $q = 0,1,2,\cdots,L$. Construct matrix function $P_i(t)$ as

$$P_i(t) = \begin{cases} (1-\alpha)P_{i,q} + \alpha P_{i,q+1}, & t \in \mathcal{N}_{k,q}, \\ P_{i,L}, & t \in [t_k + \tau_{\min}, t_{k+1}), \end{cases} \tag{10}$$

where $\alpha = (t - t_k - \theta_{k,q})/h$, $0 \leq \alpha < 1$, $q \in \mathcal{L}$.

Theorem 2: Consider the system (9), with a delay d. Given positive constants $\eta > 0, 0 < \mu < 1, \tau_{\min} > 0$, if, there exist matrices $P_{i,q} > 0$, $q = 0,1,2,\cdots,L$, $i = 1,2,\cdots,m$, $S > 0$, a constant $\tau_{\max}$, such that for any $q \in \mathcal{L}$, $i, j \in \mathcal{M}$, the following inequalities hold

$$\Xi_{1i,q} = \begin{bmatrix} \Lambda_{1i,q} & P_{i,q}A_{di} \\ * & -e^{\eta d}S \end{bmatrix} < 0, \tag{11}$$

$$\bar{\Xi}_{1i,q} = \begin{bmatrix} \bar{\Lambda}_{1i,q} & P_{i,q+1}A_{di} \\ * & -e^{\eta d}S \end{bmatrix} < 0, \tag{12}$$

$$\tilde{\Xi}_{1i,L} = \begin{bmatrix} \tilde{\Lambda}_{1i,L} & P_{i,L}A_{di} \\ * & -e^{\eta d}S \end{bmatrix} < 0, \tag{13}$$

$$P_{j,0} - \mu P_{i,L} \leq 0, i \neq j, \tag{14}$$

$$\ln\mu + \eta\tau_{\max} < 0, \tag{15}$$

where

$$\begin{aligned} \Lambda_{1i,q} &= P_{i,q}A_i + A_i^T P_{i,q} - \eta P_{i,q} + \psi_{1i,q}, \\ \bar{\Lambda}_{1i,q} &= P_{i,q+1}A_i + A_i^T P_{i,q+1} - \eta P_{i,q+1} + \psi_{1i,q}, \\ \tilde{\Lambda}_{1i,L} &= P_{i,L}A_i + A_i^T P_{i,L} - \eta P_{i,L} + S, \\ \psi_{1i,q} &= S + L/\tau_{\min}(P_{i,q+1} - P_{i,q}), \end{aligned}$$

then system (9) is UAS under the switching signal $\sigma(t) \in \mathcal{D}_{[\tau_{\min},\tau_{\max}]}$.

Proof: When $t \in [t_k, t_{k+1})$, suppose $\sigma(t) = i$, and choose the corresponding discretized Lyapunov-Krasovskii functional

$$V_i(t,x_t) = x^T(t)P_i(t)x(t) + \int_{t-d}^{t} e^{\eta(t-\xi)}x^T(\xi)Sx(\xi)d\xi, \tag{16}$$

where $P_i(t)$ is defined in form of (10), $S > 0$. Obviously, $V_i(t,x_t)$ is continuous and contains L non-differential instants, that is, $t_k + \theta_{k,q}$, $q = 1,2,\cdots,L$. According to Remark 1, we just verify that inequalities (3) holds on $\mathcal{N}_{k,q}$, $q \in \mathcal{L}$ and $[t_k + \tau_{\min}, t_{k+1})$.

When $t \in \mathcal{N}_{k,q}$, $q \in \mathcal{L}$, calculating the time derivative of (16) along the solution of system (9), we get

$$\begin{aligned} \dot{V}_i(t,x_t) =& 2x^T(t)P_i(t)\dot{x}(t) + x^T(t)\dot{P}_i(t)x(t) \\ &+ \eta\int_{t-d}^{t} e^{\eta(t-\xi)}x^T(\xi)Sx(\xi)d\xi \\ &+ x^T(t)Sx(t) - e^{\eta d}x^T(t-d)Sx(t-d). \end{aligned}$$

This, together with (16), implies

$$
\begin{aligned}
&\dot{V}_i(t,x_t)-\eta V_i(t,x_t)\\
&\quad\le 2x^T(t)P_i(t)[A_ix(t)+A_{di}x(t-d)]\\
&\quad+x^T(t)\dot{P}_i(t)x(t)+x^T(t)Sx(t)\\
&\quad-e^{\eta d}x^T(t-d)Sx(t-d)-\eta x^T(t)P_i(t)x(t)\\
&\quad\le \eta_1^T(t)\tilde{\Omega}_i(t)\eta_1(t),
\end{aligned}
$$

where

$$
\begin{aligned}
&\eta_1(t)=[\begin{array}{cc} x^T(t) & x^T(t-d)\end{array}]^T,\\
&\tilde{\Omega}_i(t)=\left[\begin{array}{cc}\tilde{\Omega}_{i1}(t) & P_i(t)A_{di}\\ * & -e^{\eta d}S\end{array}\right],\\
&\tilde{\Omega}_{i1}(t)=P_i(t)A_i+A_i^TP_i(t)-\eta P_i(t)+S+\dot{P}_i(t).
\end{aligned}
$$

Substituting $\dot{P}_i(t)=\dot{\alpha}(P_{i,q+1}-P_{i,q})=L/\tau_{\min}(P_{i,q+1}-P_{i,q})$ and (10) into matrix $\tilde{\Omega}_i(t)$, one obtains that $\tilde{\Omega}_i(t)=(1-\alpha)\Xi_{1i,q}+\alpha\bar{\Xi}_{1i,q}$. From (11) and (12), we conclude that

$$
\dot{V}_i(t,x_t)\le\eta V_i(t,x_t),\quad t\in\mathcal{N}_{k,q},\quad q\in\mathcal{L}. \tag{17}
$$

When $t\in[t_k+\tau_{\min},t_{k+1})$, similar to the above process, we get $\dot{V}_i(t,x_t)-\eta V_i(t,x_t)\le\eta_1^T(t)\tilde{\Xi}_{1i,L}\eta_1(t)$. From (13), one derives

$$
\dot{V}_i(t,x_t)\le\eta V_i(t,x_t),\quad t\in[t_k+\tau_{\min},t_{k+1}). \tag{18}
$$

Therefore, inequalities (3) holds from (17) and (18).

Furthermore, according to (14) and (16), it is easy to see that inequalities (4) holds. Since $\tau_{\max}\ge\tau_k$, $k=0,1,2,\cdots$, (15) implies (5). By Theorem 1, the proof is completed.

Remark 3: (16) constructed here is quite usual, which may lead to some conservativeness. In the following, another discretized Lyapunov-Krasovskii functional including the more time delay information can be chosen to reduce the conservativeness.

Theorem 3: Consider the system (9), with a delay d. Given positive constants $\eta>0, 0<\mu<1, \tau_{\min}>0$, if, there exist matrices $P_{i,q}>0$, $q=0,1,2,\cdots,L$, $i=1,2,\cdots,m$, $S>0$, $Q>0$, $X\ge 0$, Y, a constant $\tau_{\max}$, such that for any $q\in\mathcal{L}$, $i,j\in\mathcal{M}$, the following inequalities hold

$$
\Xi_{2i,q}=\left[\begin{array}{cc}\Lambda_{2i,q} & \Theta_{2i,q}\\ * & -e^{\eta d}S+dA_{di}^TQA_{di}\end{array}\right]<0, \tag{19}
$$

$$
\bar{\Xi}_{2i,q}=\left[\begin{array}{cc}\bar{\Lambda}_{2i,q} & \bar{\Theta}_{2i,q}\\ * & -e^{\eta d}S+dA_{di}^TQA_{di}\end{array}\right]<0, \tag{20}
$$

$$
\tilde{\Xi}_{2i,L}=\left[\begin{array}{cc}\tilde{\Lambda}_{2i,L} & \tilde{\Theta}_{2i,q}\\ * & -e^{\eta d}S+dA_{di}^TQA_{di}\end{array}\right]<0, \tag{21}
$$

$$
\left[\begin{array}{cc} X & Y\\ * & Q\end{array}\right]\ge 0, \tag{22}
$$

$$
P_{j,0}-\mu P_{i,L}\le 0,\quad i\ne j, \tag{23}
$$

$$
\ln\mu+\eta\tau_{\max}<0, \tag{24}
$$

where

$$
\begin{aligned}
\Lambda_{2i,q}&=P_{i,q}A_i+A_i^TP_{i,q}-\eta P_{i,q}+\psi_{2i,q},\\
\bar{\Lambda}_{2i,q}&=P_{i,q+1}A_i+A_i^TP_{i,q+1}-\eta P_{i,q+1}+\psi_{2i,q},\\
\tilde{\Lambda}_{2i,L}&=P_{i,L}A_i+A_i^TP_{i,L}-\eta P_{i,L}\\
&\quad+S+dA_i^TQA_i+dX+Y+Y^T,\\
\psi_{2i,q}&=S+L/\tau_{\min}(P_{i,q+1}-P_{i,q})\\
&\quad+dA_i^TQA_i+dX+Y+Y^T,\\
\Theta_{2i,q}&=P_{i,q}A_{di}+dA_i^TQA_{di}-Y,\\
\bar{\Theta}_{2i,q}&=P_{i,q+1}A_{di}+dA_i^TQA_{di}-Y,\\
\tilde{\Theta}_{2i,q}&=P_{i,L}A_{di}+dA_i^TQA_{di}-Y,
\end{aligned}
$$

then system (9) is UAS under the switching signal $\sigma(t)\in\mathcal{D}_{[\tau_{\min},\tau_{\max}]}$.

Proof: When $t\in[t_k,t_{k+1})$, suppose $\sigma(t)=i$, and choose the corresponding discretized Lyapunov-Krasovskii functional as follows

$$
\begin{aligned}
V_i(t,x_t)=&x^T(t)P_i(t)x(t)+\int_{t-d}^{t}e^{\eta(t-\xi)}x^T(\xi)Sx(\xi)d\xi\\
&+\int_{-d}^{0}\int_{t+\omega}^{t}e^{\eta(t-\xi)}\dot{x}^T(\xi)Q\dot{x}(\xi)d\xi d\omega,
\end{aligned} \tag{25}
$$

where $P_i(t)$ is defined in form of (10), $S>0$, $Q>0$. Obviously, $V_i(t,x_t)$ is continuous and contains L non-differential instants, that is, $t_k+\theta_{k,q}$, $q=1,2,\cdots,L$. According to Remark 1, we just verify that inequalities (3) holds on $\mathcal{N}_{k,q}$, $q\in\mathcal{L}$ and $[t_k+\tau_{\min},t_{k+1})$.

When $t\in\mathcal{N}_{k,q}$, $q\in\mathcal{L}$, calculating the time derivative of (25) along the solution of system (9), we get

$$
\begin{aligned}
&\dot{V}_i(t,x_t)\\
&=2x^T(t)P_i(t)\dot{x}(t)+x^T(t)\dot{P}_i(t)x(t)\\
&\quad+\eta\int_{t-d}^{t}e^{\eta(t-\xi)}x^T(\xi)Sx(\xi)d\xi\\
&\quad+x^T(t)Sx(t)-e^{\eta d}x^T(t-d)Sx(t-d)\\
&\quad+\eta\int_{-d}^{0}\int_{t+\omega}^{t}e^{\eta(t-\xi)}\dot{x}^T(\xi)Q\dot{x}(\xi)d\xi d\omega\\
&\quad+d\dot{x}^T(t)Q\dot{x}(t)-\int_{t-d}^{t}e^{-\eta(\xi-t)}\dot{x}^T(\xi)Q\dot{x}(\xi)d\xi.
\end{aligned}
$$

This, together with (25), implies

$$
\begin{aligned}
&\dot{V}_i(t,x_t)-\eta V_i(t,x_t)\\
&\le 2x^T(t)P_i(t)[(A_i+A_{di})x(t)-A_{di}\int_{t-d}^{t}\dot{x}(\xi)d\xi]\\
&\quad+x^T(t)\dot{P}_i(t)x(t)+x^T(t)Sx(t)\\
&\quad-e^{\eta d}x^T(t-d)Sx(t-d)+d\dot{x}^T(t)Q\dot{x}(t)\\
&\quad-\int_{t-d}^{t}\dot{x}^T(\xi)Q\dot{x}(\xi)d\xi-\eta x^T(t)P_i(t)x(t).
\end{aligned} \tag{26}
$$

From (22) and Moon's inequality, one derives

$$
-2x^T(t)P_i(t)A_{di}\dot{x}(\xi)
$$

$$\leq \eta_2^T(t)\begin{bmatrix} X & Y-P_i(t)A_{di} \\ * & Q \end{bmatrix}\eta_2(t), \tag{27}$$

where $\eta_2(t)=\begin{bmatrix} x^T(t) & \dot{x}^T(\xi) \end{bmatrix}^T$. Substituting (27) into (26), we obtain that

$$\begin{aligned}&\dot{V}_i(t,x_t)-\eta V_i(t,x_t)\\ &\leq 2x^T(t)P_i(t)A_ix(t)+dx^T(t)Xx(t)+2x^T(t)Yx(t)\\ &\quad -2x^T(t)(Y-P_i(t)A_{di})x(t-d)+x^T(t)\dot{P}_i(t)x(t)\\ &\quad +x^T(t)Sx(t)-e^{\eta d}x^T(t-d)Sx(t-d)+d\dot{x}^T(t)Q\dot{x}(t)\\ &\quad -\eta x^T(t)P_i(t)x(t)\leq \eta_1^T(t)\bar{\Omega}_i(t)\eta_1(t),\end{aligned}$$

where

$$\begin{aligned}&\eta_1(t)=\begin{bmatrix} x^T(t) & x^T(t-d) \end{bmatrix}^T,\\ &\bar{\Omega}_i(t)=\begin{bmatrix} \bar{\Omega}_{i1}(t) & P_i(t)A_{di}+dA_i^TQA_{di}-Y \\ * & -e^{\eta d}S+dA_{di}^TQA_{di} \end{bmatrix},\\ &\bar{\Omega}_{i1}(t)=P_i(t)A_i+A_i^TP_i(t)-\eta P_i(t)+S\\ &\qquad\qquad +\dot{P}_i(t)+dA_i^TQA_i+dX+Y+Y^T.\end{aligned}$$

Substituting $\dot{P}_i(t)=\dot{\alpha}(P_{i,q+1}-P_{i,q})=L/\tau_{\min}(P_{i,q+1}-P_{i,q})$ and (10) into matrix $\bar{\Omega}_i(t)$, one obtains that $\bar{\Omega}_i(t)=(1-\alpha)\Xi_{2i,q}+\alpha\bar{\Xi}_{2i,q}$. From (19) and (20), we conclude that

$$\dot{V}_i(t,x_t)\leq \eta V_i(t,x_t),\quad t\in\mathcal{N}_{k,q},\quad q\in\mathcal{L}. \tag{28}$$

When $t\in[t_k+\tau_{\min},t_{k+1})$, similar to the above process, we get $\dot{V}_i(t,x_t)-\eta V_i(t,x_t)\leq \eta_1^T(t)\tilde{\Xi}_{1i,L}\eta_1(t)$. From (21), one derives

$$\dot{V}_i(t,x_t)\leq \eta V_i(t,x_t),\quad t\in[t_k+\tau_{\min},t_{k+1}). \tag{29}$$

Therefore, inequalities (3) holds from (28) and (29).

Furthermore, according to (23) and (25), it is easy to see that inequalities (4) holds. Since $\tau_{\max}\geq\tau_k$, $k=0,1,2,\cdots$, (24) implies (5). By Theorem 1, the proof is completed.

3.3. Stability analysis of the linear switched time-varying delay system with all subsystems unstable

In this section, based on Theorem 1, we further discuss the stability problem of the linear switched time-varying delay systems with all subsystems unstable.

Consider the linear switched time-varying delay system given by

$$\begin{aligned}\Sigma_{(3)}:\dot{x}(t)&=A_{\sigma(t)}x(t)+A_{d\sigma(t)}x(t-d(t)),\\ x(t)&=\varphi(t),\qquad t\in[-h_1,0],\end{aligned} \tag{30}$$

where the definitions of $x(t)$ and $\sigma(t)$ are the same with (1). A_i, A_{di} are known real constant matrices with appropriate dimensions. $d(t)$ satisfies $0\leq d(t)\leq h_1$ and $\dot{d}(t)\leq h_2<1$.

Similar to the discussion in section 3.2, the stability criterion for (30) is derived by constructing the corresponding discretized Lyapunov-Krasovskii functional

$$\begin{aligned}V_i(t,x_t)=&x^T(t)P_i(t)x(t)+\int_{t-d(t)}^t e^{\eta(t-\xi)}x^T(\xi)Sx(\xi)d\xi\\ &+\int_{-h_1}^0\int_{t+\omega}^t e^{\eta(t-\xi)}\dot{x}^T(\xi)Q\dot{x}(\xi)d\xi d\omega,\end{aligned}$$

where $P_i(t)$ is defined in form of (10), $S>0$, $Q>0$.

Theorem 4: Consider system (30). Given positive constants $h_1>0$, $h_2<1$ $\eta>0$, $0<\mu<1$, $\tau_{\min}>0$, if, there exist matrices $P_{i,q}>0$, $q=0,1,2,\cdots,L$, $i=1,2,\cdots,m$, $S>0$, $Q>0$, $X\geq 0$, Y, a constant $\tau_{\max}$, such that for any $q\in\mathcal{L}$, $i,j\in\mathcal{M}$, the following inequalities hold

$$\begin{aligned}&\Xi_{2i,q}=\begin{bmatrix} \Lambda_{2i,q} & \Theta_{2i,q} \\ * & -(1-h_2)S+h_1A_{di}^TQA_{di} \end{bmatrix}<0,\\ &\bar{\Xi}_{2i,q}=\begin{bmatrix} \bar{\Lambda}_{2i,q} & \bar{\Theta}_{2i,q} \\ * & -(1-h_2)S+h_1A_{di}^TQA_{di} \end{bmatrix}<0,\\ &\tilde{\Xi}_{2i,L}=\begin{bmatrix} \tilde{\Lambda}_{2i,L} & \tilde{\Theta}_{2i,q} \\ * & -(1-h_2)S+h_1A_{di}^TQA_{di} \end{bmatrix}<0,\\ &\begin{bmatrix} X & Y \\ * & Q \end{bmatrix}\geq 0,\\ &P_{j,0}-\mu P_{i,L}\leq 0,\quad i\neq j,\\ &\ln\mu+\eta\tau_{\max}<0,\end{aligned}$$

where

$$\begin{aligned}\Lambda_{2i,q}&=P_{i,q}A_i+A_i^TP_{i,q}-\eta P_{i,q}+\psi_{2i,q},\\ \bar{\Lambda}_{2i,q}&=P_{i,q+1}A_i+A_i^TP_{i,q+1}-\eta P_{i,q+1}+\psi_{2i,q},\\ \tilde{\Lambda}_{2i,L}&=P_{i,L}A_i+A_i^TP_{i,L}-\eta P_{i,L}\\ &\quad +S+h_1A_i^TQA_i+h_1X+Y+Y^T,\\ \psi_{2i,q}&=S+L/\tau_{\min}(P_{i,q+1}-P_{i,q})\\ &\quad +h_1A_i^TQA_i+h_1X+Y+Y^T,\\ \Theta_{2i,q}&=P_{i,q}A_{di}+h_1A_i^TQA_{di}-Y,\\ \bar{\Theta}_{2i,q}&=P_{i,q+1}A_{di}+h_1A_i^TQA_{di}-Y,\\ \tilde{\Theta}_{2i,q}&=P_{i,L}A_{di}+h_1A_i^TQA_{di}-Y,\end{aligned}$$

then system (30) is UAS under the switching signal $\sigma(t)\in\mathcal{D}_{[\tau_{\min},\tau_{\max}]}$.

Proof: Similar to the proof of Theorem 3, we just omit it.

4. EXAMPLES

Two examples are provided to show the feasibility, superiority, and application of the proposed approach.

Example 1: Consider the switched delay system with two subsystems. The parameters are listed as follows:
Subsystem 1

$$A_1=\begin{bmatrix} -2 & 0.6 \\ 0.5 & -0.1 \end{bmatrix},\qquad A_{d1}=\begin{bmatrix} 0.1 & 0 \\ 0 & 0.06 \end{bmatrix},$$

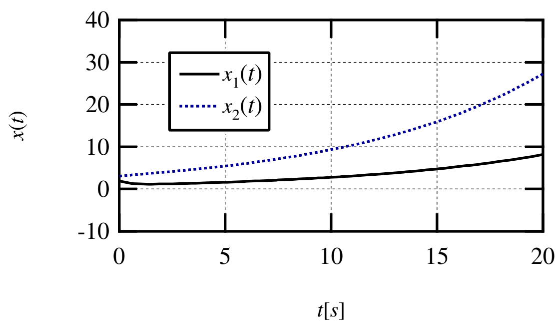

Fig. 1. The state trajectories of subsystems 1.

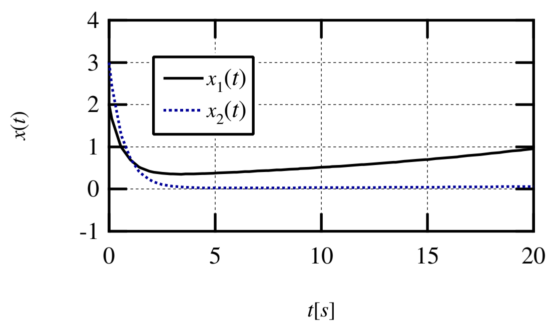

Fig. 2. The state trajectories of subsystem 2.

Subsystem 2

$$A_2 = \begin{bmatrix} 0.12 & -1 \\ 0.1 & -1.6 \end{bmatrix}, \qquad A_{d2} = \begin{bmatrix} 0 & 0.1 \\ 0 & 0.1 \end{bmatrix}.$$

The initial state function is $x(\theta) \equiv \begin{bmatrix} 2 & 3 \end{bmatrix}^T$, $\theta \in [-d, 0]$, $d = 0.3$. Assume that the system states are not measurable.

In the numerical example, system states are not measurable, thus the state-dependent switching signals in [22,23] do not effectively work. In addition, because the two subsystems are unstable from Fig. 1 and Fig. 2, the proposed methods based on time-dependent switching signals [16–21] can not stabilize this class of switched systems. Furthermore, in [24], the control scheme has been proposed to guarantee stability of switched system with unstable subsystem, but the considered switched system does not include time delay.

Here, we apply Theorem 3 to judge the stability of the given system. Let $L = 1$, $\mu = 0.7$, $\eta = 0.3$, $d = 0.3$, $\tau_{\min} = 0.6$. By Theorem 3, we can obtain the following feasible solution

$$\tau_{\max} = 1.1889,$$

$$P_{1,0} = \begin{bmatrix} 113.9662 & -11.0598 \\ * & 207.6202 \end{bmatrix},$$

$$P_{1,1} = \begin{bmatrix} 283.4615 & -84.8508 \\ * & 225.9784 \end{bmatrix},$$

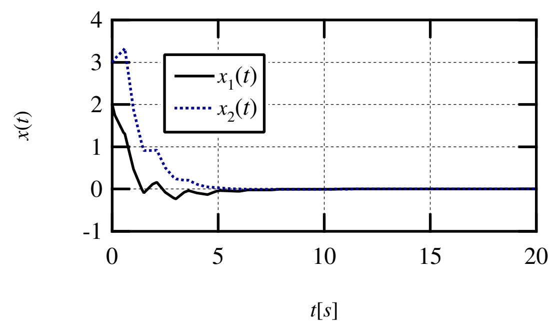

Fig. 3. The state trajectories of the whole switched delay system.

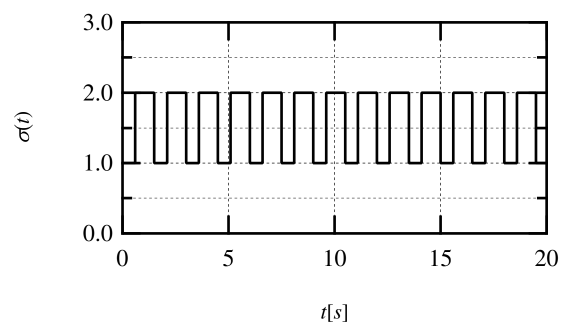

Fig. 4. The designed signal $\sigma(t) \in \mathcal{D}_{[0.6,\ 1.1889]}$.

$$P_{2,0} = \begin{bmatrix} 187.0579 & -60.0641 \\ * & 152.7374 \end{bmatrix},$$

$$P_{2,1} = \begin{bmatrix} 177.1136 & -30.0576 \\ * & 324.6626 \end{bmatrix},$$

$$S = \begin{bmatrix} 53.4102 & -5.9524 \\ * & 56.8630 \end{bmatrix},$$

$$Q = \begin{bmatrix} 116.2253 & -26.8837 \\ * & 104.4212 \end{bmatrix}.$$

The state trajectories of the whole switched delay system and the designed switching signal are shown in Fig. 3 and Fig. 4. From them, the states of the given switched delay system tend to the origin under the designed switching signal $\sigma(t) \in \mathcal{D}_{[0.6,\ 1.1889]}$, which shows the validity of the proposed method.

Example 2: A model of combustion in rocket motor chambers is considered. This model is a liquid monopropellant rocket motor with a pressure feeding system. Under the assumption of non-steady flow, lumped lag factor, $u(t) = 0$, and $\omega(t) = 0$, an appropriate linearized model can be described in the form $\dot{x}(t) = A_{\sigma(t)}x(t) + A_{d\sigma(t)}x(t-d)$. The values of these parameters are listed as follows [25]

$$A_1 = \begin{bmatrix} \rho_1 - 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\frac{1}{\zeta_1 J} \\ -\frac{p}{2J(1-\zeta_1)} & 0 & -\frac{1}{J(1-\zeta_1)} & -\frac{1}{J(1-\zeta_1)} \\ 0 & \frac{1}{E_e} & -\frac{1}{E_e} & 0 \end{bmatrix},$$

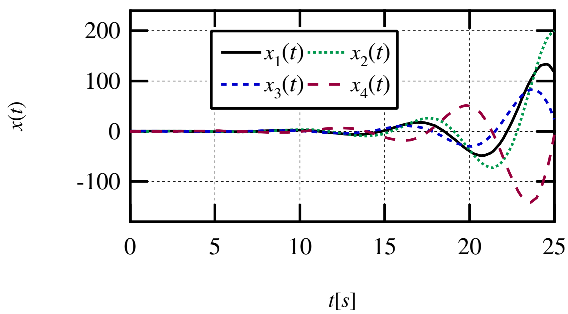

Fig. 5. The state trajectories of subsystems 1.

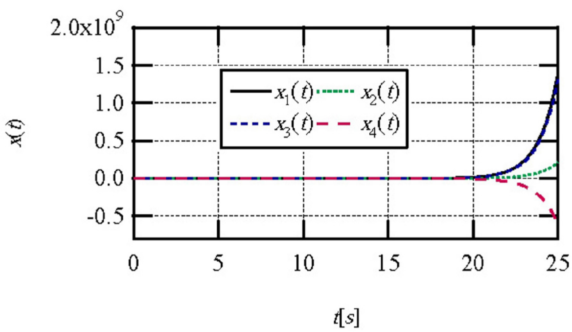

Fig. 6. The state trajectories of subsystem 2.

$$A_2 = \begin{bmatrix} \rho_2 - 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\frac{1}{\zeta_2 J} \\ -\frac{p}{2J(1-\zeta_2)} & 0 & -\frac{1}{J(1-\zeta_2)} & -\frac{1}{J(1-\zeta_2)} \\ 0 & \frac{1}{E_e} & -\frac{1}{E_e} & -1 \end{bmatrix},$$

$$A_{di} = \begin{bmatrix} -\rho_i & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad i = 1, 2,$$

where ζ_i is the fractional length for pressure supply, J is the line inertia, E_e is the line elasticity parameter, p is the ratio of steady-state pressure and steady-state injector pressure drop, and ρ_i is the pressure exponent of the combustion process. The initial state function is $x(\theta) \equiv \begin{bmatrix} 0.2 & -0.1 & 0.3 & -0.1 \end{bmatrix}^T$, $\theta \in [-d, 0]$, $d = 0.1$.

For $p = 1.02$, $J = 2$, $E_e = 0.95$, $\rho_1 = 0.6$, $\zeta_1 = 0.6$, $\rho_2 = 1.12$, $\zeta_2 = 1.6$, two subsystems are unstable from Fig. 5 and Fig. 6.

Choosing $L = 1$, $\mu = 0.1$, $\eta = 4$, $d = 0.1$, $\tau_{\min} = 0.1$, inequalities (19)-(24) of Theorem 3 have a feasible solution, where $\tau_{\max}$ is calculated as 0.5756. From Fig. 7 and Fig. 8, the states of the given switched delay system converge to the origin with $\sigma(t) \in \mathcal{D}_{[0.1,\ 0.5756]}$, which shows the effectiveness of the scheme.

5. CONCLUSIONS

Stability analysis has been studied for switched delay systems with all subsystems unstable in this paper. The sufficient criterion has been proposed for nonlinear

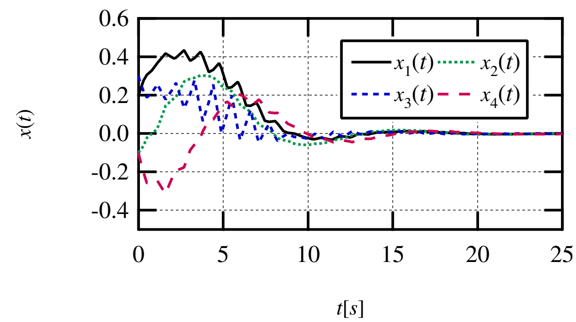

Fig. 7. The state trajectories of the whole switched delay system.

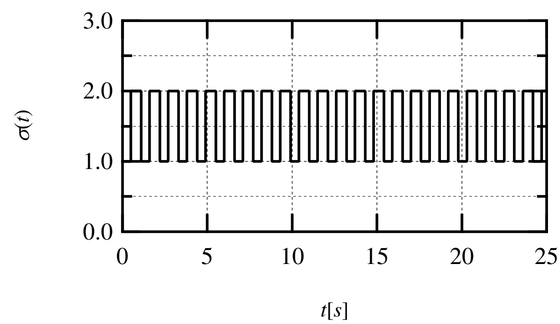

Fig. 8. The designed signal $\sigma(t) \in \mathcal{D}_{[0.1,\ 0.5756]}$.

switched delay systems to ensure asymptomatic stability. Based on it, multiple discretized Lyapunov-Krasovskii functionals have been constructed to obtain stability criteria for linear switched time-invariant and time-varying delay systems with all subsystems unstable. Finally, two examples have been shown to demonstrate the effectiveness and superiority of the proposed results.

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