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Output-Feedback Control of Underwater Gliders by Buoyancy and Pitching Moment Control: Feedback Linearization Approach

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Abstract: This paper addresses an output-feedback control problem of an underwater glider. The buoyancy and the pitching moment that are generated by the net mass variation and the elevator control, respectively, are used as control inputs. Additional forces induced by the elevator control increase nonlinearity of the plant dynamics, which make controller design difficult. By using the feedback linearization technique, we convert the concerned nonlinear dynamics to a linear time-invariant model, and based on this, design an observer-based output-feedback controller. A simulation result is shown to verify the effectiveness of the proposed technique.

Keywords: Feedback linearlization, Lie-derivative, observer-based output-feedback, relative degree, underwater glider.

1. INTRODUCTION

An underwater glider is an unmanned ocean exploration robot which is planned to oscillate between the surface and the deep layer of sea, without a fuel-based propellant [1]. It is suitable to explore the ocean for a long duration [1, 2], because energy consumption of the underwater glider is much less than the fuel-based propelled autonomous underwater vehicles. It navigates by two actions: change of buoyancy generated by the mass variation due to the inflow and emission of the seawater and change of attitude generated by the pitching moment through the movement of center-of-gravity or an elevator adjustment [3]. As a result, a sawtooth-like gliding occurs in a vertical plane, in which the steady gliding phase switches between a dive (with negative buoyancy and pitched down) and an upwards glide (positive buoyancy and pitched up) [4, 5]. The stability of the desired path and fast convergence to it are very important to reduce the energy expenditure.

Researches on the underwater glider controller design begin to gather considerable attentions recently. Essential dynamic features of underwater gliding are captured in [6]. Seo [7] develops a test-bed underwater glider. In [8], an underwater glider model is identified in a discrete-time transfer function format for buoyancy, depth, and pitch control. The dynamics of the underwater glider is analyzed through the singular perturbation technique by reducing the full-order model to a second-order one in [9–11]. In [12], a feedback linearization technique is applied to the underwater glider for synthesis. However the nonlinear model is *partially* linearized and the approximation error is ignored, thus the methods do not guarantee the (semi-)global stability. Furthermore all nonlinear controllers suggested above require full state feedback for implementation, which is generally difficult, if not impossible, to fulfill.

This paper focuses on the output-feedback stabilization problem for the nonlinear model of the underwater glider restricted to the vertical plane. Both the buoyancy change by the mass variation and pitching moment adjustment by elevator are considered as control inputs. By using the feedback linearization technique, the concerned nonlinear complex model is *fully* linearized. Throughout this process the pitching moment control law and the buoyancy control law are designed. The nonlinear observer for output feedback is designed based on the linearization, which guarantees asymptotic convergence of the estimation error. A simulation example shows the effectiveness of the present development.

2. MODELING OF UNDERWATER GLIDERS

Fig. 1 shows the motion of the underwater glider for a short distance. Subscripts ‘1’ and ‘2’ are applied to dis-

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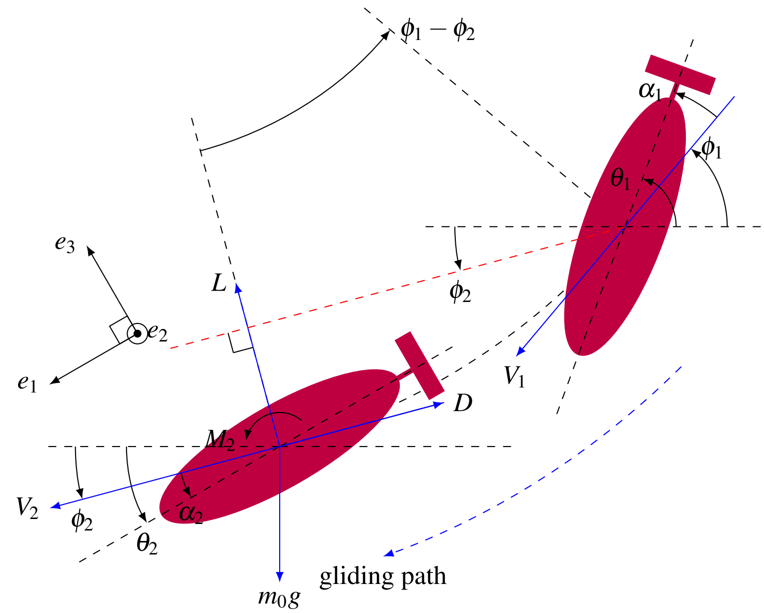

Fig. 1. Forces and moments applied to the underwater glider.

tinguish instantaneous positions of the movement. Some assumptions are introduced for modeling.

Assumption 1: The underwater glider is rigid with an ellipsoidal shape and moves in the sagittal plane.

In modeling, the body-fixed reference frame is used. The frame consists of the vectors $e_i \in \mathbb{R}^3$, $i \in \mathscr{I}_3 := \{1,2,3\}$, which has its origin at geometric center of the underwater glider. Axis e_1 is along the body's longitudinal axis pointing to the head. Axis e_2 lies in the sagittal plane perpendicular to e_1, and points out of the page. Axis e_3 lies along the direction defined by the right-hand orthonormal principle. Instantiating with dropping the subscript in the instantaneous position 2, $\alpha \in \mathbb{R}$ is the attack angle, $\phi \in \mathbb{R}$ is the flight path angle, $\theta \in \mathbb{R}$ is the pitch angle, and $\Omega \in \mathbb{R}$ is the pitch rate. $V \in \mathbb{R}$ denotes the speed in the path direction and $M \in \mathbb{R}_{\geqslant 0}$ is the pitching moment with the direction of e_2. L, $D \in \mathbb{R}_{>0}$ are the lift and the drag force, respectively, that are often modeled dependent on α and V as follows [10]:

$$D := (\kappa_{D_0} + \kappa_D \alpha^2)V^2, \quad L := (\kappa_{L_0} + \kappa_L \alpha)V^2,$$

where κ_{D_0} and κ_D are the drag coefficients, and κ_{L_0} are κ_L are the lift coefficients.

Assumption 2: The underwater glider moves along the trajectory of the arc with the central angle $\phi_1 - \phi_2$.

Assumption 3 [9,11]**:** For a tiny distance, $\phi_1 \approx \phi_2 \approx \phi$, $V_1 \approx V_2 \approx V$ and the effect from the inertia of the surrounding fluid on the dynamics of the underwater glider is negligible.

Remark 1: Due to Assumption 3, the added mass in the e_1 and e_3 directions and the added moment of inertia about the axis in the e_2 direction are excluded.

The underwater glider dynamics derived by Newton's second law of motion can be expressed as the following state-space equation [11]:

$$\dot{x} = f(x) + g(x)u \tag{1}$$

where $x \in \mathbb{R}^5$ is the state, $u \in \mathbb{R}^2$ is the control input. In detail, x is further defined by

$$x_1 := V - V_e, \quad x_2 := \phi - \phi_e, \quad x_3 := m_0 - m_{0_e},$$
$$x_4 := \alpha - \alpha_e, \quad x_5 := \frac{\kappa_q}{\kappa_M}\Omega - \Omega_e,$$

where $m_0 \in \mathbb{R}$ is the mass of the underwater glider minus the mass of the displaced fluid that is adjusted by the buoyancy control, and the subscript "e" means the equilibrium point of each state variable. Since the underwater glider dives in a sawtooth pattern [8], fixing $\Omega_e = 0$ reasonably simplifies the discussion. $\kappa_M \in \mathbb{R}$ is the pitching moment coefficient and $\kappa_q \in \mathbb{R}$ is the pitching damping coefficient. The control input consists of buoyancy control u_1 and pitching moment control u_2, which are defined as follows:

$$u = \begin{bmatrix} u_1 \\ \bar{u}_2 \end{bmatrix} := \begin{bmatrix} u_1 \\ u_2 - u_{2_e} \end{bmatrix},$$

where

$$u_{2_e} := \frac{\kappa_{M_0}}{\kappa_M} + \alpha_e,$$

where $\kappa_{M_0} \in \mathbb{R}$ is the pitching moment coefficient subject to $\kappa_{M_0} \neq \kappa_M$. Then the vector fields $f(x)$ and $g(x)$ are represented by

$$f = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{bmatrix}, \quad g = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \\ g_{31} & g_{32} \\ g_{41} & g_{42} \\ g_{51} & g_{52} \end{bmatrix},$$

where

$$\begin{aligned}
f_1 &:= -\frac{1}{m}((x_3 + m_{0_e})g\sin(x_2+\phi_e) + D \\
&\quad - \delta u_{2_e}(x_1+V_e)^2 \sin(x_4+\alpha_e)), \\
f_2 &:= \frac{1}{m(x_1+V_e)}(-(x_3+m_{0_e})g\cos(x_2+\phi_e) + L \\
&\quad + \delta u_{2_e}(x_1+V_e)^2\cos(x_4+\alpha_e)), \\
f_3 &:= 0, \\
f_4 &:= \frac{\kappa_M}{\kappa_q}x_5 - f_2, \\
f_5 &:= \frac{\kappa_q}{J}(x_4+x_5)(x_1+V_e)^2, \\
g_{11} &= g_{21} = g_{41} = g_{51} = g_{32} := 0, \\
g_{31} &:= 1, \\
g_{12} &:= \frac{1}{m}\delta(x_1+V_e)^2\sin(x_4+\alpha_e), \\
g_{22} &:= \frac{1}{m}\delta(x_1+V_e)\cos(x_4+\alpha_e), \\
g_{42} &:= -g_{22}, \\
g_{52} &:= -\frac{\kappa_q}{J}(x_1+V_e)^2,
\end{aligned}$$

where m is the mass of the underwater glider, g is the gravitational acceleration, J is the moment of inertia, and δ is the coupling factor that represents the additional force V^2u_2 induced by the elevator action.

3. FEEDBACK LINEARIZATION OF THE MODEL

Before proceeding, recall the following preliminaries.

Definition 1: An operator $z = T(x)$ is called a diffeomorphism of a region $\mathscr{D} \subset \mathbb{R}^n$, if it satisfies the following condition

(i) For all $x \in \mathscr{D}$, T is continuously differentiable.
(ii) There exists a continuously differentiable inverse T^{-1} such that

$$T^{-1}(T(x)) = x, \quad \forall x \in \mathscr{D}.$$

Definition 2: The Lie derivatives of a smooth function $h(x) : \mathbb{R}^n \to \mathbb{R}$ with respect to a vector field $f(x) : \mathbb{R}^n \to \mathbb{R}^n$ and $g(x) : \mathbb{R}^n \to \mathbb{R}^n$ are given by

$$\mathscr{L}_f h := \frac{\partial h}{\partial x} f$$

and

$$\begin{aligned}
\mathscr{L}_f^i h &:= \frac{\partial(\mathscr{L}_f^{i-1}h)}{\partial x} f \\
\mathscr{L}_g\mathscr{L}_f^{i-1} h &:= \frac{\partial(\mathscr{L}_f^{i-1}h)}{\partial x} g, \quad i \in \mathbb{Z}_{>0}
\end{aligned}$$

in a recursive form.

Definition 3 [13]**:** System (1) is said to have the relative degree $r \in \mathbb{Z}_{>0}$, for all $x \in \mathscr{D}$ if the following conditions are satisfied:

$$\begin{aligned}
&\mathscr{L}_g\mathscr{L}_f^i h(x) = 0, \quad i \in \mathbb{Z}_{[0,r-2]}, \\
&\mathscr{L}_g\mathscr{L}_f^{r-1} h(x) \neq 0.
\end{aligned}$$

Now, let us consider the following output function for (1).

$$\xi_1(x) := x_2 + x_4. \tag{2}$$

Using the Lie derivative, we know that the relative degree is $r_2 = 2$ from the following computations

$$\begin{aligned}
\dot{\xi}_1 &= \mathscr{L}_f\xi_1 + \mathscr{L}_g\xi_1 u \\
&= \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \end{bmatrix} f + \underbrace{\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \end{bmatrix} g}_{=0 \Rightarrow \mathscr{L}_g\xi_1 = 0} u \\
&= \frac{\kappa_M}{\kappa_q}x_5 =: \xi_2 \\
\dot{\xi}_2 &= \mathscr{L}_f^2\xi_1 + \mathscr{L}_g\mathscr{L}_f\xi_1 u \\
&= \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{\kappa_M}{\kappa_q} \end{bmatrix} f + \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 & \frac{\kappa_M}{\kappa_q} \end{bmatrix} g}_{\neq 0 \Rightarrow \mathscr{L}_g\mathscr{L}_f\xi_1 \neq 0} u \\
&= \frac{\kappa_M}{\kappa_q} f_5 + \frac{\kappa_M}{\kappa_q} g_{52}\bar{u}_2.
\end{aligned}$$

Setting of

$$v_2 := \frac{\kappa_M}{\kappa_q} f_5 + \frac{\kappa_M}{\kappa_q} g_{52}\bar{u}_2 \tag{3}$$

allows one to partially feedback-linearize into a second-order dynamics for $\xi := (\xi_1, \xi_2)$

$$\begin{aligned}
\dot{\xi} &= A_2\xi + B_2 v_2, \\
y_2 &= \xi_1,
\end{aligned} \tag{4}$$

where

$$A_2 := \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B_2 := \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Remark 2: The asymptotic stability of ξ-dynamics does not imply that of x_2 and x_4 but of $x_2 + x_4$. So either one of x_2 and x_4 further needs to be asymptotically stable. Thus the state variables that are unobservable from the output [14] can be defined by $\bar{x} := (x_1, x_2, x_3)$ or $\bar{x} := (x_1, x_3, x_4)$. This paper chooses the former for easier analysis.

The linear transformation

$$T_L(x) = \begin{bmatrix} \times \\ \hline \tilde{T}_L \end{bmatrix}(x) := \begin{bmatrix} \bar{x} \\ \hline \xi \end{bmatrix}$$

with (3) converts (1) to

$$\begin{cases} \dot{\bar{x}} = \bar{f}(\bar{x},\xi) + \bar{g}u_1 \\ \dot{\xi} = A_2\xi + B_2 v_2 \end{cases} \tag{5}$$

where

$$\bar{f} = \begin{bmatrix} f_1 + g_{12}\bar{u}_2 \\ f_2 + g_{22}\bar{u}_2 \\ f_3 \end{bmatrix}, \quad \bar{g} = \begin{bmatrix} g_{11} \\ g_{21} \\ g_{31} \end{bmatrix}$$

and $\times$ denotes don't care.

Remark 3: The stability of $\bar{x}$-dynamics in (5) is not guaranteed even though $\bar{u}_2$ in (3) is designed to asymptotically stabilize (4). If u_1 is further designed to asymptotically stabilize $\bar{x}$-dynamics, then (5) (equally (1)) becomes asymptotically stabilized.

Now, let us feedback-linealize the $\bar{x}$-dynamics. To that end, consider the following output function

$$\eta_1(\bar{x}) := \ln\left(\frac{(x_1+V_e)\cos(x_2+\phi_e)}{V_e\cos\phi_e}\right).$$

One can calculate the higher-order time derivative of η_1 until there exists $r_1 \in \mathbb{Z}_{>0}$ such that $\mathscr{L}_{\bar{g}}\mathscr{L}_{\bar{f}}^{r_1-1}\eta_1 \neq 0$ that results in

$$\begin{aligned}
\dot{\eta}_1 &= \mathscr{L}_{\bar{f}}\eta_1 + \mathscr{L}_{\bar{g}}\eta_1 u_1 \\
&= \begin{bmatrix} \frac{1}{x_1+V_e} & \tan(x_2+\phi_e) & 0 \end{bmatrix}(\bar{f}+\bar{g}u_1) \\
&= \left(\frac{1}{x_1+V_e}\right)\bar{f}_1 - \tan(x_2+\phi_e)\bar{f}_2 =: \eta_2, \\
\dot{\eta}_2 &= \mathscr{L}_{\bar{f}}^2\eta_1 + \mathscr{L}_{\bar{g}}\mathscr{L}_{\bar{f}}\eta_1 u_1 \\
&= \begin{bmatrix} -\begin{pmatrix} \frac{D}{m(x_1+V_e)^2} \\ +\frac{\tan(x_2+\phi_e)L}{m(x_1+V_e)^2} \end{pmatrix} & -\frac{\sec^2(x_2+\phi_e)L}{m(x_1+V_e)} & 0 \end{bmatrix} \\
&\quad \times(\bar{f}+\bar{g}u_1) =: \eta_3 \\
&= -\left(\frac{D}{m(x_1+V_e)^2} + \frac{\tan(x_2+\phi_e)L}{m(x_1+V_e)^2}\right)\bar{f}_1 \\
&\quad - \left(\frac{\sec^2(x_2+\phi_e)L}{m(x_1+V_e)}\right)\bar{f}_2 =: \eta_3, \\
\dot{\eta}_3 &= \mathscr{L}_{\bar{f}}^3\eta_1 + \mathscr{L}_{\bar{g}}\mathscr{L}_{\bar{f}}^2\eta_1 u_1,
\end{aligned} \tag{6}$$

where $\mathscr{L}_{\bar{g}}\mathscr{L}_{\bar{f}}^2\eta_1$ in (6) is computed as

$$\begin{aligned}
\mathscr{L}_{\bar{g}}\mathscr{L}_{\bar{f}}^2\eta_1 &= \underbrace{\begin{bmatrix} \frac{\partial(\mathscr{L}_{\bar{f}}^2\eta_1)}{\partial x_1} & \frac{\partial(\mathscr{L}_{\bar{f}}^2\eta_1)}{\partial x_2} & \frac{\partial(\mathscr{L}_{\bar{f}}^2\eta_1)}{\partial x_3} \end{bmatrix}}_{=\frac{\partial(\mathscr{L}_{\bar{f}}^2\eta_1)}{\partial x}} \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{=\bar{g}} \\
&= \frac{\partial(\mathscr{L}_{\bar{f}}^2\eta_1)}{\partial x_3} \\
&= \frac{g\sin(x_2+\phi_e)(D+\tan(x_2+\phi_e)L)}{m^2(x_1+V_e)^2} \\
&\quad + \frac{g\cos(x_2+\phi_e)\sec^2(x_2+\phi_e)L}{m^2(x_1+V_e)^2} \\
&\neq 0.
\end{aligned}$$

Thus, we see that $r_1 = 3$ is the relative degree and the resulting third-order linear dynamics for $\eta := (\eta_1, \eta_2, \eta_3)$

$$\begin{aligned} \dot{\eta} &= A_1\eta + B_1 v_1 \\ y_1 &= \eta_1 \end{aligned} \tag{7}$$

is constructed via the nonlinear transformation $T_x(\bar{x}) = \eta$, where

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

and

$$v_1 := \mathscr{L}_{\bar{f}}^3\eta_1 + \mathscr{L}_{\bar{g}}\mathscr{L}_{\bar{f}}^2\eta_1 u_1. \tag{8}$$

If one designs

$$\bar{u}_2 := \frac{1}{g_{52}}\left(\frac{\kappa_q}{\kappa_M}K_2\xi - f_5\right) \tag{9}$$

for the pitching moment control and

$$u_1 = \frac{K_1\eta - \mathscr{L}_{\bar{f}}^3\eta_1}{\mathscr{L}_{\bar{g}}\mathscr{L}_{\bar{f}}^2\eta_1} \tag{10}$$

for the buoyancy control, the overall closed-loop feedback-linearized system is described by

$$\begin{aligned} \dot{z} &= (A+BK)z, \\ y &= Cz, \end{aligned} \tag{11}$$

where

$$z = \begin{bmatrix} \eta \\ \xi \end{bmatrix}, \quad A = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}, \quad B = \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix},$$

$$K = \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

where $K_2 \in \mathbb{R}^{1\times 2}$ and $K_1 \in \mathbb{R}^{1\times 3}$ are to be selected so that $A+BK$ is Hurwitz.

Since the system (11) is equivalent to (1), the asymptotic stability of (11) implies that (1) closed by (9) and (10).

4. NONLINEAR OBSERVER-BASED OUTPUT-FEEDBACK CONTROLLER DESIGN

Assuming that only y is available for control, a way is to employ an observer-based output-feedback controller in the form of

$$\begin{aligned} \dot{\hat{x}} &= f(\hat{x}) + g(\hat{x})u + l(\hat{x})(y-\hat{y}), \\ \hat{y} &= \begin{bmatrix} \xi_1(\hat{x}) \\ \eta_1(\hat{x}) \end{bmatrix}, \end{aligned} \tag{12}$$

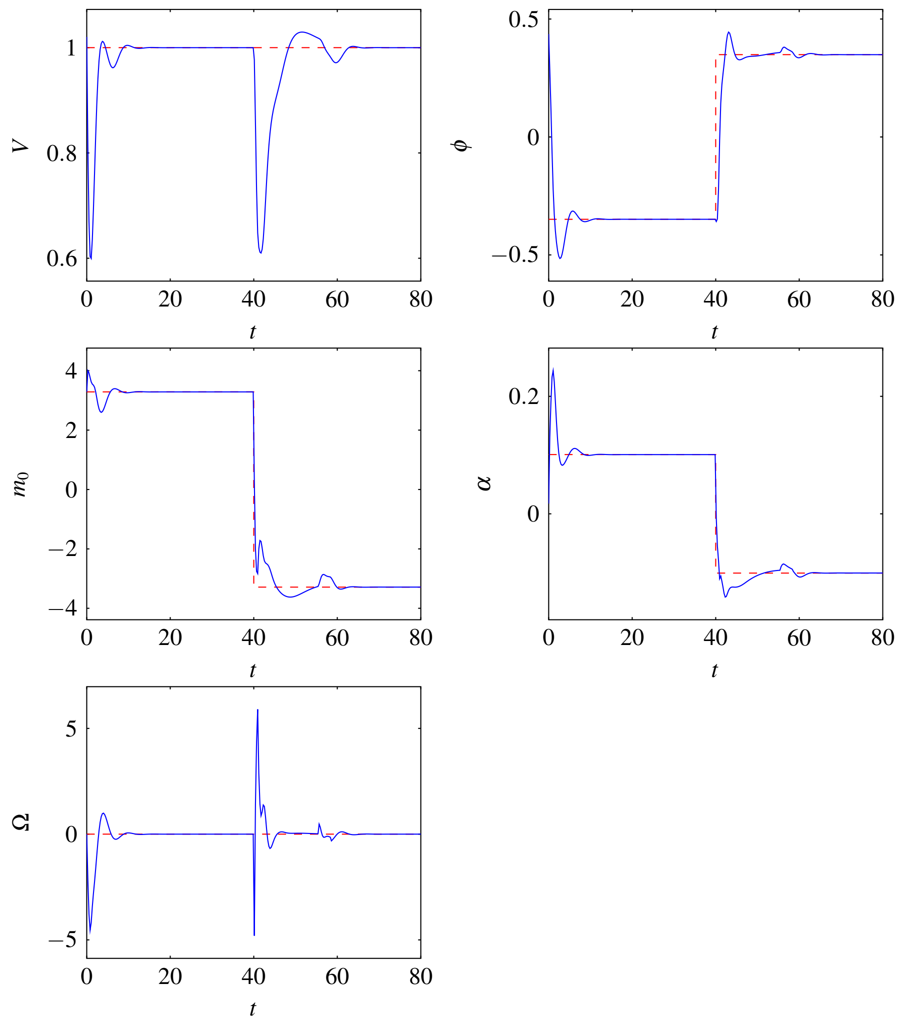

Fig. 2. Time responses of the underwater glider (blue: controlled; red-dashed: equilibrium).

$$u = \begin{bmatrix} \frac{K_1\hat{\eta} - \mathscr{L}^3_{\tilde{f}}\hat{\eta}_1}{\mathscr{L}_g\mathscr{L}^2_{\tilde{f}}\hat{\eta}_1} \\ \frac{1}{g_{52}}\left(\frac{\kappa_q}{\kappa_M}K_2\hat{\xi} - f_5(\hat{x})\right) \end{bmatrix}, \tag{13}$$

where $l(\hat{x})$ is an observer-gain function matrix, which may be difficult to design because of the complexity of (1), and $\hat{\eta} := \eta(\hat{x})$ and $\hat{\xi} := \xi(\hat{x})$. A simpler remedy is to use to build an linear time-invariant observer as

$$\begin{aligned} \dot{\hat{z}} &= A\hat{z} + Bv + L(y - \hat{y}), \\ \hat{y} &= C\hat{z}, \end{aligned} \tag{14}$$

where L is the observer gain matrix such that $A - LC$ is Hurwitz. Let $e := z - \hat{z}$. Then the augmented closed-loop dynamics with (11) and (14) is constructed as

$$\begin{bmatrix} \dot{z} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A + BK & -BK \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} z \\ e \end{bmatrix}. \tag{15}$$

It remains to show that (12) is diffeomorphic to (14) via

$$T(\hat{x}) = \begin{bmatrix} T_N(\hat{x}) \\ \hline \tilde{T}_L(\hat{x}) \end{bmatrix} = \hat{z} = \begin{bmatrix} \hat{\eta} \\ \hline \hat{\xi} \end{bmatrix} := \begin{bmatrix} \eta_1(\hat{x}) \\ \mathscr{L}_{\tilde{f}(\hat{x})}\eta_1(\hat{x}) \\ \mathscr{L}^2_{\tilde{f}(\hat{x})}\eta_1(\hat{x})) \\ \hline \xi_1(\hat{x}) \\ \mathscr{L}_{f(\hat{x})}\xi_1(\hat{x}) \end{bmatrix}$$

is asymptotically stable.

Remark 4: Since $T(\hat{x}) \in \mathbb{R}^5$ is the diffeomorphism on $\mathscr{D}$, there exist a Jacobian $J_T(\hat{x}) \in \mathbb{R}^{5\times5}$ and its inverse $J_T^{-1}(\hat{z})$. It is noted that the feedback linearization process is carried out twice. Therefore $J_T(\hat{x})$ has the structure of

$$\begin{aligned} J_T(\hat{x}) &= \begin{bmatrix} \frac{\partial T_N(\hat{x})}{\partial \hat{x}} & 0_{3\times2} \\ 0_{2\times3} & I_{2\times2} \end{bmatrix} \begin{bmatrix} \frac{\partial T_L(\hat{x})}{\partial \hat{x}} \end{bmatrix} \\ &= \begin{bmatrix} \frac{\partial T_N(\hat{x})}{\partial \hat{x}} \quad 0_{3\times2} \\ \hline \frac{\partial \tilde{T}_L(\hat{x})}{\partial \hat{x}} \end{bmatrix}. \end{aligned}$$

Theorem 1: The nonlinear observer in (12) is equivalent to (14), where $\hat{x} := T^{-1}(\hat{z})$ and $l(\hat{x}) := J_T^{-1}(\hat{z})L$.

Proof: Since $\hat{z} = T(\hat{x})$, one has

$$\begin{aligned}
\dot{\hat{z}} &= \frac{\partial T}{\partial \hat{x}} \dot{\hat{x}} \\
&= \left[\begin{array}{c|c} \frac{\partial T_N(\hat{x})}{\partial \hat{x}} & 0_{3\times 2} \\ \hline \multicolumn{2}{c}{\frac{\partial T_L(\hat{x})}{\partial \hat{x}}} \end{array} \right] \\
&\quad \times \left(f(\hat{x}) + g(\hat{x})u + J_T^{-1}(\hat{z}) L(y - \hat{y}) \right) \\
&= \begin{bmatrix} \frac{\partial T_N(\hat{x})}{\partial \hat{x}} \bar{f}(\hat{x}) \\ \frac{\partial T_L(\hat{x})}{\partial \hat{x}} f(\hat{x}) \end{bmatrix} + \begin{bmatrix} \frac{\partial T_N(\hat{x})}{\partial \hat{x}} \bar{g}(\hat{x}) u_1 \\ \frac{\partial T_L(\hat{x})}{\partial \hat{x}} g(\hat{x}) u \end{bmatrix} + L(y - \hat{y}) \\
&= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \hat{z} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} v + L(y - \hat{y}) \\
&= A\hat{z} + Bv + L(y - \hat{y})
\end{aligned}$$

□

Theorem 2: The state estimation error $\tilde{e} := x - \hat{x}$ between (1) and (12) satisfies

$$\lim_{t\to\infty} \tilde{e} = 0.$$

Proof: Since the fact that $A - LC$ is Hurwitz implies $\lim_{t\to 0} e = 0$, it is readily seen that

$$\begin{aligned}
\lim_{t\to\infty} \tilde{e} &= \lim_{t\to\infty} (x - \hat{x}) \\
&= \lim_{t\to\infty} (T^{-1}(z) - T^{-1}(z - e)) \\
&= 0.
\end{aligned}$$

□

5. SIMULATION

Let us borrow the following parameters in [11] for (1)

$$\begin{aligned}
&g = 9.8\ \text{m/s}^2, \quad m = 28\ \text{kg}, \quad J = 0.1\ \text{kg m}^2 \\
&\kappa_{L_0} = 0\ \text{kg/m}, \quad \kappa_L = 300\ \text{kg/m}, \quad \kappa_q = -5\ \text{Nms} \\
&\kappa_{D_0} = 10\ \text{kg/m}, \quad \kappa_D = 100\ \text{kg/m}, \quad \kappa_{M_0} = 1\ \text{Nm} \\
&\kappa_M = -40\ \text{Nm}, \quad \delta = 0.3.
\end{aligned}$$

The closed-loop system matrix in (15) can be Hurwitz by solving the following linear matrix inequalities:

$$\begin{aligned}
&PA^T + M^T B^T + AP + BM \prec 0 \\
&A^T Q + C^T N^T + QA + NC \prec 0
\end{aligned}$$

where $P := \text{blockdiag}\{P_1, P_2\} \succ 0$, $Q := \text{blockdiag}\{Q_1, Q_2\} \succ 0$, $M := \text{blockdiag}\{M_1, M_2\}$, and $N := \text{blockdiag}\{N_1, N_2\}$. In this case, $K = MP^{-1}$ and $L = Q^{-1}N$. In specific

$$\begin{aligned}
K_1 &= \begin{bmatrix} -2.1212 & -5.2727 & -2.7879 \end{bmatrix} \\
K_2 &= \begin{bmatrix} -1.3125 & -0.9375 \end{bmatrix}
\end{aligned}$$

$$L_1 = \begin{bmatrix} -2.7879 \\ -5.2727 \\ -2.1212 \end{bmatrix}, \quad L_2 = \begin{bmatrix} -0.9375 \\ -1.3125 \end{bmatrix}.$$

The following equilibria

$$\begin{aligned}
&(V_e, \phi_e, m_{0_e}, \alpha_e, \Omega_e) \\
&= (1\ \text{m/s}, -0.7854\ \text{rad}, 1.46\ \text{kg}, 0.0337\ \text{rad}, 0\ \text{rad/s})
\end{aligned}$$

at $t = 0$ and

$$\begin{aligned}
&(V_e, \phi_e, m_{0_e}, \alpha_e, \Omega_e) \\
&= (1\ \text{m/s}, 0.7854\ \text{rad}, -1.46\ \text{kg}, -0.0337\ \text{rad}, 0\ \text{rad/s})
\end{aligned}$$

at $t = 40$ are chosen, each of which corresponds to the descending and the ascending gliding, respectively. We set the initial data for (1) as

$$(V, \phi, m_0, \alpha, \Omega)(0) = (1.0204, 0.4363, 0, 0.0175, 0)$$

and for (12) as

$$(\hat{V}, \hat{\phi}, \hat{m}_0, \hat{\alpha}, \hat{\Omega})(0) = (1.0204, 0.3840, 0, 0.0197, 0).$$

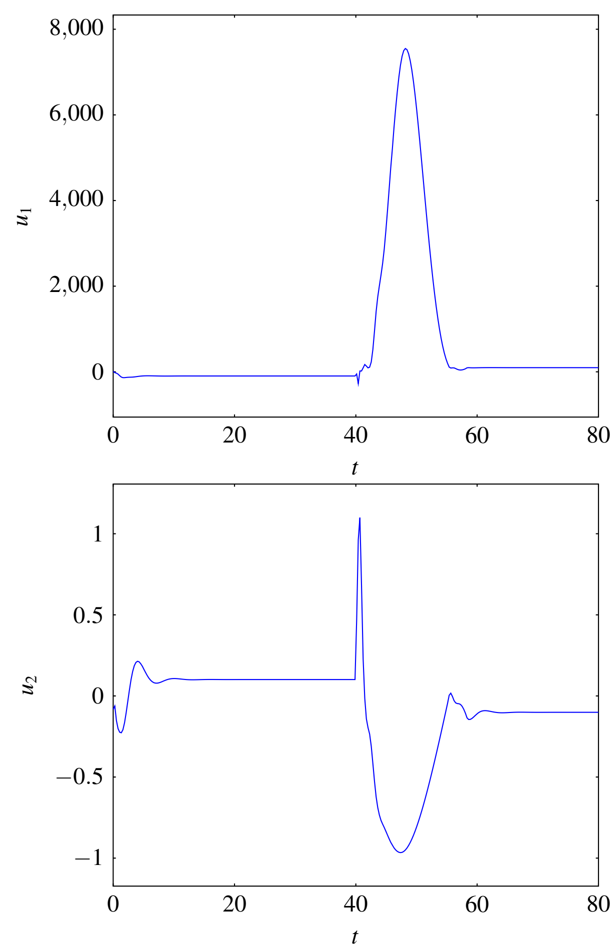

Fig. 3. Control inputs.

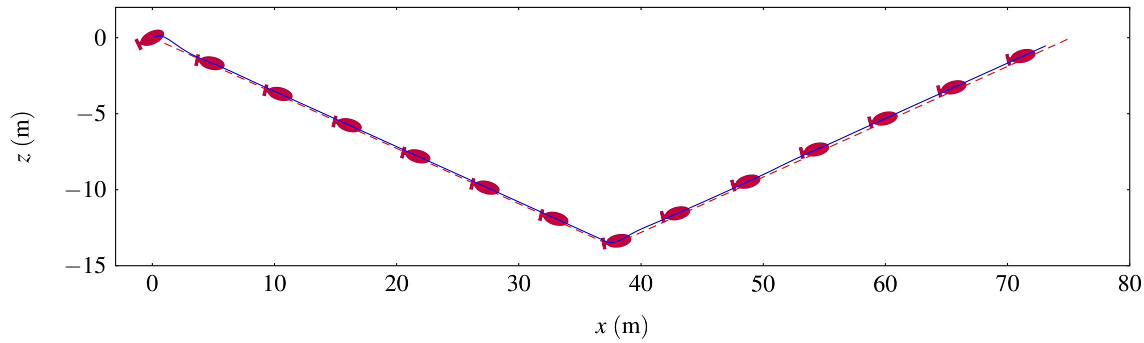

Fig. 4. Moving pathes (blue: controlled; red-dashed: on equilibrium).

The variables $\hat{m}_0$ and $\hat{\alpha}$ are replaced by their saturated versions such that our region of interest is included in the system's region of attraction. This excludes the chance that the observer suffers from peaking [15]. The time-response of (1) closed by (13) with (12) is shown in Fig. 2, from which we know that each state asymptotically converges to its equilibrium. The control inputs are depicted in Fig. 3. Fig. 4 shows that the lateral trajectory of the underwater glider controlled by the proposed controller asymptotically follows that with the equilibrium, where each underwater glider is marked every 6 s.

6. CONCLUSIONS

In this paper, the observer-based output-feedback control problem of the underwater glider by the buoyancy and the pitching moment control was discussed. The pitching moment controller is designed in such a way that the concerned fifth-order nonlinear model is partially feedback-linearized to the second-order model. The remaining nonlinear dynamics is consecutively feedback-linearized to obtain the buoyancy controller. The nonlinear observer for output feedback is designed based on the feedback-linearizated model. Numerical simulation convincingly verified our theoretical discussions.

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