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Invited Paper

A Survey on Markovian Jump Systems: Modeling and Design

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Abstract: Markovian jump systems are a special class of hybrid and stochastic systems which can be used to describe many real world applications, such as manufacturing systems, power systems, chemical systems, economic systems, communication and control, etc. In this paper, a survey on recent developments of modeling, analysis and design of Markovian jump systems is presented. First, stability issues on Markovian jump systems are addressed. Then a variety of control and filter design methods are systematically recalled. Furthermore, the new trends of Markovian jump systems with uncertain transition rates as well as semi-Markovian jump systems are also discussed.

Keywords: Control and filtering, Markovian jump system, semi-Markovian jump system, stability, stabilization.

1. INTRODUCTION

Markovian jump systems (MJSs) are a special class of parameter-switching systems, and they are modeled by a set of linear or nonlinear systems with the transitions between the models determined by a Markov chain taking values in a finite set [1]. Some of the earliest works with these features include [2-6]. MJSs can also be considered as special case of switched hybrid systems with the switching signals governed by a Markovian chain. From a mathematical point of view, MJSs can be regarded as a special class of stochastic systems with system matrices changing randomly at discrete-time points governed by a Markov process and remaining time-invariant between random jumps. Over the past decades, a great amount of attention has been paid to MJSs, due their wide applications in practical systems.

Applications of MJSs can be found in many real world applications, such as economic systems [7-9], flight systems [10], power systems [11-13], communication systems [14] and networked control systems [15,16]. In the following, we will give a brief exposition of some selected topics regarding applications of MJSs.

Economics: The economy model has been studied in [7] which assumed that the state of the economy could be roughly lumped into three possible operation modes ("normal", "boom", and "slump") and that the switching between them could be modeled as a homogeneous Markov chain. The subsequent problem considered in [7] corresponds to MJSs version of the optimal linear quadratic control setup.

Flight systems: MJSs have been employed in [10] for the stability analysis of controlled flight systems whose on-board electronic devices fails randomly under external disturbances such as lightning, thermal noise, and radio signals. The accumulative effect of these disturbances are modeled as a Markov process, rendering the arrival of new disturbances as a Poisson process with exponentially distributed sojourn times.

Power systems: The modeling and control of power systems subject to Markov jumps has been addressed in [12] which uses the switching mechanism to model random load changes, generating unit outages and transmission line faults. The theoretic findings have been applied to solve the problem of dynamic security assessment that determines whether certain parameters of the electrical system will remain within a safe region of operation at a given period. A security measure has been defined to quantify the vulnerability of the current system state and network topology to two types of stochastic contingency events: primary ones, driven by a continuous-time Markov process taking values in a finite set, and secondary ones, modeled by state-dependent jumps. Recent advances in power systems control using decentralized design methods and the S-procedure have also been reported in [11,13].

Communication systems: Applying MJSs in communication systems modeling is by now another promising trend boldly that is motivated, in discrete-time by the connection between MJSs and the Gilbert-Elliott model for burst communication channels [14], which in its simplest form corresponds to a two-state Markov chain. A convenient feature of these models (relative

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simplicity aside) is their ability to describe the fact that eventual packet losses typically occur during intervals of time. As pointed out in [14], large packet-loss rates imply poor performance or even instability, and controllers implemented within a network may provide considerably better results if their design takes into account the probabilistic nature of the network.

Networked control systems: With packet loss rate known and constant, networked control systems have been formulated as MJSs in [16] with two operation modes. The approach in [15] considers that sudden variations, modeled by MJSs, occur in the network due to, e.g., mobility and topological variations. The consideration of time delay in the underlying model is of major importance, owing to the time that packets must wait on a queue before being processed, together with the fact that the network nodes are geographically separated. Furthermore, this latter constraint motivated the consideration of a decentralized control scheme. The ultimate problem considered in [15] was the minimization of the worst-case queuing length, with the aid of H_∞ control method.

In recent years, to ease the practical application of MJSs, considerable efforts have been made, and a lot of progresses have been made on topics such as: 1) modeling of MJSs; 2) stability and performance analysis; 3) control and filtering; 4) fault detection and fault tolerance; 5) identification via networks; and so on. In this survey, we will recall recent development on Markovian jump systems, paying particular attention to those carried out since the publishing dates of the surveys referred to. The interested readers may go to the original source for the full development of the ideas involved. We attempt to present all the important results on all the aspects here but not emphasize their applicability or their limitations, if any, since the encountered scenario varies. Clearly covering all the contributions on the topic in this paper is impossible, we devote ourselves to identifying explicit research lines and helping categorize the methodologies. But, if we can generate the feeling that, after having read this paper, the reader has a better understanding of what the subject is about and what the problems of the subject are, then we achieved our goals. This paper is organized as follows. Section 2 recalls a variety of stability properties on MJSs; and one more general MJSs are discussed, that is, MJSs with partially known jump rates. Control and filtering design techniques on Markovian jump systems and semi-Markovian jump systems are presented in Section 3, and Section 4 provides the conclusion and lists the expected future lines of research.

Notation: Throughout this paper, $\mathbb{R}^n$ denotes the n dimensional Euclidean space; $\mathbb{R}^{m\times n}$ is the space of real matrices of dimension $m\times n$. The notation $(\Omega,\mathscr{F},\Pr)$ represents the probability space with the sample space Ω, σ-algebra $\mathscr{F}$ of subsets of the sample space, and probability measure Pr; $\mathbf{E}\{\cdot\}$ denotes the expectation operator; $\ell_2[0,\infty)$ refers to the space of square-summated infinite vector sequences over $[0,\infty)$; $|\cdot|$ refers to the Euclidean vector norm. For continuous-time systems, $\{\eta_t, t\geq 0\}$ is a time homogeneous Markov process with right continuous trajectories and taking values in a finite set $\mathscr{M}=\{1,2,\cdots,M\}$ with stationary transition probabilities

$$\Pr(\eta_{t+h}=j\mid\eta_t=i)=\begin{cases}\pi_{ij}h+o(h), & i\neq j,\\ 1+\pi_{ij}h+o(h), & i=j,\end{cases} \tag{1}$$

where $h>0$, $\lim_{h\to 0}o(h)/h=0$ and $\pi_{ij}\geq 0$ is the transition rate from mode i at time t to mode j at time $t+h$, and $\pi_{ii}=-\sum_{j=1,j\neq i}^{M}\pi_{ij}$. For discrete-time systems, $\{\gamma_k, k=0,1,\cdots\}$ is a time homogeneous Markov chain taking values in a finite set $\mathscr{M}=\{1,2,\cdots,M\}$ with stationary transition probabilities

$$\lambda_{ij}=\Pr(\gamma_{k+1}=j\mid\gamma_k=i), \tag{2}$$

where $\lambda_{ij}\geq 0$ is the transition probability from mode i at time k to mode j at time $k+1$ and $\sum_{j=1}^{M}\lambda_{ij}=1$.

2. STABILITY ANALYSIS FOR MARKOVIAN JUMP SYSTEMS

It is common knowledge that the stability of a dynamical system is one of the primary concerns in the design and synthesis of a control system. The study of stability of jump linear systems has attracted the attention of many researchers. The stability analysis and stabilization problems for MJSs have been addressed in [17-29]. Specifically, Cao and Lam investigated the stochastic stabilizability for discrete-time jump linear systems with time delay in [19]; de Souza studied the robust stability and stabilization problems for uncertain discrete-time MJSs in [20]; Gao *et al.* considered the stabilization problem for two-dimensional (2-D) MJSs in [21]; and Sun *et al.* discussed the robust exponential stabilization for MJSs with mode-dependent input delay in [26]. In addition, there have been many progresses on the stability and stabilization for stochastic systems with Markovian jump parameters. To mention a few, Boukas and Yang proposed an exponential stabilizability condition for stochastic systems with Markovian jump parameters in [17]; while Wang *et al.* solved the stabilization problem for bilinear uncertain time-delay stochastic systems with Markovian jump parameters [27]; and some other results on Markovian jump stochastic systems can be found, for example [23], and the references therein.

2.1. Stability analysis of Markovian jump stochastic differential equations

Following the development of stochastic differential equations, stochastic differential equations with Markovian switching have become an active area of stochastic analysis in the past 30 years. In [22] and [30] the stability of such jump linear systems were studied. Basak *et al.* discussed the stability of a semi-linear stochastic differential equation with Markovian switching in [31], while the stability of a nonlinear stochastic differential equation with Markovian switching was investigated in [32]. Taking a further step, it is realized that, in real systems, the future state is usually dependent not only on the present state but also on the past states, so they should be modeled by differential equations with time delays. Shaikhet took the time delay into account in [33] and considered the stability of a semi-linear stochastic differential delay equation with Markovian switching, while Mao *et al.* addressed the stability of a nonlinear stochastic differential delay equation with Markovian switching [34].

Consider stochastic differential equations with Markovian switching [32]:

$$\begin{aligned} \mathrm{d}x(t) &= f(x(t),\eta_t)\mathrm{d}t + g(x(t),\eta_t)\mathrm{d}B(t), \quad t \ge t_0, \\ x(t_0) &= x_0, \end{aligned} \tag{3}$$

with solutions defined on $t \ge 0$ with initial values $x_0 \in \mathbb{R}^n$ and η_0. Here $f(\cdot): \mathbb{R}^n \times \mathscr{M} \to \mathbb{R}^n$, $g(\cdot): \mathbb{R}^n \times \mathscr{M} \to \mathbb{R}^{n\times m}$ and $B(\cdot)$ is an m-dimensional Brownian motion defined on the underlying probability space and independent of η_t. Both $f(\cdot)$ and $g(\cdot)$ satisfy the local Lipschitz condition and grow at most linearly. Under these conditions, (3) has a unique solution; see [34] for more details.

Definition 1 [23,35]**:** Consider stochastic differential equations (3) with Markovian switching, the equilibrium point is

i) stochastically stable if for any initial state η_0 and $t_0 \ge 0$, there exists $\rho > 0$ and $\varepsilon \in (0,1)$ such that

$$\Pr\{|x(t_0,x_0,\eta_0)| < \rho, \text{ for all } t \ge t_0\} \ge 1-\varepsilon.$$

ii) stochastically asymptotically stable in the large if it is stochastically stable and, moreover,

$$\Pr\left\{\lim_{t\to\infty} x(t_0,x_0,\eta_0) = 0\right\} = 1.$$

iii) almost surely exponential stable if for any initial state η_0 and $t_0 \ge 0$,

$$\lim\sup \frac{1}{t}\log(|x(t_0,x_0,\eta_0)|) < 0.$$

iv) pth moment stable, if for any initial state η_0 and $t_0 \ge 0$, there exists $\varepsilon > 0$ such that

$$\mathbf{E}\{|x(t,t_0,\eta_0)|^p\} < \varepsilon.$$

v) exponentially stable in mean square if there exist constants $\epsilon_1 > 0$ and $\epsilon_2 > 0$ such that for all $t \ge 0$

$$\mathbf{E}\{|x(0,\eta_0)|^2\} \le \epsilon_1 |x_0|^2 \exp(-\epsilon_2 t).$$

Remark 1: Note that necessary and sufficient conditions are presented in [36] for the three equivalent moment stability properties i), ii) and v). A similar condition is established in [22] for stochastic stabilizability in a different format. As mentioned in [22,36], stochastic stability or exponential mean square stability of (3) implies almost sure stability. It is well known that the converse statement is not true, i.e., second moment stability is stronger than sample stability. However, as observed in [37], for a certain class of stochastic systems, sample stability properties are inherited by pth moment stability properties for small p.

In the following, using the comparison principle, some stochastic stability criteria are presented for stochastic differential equations with Markovian switching.

Let $C^{2,1}(\mathbb{R}^n \times [0,+\infty) \times \mathscr{M}; \mathbb{R}_+)$ denote the family of all nonnegative function $V(x,t,i)$ on $\mathbb{R}^n \times [0,+\infty) \times \mathscr{M}$ which are continuously twice differentiable in $x(t)$ and once differentiable in t. For any $(x,t,i) \in \mathbb{R}^n \times [0,+\infty) \times \mathscr{M}$, define an operator $\mathscr{L}$ by

$$\begin{aligned} \mathscr{L}V(x,t,i) &= V_t(x,t,i) + V_x(x,t,i)f(x,t,i) \\ &\quad + \frac{1}{2}\mathrm{trace}\left[g^T(x,t,i)V_{xx}(x,t,i)g(x,t,i)\right] \\ &\quad + \sum_{j=1}^{M} \pi_{ij}V(x,t,j), \end{aligned}$$

where π_{ij} is defined in (1) and

$$V_t(x,t,i) \triangleq \frac{\partial V(x,t,i)}{\partial t},$$

$$V_x(x,t,i) \triangleq \left(\frac{\partial V(x,t,i)}{\partial x_1}, \cdots, \frac{\partial V(x,t,i)}{\partial x_n}\right),$$

$$V_{xx}(x,t,i) \triangleq \left(\frac{\partial^2 V(x,t,i)}{\partial x_i \partial x_j}\right)_{n\times n}.$$

Theorem 1 [38]**:** Consider the stochastic differential equation (4) and the following ordinary differential equation

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = h(t,x(t)), \quad t \ge 0. \tag{4}$$

where $h(\cdot): \mathbb{R} \times \mathbb{R}^+ \to \mathbb{R}$ is a continuous mapping, $h(t,0) \equiv 0$.

i) Assume that there exists a nonnegative function $V(x,t,i)$ such that

$$\mathscr{L}V(x,t,i) \le h(t,V(x,t,i)), \quad t \ge 0.$$

ii) $\mathbf{E}\{h(t,\xi)\} \le h(t,\mathbf{E}\{\xi\})$ for any n-dimensional stochastic vector ξ on the probability space $(\Omega, \mathscr{F}, \Pr)$.

iii) There exist a function $b(x)$ and a function $a(t,x)$ such that

$$b(\|x(t)\|) \le V(x,t,i) \le a(t,\|x(t)\|). \tag{5}$$

Then, the following hold.

i) The trivial solution of (4) is stable (asymptotically stable) implies that the trivial solution of (3) is stable in probability (asymptotically stable in probability). In particular, If $a(t,x) \equiv a(x)$ in (5), then the trivial solution of (4) is uniformly stable (uniformly asymptotic stable) implies that the trivial solution of (3) is uniformly stable in probability (uniformly asymptotic stable in probability).

ii) If condition (5) is replaced by

$$b(\| x(t) \|^p) \le V(x,t,i) \le a(t,\| x(t) \|^p), \tag{6}$$

then the trivial solution of (4) is stable (asymptotically stable, exponentially stable) implies that the trivial solution of (3) is the pth moment stable (pth moment asymptotic stable, pth moment exponentially stable). In particular, If $a(t,x) \equiv a(x)$ in (6), then the trivial solution of (4) is uniformly stable (uniformly asymptotically stable, uniformly exponentially stable) implies that the trivial solution of (3) is the pth moment uniformly stable (pth moment uniformly asymptotic stable, pth moment uniformly exponentially stable).

On the other hand, time delays and nonlinearities are frequently encountered in practical systems, which are two of the main sources for causing instability and poor performances. Due to the two facts just aforesaid, Markovian jump systems with time delay and nonlinearities are of great significance in efficiently modeling many practical dynamics systems, and have received considerable attentions, see for example, [25-27] and the references therein.

2.2. Stability of continuous time linear Markovian jump systems

Stability properties of systems described by multiple models switching according to Markov processes/chains can be analyzed via the notion of stochastic stability introduced in [22]. When parameters of Markov process/chain describing the transition between different models are not completely known, it is important to know how much uncertainty can be tolerated for the system to be stochastically stable. In [36], the problem of almost sure instability has studied of the random harmonic oscillator.

One convenient way to classify Markovian jump systems is based on the dynamics of their subsystems, for example continuous-time or discrete-time, linear or nonlinear and so on. Consider the following continuous- and discrete-time MJSs with state vectors $x(t) \in \mathbb{R}^n$ and $x(k) \in \mathbb{R}^n$, respectively.

$$\dot{x}(t) = A(\eta_t)x(t), \tag{7}$$

$$x(k+1) = A(\gamma_k)x(k). \tag{8}$$

Much work has been done, for example [22,39], studied the stability issue of such jump linear systems.

Definition 2 [40,41]**:** For system (7) with transition probabilities satisfied (1), the equilibrium point is

i) *asymptotically mean square stable*, if for any initial state x_0 and initial distribution η_0

$$\lim_{t\to+\infty} \mathbf{E}\left\{\| x(t,x_0,\eta_0) \|^2\right\} = 0,$$

ii) *exponentially mean square stable*, if for every initial state x_0 and initial distribution η_0, there exist constants $\alpha > 0$ and $\beta > 0$ such that

$$\mathbf{E}\left\{\| x(t,x_0,\eta_0) \|^2\right\} < \alpha \| x_0 \|^2 \exp(-\beta t), \quad \forall t > 0.$$

iii) *stochastically stable*, if for every initial state x_0 and initial distribution η_0,

$$\mathbf{E}\left\{\int_0^{+\infty} \| x(t,x_0,\eta_0) \|^2 \, \mathrm{d}t\right\} < +\infty.$$

iv) *almost surely (asymptotically) stable*, if for any initial state x_0 and initial distribution η_0

$$\Pr\left\{\lim_{t\to+\infty} \mathbf{E}\{\| x(t,x_0,\eta_0) \|\} = 0\right\} = 1.$$

Remark 2: By analyzing the stochastic properties of the transition matrix for jump linear systems, it has been shown in [36] that the second moment stability concepts, namely, mean square stability, stochastic stability and exponential mean square stability are all equivalent, and any one of them implies almost sure stability.

Remark 3: The problem establishes the equivalence of second moment stability properties and the fact that each of them is sufficient for almost sure stability of (7). The fact that moment stability implies almost sure sample stability was observed earlier in [42], for a special class of randomly switched systems. A general result for systems in the form (7) with η_t an arbitrary stationary random process satisfying certain separability and boundness conditions was obtained by [41]. In particular, for (7), the results in [41] give the fact that ii) or iii) implies iv).

Remark 4: For discrete-time MJSs (8) with a time homogeneous Markov chain γ_k satisfying (2), the definitions of stochastically stable, mean square stable and exponentially mean square stable are given in [43,44]. Also, the work in [43] established the equivalence of these three stability concepts. That is, stochastic, mean square and exponential mean square stability are the same for discrete-time MJSs. Moreover, the almost sure stability for systems (8) and (2) are given in [43]. In particular, stochastic stability, mean square stability and exponentially mean square stability are each sufficient but not necessary for almost sure stability.

The following Theorems on the stochastic stability of systems (7) and (8) are recalled based on Lyapunov method and their proofs can be found in the cited references.

Theorem 2 [18]**:** System (7) is stochastically stable if, and only if, there exists a set of matrices $P_i,\ i \in \mathscr{M}$ satisfying

$$A_i^T P_i + P_i A_i + \mathscr{P}_i < 0, \tag{9}$$

where $\mathscr{P}_i \triangleq \sum_{j\in\mathscr{M}} \pi_{ij} P_j$ and $A_i \triangleq A(\eta_t),\ \eta_t \in \mathscr{M}$.

Theorem 3 [45]**:** System (8) is stochastically stable if, and only if, there exists a set of matrices P_i, $i \in \mathscr{M}$ satisfying

$$A_i^T \mathscr{P}_i A_i - P_i < 0, \tag{10}$$

where $\mathscr{P}_i \triangleq \sum_{j \in \mathscr{M}} \lambda_{ij} P_j$.

Remark 5: It should be emphasized that Theorems 1 and 2 play an important role in determining the stability of Markovian jump systems, and are given in the form of strict linear matrix inequalities that allow solutions to be readily obtained via available optimization techniques.

2.3. Stability of MJSs with partially known transition rates

In previous subsections, all the transition rates in the corresponding Markov jumping process, as a crucial factor, are assumed to be completely accessible. In practice, it is difficult to obtain the exact value of the switching probabilities. For instance, in the soft landing process of a reentry body [46], the probability of opening the parachute is determined by the altitude as well as its rate of change. Another example refers to internet based networked control systems, where the packets dropouts and channel delays can be modeled by Markov chains [47], but the delay or packet loss is distinct at different periods, which leads to the resulting transition probability (TP) matrix changing throughout the running time. Similar phenomenon also arises in other systems, such as electronic circuits, mental health analysis, and manpower systems. To overcome the above issues, MJSs with uncertain transition probabilities have been studied in [28,48,49], in which robust approaches were adopted to cope with some compact sets with polytopic-type or norm-bounded structure in the transition probability matrix.

In [50-52], the transition rates or probabilities of the jump are considered to be partially accessed, i.e., some elements in matrix $\pi \triangleq [\pi_{ij}]$ or $\lambda \triangleq [\lambda_{ij}]$ are unknown. For notational clarity, denote $l = l_K^i + l_{UK}^i$ for any $i \in \mathscr{M}$ with

$$l_K^i \triangleq \{j : \pi_{ij} (or\ \lambda_{ij}) \text{ is known}\}, \tag{11}$$

$$l_{UK}^i \triangleq \{j : \pi_{ij} (or\ \lambda_{ij}) \text{ is unknown}\}. \tag{12}$$

Also, we denote $\pi_K^i \triangleq \sum_{j \in l_K^i} \pi_{ij}$ and $\lambda_K^i \triangleq \sum_{j \in l_K^i} \lambda_{ij}$, respectively.

Remark 6: The accessibility of the jumping process $\{\eta_t, t \geq 0\}$ (or $\{\gamma_k, k = 0,1,\cdots\}$) in literature is commonly assumed to be completely accessible ($l_{UK} = \varnothing$, $l_K = l$) or completely inaccessible ($l_{UK} = l$, $l_K = \varnothing$). Moreover, the transition rates or probabilities with polytopic or norm-bounded uncertainties require the knowledge of bounds or structure of uncertainties, which can still be viewed as accessible. Therefore, the transition rates or probabilities matrix considered in [50-53] is a more natural assumption to Markovian jump systems, thus have more practical potentials.

The following theorems present sufficient conditions on the stochastic stability of the considered system with partially known transition probabilities (11) and (12), respectively.

Theorem 4 [52]**:** Consider system (7) with partially known transition probabilities (11). The corresponding system is stochastically stable if there exist matrix P_i, $i \in \mathscr{M}$, such that

$$(1 + \pi_K^i)(A_i^T P_i + P_i A_i) + \mathscr{P}_K^i < 0,$$
$$A_i^T P_i + P_i A_i + P_j \geq 0, \quad \forall j \in l_{UK}^i, \quad j = i,$$
$$A_i^T P_i + P_i A_i + P_j \leq 0, \quad \forall j \in l_{UK}^i, \quad j \neq i,$$

where $\mathscr{P}_K^i \triangleq \sum_{j \in l_K^i} \pi_{ij} P_j$.

Theorem 5 [52]**:** Consider system (8) with partially known transition probabilities (12). The corresponding system is stochastically stable if there exist matrix P_i, $i \in \mathscr{M}$, such that

$$A_i^T \mathscr{P}_K^i A_i - \lambda_K^i P_i < 0,$$
$$A_i^T P_j A_i - P_i < 0, \quad \forall j \in l_{UK}^i,$$

where $\mathscr{P}_K^i \triangleq \sum_{j \in l_K^i} \lambda_{ij} P_j$.

In [52], the stability and stabilization problems of a class of continuous-time and discrete-time MJSs with partially known transition probabilities are investigated. The system under consideration is more general, which covers the systems with completely known and completely unknown transition probabilities as two special cases-the latter is hereby the switched linear systems under arbitrary switching. Moreover, in contrast with the uncertain transition probabilities studied recently, the concept of partially known transition probabilities proposed in [52] does not require any knowledge of the unknown elements. The sufficient conditions for stochastic stability and stabilization of the underlying systems are derived via linear matrix inequality formulation, and the relation between the stability criteria currently obtained for the usual MJSs and switched linear systems under arbitrary switching, are exposed by the proposed class of hybrid systems.

3. DESIGN OF MARKOVIAN JUMP SYSTEMS

In this section, we will review some recent advancements on control design for Markovian jump systems, including robust control for systems with uncertainties. We know that robust control theory plays a very effective role against model uncertainty and external disturbances. Consequently, robust control problem for MJSs has become a hot topic. A great number of fundamental concepts and results on continuous-time MJSs have been developed, for example, optimal control [2,4,8,9,54], dissipative control [55], observer design

[56-58]. Other results related to discrete-time MJSs have also been reported in [19,25,29,60,59].

Definition 3 [22]**:** Consider the Markovian jump control system

$$\dot{x}(t) = A(\eta_t)x(t) + B(\eta_t)u(t), \tag{13}$$

$$x(k+1) = A(\gamma_k)x(k) + B(\gamma_k)u(t). \tag{14}$$

If there exists a feedback control

$$u(t) = K(\eta_t)x(t), \text{ (respectively,} u(k) = K(\gamma_k)x(k)), \tag{15}$$

such that the resulting closed-loop control system is stable, where $K(\eta_t)$ (respectively $K(\gamma_k)$) is the controller gain to be determined, respectively. Then the control system (13) (respectively (14)) is said to be stochastically stabilizable in the corresponding sense. If the resulting closed-loop system is absolutely stable, then (13) (respectively (14)) is absolutely stabilizable.

For the stabilization problem of Markovian jump systems, readers may refer to [20,21,25-27,29,61,62]. Design of control systems that can handle model uncertainties has been one of the most challenging problems and received considerable attention from academics, scientists and engineers in the past decades. There are two major issues in robust controller design. The first is concerned with the robust stability of the uncertain closed-loop system (see for example, [20] and the references therein), and another is robust performance. On the other hand, convex analysis has shown to be a powerful tool to derive numerical algorithms for control problems. For state feedback MJSs case, convex analysis had been previously considered in [39].

3.1. Control and filtering for Markovian jump systems

In order to ensure the performance of MJSs, the researchers have proposed linear quadratic control theory [22], H_2 control theory [63,64], H_∞ control theory [19,65,60,66], H_∞ filtering theory [67-69], and so on by defining accordingly performance index. Since its introduction in 1980s, the so-called H_∞ optimal control has been one of the most attractive and dominated research topics in the past 30 years [70].

Consider the following Markovian jump control system in probability space $(\Omega, \mathscr{F}, \Pr)$:

$$\begin{aligned} \dot{x}(t) &= A(\eta_t)x(t) + B(\eta_t)u(t) + G(\eta_t)w(t), \\ z(t) &= C(\eta_t)x(t) + D(\eta_t)u(t), \end{aligned} \tag{16}$$

where $(\eta_t, t \geq 0)$ are finite Markov processes satisfying (1); $x(t)$ is the system state; $u(t)$ is control input satisfying (15); $w(t)$ is the disturbance input which belongs to $l_2[0,\infty)$; and $z(t)$ is the controlled output which belongs to $l_2[0,\infty)$.

Definition 4 [65]**:** Consider system (16) with $(\eta_t, t \geq 0)$ are satisfied (1). We are concerned with designing a state feedback controller (15), such that, for all nonzero $w(t) \in l_2[0,\infty)$

$$\| z(t) \|_{E_2} < \gamma \| w(t) \|_2, \tag{17}$$

where $\gamma > 0$ is a prescribed level of disturbance attenuation to be achieved and

$$\| z(t) \|_{E_2} = \mathbf{E}\left\{ \int_0^T z^T(t)z(t)\mathrm{d}t \right\}^{1/2}.$$

When (17) is satisfied, the system (16) with controller (15) is said to have H_∞ performance (17) over the horizon [0, T].

Both the cases of continuous-time and discrete-time dynamical linear and nonlinear systems have been intensively studied. H_∞ control for MJSs has been investigated in [19,21,60,65,66,71,72], while robust H_∞ control for MJSs with unknown nonlinearities has been studied in [65]. H_∞ control has been designed in [60] for discrete-time MJSs with bounded transition probabilities; the robust H_∞ control problem has been considered in [19] for uncertain MJSs with time delay; and the delay-dependent H_∞ control problem have been discussed in [66,72] for singular Markovian jump systems with time-varying delays.

Remark 7: Note that H_∞ performance analysis plays a vital role in controller design of Markovian jump systems. Apart from this approach, other methods for performance analysis have also yielded important results. For instance, H_2 performance [64], L_1 gain performance [73]. In addition, the $L_2 - L_\infty$ performance, which is also referred to as the energy-to-peak performance [74] and H_2 extended performance, is an important index and has received considerable attention.

As is well known, filtering technique has been playing an important role in a variety of application areas including signal processing, target tracking, and image processing ([75,76]). Up to now, many important developments on the filtering problem have been made for MJSs. The designed filters can be classified into two types: mode-dependent filters ([48,67,68,77-80]) and mode-independent filters ([69, 76, 81-83]).

Consider the following Markovian jump control system in probability space $(\Omega, \mathscr{F}, \Pr)$:

$$\begin{aligned} \dot{x}(t) &= A(\eta_t)x(t) + B(\eta_t)w(t), \\ y(t) &= C(\eta_t)x(t) + D(\eta_t)w(t), \\ z(t) &= L(\eta_t)x(t), \end{aligned} \tag{18}$$

where $(\eta_t, t \geq 0)$ is a finite Markov process satisfying (1); $x(t) \in \mathbb{R}^n$ is the state; $w(t) \in \mathbb{R}^m$ is the noise signal (including process and measurement noises), which is assumed to be an arbitrary signal in $l_2[0,\infty)$; $y(t) \in \mathbb{R}^l$ is the measurement; and $z(t) \in \mathbb{R}^s$ is the signal to be estimated.

The filtering problem to be addressed is to obtain an estimate $\hat{z}(t)$ of $z(t)$ via a causal mode-dependent linear filter which provides a uniformly small estimation error, $\tilde{z}(t) \triangleq z(t) - \hat{z}(t)$, for all $w(t) \in l_2[0, \infty)$. Attention is focused on the design of a linear time-invariant, asymptotically stable, filter with state space-realization

$$\begin{aligned} \dot{\hat{x}}(t) &= A_f(\eta_t)\hat{x}(t) + B_f(\eta_t)y(t), \\ z(t) &= C_f(\eta_t)\hat{x}(t), \end{aligned} \tag{19}$$

where the matrices $A_f(\eta_t) \in \mathbb{R}^{n \times n}$, $B_f(\eta_t) \in \mathbb{R}^{n \times l}$ and $C_f(\eta_t) \in \mathbb{R}^{s \times n}$ are to be designed.

It follows from (18)-(19) that the dynamics of the estimation error $\tilde{z}(t)$ can be described by the following state-space model:

$$\begin{aligned} \dot{\xi}(t) &= \tilde{A}(\eta_t)\xi(t) + \tilde{B}(\eta_t)w(t), \\ \tilde{z}(t) &= \tilde{C}(\eta_t)\xi(t), \end{aligned} \tag{20}$$

where

$$\begin{aligned} \tilde{A}(\eta_t) &\triangleq \begin{bmatrix} A(\eta_t) & 0 \\ B_f(\eta_t)C(\eta_t) & A_f(\eta_t) \end{bmatrix}, \\ \tilde{B}(\eta_t) &\triangleq \begin{bmatrix} B(\eta_t) \\ B_f(\eta_t)D(\eta_t) \end{bmatrix}, \quad \xi(t) \triangleq \begin{bmatrix} x^T(t) & \hat{x}^T(t) \end{bmatrix}^T, \\ \tilde{C}(\eta_t) &\triangleq \begin{bmatrix} L(\eta_t) & -C_f(\eta_t) \end{bmatrix}. \end{aligned}$$

Then, the robust H_∞ filtering problem addressed is formulated as follows: given MJSs (18), determine a filter system (19) such that the filtering error system (20) is stochastically stability (exponential mean-square stability), and satisfies a prescribed H_∞ performance index. The problem of H_∞ filter for Markov jump systems has been investigated in [48], where a method for designing a mode-independent filter has been proposed. The results reported in [48] have been further improved in [84]. The problem of mode-independent H_∞ filtering has been discussed in [85] for singular Markov jump systems, and the full-order and reduced-order filters have been designed in a unified framework. In [86], the problem of mode-independent H_∞ filter has been addressed for discrete-time Markov jump systems, and a design procedure has been proposed. It is noted that the mode-independent filter is very useful when the system mode information is completely unaccessible. However, it should be pointed out that the mode-independent filters cannot deal with the complex asynchronous phenomenon between filter modes and system modes, all the available modes information are neglected, which inevitably leads to conservatism to some extent. On the other hand, Kalman filtering for continuous-time uncertain MJSs has been considered in [78]; the H_∞ filter problem for continuous- and discrete-time MJSs have been studied in [67,68], respectively. In the mean time, Wu *et al.* have extended the H_∞ filtering problem to 2-D Markovian jump systems; and the quantized H_∞ filtering for Markovian jump linear parameter varying systems has been studied with intermittent measurements [83].

In addition of filtering design, the fault detection problem for MJSs has been investigated in [87-93]. Specifically, Meskin and Khorasani considered the fault detection and isolation problems in [87] for discrete-time MJSs with application to a network of multi-agent systems with imperfect communication channels; while Nader *et al.* proposed a geometric approach to fault detection and isolation for continuous-time MJSs in [88]. The work in [89] studied the problem of generalized H_2 fault detection for Markovian jumping two-dimensional systems; while the work in [90] developed fault detection filter design for Markovian jump singular systems with intermittent measurements. The problems of robust fault detection were addressed in [91,92] respectively.

Apart from the above-mentioned synthesis problems for MJSs, the model reduction problem for such systems has also been investigated, see for example, [94,95]. in [94], the model reduction problem was considered for discrete-time MJSs; and Zhang *et al.* considered H_∞ model reduction for both continuous- and discrete-time MJSs [95].

3.2. Sliding mode control for Markovian jump systems

Sliding-mode control (SMC) has received noticeable attention since it has various attractive features such as fast response, good transient performance, order reduction and so on. In particular, SMC laws are robust with respect to the so-called matched uncertainty, see for example, [96-98]. Recently, sliding mode control is proposed to stabilize MJSs with matched uncertainties and disturbances [99]. However, due to the system is switching stochastically between different subsystems, the dynamics of the jump systems can not stay on each sliding surface of subsystems forever, therefore, it can not be determined whether the closed-loop system is stochastically stable or not.

Consider the following Markovian jump systems in probability space $(\Omega, \mathscr{F}, \Pr)$

$$\dot{x}(t) = A(\eta_t)x(t) + B(\eta_t)[u(t) + F(\eta_t)\omega(t)], \tag{21}$$

where $x(t) \in \mathbb{R}^n$ is the system state vector; $u(t) \in \mathbb{R}^m$ is the control input; $\omega(t) \in \mathbb{R}^l$ is the disturbance, and $\{\eta_t, t \geq 0\}$ is a time homogeneous Markov process satisfying (1).

We will design a sliding surface

$$s(x, \eta_t, t) = 0,$$

where $s(x, \eta_t, t)$ is the switching function, and its order is usually equivalent to that of the control input.

Following the switching function, a sliding mode controller

$$u(t) = [u_1(t) \quad u_2(t) \quad \cdots \quad u_m(t)]^T,$$

is designed in the form of

$$u_i(t) = \begin{cases} u_i^+(t), & \text{when } s_i(x, \eta_t, t) > 0, \\ u_i^-(t), & \text{when } s_i(x, \eta_t, t) < 0, \end{cases} \quad i = 1, 2, \cdots, m,$$

where $u_i^+(t) \neq u_i^-(t)$, such that the following two conditions hold:

i) The sliding mode is reached in a finite time and subsequently maintained, that is to say, the system state trajectories can be driven onto the specified sliding surface $s(x,\eta_t,t)=0$ by the sliding mode controller in a finite time and maintained there for all subsequent time; and

ii) The dynamics in sliding surface $s(x,\eta_t,t)=0$, that is, the sliding mode dynamics, is stable with some specified performances.

Next step, a sliding mode controller will be designed such that the system state trajectories can be driven onto the specified sliding surface in a finite time and maintained there for all subsequent time. Some commonly used methods to the sliding mode controller design include 1) equivalent control design; 2) reaching condition approach; 3) Lyapunov function approach; and 4) reaching law approach.

Remark 8: It should be noted that under Markov switching, all modes of systems (13) are not independent, but dependent of each other via Markov process η_t. This implies that the sliding function corresponding to every mode is also not independent, but dependent of each other. On the other hand, the proposed SMC law still depends on the transition rates π_{ij} that reflects the effect of Markovian switching from one mode to another. Finally, the connections among sliding surfaces is also reflected in the proposed SMC law. Hence, the present SMC method can effectively deal with the effect of Markovian switching, and achieves the desired dynamic performance of SMC systems.

SMC design problem has been addressed for MJSs in [73,75,76,99-104]. SMC of MJSs with actuator nonlinearities was considered in [100]; Ma and Boukas proposed a singular system approach to robust SMC for uncertain MJSs [102]; SMC problem for Markovian jump singular stochastic hybrid systems was solved in [76]; the problems of state estimation and SMC of Markovian jump singular systems were discussed in [75]; and also SMC design with bounded l_2 gain performance for Markovian jump singular time-delay systems in [73]. Recently, Shi *et al.* designed the sliding-mode control of Markovian jump systems [99]. Motivated by the work in [99], Wu *et al.* further studied the sliding-mode control problem for Markovian jump singular systems where the main difficulties come from the sliding surface function design and the stochastic admissibility analysis for the resulting sliding-mode dynamics [75].

3.3. Networked control systems with Markovian delay and/or Markovian packet losses

Networked control systems (NCSs) are a type of distributed control systems, where the information of control system components is exchanged via communication networks. The introduction of networks also presents some constraints such as time delays and packet dropouts which bring difficulties for analysis and design of NCSs. Nowadays, various methodologies have been proposed for modeling, stability analysis, and controller design for NCSs in the presence of network-induced time delays and packet dropouts. The Markov chain, a discrete-time stochastic process with the Markov property, can be effectively used to model the network-induced delays in NCSs. In [105,106], the time delays in NCSs are modeled by using the Markov chains, and further an linear quadratic Gaussian optimal controller design method is proposed. Xiao *et al.* developed two types of controller design methods for NCSs modeled as finite dimensional [107], discrete-time jump linear systems: One is the state feedback controller that only depends on delays from sensor to controller (S-C delays), and is called the one-mode-dependent controller; the other is the output feedback controller that does not depend on either the S-C delays or the C-A delays (delays from controller to actuator), and called the mode-independent controller.

Zhang *et al.* extended the idea and used two Markov chains to model the delays in both feedback and forward channels [108]. It is assumed that at each sampling instant, the current S-C delay (τ_{sc}^k) and previous C-A delay (τ_{ca}^k) can be obtained by the time-stamping technique. However, practically the previous C-A delay is not always available because the information about C-A delays needs to be transmitted through the S-C communication link before reaching the controller. More precisely, the discretized controlled plant is considered as

$$x(k+1)=Ax(k)+Bu(k), \tag{22}$$

$$x(k)=\phi(k),\quad k\in[-\overline{\tau}-\overline{d},-\overline{\tau}-\overline{d}+1,\ldots,0]. \tag{23}$$

The control input can be obtained as

$$u(k)=K(\tau_{sc}^k,\tau_{ca}^{k-1})x(k-\tau_{sc}^k-\tau_{ca}^k),$$

where the delays τ_{ca}^k and τ_{ca}^k are subject to Markov chain with

$$\hat{\lambda}_{ij}=\Pr(\tau_{sc}^{k+1}=j\mid\tau_{sc}^k=i),$$

$$\lambda_{rs}=\Pr(\tau_{ca}^{k+1}=s\mid\tau_{ca}^k=r),$$

where $\hat{\lambda}_{ij}\geq 0$, $\lambda_{rs}\geq 0$, $\sum_{j=0}^{\overline{\tau}}\hat{\lambda}_{ij}=1$, and $\sum_{s=0}^{\overline{d}}\lambda_{rs}=1$, $\forall i,j,r,s\in\mathscr{M}$.

Remark 9: Note that in [108], the designed controller gain $(K(\tau_{sc}^k,\tau_{ca}^{k-1}))$ depends on the current feedback channel delay and the previous forward channel delay. Yet, the property of NCSs listed above enables a set of control commands to be sent from the controller site, by which the control command can be selected at the smart actuator according to the current forward channel delay. In addition to the stochastic description of delay variations, nondeterministic descriptions can be considered within the framework, i.e., assuming the transitions probabilities are completely unknown and the variations of the time delays are state dependent or time dependent.

The work for network-induced delays issue is classified whether the methodology is dependent on the delay information online or not. Similar thought is also applicable to the context of packet losses problem in NCSs.

Consider system (22). Let $\mathscr{J} \triangleq \{i_1, i_2, \ldots\}$, a subsequence of $\{1,2,3,\ldots\}$, denote the sequence of time points of successful data transmissions from the sampler to the zero-order hold, and $s \triangleq \max_{i_k \in \mathscr{J}}(i_{k+1} - i_k)$ be the maximum packet-loss upper bound. Then the following concept and mathematical models are introduced to capture the nature of packet losses.

Definition 5 [109]**:** Packet-loss process is defined as

$$\{\eta(i_k) \triangleq i_{k+1} - i_k : i_k \in \mathscr{L}\}, \tag{24}$$

which takes values in the finite state space $\mathscr{M} = \{1, \cdots, M\}$.

Definition 6 [109]**:** Packet-loss process (24) is said to be Markovian if it is a discrete-time homogeneous Markov chain on a complete probability space $(\Omega, \mathscr{F}, \Pr)$, and takes values in $\mathscr{M}$ with known transition probability matrix $\Lambda \triangleq (\lambda_{ij}) \in \mathbb{R}^{M \times M}$, where

$$\lambda_{ij} = \Pr(\eta(i_{k+1}) = j \mid \eta(i_k) = i), \tag{25}$$

for any $i, j \in \mathscr{M}$, and $\sum_{j=1}^{M} \lambda_{ij} = 1$.

From the viewpoint of the zero-order hold, the control input is

$$u(l) \triangleq u(i_k) = Kx(i_k), \tag{26}$$

for $i_k \le l \le i_{k+1} - 1$. The initial inputs are set to zeros: $u(l) = 0,\ 0 \le l \le i_1 - 1$. Hence the closed-loop system becomes

$$x(l+1) = Ax(l) + BKx(i_k), \tag{27}$$

for $i_k \le l \le i_{k+1} - 1,\ i_k \in \mathscr{M}$. The objective of analysis and design of NCSs with packet losses is to construct controller (26) such that NCSs (27) is stable.

Next, sufficient conditions for stochastic stability of the closed-loop NCSs are obtained via Markovian theories and the packet-loss-dependent Lyapunov function approach.

Theorem 6 [109]**:** The closed-loop system (27) with a Markovian packet-loss process defined as in (25) is stochastically stable, if there exist positive symmetry matrices $P_i > 0,\ i \in \mathscr{M}$, such that

$$(A+BK)^T \left(\sum_{j=1}^{M} \lambda_{ij} P_j \right) (A+BK) - P_i < 0, \quad \forall i \in \mathscr{M}.$$

One framework for the analysis and design of NCSs with packet losses is the offline framework, where the controller is designed despite any situation of real packet losses[109-111]. However, there exist the effects of the current packet cases (dropped and received successfully) on the future packet cases. A Markov process is able to represent such effect [109,112]. In [109], the time interval between packet successful transmissions is used as a state in a Markov chain. The relations among the amount of consecutive packet dropouts are employed to establish the transition probability matrix. In [113], a Markov process is used to describe the quantity of packet dropouts between the current time instant and the latest successful transmission.

3.4. Control and filtering for MJSs with incomplete transition descriptions

The ideal knowledge on the transition probabilities are definitely expected to simplify the system analysis and design. However, the likelihood of obtaining such available knowledge is actually questionable, and the cost is probably expensive. Therefore, rather than having a large complexity to measure or estimate all the transition probabilities, it is significant and necessary, from control perspectives, to further study more general jump systems with partially unknown transition probabilities.

The work in [28] and [82], the uncertainties in TPs were represented by norm-bounded or polytopic description. Then, robust control and filtering methods are utilized to deal with the uncertainties presumed in the TPs. Considering a more realistic situation that some parts of the elements in the desired TPs matrix are hard to obtain, Zhang studied the stability, stabilization, and H_∞ filtering problems for MJSs with partially known TPs in [50-52]. The significance of this hypothesis lies in that rather than having a large complexity to measure all the TPs, it is more meaningful to directly study MJSs with partially unknown TPs. The transient and steady performance for MJSs with partially known TPs were considered in [114,115] in time domain.

On the other hand, there is a general lack of online sensors in many fields, such as pharmacy industry, fermentation process, and petrochemical industry. Therefore, state estimation problem is an important research issue in control discipline. Meanwhile, considering the practical applications that TPs are generally determined by physical experiments or numerical simulations that lead to the TPs with stochastic features, H_∞ filtering problem for MJSs, viewed from the stochastic standpoint, is studied in [115]. It assumes that the exact value of TPs is unknown, but the distribution can be approximated by Gaussian process. To obtain the expectation of unknown TPs from Gaussian probability density function (PDF), a discretization method is developed. On this basis, a H_∞ filter is designed such that the worst-case induced l_2 gain from process noise to estimation error is minimized. Different from the existing results in literatures, a Gaussian PDF is utilized to characterize the relative likelihood for unknown TPs to occur at a given constant. With the Gaussian distribution of TPs, we can obtain the expectation of unknown TPs. Moreover, the considered systems are more general than the systems with completely known and partially known TPs, which can be viewed as two special cases of the ones tackled here.

3.5. Control of Semi-Markovian jump systems

Markovian jump systems, although important in theory and useful to describe many practical systems, have many limitations in applications, since the jump time of a Markov chain is, in general, exponentially distributed, and the results obtained for the MJSs are intrinsically conservative due to constant transition rates. Different from the MJSs, semi-Markovian jump systems (S-MJS) are characterized by a fixed matrix of transition probabilities and a matrix of sojourn time probability density functions. Due to their relaxed conditions on the probability distributions, S-MJS have much broader applications than the conventional MJS. Indeed, most of the modeling, analysis, and design results for MJS would be special cases of S-MJS. Thus, this area of research is significant not only in theory, but also in practice.

Definition 7 [116,117]**:** The evolution of the semi-Markov process $\{r_t, t \geq 0\}$ is governed by the following probability transitions:

$$\Pr(r_{t+h} = j \mid r_t = i) = \begin{cases} \pi_{ij}(h)h + o(h), & i \neq j, \\ 1 + \pi_{ij}(h)h + o(h), & i = j, \end{cases} \tag{28}$$

where $h > 0$, $\lim_{h \to 0} o(h)/h = 0$ and $\pi_{ij} \geq 0$ is the transition rate from mode i at time t to mode j at time $t+h$, and $\pi_{ii}(h) = -\sum_{j=1, j\neq i}^{M} \pi_{ij}(h)$.

Remark 10: In [116,117], probability distributions of sojourn-time on Markovian processes, from an exponential distribution to a Weibull one, are discussed. Therefore, the transition rate is time-varying instead of constant. It has been moved one step further towards the numerically solvable conditions by making use of the upper and lower bounds of the transition rate. However, the main proposition in [117] to partition the sojourn-time into M sections in each working mode is relatively conservative and practically infeasible. Stochastic stability of systems with semi-Markovian jump parameters is studied in [38,118,119].

In [118,120], by a supplementary variable technique and a novel transformation, a finite phase-type (PH) semi-Markov process has been transformed into a finite Markov chain, which is called its associated Markov chain. Consequently, a PH semi-Markovian jump system can be equivalently expressed as its associated Markovian system. Having the density property of PH distributions allows us to choose a PH distribution that approximates the original distribution to any accuracy. So, the new design methods presented in [118,120] are less conservative and bear a more practical value.

Remark 11: The definition of phase-type semi-Markov chain, readers may refer to [38,118,120] for more details. It is worth noting that the phase-type distribution is a generalization of the exponential distribution while still preserving much of its analytic tractability, and has been used in a wide range of stochastic modeling applications in areas as diverse as reliability theory, queuing theory and biostatistics. Furthermore, the family of PH distribution is dense in all the families of distributions on $[0,+\infty)$. So, for every probability distribution on $[0,+\infty)$, we may choose a PH distribution to approximate the original distribution in any accuracy.

Consider a class of stochastic differential equations with semi-Markovian jump parameters in the probability space $(\Omega, \mathscr{F}, \Pr)$ for $t > 0$

$$\begin{aligned} &\mathrm{d}x(t) = \hat{f}(x(t), t, \hat{r}_t)\mathrm{d}t + \hat{g}(x(t), t, \hat{r}_t)\mathrm{d}\omega(t), \\ &x(0) = x_0, \end{aligned} \tag{29}$$

where $\omega(t)$ be an m-dimensional Brownian motion defined on the probability space, and we assume that the semi-Markov chain $\{\hat{r}_t, t \geq 0\}$ is independent of the Brownian motion $\omega(t)$. The initial state $x_0 \in \mathbb{R}^n$ is a fixed constant vector. $f(\cdot): \mathbb{R}^n \times \mathbb{R} \times \mathscr{M} \to \mathbb{R}^n$ and $g(\cdot): \mathbb{R}^n \times \mathbb{R} \times \mathscr{M} \to \mathbb{R}^{n\times m}$.

In the following, using a supplementary variable technique and a plant transformation, a finite phase-type semi-Markov process has been transformed into a finite Markov chain, which is called its associated Markov chain. As a result, phase-type semi-Markovian jump systems can be equivalently expressed as its associated Markovian jump systems. For the proof, the reader may refer to [38].

Theorem 7 [38]**:** System (29) is equivalent to the following system for $t > 0$

$$\begin{aligned} &\mathrm{d}x(t) = f(x(t), t, r_t)\mathrm{d}t + g(x(t), t, r_t)\mathrm{d}\omega(t), \\ &x(0) = x_0, \end{aligned}$$

where $\{r_t, t \geq 0\}$ is the associated Markov chain of PH semi-Markovian chain $\{\hat{r}_t, t \geq 0\}$.

Remark 12: It is worth to mention that the advantages of Theorem 7 is that when we study stochastic stability and control problems, we can replace Markovian jump systems with semi-Markovian jump systems, and achieve the same results, while semi-Markovian jump systems are much less restrictive and it can be widely found and used in many real system applications [38]. Furthermore, more importantly, almost all the nice results obtained so far on Markovian jump systems, for example, [38,61, 121-123] are also true in semi-Markovian jump systems.

Moreover, a sliding surface is then constructed in [118] and a sliding mode controller is synthesized to ensure that the associated Markovian jump systems satisfy the reaching condition. A sufficient condition for associated Markovian jump systems is developed in [120]. This condition guarantees that the corresponding closed-loop system is stochastically stable and has a prescribed H_∞ performance. The existence conditions for full- and reduced-order dynamic output feedback controllers are proposed, and the cone complementarity linearization procedure is employed to cast the controller design problem into a sequential minimization one, which can be solved efficiently with available optimization techniques. Stochastic stability of linear systems with PH semi-Markovian jump parameters is studied in [38].

The application of semi-Markov process in fault-tolerant control systems has been discussed in [124], and it was shown that when a practical system does not satisfy the so-called memoryless restriction, the widely used Markov switching scheme would not be applicable. A typical transition rate in the bathtub shape in the reliability analysis has been reported in [125]. Also, the work in [126] considered the control problem of singularly perturbed Markov and semi-Markov jump linear systems, as well as a particular control problem in accelerator physics known as the bunch-train cavity interaction (BTCI). It is shown that the BTCI is in fact a physical model example of a semi-Markov jump linear system.

4. CONCLUSION AND FUTURE WORKS

In this paper, we have attempted to present a survey of some major problems, results and trends in the subject of modeling, analysis and design (control and filtering) on Markovian jump systems. The aim is to present the background and new developments in the fields. Clearly it is difficult, if not impossible, to cover all the contributions in the area; therefore, our emphasis is placed on the categorizations of the advancements in literature as much as we possible could. The applicability or the limitations of the developed approaches have not thoroughly been commented on, and we believe readers can figure out these by their own interests, and make further improvements.

Despite diverse results, there are still numerous points that should be further considered in future works. We highlight some of them as follows.

i) For state-delayed Markovian jump systems, the results on stability have some conservativeness. Some recently developed methods such as delay-partitioning method, small gain based input-output method, and reciprocally convex method can be utilized to further reduce the conservativeness caused by time-delay.

ii) The work has studied an NCSs architecture where a predictive controller uses an unreliable network affected by Markovian packet-dropouts to control a nonlinear plant with unbounded disturbances. It has been shown that, provided that the plant and network satisfy suitable conditions, stochastic stability of the closed-loop can be ensured by appropriate choice of tuning parameters. Future research could include the study of more general NCSs, including where the controller does not have access to the plant state.

iii) Chattering problem is one of the most common handicaps for applying SMC to real applications. The chattering in SMC systems is usually caused by 1) the dynamics with small time constants, which are often neglected in the ideal model; and 2) utilization of digital controllers with finite sampling rate, which causes so called 'discretization chattering'. The discontinuity leads to control chattering in practice, and involves high frequency dynamics. How to reduce chattering will be a research topic in future studies.

iv) Another future research direction is to investigate 2-D semi-Markovian jump systems, which consist of a family of subsystems described by discrete-time 2-D dynamical systems, and a rule specifying the switching among them. Some advanced techniques (such as quadratic Lyapunov functions and piecewise Lyapunov functions) used in analyzing and designing for 1-D Markovian jump systems can be extended to deal with 2-D semi-Markovian jump systems.

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