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H_∞ Control for Sochastic Time-delayed Markovian Switching Systems with Partly Known Transition Rates and Input Saturation

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Abstract: The paper is concerned with the problem of H_∞ control for stochastic time-delayed Markovian switching systems with partly known transition rates and input saturation. By employing more appropriate Lyapunov-Krasovskii functional, a state feedback controller is designed to guarantee stochastic stability of the corresponding closed-loop system with H_∞ performance. A linear matrix inequality approach is employed to obtain the controller gain matrix. Two illustrative examples are provided to show the potential of the proposed techniques.

Keywords: Input saturation, Markovian switching systems, partly known transition rates, stochastic stability.

1. INTRODUCTION

As a special class of hybrid systems, the study of Markovian switching systems have drawn more and more attention such as failures and repairs of machine in manufacturing systems, packet dropout in networked control systems, etc. Due to its extensive applications, there are many achievements about Markovian switching systems [1–9]. However, most of the results for Markovian switching systems have been reported under the assumption of completely known transition rates. Due to the complicated factors, the exact values of transition rates can not be completely available. Recently, there are some pioneer works with respect to partly known transition rates covered [10–19]. To mention a few, by use of fixed-weighting matrices, the problem of continuous-time Markovian jump linear systems with partly unknown transition rates was firstly discussed in [11]. To reduce some conservativeness, the method of free-connection weighting matrices was firstly proposed in [12].

On the other hand, time delay, which often encounters in various practical systems, will cause undesirable performance and instability of dynamic system, such as chemical processes, neural networks and long transmission lines in pneumatic systems [20–26]. At the same time, as a physical phenomenon, actuator saturation often arises in practical systems due to physical constraints, which can severely degrade the performance of closed-loop system and even make a stable closed-loop system unstable. As a result, the research on actuator saturation has attracted considerable attention due to its importance from both theoretical and practical viewpoints, such as balancing pointer [27,28], cart-spring-pendulum system [27,29], F-8 aircraft system [27, 30] and RLC series circuit [18]. Saturated systems subject to random abrupt variations can be modeled as saturated Markovian jump systems. Recently, there are some results on Markovian jump systems with actuator saturation reported [15–18, 31]. The problem of stability and stabilization as well as the domain of attraction in mean square sense for Markovian jump systems subject to actuator saturation was firstly discussed in [31].

As stochastic systems play an important role in many branches of science and engineering applications, the problem of stability and stabilization analysis for stochastic systems has been an important topic of control theory. Recently, there are some papers conducted on stochastic system in [32–35]. To mention a few, when considering the exogenous disturbance, sufficient conditions for stochastic stability with H_∞ performance were proposed in terms of linear matrix inequalities in [34, 35].

However, above all, for Markovian switching system with partly known transition rates and actuator saturation, there is further room for investigation. As pointed out in [16], the Lyapunov-Krasovskii functional was given as $V(x(t),i) = x^T(t)P_i x(t) + \int_{t-\tau(t)}^{t} x^T(s)Q_1 x(s)ds$, which may have some conservativeness. And time-delay in [19] was constant, which may have some conservativeness of dynamic systems. To the best of our knowledge, most of the existing results on Markovian switching systems with partly known transition rates and actuator saturation are based on nominal system without taking the It$\hat{o}$-type stochastic disturbance and time-varying delay into account. As stochastic disturbance and time-varying de-

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lay are frequently encountered in practice, it is necessary and significant to further consider stochastic saturated Markovian switching system with those two kinds of phenomena. When taking the Itô-type stochastic disturbance and time-varying delay into account, the problem of choosing an appropriate mode-dependent co-positive Lyapunov-Krasovskii functional candidate that is different from that in the existing literatures, and how to reduce some conservativeness of Lyapunov-Krasovskii functional will be more complicated and challenging. However, up to now, no relevant work that considers this kind of system has been published, which motivates our investigation.

In this paper, the problem of H_∞ control for stochastic time-delayed Markovian switching systems with partly known transition rates and input saturation is addressed. The main contributions of this paper include: (i) By constructing appropriate Lyapunov-Krasovskii functional and some free-weighting matrices, sufficient conditions for stochastic stability of the close-loop time-delay system with partly known transition rates and input saturation are proposed; (ii) Based on the results in (i), H_∞ performance is analyzed; (iii) A state feedback controller is designed to ensure the closed-loop system with H_∞ performance stochastically stable.

Notations: Throughout this paper, N^T represents the transpose of N. Symbol $\mathbb{R}^n$ stands for the n dimensional Euclidean space, $\mathbb{R}^{n\times m}$ is the set of $n\times m$ real matrices, and $S=\{1,2,\ldots,N\}$ means a set of positive numbers. The superscript I denotes identity matrix with appropriate dimensions. Given a probability space (Ξ,Υ,Θ), Ξ is the sample space, Υ is the σ-algebra of subsets of the sample space and Θ is the probability measure on Υ. Symbol $E\{\cdot\}$ represents the mathematical expectation, $\|*\|$ means the Euclidean norm of vectors and $L_2^n[0,+\infty)$ is space of n-dimensional square integral function vector on $[0,+\infty)$. $P>0$ (≥ 0) means P is real symmetric positive (semi-positive) definite. For simplicity, symbol $*$ is represented as an ellipsis for symmetry. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. PROBLEM STATEMENT AND PRELIMINARIES

Consider the following stochastic Markovian switching system with input saturation and time-varying delay on the probability space(Ξ,Υ,Θ):

$$\begin{aligned} dx(t) =&[A(r_t)x(t)+A_d(r_t)x(t-\tau(t)) \\ &+B(r_t)sat(u(t))+G(r_t)v(t)]dt \\ &+[W(r_t)x(t)+W_d(r_t)x(t-\tau(t))]d\omega(t), \\ z(t) =&C(r_t)x(t)+D(r_t)v(t), \\ x(t+\theta) =&\varphi(\theta),\forall\theta\in[-\tau,0], \end{aligned} \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state vector; $u(t)\in\mathbb{R}^m$ is the control input; $v(t)\in\mathbb{R}^l$ is the disturbance input which belongs to $L_2^n[0,+\infty)$; $z(t)\in\mathbb{R}^q$ is the controlled out; $\omega(t)$ is a standard Wiener process; $\tau(t)$ denotes the time-varying delay which is everywhere time-differentiable and satisfies $0<\tau(t)\leq\tau$ and $\dot{\tau}(t)\leq h$ for known constants τ and h; $\varphi(\theta)$ is a vector-valued initial continuous function which is defined on interval $[-\tau,0]$; $\{r_t,t\geq 0\}$ is a time-homogeneous stochastic Markovian process with right continuous trajectories and takes values in a finite set $S=\{1,2,\ldots,N\}$ with transition rate matrix $\Pi=\{\pi_{ij}\}$, i, $j\in S$. The transition rate from mode i at time t to mode j at time $t+\Delta t$ is given by:

$$P\{r_{t+\Delta t}=j|r_t=j\}=\begin{cases}\pi_{ij}\Delta t+o(\Delta t), & i\neq j,\\ 1+\pi_{ij}\Delta t+o(\Delta t), & i=j,\end{cases}$$

where Δt, $\lim\limits_{\Delta t\to 0}(o(\Delta t)/\Delta t)=0$ and $\pi_{ij}\geq 0$,for $i\neq j$ and $\sum\limits_{j=1,i\neq j}^{N}\pi_{ij}=-\pi_{ii}$.

Throughout the paper, the transition rates are built to be partly known, that means, there are only some elements to be obtained in matrix $\Pi=\{\pi_{ij}\}$. For $\forall i\in S$, the set S^i represents $S^i=S_k^i\bigcup S_{uk}^i$, with

$$\begin{aligned} &S_k^i\triangleq\{j:\pi_{ij} is known, for j\in S\},\\ &S_{uk}^i\triangleq\{j:\pi_{ij} is unknown, for j\in S\}. \end{aligned} \tag{2}$$

Moreover, if $S^i\neq\varnothing$, it is further given by

$$S_k^i\triangleq\{k_1^i,k_2^i,\ldots,k_m^i\},1\leq m\leq N, \tag{3}$$

where $k_m^i\in S$ means the mth known transition rate of S_k^i in the ith row of the matrix Π.

The plant inputs are assumed to be bounded as follows:

$$-u_{0(i)}\leq u_{(i)}\leq u_{0(i)},\;\; u_{(i)}>0,\;\; i=1,2,\cdots,m. \tag{4}$$

In this paper, we will design the state feedback controller:

$$u(t)=K(r_t)x(t). \tag{5}$$

where $K(r_t)$ are controller gain matrices.

For $r_t=i\in S$, the matrices $A(r_t)$, $A_d(r_t)$, $B(r_t)$, $G(r_t)$, $W(r_t)$, $W_d(r_t)$, $C(r_t)$, $D(r_t)$ and $K(r_t)$ are denoted by A_i, A_{di}, B_i, G_i, W_i, W_{di}, C_i, D_i and K_i.

With the state feedback controller (5), the closed-loop system is denoted as:

$$\begin{aligned} dx(t) =& [A_ix(t)+A_{di}x(t-\tau(t)) \\ &+B_isat(K_ix(t))+G_iv(t)]dt \\ &+[W_ix(t)+W_{di}x(t-\tau(t))]d\omega(t), \\ z(t) =& C_ix(t)+D_iv(t), \\ x(t+\theta) =& \varphi(\theta),\forall\theta\in[-\tau,0]. \end{aligned} \tag{6}$$

The following definitions and lemmas will be adopted in the rest of the paper.

Definition 1 [31]**:** System (6) ($v(t)=0$) is said to stochastically stable, if for any initial mode $r_0 \in S$, and initial condition $\varphi(\theta)$, the solution $x(t,\varphi(\theta),r_0)$ satisfies

$$\lim_{T_f\to\infty} E\left\{\int_0^{T_f} x^T(t,\varphi(\theta),r_0)x(t,\varphi(\theta),r_0)dt|(\varphi(\theta),r_0)\right\} < \infty.$$

Definition 2 [16]**:** For a scalar $\gamma>0$, system (6) is said to be stochastically stable with γ-disturbance attenuation, if the following conditions are satisfied:
(i) System (6) ($v(t)=0$) is stochastically stable;
(ii) Under zero-initial condition, system (6) satisfies:

$$E\left\{\int_0^\infty z^T(t)z(t)dt\right\} \le \gamma^2 E\left\{\int_0^\infty v^T(t)v(t)dt\right\},$$

when $v(t)\neq 0$.

For any matrices $P_i>0$, an ellipsoid is defined as: $\varepsilon(P_i)=\{x(t)\in\mathbb{R}^n : x^T(t)P_ix(t)\le 1\}$.

Definition 3 [5]**:** Considering $V(x(t),i)$ as the Lyapunov-Krasovskii functional for the closed-loop system (6), we define the weak infinitesimal operator as follows:

$$\begin{aligned}&LV(x(t),i)\\&\quad=\lim_{\Delta t\to 0}\frac{1}{\Delta t}[E\{V(x(t+\Delta t),r(t+\Delta t))|x(t),r(t)=i)\}\\&\qquad -V(x(t),r(t)=i)].\end{aligned}$$

Lemma 1 [18]**:** For the matrix K_i and system (6), the appropriate matrix $L_i\in\mathbb{R}^{m\times n}$ is given, if $x(t)$ is in the set $D(u_0)$, where $D(u_0)$ is defined as follows:

$$\begin{aligned}D(u_0)=\{&x(t)\in\mathbb{R}^n; |(K_{i(k)}-L_{i(k)})x(t)|\le u_{0(k)},\\&u_{0(k)}\ge 0, k=1,2,\cdots,m\}.\end{aligned}$$

For any diagonal positive matrices $T_i\in\mathbb{R}^{m\times m}$, we drive:

$$\psi^T(u(t))T_i[\psi(u(t))-L_ix(t)]\le 0. \tag{7}$$

Lemma 2 [18]**:** For any given symmetrical positive constant matrix M with appropriate dimensions, scalar $r>0$, and vector function $x:[0,r]\to\mathbb{R}^n$ make the following integrals meaningful, then we drive the following inequality:

$$-r\int_0^r x^T(s)Mx(s)ds \le -\left(\int_0^r x(s)ds\right)^T M\int_0^r x(s)ds.$$

3. MAIN RESULTS

In this section, we will consider the problem of stochastic stability analysis, H_∞ performance analysis, state feedback controller design for system (6).

3.1. Stochastic stability analysis

In this subsection, the problem of stochastic stability for the closed-loop system (6) ($v(t)=0$) is addressed.

Theorem 1: For the given bound of the input u_0, if there exist symmetric positive definite matrices P_i, Q_1, Q_2, Q_3, symmetric matrices Q_{4i}, diagonal positive matrices T_i and appropriate matrices L_i, L_{1i}, L_{2i}, such that for $i=1,2,\ldots,N$,

$$\begin{bmatrix}\Pi_{1i} & \Xi_{1i}^T & \sqrt{\tau}\Xi_{1i}^T & \tau\Xi_{2i}^T & \Xi_{3i}^T\\ * & -P_i^{-1} & 0 & 0 & 0\\ * & * & -Q_3^{-1} & 0 & 0\\ * & * & * & -Q_2^{-1} & 0\\ * & * & * & * & -Q_3\end{bmatrix}<0. \tag{8}$$

$$P_j-Q_{4i}\le 0, j\in S_{uk}^i, j\neq i, \tag{9}$$

$$P_j-Q_{4i}\ge 0, j\in S_{uk}^i, j=i, \tag{10}$$

$$\varepsilon(P_i)\in D(u_0), \tag{11}$$

where

$$\begin{aligned}\Xi_{1i}=&[W_i\ \ W_{di}\ \ 0\ \ 0], \Xi_{2i}=[A_i+B_iK_i\ \ A_{di}\ \ 0\ \ B_i],\\ \Xi_{3i}=&[L_{1i}\ \ L_{2i}\ \ 0\ \ 0],\\ \Pi_{1i}=&\begin{bmatrix}\Pi_{1i}^{11} & \Pi_{1i}^{12} & -L_{1i} & \Pi_{1i}^{14}\\ * & \Pi_{1i}^{22} & -L_{2i} & 0\\ * & * & -Q_2 & 0\\ * & * & * & -2T_i\end{bmatrix},\\ \Pi_{1i}^{11}=&P_i(A_i+B_iK_i)+(A_i+B_iK_i)^TP_i+\sum_{j\in S_k^i}\pi_{ij}(P_j-Q_{4i})\\ &+Q_1+\tau^2Q_2+L_{1i}+L_{1i}^T,\\ \Pi_{1i}^{12}=&P_iA_{di}+L_{2i}^T-L_{1i}, \Pi_{1i}^{14}=P_iB_i+L_i^TT_i,\\ \Pi_{1i}^{22}=&-(1-h)Q_1-L_{2i}-L_{2i}^T.\end{aligned}$$

The system (6) ($v(t)=0$) with partly known transition rates is locally stochastically stable for every initial condition belong to $\varepsilon(P_i)$.

Proof: System (6) ($v(t)=0$) is rewritten as follows:

$$\begin{aligned}&dx(t)=\zeta(t)dt+\rho(t)d\omega(t),\\&z(t)=C_ix(t)+D_iv(t),\\&x(t+\theta)=\varphi(\theta), \forall\theta\in[-\tau,0],\end{aligned} \tag{12}$$

where

$$\begin{aligned}&\zeta(t)=(A_i+B_iK_i)x(t)+A_{di}x(t-\tau(t))+B_i\psi(u(t)),\\&\rho(t)=W_ix(t)+W_{di}x(t-\tau(t)),\\&\psi(u(t))=sat(u(t))-u(t).\end{aligned}$$

For system (12) ($v(t)=0$), choose a Lyapunov-Krasovskii functional candidate as

$$\begin{aligned}V(x(t),i)=&x(t)^TP_ix(t)+\int_{t-\tau(t)}^t x^T(s)Q_1x(s)ds\\&+\tau\int_{-\tau}^0\int_{t+\theta}^t \zeta^T(s)Q_2\zeta(s)dsd\theta\end{aligned}$$

$$+\int_{-\tau}^{0}\int_{t+\theta}^{t}\rho^T(s)Q_3\rho(s)dsd\theta, \tag{13}$$

where P_i, Q_1, Q_2, $Q_3 > 0$. According to definition 3, we achieve

$$dV(x(t),i) = LV(x(t),i)dt + 2x^T(t)P_i\rho(t)dw(t), \tag{14}$$

where

$$\begin{aligned}
&LV(x(t),i)\\
=&2x^T(t)P_i\zeta(t)+\rho^T(t)P_i\rho(t)+x^T(t)\sum_{j=1}^{N}\pi_{ij}P_jx(t)\\
&+x^T(t)Q_1x(t)-(1-\dot{\tau}(t))x^T(t-\tau(t))Q_1x(t-\tau(t))\\
&+\tau^2\zeta^T(t)Q_2\zeta(t)-\tau\int_{t-\tau}^{t}\zeta^T(s)Q_2\zeta(s)ds\\
&+\tau\rho^T(t)Q_3\rho(t)-\int_{t-\tau}^{t}\rho^T(s)Q_3\rho(s)ds\\
\le&2x^T(t)P_i\zeta(t)+\rho^T(t)P_i\rho(t)+x^T(t)\sum_{j=1}^{N}\pi_{ij}P_jx(t)\\
&+x^T(t)Q_1x(t)-(1-h)x^T(t-\tau(t))Q_1x(t-\tau(t))\\
&+\tau^2\zeta^T(t)Q_2\zeta(t)-\tau\int_{t-\tau(t)}^{t}\zeta^T(s)Q_2\zeta(s)ds\\
&+\tau\rho^T(t)Q_3\rho(t)-\int_{t-\tau(t)}^{t}\rho^T(s)Q_3\rho(s)ds.
\end{aligned}\tag{15}$$

Based on Lemma 2, it is clear that

$$\begin{aligned}
&-\tau\int_{t-\tau}^{t}\zeta^T(s)Q_2\zeta(s)ds\\
\le&-\left(\int_{t-\tau}^{t}\zeta(s)ds\right)^T Q_2\int_{t-\tau}^{t}\zeta(s)ds\\
\le&-\left(\int_{t-\tau(t)}^{t}\zeta(s)ds\right)^T Q_2\int_{t-\tau(t)}^{t}\zeta(s)ds.
\end{aligned}\tag{16}$$

Let us consider the following equality which comes from system (12):

$$\begin{aligned}
&2[x^T(t)L_{1i}+x^T(t-\tau(t))L_{2i}][x(t)-x(t-\tau(t))\\
&-\int_{t-\tau(t)}^{t}\zeta(s)ds-\int_{t-\tau(t)}^{t}\rho(s)dw(s)]=0,
\end{aligned}\tag{17}$$

where L_{1i}, L_{2i} are matrices with appropriate dimensions and for each $i\in S$, we have

$$\begin{aligned}
&-2\left[x^T(t)L_{1i}+x^T(t-\tau(t))L_{2i}\right]\int_{t-\tau(t)}^{t}\rho(s)dw(s)\\
\le&\left[\int_{t-\tau(t)}^{t}\rho(s)dw(s)\right]^T Q_3\left[\int_{t-\tau(t)}^{t}\rho(s)dw(s)\right]\\
&+\left[x^T(t)L_{1i}+x^T(t-\tau(t))L_{2i}\right]Q_3^{-1}\\
&\left[x^T(t)L_{1i}+x^T(t-\tau(t))L_{2i}\right]^T.
\end{aligned}\tag{18}$$

By employing it$\hat{o}$ formula, we drive

$$\begin{aligned}
&E\left\{\left[\int_{t-\tau(t)}^{t}\rho(s)dw(s)\right]^T Q_3\left[\int_{t-\tau(t)}^{t}\rho(s)dw(s)\right]\right\}\\
=&E\left\{\int_{t-\tau(t)}^{t}\rho^T(s)Q_3\rho(s)ds\right\}.
\end{aligned}\tag{19}$$

Since $\sum_{j=1}^{N}\pi_{ij}=0$, there exists

$$\sum_{j=1}^{N}\pi_{ij}Q_{4i}=0, Q_{4i}=Q_{4i}^T>0. \tag{20}$$

It follows from (7), (15)-(20) that

$$E\{LV(x(t),i)\}\le\xi^T(t)\Pi_{3i}\xi(t), \tag{21}$$

where

$$\begin{aligned}
\xi(t)=&\left[x^T(t)\ \ x^T(t-\tau(t))\ \ \int_{t-\tau(t)}^{t}\zeta^T(s)ds\ \ \psi^T(u(t))\right]^T,\\
\Pi_{3i}=&\Pi_{2i}+\Xi_{1i}^T(P_i+\tau Q_3)\Xi_{1i}+\tau^2\Xi_{2i}^TQ_2\Xi_{2i}+\Xi_{3i}^TQ_3^{-1}\Xi_{3i},\\
\Pi_{4i}=&\begin{bmatrix}\Pi_{5i} & \Pi_{1i}^{12} & -L_{1i} & \Pi_{1i}^{14}\\ * & \Pi_{1i}^{22} & -L_{2i} & 0\\ * & * & -Q_2 & 0\\ * & * & * & -2T_i\end{bmatrix},\\
\Pi_{5i}=&P_i(A_i+B_iK_i)+(A_i+B_iK_i)^TP_i+\sum_{j\in S_k^i}\pi_{ij}(P_j-Q_{4i})\\
&+\sum_{j\in S_{uk}^i}\pi_{ij}(P_j-Q_{4i})+Q_1+\tau^2Q_2+L_{1i}+L_{1i}^T,
\end{aligned}$$

where Ξ_{1i}, Ξ_{2i} and Ξ_{3i} described in Theorem 1.

Applying the Schur complements to Π_{3i}, Π_{3i} is equivalent to

$$\Pi_{6i}=\begin{bmatrix}\Pi_{4i} & \Xi_{1i}^T & \sqrt{\tau}\Xi_{1i}^T & \tau\Xi_{2i}^T & \Xi_{3i}^T\\ * & -P_i^{-1} & 0 & 0 & 0\\ * & * & -Q_3^{-1} & 0 & 0\\ * & * & * & -Q_2^{-1} & 0\\ * & * & * & * & -Q_3\end{bmatrix}<0. \tag{22}$$

Notice that if $i\in S_k^i$, we can get $\Pi_{3i}<0$ by inequalities (8)-(9) and the fact $\pi_{ij}\ge 0$ $(\forall i,j\in S, i\ne j)$. On the other hand, if $\forall i\in S_{uk}^i$, from the fact $\pi_{ii}=-\sum_{j=1,i\ne j}^{N}\pi_{ij}<0$ and inequalities (8)$-$(10), we can also get $\Pi_{3i}<0$. Therefore, we have

$$LV(x(t),i)<0. \tag{23}$$

Therefore,

$$\lim_{T_f\to\infty}E\left\{\int_0^{T_f}x^T(t,\varphi(\theta),r_0)x(t,\varphi(\theta),r_0)dt|(\varphi(\theta),r_0)\right\}<\infty.$$

From condition (11), it follows that if $x(t) \in \varepsilon(P_i)$, then $\varepsilon(P_i) \in D(u_0)$.

Above all, system (6) ($v(t) = 0$) with partly known transition rates is locally stochastically stable for every initial condition belong to $\varepsilon(P_i)$. The proof is completed. □

Remark 1: Compared with the Lyapunov-Krasovskii functional in the paper [16] $V(x(t),i) = x^T(t)P_ix(t) + \int_{t-\tau(t)}^{t} x^T(s)Q_1x(s)ds$, the Lyapunov-Krasovskii functional in this paper is chosen as (13), which may reduce some conservativeness.

Remark 2: If $S^i_{uk} = \varnothing$, then system (6) becomes Markovian switching system with completely known transition probabilities. Therefore, we have the following corollaries.

Corollary 1: For the given bound of the input u_0, if there exist symmetric positive definite matrices P_i, Q_1, Q_2, Q_3, diagonal positive matrices T_i and appropriate matrices L_i, L_{1i}, L_{2i}, such that for $i = 1,2,\dots,N$, the condition (11) and the following inequality holds,

$$\begin{bmatrix} \tilde{\Pi}_{1i} & \Xi^T_{1i} & \sqrt{\tau}\Xi^T_{1i} & \tau\Xi^T_{2i} & \Xi^T_{3i} \\ * & -P_i^{-1} & 0 & 0 & 0 \\ * & * & -Q_3^{-1} & 0 & 0 \\ * & * & * & -Q_2^{-1} & 0 \\ * & * & * & * & -Q_3 \end{bmatrix} < 0,$$

where

$$\tilde{\Pi}_{1i} = \begin{bmatrix} \tilde{\Pi}^{11}_{1i} & \Pi^{12}_{1i} & -L_{1i} & \Pi^{14}_{1i} \\ * & \Pi^{22}_{1i} & -L_{2i} & 0 \\ * & * & -Q_2 & 0 \\ * & * & * & -2T_i \end{bmatrix},$$

$$\tilde{\Pi}^{11}_{1i} = P_i(A_i + B_iK_i) + (A_i + B_iK_i)^T P_i + \sum_{j=1}^{N} \pi_{ij}P_j + Q_1 + \tau^2 Q_2 + L_{1i} + L^T_{1i},$$

where Ξ_{1i}, Ξ_{2i}, Ξ_{3i}, Π^{12}_{1i}, Π^{14}_{1i}, Π^{22}_{1i} described in Theorem 1, the system (6) ($v(t) = 0$) with completely known transition rates is locally stochastically stable for every initial condition belong to $\varepsilon(P_i)$.

3.2. H_∞ performance analysis

In this subsection, we will consider the problem of H_∞ performance for the system (6).

Theorem 2: For the given bound of the input u_0 and constant γ, if there exist symmetric positive definite matrices P_i, Q_1, Q_2, Q_3, symmetric matrices Q_{4i}, diagonal positive matrices T_i and appropriate matrices L_i, L_{1i}, L_{2i}, such that for $i = 1,2,\dots,N$, the conditions (9)–(11) and the following inequality holds

$$\begin{bmatrix} \Pi_{7i} & \Xi^T_{4i} & \sqrt{\tau}\Xi^T_{4i} & \tau\Xi^T_{5i} & \Xi^T_{6i} \\ * & -P_i^{-1} & 0 & 0 & 0 \\ * & * & -Q_3^{-1} & 0 & 0 \\ * & * & * & -Q_2^{-1} & 0 \\ * & * & * & * & -Q_3 \end{bmatrix} < 0, \quad (24)$$

where

$$\Pi_{7i} = \begin{bmatrix} \Pi_{8i} & \Pi^{12}_{1i} & -L_{1i} & \Pi^{14}_{1i} & P_iG_i & C^T_i \\ * & \Pi^{22}_{1i} & -L_{2i} & 0 & 0 & 0 \\ * & * & -Q_2 & 0 & 0 & 0 \\ * & * & * & -2T_i & 0 & 0 \\ * & * & * & * & -\gamma^2 & D^T_i \\ * & * & * & * & * & -I \end{bmatrix},$$

$$\Pi_{8i} = P_i(A_i + B_iK_i) + (A_i + B_iK_i)^T P_i + \sum_{j\in S^i_k} \pi_{ij}(P_j - Q_{4i}) + Q_1 + \tau^2 Q_2 + L_{1i} + L^T_{1i},$$

$$\Xi_{4i} = [W_i \ \ W_{di} \ \ 0 \ \ 0 \ \ 0 \ \ 0],$$
$$\Xi_{5i} = [A_i + B_iK_i \ \ A_{di} \ \ 0 \ \ B_i \ \ G_i \ \ 0],$$
$$\Xi_{6i} = [L_{1i} \ \ L_{2i} \ \ 0 \ \ 0 \ \ 0 \ \ 0].$$

with Π^{12}_{1i}, Π^{14}_{1i}, Π^{22}_{1i} described in Theorem (1), the system (6) with partly known transition rates is locally stochastically stable with γ-disturbance attenuation for every initial condition belong to $\varepsilon(P_i)$.

Proof: From condition (24), it is easy to see that inequality (8) holds. Based on Theorem 1, system (6) ($v(t) = 0$) is stochastically stable.

For system (6) with Lyapunov-Krasovskii functional candidate (13), it follows from (7), (15)-(20) that

$$z^T(t)z(t) - \gamma^2 v^T(t)v(t) + LV(x(t),i) \le \Psi_1\Pi_{9i}\Psi^T_1, \quad (25)$$

where

$$\Psi_1 = [x^T(t) \ \ x^T(t-\tau(t)) \ \ \int_{t-\tau(t)}^{t} \zeta^T(s)ds \ \ \psi^T(u(t)) \ \ v^T(t)],$$

$$\Pi_{9i} = \Pi_{10i} + \Xi^T_{4i}(P_i + \tau Q_3)\Xi_{4i} + \tau^2\Xi^T_{5i}Q_2\Xi_{5i} + \Xi^T_{6i}Q_3^{-1}\Xi_{6i} + \Xi^T_{7i}\Xi_{7i},$$

$$\Pi_{10i} = \begin{bmatrix} \Pi_{11i} & \Pi^{12}_{1i} & -L_{1i} & \Pi^{14}_{1i} & P_iG_i \\ * & \Pi^{22}_{1i} & -L_{2i} & 0 & 0 \\ * & * & -Q_2 & 0 & 0 \\ * & * & * & -2T_i & 0 \\ * & * & * & * & -\gamma^2 \end{bmatrix},$$

$$\Pi_{11i} = P_i(A_i + B_iK_i) + (A_i + B_iK_i)^T P_i + \sum_{j\in S^i_k} \pi_{ij}(P_j - Q_{4i}) + Q_1 + \tau^2 Q_2 + L_{1i} + L^T_{1i},$$

$$\Xi_{4i} = [W_i \ \ W_{di} \ \ 0 \ \ 0 \ \ 0], \Xi_{5i} = [A_i + B_iK_i \ \ A_{di} \ \ 0 \ \ B_i \ \ G_i],$$
$$\Xi_{6i} = [L_{1i} \ \ L_{2i} \ \ 0 \ \ 0 \ \ 0], \Xi_{7i} = [C^T_i \ \ 0 \ \ 0 \ \ 0 \ \ D^T_i].$$

Applying the Schur complement to Π_{9i}, the inequality (24) holds .

Therefore, the inequalities (8)-(10) and (24) imply

$$z^T(t)z(t)-\gamma^2 v^T(t)v(t)+LV(x(t),i)<0. \quad (26)$$

which means

$$E\left\{\int_0^\infty z^T(t)z(t)dt\right\}\le\gamma^2 E\left\{\int_0^\infty v^T(t)v(t)dt\right\}. \quad (27)$$

That completes the proof of Theorem 2. □

3.3. State feedback controller design

In this subsection, we will consider the problem of state feedback controller design for the system (6).

Theorem 3: For the given bound of the input u_0 and constant γ, if there exist symmetric positive definite matrices X_i, U_{1i}, U_{2i}, U_2, U_3, symmetric matrices V_i, diagonal positive matrices S_i and appropriate matrices Z_i, U_{4i}, U_{5i}, Y_i, such that for $i=1,2,\dots,N$, $k=1,2,\dots,m$,

$$\begin{bmatrix}\Pi_{12i} & \Xi_{8i}^T & \sqrt{\tau}\Xi_{8i}^T & \tau\Xi_{9i}^T & \Xi_{10i}^T & \Pi_{13i}\\ * & -X_i & 0 & 0 & 0 & 0\\ * & * & -U_3 & 0 & 0 & 0\\ * & * & * & -U_2 & 0 & 0\\ * & * & * & * & -X_i & 0\\ * & * & * & * & * & -\Pi_{14i}\end{bmatrix}<0, \quad (28)$$

$$\begin{bmatrix}\tilde\Pi_{12i} & \Xi_{8i}^T & \sqrt{\tau}\Xi_{8i}^T & \tau\Xi_{9i}^T & \Xi_{10i}^T & \tilde\Pi_{13i}\\ * & -X_i & 0 & 0 & 0 & 0\\ * & * & -U_3 & 0 & 0 & 0\\ * & * & * & -U_2 & 0 & 0\\ * & * & * & * & -X_i & 0\\ * & * & * & * & * & -\tilde\Pi_{14i}\end{bmatrix}<0, \quad (29)$$

$$\begin{bmatrix}-V_i & X_i\\ * & -X_j\end{bmatrix}\le 0, j\in S_{uk}^i, j\ne i, \quad (30)$$

$$X_j-V_j\ge 0, j\in S_{uk}^i, j=i, \quad (31)$$

$$U_2-X_i\ge 0, \quad (32)$$

$$U_3-X_i\ge 0, \quad (33)$$

$$\begin{bmatrix}X_i & Y_{i(k)}^T-Z_{i(k)}^T\\ * & u_{0(k)}^2\end{bmatrix}>0, \quad (34)$$

where

$$\Pi_{12i}=\begin{bmatrix}\Pi_{15i} & \Pi_{2i}^{12} & -U_{4i} & \Pi_{2i}^{14} & G_i & X_iC_i^T\\ * & \Pi_{2i}^{22} & -U_{5i} & 0 & 0 & 0\\ * & * & -Q_2 & 0 & 0 & 0\\ * & * & * & -2S_i & 0 & 0\\ * & * & * & * & -\gamma^2 & D_i^T\\ * & * & * & * & * & -I\end{bmatrix}$$

$$\Pi_{15i}=A_iX_i+B_iY_i+X_iA_i^T+Y_i^TB_i^T-\sum_{j\in S_k^i}\pi_{ij}V_i+\pi_{ii}X_i$$
$$+U_{1i}+\tau^2U_{2i}+U_{4i}+U_{4i}^T,$$

$$\Xi_{8i}=[W_iX_i\ \ W_{di}X_i\ \ 0\ \ 0\ \ 0\ \ 0],$$
$$\Xi_{9i}=[A_iX_i+B_iY_i\ \ A_{di}X_i\ \ 0\ \ B_iS_i\ \ G_i\ \ 0],$$
$$\Xi_{10i}=[U_{4i}\ \ U_{5i}\ \ 0\ \ 0\ \ 0\ \ 0],$$
$$\Pi_{13i}=\left[\sqrt{\pi_{ik_1^i}}X_i,\dots,\sqrt{\pi_{ik_{r-1}^i}}X_i,\sqrt{\pi_{ik_{r+1}^i}}X_i,\dots,\sqrt{\pi_{ik_m^i}}X_i\right],$$
$$\Pi_{14i}=diag\left\{X_{k_1^i},\dots,X_{k_{r-1}^i},X_{k_{r+1}^i},\dots,X_{k_m^i}\right\},$$

$$\tilde\Pi_{12i}=\begin{bmatrix}\tilde\Pi_{15i} & \Pi_{2i}^{12} & -U_{4i} & \Pi_{2i}^{14} & G_i & X_iC_i^T\\ * & \Pi_{2i}^{22} & -U_{5i} & 0 & 0 & 0\\ * & * & -X_i & 0 & 0 & 0\\ * & * & * & -2S_i & 0 & 0\\ * & * & * & * & -\gamma^2 & D_i^T\\ * & * & * & * & * & -I\end{bmatrix},$$

$$\tilde\Pi_{15i}=A_iX_i+B_iY_i+X_iA_i^T+Y_i^TB_i^T-\sum_{j\in S_k^i}\pi_{ij}V_i$$
$$+U_{1i}+\tau^2U_{2i}+U_{4i}+U_{4i}^T,$$
$$\tilde\Pi_{13i}=\left[\sqrt{\pi_{ik_1^i}}X_i,\dots,\sqrt{\pi_{ik_m^i}}X_i\right],$$
$$\tilde\Pi_{14i}=diag\left\{X_{k_1^i},\dots,X_{k_m^i}\right\},\Pi_{2i}^{12}=A_{di}X_i+U_{5i}^T-U_{4i},$$
$$\Pi_{2i}^{14}=B_iS_i+Z_i^T,\Pi_{2i}^{22}=-(1-h)U_{1i}-U_{5i}-U_{5i}^T,$$

with $(K_{i(k)},Y_{i(k)})$ denoted the kth row of (K_i,Y_i), $(k_1^i,k_2^i,\dots,k_m^i)$ described in (3) and $k_r^i=i$, the system (6) with partly known transition rates is locally stochastically stable with γ-disturbance attenuation for every initial condition belong to $\varepsilon(P_i)$. Moreover, state feedback controller gain matrices are given by $K_i=Y_iX_i^{-1}$.

Proof: Let

$$X_i=P_i^{-1},Y_i=K_iX_i,U_{1i}=X_iQ_1X_i,U_2=Q_2^{-1},$$
$$U_3=Q_3^{-1},U_{4i}=X_iL_{1i}X_i,U_{5i}=X_iL_{2i}X_i,$$
$$V_i=X_iQ_{4i}X_i,S_i=T_i^{-1},Z_i=X_iL_i,U_{2i}=X_iQX_i. \quad (35)$$

Based on LMIs (32) and (33), it is obtained that $-Q_2\le -P_i$ and $-Q_3\le -P_i$, then we replace $-Q_2$ and $-Q_3$ by $-P_i$ in inequality (24). Pre- and post-multiplying inequality (24) by $diag\{X_i,X_i,X_i,S_i,I,I,I,I,I,X_i\}$, inequality (24) is equivalent to

$$\begin{bmatrix}\Pi_{16i} & \Xi_{8i}^T & \sqrt{\tau}\Xi_{8i}^T & \tau\Xi_{9i}^T & \Xi_{10i}^T\\ * & -X_i & 0 & 0 & 0\\ * & * & -U_3 & 0 & 0\\ * & * & * & -U_2 & 0\\ * & * & * & * & -X_i\end{bmatrix}<0, \quad (36)$$

where

$$\Pi_{16i}=\begin{bmatrix}\Pi_{17i} & \Pi_{2i}^{12} & -U_{4i} & \Pi_{2i}^{14} & G_i & X_iC_i^T\\ * & \Pi_{2i}^{22} & -U_{5i} & 0 & 0 & 0\\ * & * & -X_i & 0 & 0 & 0\\ * & * & * & -2S_i & 0 & 0\\ * & * & * & * & -\gamma^2 & D_i^T\\ * & * & * & * & * & -I\end{bmatrix},$$

$$\Pi_{17i} = A_iX_i + B_iY_i + X_iA_i^T + Y_i^TB_i^T + U_{4i} + U_{4i}^T + U_{1i} + \tau^2U_{2i} + \sum_{j\in S_k^i} \pi_{ij}(X_iX_j^{-1}X_i - V_i),$$

with Π_{2i}^{12}, Π_{2i}^{14}, Π_{2i}^{22}, Ξ_{8i}, Ξ_{9i}, Ξ_{10i} described in Theorem 3.

As $\pi_{ii} < 0, \forall i \in S$, the inequality (36) is discussed in the following two cases.

Case 1. For $i \in S_k^i$, applying Schur complement lemma, the inequality (28) holds.

Case 2. For $i \notin S_k^i$, applying Schur complement lemma, the inequality (29) holds.

Pre- and post-multiplying inequality (9) by X_i, inequality (30) holds. Pre- and post-multiplying inequality (10) by X_i, the inequality (31) holds.

Applying Schur complement and Lemma 1 to the constraint (10), the inequality (34) holds. □

Remark 3: In Theorem 3, we can take γ^2 as the optimized variable to obtain an optimized state feedback controller. The attenuation level ρ can be reduced to the minimum possible value that conditions (28)-(34) hold. The optimization problem can be described as follows:

$$\begin{array}{ll} \min & \rho \\ & X_i, U_{1i}, U_{2i}, U_2, U_3, \gamma, > 0, V_i, Y_i, Z_i, U_{4i}, U_{5i} \\ s.t. & Inequalities\ (28)-(34)\ with\ \rho = \gamma^2. \end{array} \quad (37)$$

4. NUMERICAL EXAMPLES

Example 1: Consider four-mode Markovian switching system with the following parameters:

$$A_1 = \begin{bmatrix} 2.5 & 0.5 \\ 0.1 & -1 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 1 \\ 0.5 & -2 \end{bmatrix}, A_3 = \begin{bmatrix} 1 & -1 \\ 0 & -4 \end{bmatrix},$$
$$A_4 = \begin{bmatrix} -5 & 0.1 \\ 3 & -1 \end{bmatrix}, A_{d1} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.2 \end{bmatrix}, A_{d2} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$A_{d3} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.1 \end{bmatrix}, A_{d4} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.4 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, B_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, B_4 = \begin{bmatrix} 2 \\ 2 \end{bmatrix},$$
$$W_1 = W_2 = W_3 = W_4 = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix},$$
$$W_{d1} = W_{d2} = W_{d3} = W_{d4} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$G_1 = G_2 = G_3 = G_4 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, C_1 = C_2 = C_3 = C_4 = \begin{bmatrix} 1 & 1 \end{bmatrix},$$
$$D_1 = D_2 = D_3 = D_4 = 1, x(0) = \begin{bmatrix} -0.2 & 0.1 \end{bmatrix}^T,$$
$$x(t) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T, t \in [-\tau, 0), r_0 = 2, u_0 = 2.$$

Let $\tau(t) = 0.5(1 - \sin(t))$, then $\tau = 1$, $h = 0.5$. The transition rate matrix is given as follows:

$$\begin{bmatrix} -1 & ? & ? & 0.2 \\ ? & ? & 0.2 & 0.6 \\ 0.5 & ? & -1.2 & ? \\ 0.5 & ? & 0.9 & ? \end{bmatrix}$$

Solving the optimization problem (37), we get $\gamma_{min} = 2.5215$ and controller gain matrices:

$$K_1 = \begin{bmatrix} -234.6995 & -21.6135 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -17.4077 & -5.4677 \end{bmatrix},$$
$$K_3 = \begin{bmatrix} -10.4304 & -0.7101 \end{bmatrix},$$
$$K_4 = \begin{bmatrix} -26.9337 & -3.9614 \end{bmatrix}.$$

Figs. 1-3 stand for system mode $r(t)$, state trajectory $x(t)$ and controlled out $z(t)$ of the closed-loop time-delayed system with partly known transition rates and input saturation. It is easily seen that the closed-loop stochastic time-delayed Markovian switching system with with partly known transition rates and input saturation is stochastically stable with γ-disturbance attenuation.

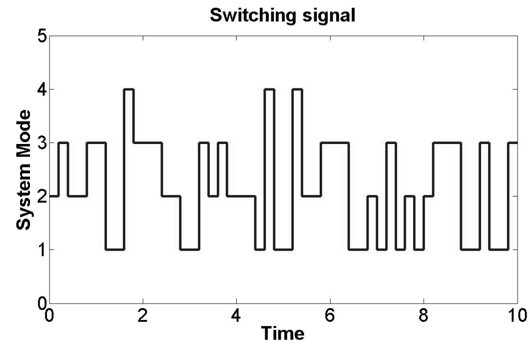

Fig. 1. System mode $r(t)$ of Example 1.

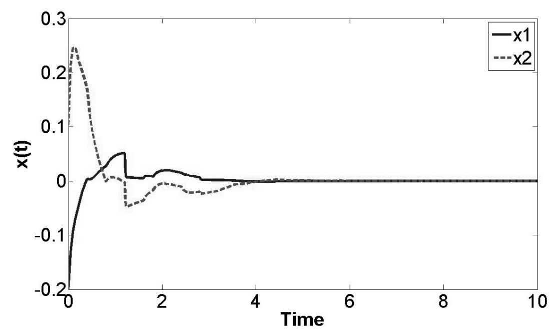

Fig. 2. State trajectory $x(t)$ of Example 1.

Remark 4: In Example 1, if the Lyapunov-Krasovskii functional is chosen as $V(x(t),i) = x^T(t)P_ix(t) + \int_{t-\tau(t)}^{t} x^T(s)$ $Q_1x(s)ds$, we can not get the state feedback controller

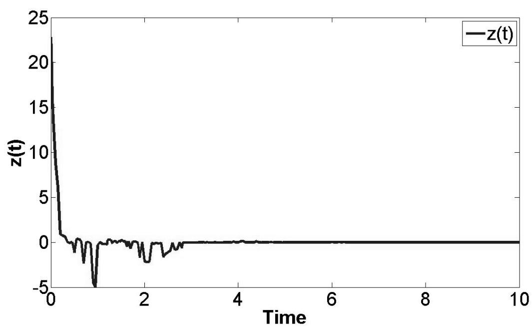

Fig. 3. Controlled out $z(t)$ of Example 1.

gain matrices, which means that the obtained results in this paper reduce some conservativeness.

Remark 5: If the completely known transition rate matrix is given as follows:

$$\begin{bmatrix} -1 & 0.5 & 0.3 & 0.2 \\ 0.7 & -1.5 & 0.2 & 0.6 \\ 0.5 & 0.4 & -1.2 & 0.3 \\ 0.5 & 0.4 & 0.9 & -1.8 \end{bmatrix}.$$

Solving the optimization problem (37), we get $\gamma_{min} = 1.5245$. From $\gamma_{min} = 1.5245$, we can easily see the completely known transition rates reduce the disturbance attenuation.

Example 2: Considering a single-link robot arm in [19], we describe the dynamic system as (1). In the dynamic system, the state variables are the angle position of the arm $x_1(t)$, the angle position of the arm change rate $x_2(t)$. Now, we assume that there exists the noise signal $v(t)$ which belongs to $L_2^n[0,+\infty)$ and Itô-type stochastic disturbance $\omega(t)$ which belongs to a standard Wiener process. The following parameters are given as follows:

$$A_i = \begin{bmatrix} 0 & 1 \\ -gl & -\frac{2}{J(i)} \end{bmatrix}, B_i = \begin{bmatrix} 0 \\ \frac{1}{J(i)} \end{bmatrix}, G_i = \begin{bmatrix} 0 \\ \frac{1}{J(i)} \end{bmatrix},$$
$$W_i = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, C_i = \begin{bmatrix} 1 & 1 \end{bmatrix}, D_i = 0,$$
$$x(0) = \begin{bmatrix} -0.5 & 1 \end{bmatrix}^T, x(t) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T, t \in [-\tau, 0),$$
$$r_0 = 2, u_0 = 16, J(1) = 1, J(2) = 5, J(3) = 10, J(4) = 15,$$
$$g = 9.80, l = 0.5, i = 1,2,3,4,$$

where g is the acceleration of gravity, l is the length of the arm, $J(i)$ depends on the jump mode i.

We consider the other parameters as follows:

$$A_{di} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, W_{di} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}.$$

Let $\tau(t) = 1.6(1-\sin(t))$, then $\tau = 3.2$, $h = 1.6$. The transition rate matrix is given as follows:

$$\begin{bmatrix} -1 & ? & ? & 0.2 \\ ? & ? & 0.2 & 0.6 \\ 0.5 & ? & -1.2 & ? \\ 0.5 & ? & 0.9 & ? \end{bmatrix}$$

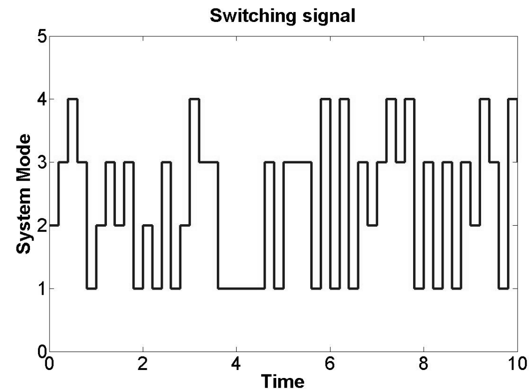

Fig. 4. System mode $r(t)$ of Example 2.

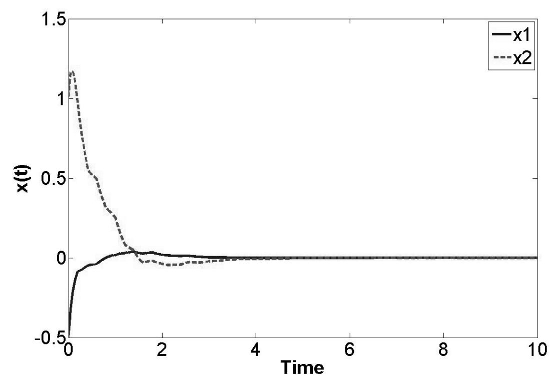

Fig. 5. State trajectory $x(t)$ of Example 2.

Solving the optimization problem (37), we get $\gamma_{min} = 1.2158$ and controller gain matrices:

$$K_1 = \begin{bmatrix} -10.1058 & -4.5189 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -5.1258 & -0.4578 \end{bmatrix},$$
$$K_3 = \begin{bmatrix} 25.0878 & 0.7951 \end{bmatrix},$$
$$K_4 = \begin{bmatrix} -4.1249 & -0.8795 \end{bmatrix}.$$

Figs, 4-5 stand for system mode $r(t)$ and state trajectory $x(t)$ of the closed-loop time-delay system with partly known transition rates and input saturation. It is easily seen that the closed-loop stochastic time-delayed Markovian switching system with partly known transition rates and input saturation is stochastically stable with γ-disturbance attenuation.

Remark 6: In Example 2, if the Lyapunov-Krasovskii functional is chosen as $V(x(t),i) = x^T(t)P_i x(t) + \int_{t-\tau(t)}^{t}$

$x^T(s)Q_1x(s)ds$, the state feedback controller gain matrices can not be obtained, which means that the proposed results in this paper may reduce some conservativeness.

5. CONCLUSIONS

In this paper, we have given an approach design of H_∞ state feedback controller for stochastic time-delayed Markovian switching system with partly known transition rates and input saturation. Such a problem has been proposed in an optimization problem with LMI conditions. Simulations are given to demonstrate the validity of the main results.

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