International Journal of Control, Automation and Systems 15(1) (2017) 385-393
http://dx.doi.org/10.1007/s12555-014-0575-5
ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Local Stabilization of Polynomial Fuzzy Model with Time Delay: SOS Approach

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Abstract: In this paper, a design method of control for Polynomial Fuzzy Models (PFM) with time delay is developed. By using a Polynomial Lyapunov Krasovskii Functional (PLKF) with double integral and by imposing bounds on the derivatives of each state, less conservative sufficient conditions are established to ensure the local stability of the closed loop system. Furthermore, a Domain Of Attraction (DOA) in which the initial states are ensured to converge asymptotically to the origin is estimated. The resulting conditions are formulated in terms of Sum-Of-Squares (SOS) which can be numerically (partially symbolically) solved via the recently developed SOSTOOLS. Some examples are provided to show the effectiveness and the merit of the design procedure.

Keywords: Domain Of Attraction (DOA), local stability, polynomial fuzzy systems, polynomial Lyapunov Krasovskii functional, sum of squares (SOS), time delay.

1. INTRODUCTION

The Takagi Sugeno (TS) fuzzy model [1] has been recognized as a powerful tool in describing the dynamics of a nonlinear system. A general TS fuzzy model combines some local linear models according to the nonlinear membership functions. So far a flurry of research activities have been presented for investigating TS fuzzy systems based on Linear Matrix Inequalities (LMI). A feasible solution to the LMI conditions can be found numerically using LMI toolbox of Matlab software [2].

Recently, Polynomial Fuzzy Models (PFM) have been appearing [3] for modeling of nonlinear systems using polynomials matrices in the consequence part. The analysis and control design methods developed for PFM aim to seek conditions in terms of sum of squares (SOS) conditions, which can be symbolically and numerically solved via SOSTOOLS [4]. It is well known that the SOS approach applied to polynomial fuzzy model provided more relaxed results than the LMI approach applied to TS fuzzy model. Different methodologies have been proposed for investigation of PFM in wide research topics, e.g., global stability analysis using Polynomial Lyapunov function (PLF) [3], multiple PLF [5], switching PLF [6]; Robust stability [7]; local stability analysis [8]; stabilization [9]-[10]; observer-based control [11]; output regulation [12]; fault tolerant control [13].

All the results cited previously are proposed for time-delay free systems. In practice, time-delay phenomenon appears commonly in many Engineering processes, such as chemical processes and long transmission lines in pneumatic, hydraulic, or rolling mill systems. It is shown that the presence of delays usually becomes the source of instability and deteriorating performance of systems. In recent years, PFM has been extended to tackle the analysis and control problems of nonlinear systems with time delay. Concerning the stabilization of PFM with time delay, limited works have been reported in the literature. Thus, in [14], the authors have provided a delay independent method of guaranteed cost control design. The disadvantage of this work is linked to the fact that the polynomial Lyapunov matrix is composed only of states whose dynamics are not directly affected by the control input and the delay.

Motivated by the aforementioned observation, in this paper, we study the stabilization of PFM with time delay. We aim to reduce the conservatism caused by the restriction on the construction of PLKF. To achieve this objective, we use a PLKF in order to obtain delay dependent SOS conditions. Moreover, we confine the state variation within a bounded interval in order to obtain a PLKF in which the Lyapunov matrix is not restricted to be dependent only of the states corresponding to zero rows of input polynomial matrices and delayed state polynomial matrices. Consequently, local stabilization problem is reformulated. The proposed stabilization conditions are given in

Manuscript received December 30, 2014; revised July 27, 2015; accepted December 26, 2015. Recommended by Associate Editor Ho Jae Lee under the direction of Editor Euntai Kim. This journal was supported by the Korean Federation of Science and Technology Societies Grant.

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terms of SOS.

The rest of the paper is structured as follows. Section 2. gives the description of PFM with time delay and the Polynomial Fuzzy Controller (PFC). We present also some definitions and lemmas to be used in this paper. In Section 3. we propose a delay-dependent design method of a PFC which ensures the stability of the closed loop system. The stabilization conditions are given in terms of SOS. In section 4., numerical examples are used to illustrate the relaxation and validity of the proposed local SOS stabilization conditions. Finally, some conclusions are given.

Notations: $\mathcal{C}_{n,\tau}$ denotes the Banach space of continuous functions mapping $[-\tau,0]$, $\|.\|$ refers to the Euclidean vector norm or spectral matrix norm and $\|\psi\|_c = sup_{-\tau\leq t\leq 0}\|\psi(t)\|$ stands for the norm of a function $\psi(t)\in\mathcal{C}_{n,\tau}$. A monomial in $x(t)=[x_1,...,x_{n_u}]$ is a function of the form $x_1^{d_1}x_2^{d_2}...x_{n_u}^{d_{n_u}}$ where $d_i, i=1,...,n_u$ are nonnegative integers. The degree of a monomial is defined as $d=\sum_{i=1}^{n_u} d_i$. $\lambda_{max}(P)$ denotes the maximum eigenvalues of the corresponding real symmetric matrices.

2. PROBLEM FORMULATION

2.1. Polynomial fuzzy model with state delay

Consider a nonlinear time-delay system which could be represented by the following PFM with time delay:
Plant Rule $i(i=1,2,\cdots,r)$:
If θ_1 is μ_{i1} and $\cdots$ and θ_p is μ_{ip} THEN

$$\begin{cases}\dot{x}(t) = A_i(x(t))\tilde{x}(x(t)) + A_{\tau i}(x(t))\tilde{x}(x(t-\tau)) \\ \qquad + B_i(x(t))u(t) \\ x(t)=\psi(t), t\in[-\tau,0],\end{cases} \tag{1}$$

where $\theta_j(x(t))$ and $\mu_{ij}(i=1,...,r, j=1,...,p)$ are the premise variable and the fuzzy sets respectively; $\psi(t)$ is the initial conditions; $x(t)\in\Re^{n_x}$ is the state; $u(t)\in\Re^{n_u}$ is the control input; r is the number of IF-THEN rules; $A_i(x(t))$, $B_i(x(t))$, $A_{\tau i}(x(t))$ are polynomial matrices in $x(t)$; $\tilde{x}(x(t))$ and $\tilde{x}(x(t-\tau))$ are column vectors whose entries are all monomials in $x(t)$ and $x(t-\tau(t))$ respectively; τ is a real positive constant representing the time delay.

By using singleton fuzzifier, the common used center-average defuzzifier and product interference, fuzzy model (1) can be represented as :

$$\begin{cases}\dot{x}(t) = \sum_{i=1}^{r} h_i(\theta(x(t)))\{A_i(x(t))\tilde{x}(x(t)) \\ \qquad + A_{\tau i}(x(t))\tilde{x}(x(t-\tau(t))) + B_i(x(t))u(t)\} \\ x(t)=\psi(t), t\in[-\tau,0],\end{cases} \tag{2}$$

The Polynomial Fuzzy Controller (PFC) that mirrors the structure of the PFM is presented as follows
Controller Rule $i(i=1,2,..,r)$:
If θ_1 is μ_{i1} and $\cdots$ and θ_p is μ_{ip} THEN

$$u(t) = K_i(x(t))\tilde{x}(x(t)). \tag{3}$$

The PFC is inferred as

$$u(t) = \sum_{i=1}^{r} h_i(\theta(x(t)))K_i(x(t))\tilde{x}(x(t)). \tag{4}$$

In the sequel, for brevity we use h_i, $\tilde{x}(t)$, $P(t)$ and $A_i(t)$ to denote respectively $h_i(\theta(x(t)))$, a monomial $\tilde{x}(x(t))$, a polynomial matrix $P(x(t))$ and a matrix $A_i(x(t))$.

Combining (2) and (4), the closed-loop fuzzy system can be expressed as follows:

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j[A_{ij}(t)\tilde{x}(t) + A_{\tau i}(t)\tilde{x}(t-\tau)] \tag{5}$$

with

$$A_{ij}(t) := A_i(t) + B_i(t)K_j(t). \tag{6}$$

2.2. Domain of attraction

Denoting the state trajectory of system (5) with initial condition $x_0=\psi\in\mathcal{C}[-\overline{\tau},0]$ by $x(t,\psi)$. Then the domain of attraction of the origin is set

$$\mathcal{A} = \{\psi\in\mathcal{C}_{n,\tau} : \lim_{t\to\infty} x(t,\psi)=0\}.$$

The determination of the exact domain of attraction is practically impossible [15]. An estimate $\chi_\delta\subset\mathcal{A}$ of the domain of attraction is given by

$$\chi_\delta = \{\psi\in\mathcal{C}_{n,\overline{\tau}} : max_{[-\tau,0]}|\psi|\leq\delta_1, max_{[-\tau,0]}|\dot{\psi}|\leq\delta_2\},$$

where $\delta_1>0$ and $\delta_2>0$ are scalars.

Definition 1: Define the following subsets of $\mathbb{R}^{n_x}$.

$$\varepsilon(X^{-1}(t),1) = \{x(t)\in\mathbb{R}^{n_x}; \tilde{x}^T(t)X^{-1}(t)\tilde{x}(t)\leq 1\}, \tag{7}$$

where $X(t)$ is a symmetric positive polynomial matrix.
Note that the symmetric positive polynomial matrix $X(t)$ is not constant but state-dependent. Therefore the resulting domains of attraction are not necessarily standard ellipsoids but could be more general to facilitate exploiting stability region of nonlinear systems.

$$\mathcal{L}(v) = \{x\in\mathbb{R}^{n_x}; |\dot{x}_k|\leq v_k, k=1,\ldots,n_x\}, \tag{8}$$

where $\dot{x}_k$ is the k^{th} row of $\dot{x}(t)$.

2.3. Sum of squares

A multivariate polynomial $f(x(t))$ (where $x(t)\in\Re^{n_x}$) is an SOS, if there exist polynomials $f_1(x(t)),\ldots, f_m(x(t))$ such that $f(x(t))=\sum_{i=1}^{m} f_i^2(x(t))$. It is clear that $f(x(t))$ being an SOS implies $f(x(t))>0$ for all $x(t)\in\Re^{n_x}$. This can be shown equivalent to the existence of a special quadratic form stated in the following lemma.

Lemma 1 [3]**:** Let $f(x(t))$ be a polynomial in $x(t) \in \Re^n$ of degree $2d$ and $\tilde{x}(t)$ be a column vector whose entries are all monomials in $x(t)$ with degree no greater than d. Then, $f(x(t))$ is an SOS if there exists a positive semi-definite matrix $P(t)$ such that

$$f(x(t)) = \tilde{x}(x(t))^T P(t) \tilde{x}(x(t)).$$

Hence, it is clear that $f(x(t))$ being an SOS implies that $f(x(t)) > 0$.

Lemma 2 [10]**:** Let $F(x(t))$ be an $N \times N$ symmetric polynomial matrix of degree $2d$ in $x(t)$. $\tilde{x}(x(t))$ is as defined in the above definition. Consider the following conditions.

1. $F(x(t)) \geq 0$ for all $x(t) \in \Re^n$.
2. $w^T(t)F(x(t))w(t)$ is a sum of squares, where $w(t) \in \Re^N$.
3. There exists a positive semi-definite matrix Q such that $w^T(t)F(x(t))w(t) = (w(t) \otimes \tilde{x}(x(t)))^T Q(v \otimes \tilde{x}(x(t)))$, where $\otimes$ denotes the Kronecker product.

Then $(1) \Longleftarrow (2)$ and $(2) \Longleftrightarrow (3)$.

Lemma 3 [10]**:** For a symmetric polynomial matrix $P(t)$ which is nonsingular for all $x(t)$, then

$$\frac{\partial P(t)}{\partial x_k} = -P(t) \frac{\partial P(t)^{-1}}{\partial x_k} P(t).$$

Lemma 4: Let us consider a negative-definite matrix Π. Given a polynomial matrix $X(t)$ of appropriate dimension, the two following inequalities are equivalent:

$$(X(t) + \Pi^{-1})^T \Pi (X(t) + \Pi^{-1}) \leq 0, \tag{9}$$

$$X(t)^T \Pi X(t) \leq -(X^T(t) + X(t)) - \Pi^{-1}. \tag{10}$$

3. MAIN RESULTS

In the following theorem we present the conditions for which closed loop system (5) is locally asymptotically stable.

Theorem 1: Closed loop system (5) is locally asymptotically stable within set $\varepsilon(X^{-1}(t), 1)$ if, for given positive scalar τ, there exist symmetric polynomial matrices $X^{-1}(t)$, S^{-1} and Z^{-1} satisfying the following conditions:

$$w_1^T(X^{-1}(t) - \varepsilon_1(t)I)w_1 \text{ is SOS}, \tag{11}$$

$$w_1^T(S^{-1} - \varepsilon_2(t)I)w_1 \text{ is SOS}, \tag{12}$$

$$w_1^T(Z^{-1} - \varepsilon_3(t)I)w_1 \text{ is SOS}, \tag{13}$$

$$\Phi_{ij}(t) + \Phi_{ji}(t) < 0, \quad i \leq j \tag{14}$$

$$(\lambda_{max}(X^{-1}(0)) + \tau \lambda_{max}(S^{-1}))\delta_1^2 + \frac{1}{2}\tau^2 \lambda_{max}(Z^{-1})\delta_2^2 \leq 1, \tag{15}$$

$$\varepsilon(X^{-1}(t), 1) \subset \mathcal{L}(v), \tag{16}$$

where:
w_1 is arbitrary vector.
$T(t)$ is a polynomial matrix whose (i,j)-th entry is given by $T^{ij}(t) = \frac{\partial \tilde{x}_i}{\partial x_j}(t)$, $\varepsilon_l(x)$ are nonnegative polynomials, for $l = 1,2,3$.

$$\Phi_{ij}(t) = \begin{bmatrix} \Phi_{ij}^{11}(t) & \Phi_{ij}^{12}(t) & \Phi_{ij}^{13}(t) \\ * & -S^{-1} & \Phi_{ij}^{23}(t) \\ * & * & -\frac{1}{\tau}Z^{-1} \end{bmatrix} < 0 \tag{17}$$

in which

$$\begin{aligned} \Phi_{ij}^{11}(t) &= X^{-1}(t)T(t)A_{ij}(t) + A_{ij}^T(t)T^T(t)X^{-1}(t) \\ &\quad + S^{-1} \pm \sum_{k=1}^{n} \frac{\partial X^{-1}(t)}{\partial x_k} v_k, \\ \Phi_{ij}^{12}(t) &= X^{-1}(t)T(t)A_{\tau i}(t), \\ \Phi_{ij}^{13}(t) &= A_{ij}(t)^T T^T(t) Z^{-1}, \\ \Phi_{ij}^{23}(t) &= A_{\tau i}(t)^T T^T(t) Z^{-1}. \end{aligned}$$

Proof : Choose the LKF as

$$\begin{aligned} V(t) = \; & \tilde{x}^T(t)X^{-1}(t)\tilde{x}(t) + \int_{t-\tau}^{t} \tilde{x}^T(\alpha)S^{-1}\tilde{x}(\alpha)d\alpha \\ & + \int_{-\tau}^{0}\int_{t+\sigma}^{t} \dot{\tilde{x}}^T(\alpha)Z^{-1}\dot{\tilde{x}}(\alpha)d\alpha d\sigma. \end{aligned} \tag{18}$$

The time derivative of this LKF (18) along the trajectory of system (1) is obtained as

$$\begin{aligned} \dot{V}(t) =& \dot{\tilde{x}}^T(t)X^{-1}(t)\tilde{x}(t) \\ & + \tilde{x}^T(t)X^{-1}(t)\dot{\tilde{x}}(t) + \tilde{x}^T(t)\dot{X}^{-1}(t)\tilde{x}(t) \\ & + \tilde{x}^T(t)S^{-1}\tilde{x}(t) - \tilde{x}^T(t-\tau)S^{-1}\tilde{x}(t-\tau) \\ & + \tau\dot{\tilde{x}}^T(t)Z^{-1}\dot{\tilde{x}}(t) - \int_{t-\tau}^{t} \dot{\tilde{x}}^T(s)Z^{-1}\dot{\tilde{x}}(s)ds \\ =& \dot{x}^T(t)T^T(t)X^{-1}(t)\tilde{x}(t) + \tilde{x}^T(t)X^{-1}(t)T(t)\dot{x}(t) \\ & + \tilde{x}^T(t)\left(\sum_{k=1}^{n} \frac{\partial X^{-1}(t)}{\partial x_k}\dot{x}_k\right)\tilde{x}(t) \\ & + \tilde{x}^T(t)S^{-1}\tilde{x}(t) - \tilde{x}^T(t-\tau)S^{-1}\tilde{x}(t-\tau) \\ & + \tau\dot{x}^T(t)T^T(t)Z^{-1}T(t)\dot{x}(t) - \int_{t-\tau}^{t} \dot{\tilde{x}}^T(s)Z^{-1}\dot{\tilde{x}}(s)ds. \end{aligned}$$

As it is shown in [16]- [17]

$$\dot{x}^T(t)T^T(t)Z^{-1}T(t)\dot{x}(t) \leq \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \tilde{\eta}(t)^T \times \begin{bmatrix} \Upsilon_{ij}^{11}(t) & \Upsilon_{ij}^{12}(t) \\ * & \Upsilon_{ij}^{22}(t) \end{bmatrix} \tilde{\eta}(t),$$

where

$$\eta^T(t) = [\tilde{x}^T(t), \tilde{x}^T(t-\tau(t))^T],$$

$$\Upsilon_{ij}^{11}(t) = \frac{\Delta_{ij}(t)^T}{2} T^T(t)Z^{-1}T(t)\frac{\Delta_{ij}(t)}{2},$$

$$\Upsilon_{ij}^{12}(t) = \frac{\Delta_{ij}(t)^T}{2} T^T(t)Z^{-1}T(t)\frac{\Delta_{\tau ij}(t)}{2},$$

$$\Upsilon_{ij}^{22}(t) = \frac{\Delta_{\tau ij}(t))^T}{2} T^T(t) Z^{-1} T(t) \frac{\Delta_{\tau ij}(t)}{2}$$

in which:

$$\Delta_{ij}(t) = A_{ij}(t) + A_{ji}(t),$$

$$\Delta_{\tau ij}(t) = A_{\tau i}(t) + A_{\tau j}(t).$$

Therefore, we get

$$\dot{V}(t) \leq \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \tilde{\eta}^T(t) \tilde{\Phi}_{ij}(t) \tilde{\eta}(t), \tag{19}$$

where:

$$\tilde{\Phi}_{ij}(t) = \begin{bmatrix} \tilde{\Phi}_{ij}^{11}(t) & \tilde{\Phi}_{ij}^{12}(t) \\ * & \tilde{\Phi}_{ij}^{22}(t) \end{bmatrix} \tag{20}$$

in which

$$\begin{aligned} \tilde{\Phi}_{ij}^{11}(t) &= X^{-1}(t)T(t)A_{ij}(t) + A_{ij}^T(t)T^T(t)X^{-1}(t) \\ &\quad + S + \sum_{k=1}^{n} \frac{\partial X^{-1}(t)}{\partial x_k}\dot{x}_k + \tau\Upsilon_{ij}^{11}(t), \\ \tilde{\Phi}_{ij}^{12}(t) &= X^{-1}(t)T(t)A_{\tau i}(t) + \tau\Upsilon_{ij}^{12}(t), \\ \tilde{\Phi}_{ij}^{22}(t) &= -S + \tau\Upsilon_{ij}^{22}(t). \end{aligned}$$

By applying schur complement $\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \tilde{\Phi}_{ij}(t) < 0$ is equivalent to

$$\begin{aligned} \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \widehat{\Phi}_{ij} &= \frac{1}{2}\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j (\widehat{\Phi}_{ij} + \widehat{\Phi}_{ji}) \\ &= \tfrac{1}{2}\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j (\bar{\Phi}_{ij} + \bar{\Phi}_{ji}) < 0, \end{aligned} \tag{21}$$

where

$$\widehat{\Phi}_{ij}(t) = \begin{bmatrix} \widehat{\Phi}_{ij}^{11}(t) & \Phi_{ij}^{12}(t) & \widehat{\Phi}_{ij}^{13}(t) \\ * & -S^{-1} & \widehat{\Phi}_{ij}^{23}(t) \\ * & * & -\frac{1}{\tau}Z^{-1} \end{bmatrix} < 0 \tag{22}$$

in which

$$\begin{aligned} \widehat{\Phi}_{ij}^{11}(t) &= X^{-1}(t)T(t)A_{ij}(t) + A_{ij}^T(t)T^T(t)X^{-1}(t) \\ &\quad + S + \sum_{k=1}^{n} \frac{\partial X^{-1}(t)}{\partial x_k}\dot{x}_k, \\ \widehat{\Phi}_{ij}^{13}(t) &= \frac{\Delta_{ij}(t)^T}{2} T^T(t) Z^{-1}, \\ \widehat{\Phi}_{ij}^{23}(t) &= \frac{\Delta_{\tau ij}(t)^T}{2} T^T(t) Z^{-1}, \end{aligned} \tag{23}$$

and

$$\bar{\Phi}_{ij}(t) = \begin{bmatrix} \widehat{\Phi}_{ij}^{11}(t) & \Phi_{ij}^{12}(t) & \Phi_{ij}^{13}(t) \\ * & -S^{-1} & \Phi_{ij}^{23}(t) \\ * & * & -\frac{1}{\tau}Z^{-1} \end{bmatrix} < 0. \tag{24}$$

To avoid the construction of a restrictive Polynomial Lyapunov Matrix (PLM) depending only of states whose dynamics are not directly affected by the control input and delayed state, we will specify the bound of the state derivative $x(t) \in \mathcal{L}(v)$. Then, (14) implies that $\dot{\mathbf{V}}(t) \leq 0$.

From $\dot{\mathbf{V}}(t) \leq 0$ it follows that $\mathbf{V}(t) \leq \mathbf{V}(0)$ and therefore

$$\begin{aligned} &\tilde{x}^T(t)X^{-1}(t)\tilde{x}(t) \leq V(t) \leq V(0) \leq \\ &(\lambda_{max}(X^{-1}(0)) + \tau\lambda_{max}(S^{-1}))\delta_1^2 + \frac{1}{2}\tau^2\lambda_{max}(Z)\delta_2^2. \end{aligned}$$

Inequality (15) then guarantees that for all initial functions $\tilde{\psi}(t)$, the trajectories of $x(t)$ remain within $\varepsilon(X^{-1}(t),1)$. In the following Theorem, we transform the matrix inequalities in Theorem 1 into SOS.

Theorem 2: Closed loop system (5) is locally asymptotically stable within the set $\varepsilon(X^{-1}(t),1)$ if, for given positive scalar τ, there exist a symmetric polynomial matrix $X(t)$, symmetric matrices S, Z and polynomial matrices $M_i(x)$ satisfying (15) and the following SOS conditions:

$$w_1^T(X(t) - \tilde{\varepsilon}_1(t)I)w_1 \text{ is SOS,} \tag{25}$$

$$w_1^T(S - \tilde{\varepsilon}_2(t)I)w_1 \text{ is SOS,} \tag{26}$$

$$w_1^T(Z - \tilde{\varepsilon}_3(t)I)w_1 \text{ is SOS,} \tag{27}$$

$$w_2^T(-\Omega_{ij}(t) - \Omega_{ji}(t) - \tilde{\varepsilon}_{4ij}(t)I)w_2 \text{ is SOS,} \tag{28}$$

$$w_3^T(\Xi_{ij}^k(t) - \tilde{\varepsilon}_{5ij}^k(t)I)w_3 \text{ is SOS,} \tag{29}$$

$$w_4^T(\begin{bmatrix} X(t) & X(t) \\ * & \frac{1}{\tau}S \end{bmatrix} - \tilde{\varepsilon}_6(t)I)w_4 \text{ is SOS,} \tag{30}$$

where:
w_1, w_2, w_3 and w_4 are arbitrary vectors.
$\tilde{\varepsilon}_l(x)$, $\tilde{\varepsilon}_{4ij}(t)$, $\tilde{\varepsilon}_{5ij}^k$ and $\tilde{\varepsilon}_6(t)$ are nonnegative polynomials, for $l = 1,2,3$, $i,j = 1,...,r$, $k = 1,....,n_x$

$$\Omega_{ij}(t) = \begin{bmatrix} \Omega_{ij}^{11}(t) & \Omega_{ij}^{12}(t) & \Omega^{13}(t) & X(t) \\ * & \Omega_{ij}^{22}(t) & X(t)A_{\tau i}(t)^T & 0 \\ * & * & -\frac{1}{\tau}Z & 0 \\ * & * & * & -S \end{bmatrix} \tag{31}$$

in which

$$\begin{aligned} \Omega_{ij}^{11}(t) &= T(t)A_i(t)X(t) + T(t)B_i(t)M_j(t) \\ &\quad + X(t)A_i(t)^T T^T(t) + M_j(t)^T B_i(t)^T T^T(t) \\ &\quad \mp \sum_{k=1}^{n} \frac{\partial X(t)}{\partial x_k} v_k, \\ \Omega_{ij}^{12}(t) &= T(t)A_{\tau i}(t)X(t), \\ \Omega_{ij}^{13}(t) &= X(t)A_i(t)^T T^T(t) + M_j(t)^T B_i(t)^T T^T(t), \\ \Omega_{ij}^{22}(t) &= -2X(t) + S, \end{aligned}$$

and

$$\Xi_{ij}^{k}(t)=\begin{bmatrix}\frac{v_k^2}{2} & A_i^k(t)X(t)+B_i^k(t)M_j(t) & A_{\tau i}^k(t)S\\ * & X(t) & 0\\ * & * & \tau S\end{bmatrix}. \quad (32)$$

In this case, stabilizing feedback gains $K_i(t)$ can be obtained from $X(t)$ and $M_i(t)$ as $K_i(t)=M_i(t)X(t)^{-1}$.

Proof :
Multiplying both sides of $\Phi_{ij}(t)$ by $diag\Big(X(t),X(t),X(t)\Big)$ and pre-and-post multiplying the obtained matrix with $diag[I,I,ZX^{-1}(t)]$ and its transpose, we get:

$$\Theta_{ij}(t)=\begin{bmatrix}\Theta_{ij}^{11}(t) & \Theta_{ij}^{12}(t) & \Theta_{ij}^{13}(t)\\ * & -X(t)S^{-1}X(t) & \Theta_{ij}^{23}(t)\\ * & * & -\frac{1}{\tau}Z\end{bmatrix}, \quad (33)$$

where:

$$\Theta_{ij}^{11}(t)=T(t)A_{ij}(t)X(t)+X(t)A_{ij}^T(t)T^T(t) + X(t)S^{-1}X(t)\mp\sum_{k=1}^{n}\frac{\partial X(t)}{\partial x_k}v_k,$$
$$\Theta_{ij}^{12}(t)=T(t)A_{\tau i}(t)X(t),$$
$$\Theta_{ij}^{13}(t)=X(t)A_{ij}^T(t)T^T(t),$$
$$\Theta_{ij}^{23}(t)=X(t)A_{\tau i}^T(t)T^T(t).$$

By applying Schur complement and lemma 3, condition (28) implies that $\Phi_{ij}(t)+\Phi_{ji}(t)<0$.

In the next, we prove that SOS (29)-(30) guarantee that $|\dot{x}_k|\le v_k$, $\forall x(t)\in\varepsilon(X^{-1}(t),1)$.
On one hand, by applying Schur complement, SOS (30) is equivalent to

$$\tau\tilde{x}(t)^TS^{-1}\tilde{x}(t)\le\tilde{x}^T(t)X^{-1}(t)\tilde{x}(t).$$

Then, the following inequalities hold $\forall x(t)\in\varepsilon(X^{-1}(t),1)$

$$\tilde{x}^T(t)X^{-1}(t)\tilde{x}(t)\le 1, \quad (34)$$

and

$$\tau\tilde{x}(t)^TS^{-1}\tilde{x}(t)\le 1. \quad (35)$$

Furthermore, for all initial functions $\tilde{\psi}(t)$ satisfying (15), it follows that:

$$\tau\psi(t)^TS^{-1}\psi(t)\le\tau\lambda_{max}(S^{-1})\delta_1^2\le 1. \quad (36)$$

By considering (35) and (36), we can verify that the following inequality holds :

$$\tau\tilde{x}(t-\tau)^TS^{-1}\tilde{x}(t-\tau)\le 1.$$

Therefore

$$2v_k\ge v_k(1+\frac{1}{2}\tilde{x}^T(t)X^{-1}(t)\tilde{x}(t) +\frac{1}{2}\tau\tilde{x}(t-\tau)^TS^{-1}\tilde{x}(t-\tau)). \quad (37)$$

On the other hand, Pre- and post-multiplying both sides of $\Xi_{ij}^k(t)$ by $diag(\sqrt{\frac{2}{v_k}},\sqrt{\frac{v_k}{2}}X^{-1}(t),\sqrt{\frac{v_k}{2}}S^{-1})$ and its transpose, we obtain

$$\begin{bmatrix}v_k & A_i^k(t)+B_i^k(t)K_j(t) & A_{\tau i}^k(t)\\ * & \frac{v_k}{2}X^{-1}(t) & 0\\ * & * & \frac{v_k}{2}\tau S^{-1}\end{bmatrix}\ge 0,$$

which implies that

$$\vartheta^T(t)\begin{bmatrix}v_k & A_i^k(t)+B_i^k(t)K_j(t) & A_{\tau i}^k(t)\\ * & \frac{v_k}{2}X^{-1}(t) & 0\\ * & * & \frac{v_k}{2}v_k\tau S^{-1}\end{bmatrix}\vartheta(t)\ge 0, \quad (38)$$

where:

$$\vartheta^T(t)=[1,\pm\tilde{x}^T(t),\pm\tilde{x}(t-\tau)^T].$$

The last inequality can be rewritten as

$$v_k(1+\frac{1}{2}\tilde{x}^T(t)X^{-1}(t)\tilde{x}(t)+\frac{1}{2}\tau\tilde{x}(t-\tau)^TS^{-1}\tilde{x}(t-\tau)) \ge 2|(A_i^k(t)+B_i^k(t)K_j(t))\tilde{x}(t)|+2|A_{\tau i}^k(t)\tilde{x}(t-\tau)|. \quad (39)$$

By considering (37) and (39), we get

$$2v_k\ge 2|(A_i^k(t)+B_i^k(t)K_j(t))\tilde{x}(t)|+2|A_{\tau i}^k(t)\tilde{x}(t-\tau)| \ge 2|(A_i^k(t)+B_i^k(t)K_j(t))\tilde{x}(t)+A_{\tau i}^k(t)\tilde{x}(t-\tau)|$$

since $h_i\ge 0$ and $\sum_{i=1}^{r}h_i=1$, then $|\dot{x}_k|\le v_k$. This complete the proof of Theorem 2.

Remark 1: There are two approaches proposed in the literature to measure the largeness of the attraction set. The first one is given in [18] where a largeness of a set is measured by its volume. The second one takes the shape of the attraction set into consideration [19]. In what follows, we will measure the size of $\varepsilon(X^{-1}(t),1)$ with respect to a shape reference set $\mathcal{X}_R$ by the largest γ such that $\gamma\mathcal{X}_R\subset\varepsilon(X^{-1}(t),1)$. Thus, the determination of largest $\varepsilon(X^{-1}(t),1)$ can be formulated into the following optimization problem:
Sup γ
s.t.

$$\begin{cases}\gamma\mathcal{X}_R\subset\varepsilon(X^{-1}(t),1)\\ (44)-(26)-(27)-(28)-(29)-(30)\end{cases} \quad (40)$$

If $\mathcal{X}_R = \{x(t) \in \mathbb{R}^{n_x}; \tilde{x}^T(t)R\tilde{x}(t) \leq 1\}$, then constraint $\gamma\mathcal{X}_R \subset \varepsilon(X^{-1}(t),1)$ is converted to $\gamma^2 X^{-1}(t) \leq R$. Note that this inequality holds if and only if

$$\begin{bmatrix} \frac{1}{\gamma^2}R & I \\ I & X(t) \end{bmatrix} \geq 0. \tag{41}$$

Let $\mu = \frac{1}{\gamma^2}$. Then the optimization problem (40), with $\mathcal{X}_R$ being an ellipsoid, can be formulated as
min μ
s.t.

$$\begin{cases} \begin{bmatrix} \mu R & I \\ I & X(t) \end{bmatrix} \geq 0 \\ (44)-(26)-(27)-(28)-(29)-(30) \end{cases} \tag{42}$$

Now, we consider a simple case, i.e., delay free case. We rewrite the overall polynomial fuzzy model without delay terms as:

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j A_{ij}(t)\tilde{x}(t). \tag{43}$$

In this case, we have the following result which is a direct corollary of Theorem 2.

Corollary 1: Closed loop system (43) is locally asymptotically stable within set $\varepsilon(X^{-1}(t),1)$ if, for given positive scalar τ, there exists a symmetric polynomial matrix $X(t)$ and polynomial matrices $M_i(x)$ satisfying the following conditions:

$$w_1^T(X(t)-\tilde{\varepsilon}_1(t)I)w_1 \ \text{ is SOS}, \tag{44}$$
$$w_2^T(-\Omega_{ij}^{11}(t)-\Omega_{ji}(t)-\tilde{\varepsilon}_{4ij}(t)I)w_2 \ \text{ is SOS}, \tag{45}$$
$$w_3^T(\Gamma_{ij}^k(t)-\tilde{\varepsilon}_{5ij}^k(t)I)w_3 \ \text{ is SOS}, \tag{46}$$
$$(\lambda_{max}(X^{-1}(0)))\delta_1^2 \leq 1, \tag{47}$$

where

$$\Gamma_{ij}^k(t) = \begin{bmatrix} \frac{v_k^2}{2} & A_i^k(t)X(t)+B_i^k(t)M_j(t) \\ * & X(t) \end{bmatrix}. \tag{48}$$

In this case, a stabilizing feedback gain $K_i(t)$ can be obtained from $X(t)$ and $M_i(t)$ as $K_i(t) = M_i(t)X(t)^{-1}$.

Remark 2: Tanaka *et al.* [10] proposed a stabilizing criterion in terms of SOS conditions by using a polynomial Lyapunov function in the form of:

$$V(t) = \tilde{x}^T(t)X^{-1}(\bar{x}(t))\tilde{x}(t), \tag{49}$$

where $\bar{x}(t)$ is composed of states whose dynamics is not directly affected by the control input.

The disadvantage of this result is linked to the fact that the Lyapunov polynomial matrix $X^{-1}(\bar{x}(t))$ is composed only of the states corresponding to the zeros rows of $B_i(t)$. For example, if the input matrices $B_i(t)$ not have zero rows, the matrix $X^{-1}(\bar{x}(t))$ can only be a constant. Corollary 1 allows to overcome this restriction by employing polynomial Lyapunov matrix $X^{-1}(t)$ depending on all states.

Remark 3: In this paper, the SOS conditions are solved via SeDuMi in addition to SOSTOOLS. For more details of how to solve the SDPs using SeDuMi, see [4].

Remark 4: Li *et al.* have presented in [14] a guaranteed cost controller design for PFM with time delay. The restrictions of this approach are: 1) the result is delay independent, 2) The PLM $X(t)$ depends on states whose corresponding rows in $B_i(t)$ and $A_{\tau i}(t)$ are zeros. In our approach, we have overcome these great disadvantages by presenting delay dependent SOS and using a PLM $X(t)$ depends on all states.

4. ILLUSTRATIVE EXAMPLES

4.1. Example 1

Consider a delay free polynomial fuzzy model with two IF-THEN rules where:

$$A_1(t) = \begin{bmatrix} -1+x_1+x_1^2+x_1x_2-x_2^2 & 1 \\ -1 & -1 \end{bmatrix}, \tag{50}$$
$$A_2(t) = \begin{bmatrix} -1+x_1+x_1^2+x_1x_2-x_2^2 & 1 \\ 0.2172 & -1 \end{bmatrix}, \tag{51}$$
$$B_1(t) = \begin{bmatrix} x_1 \\ 0.1 \end{bmatrix}, \quad B_2(t) = \begin{bmatrix} x_1 \\ 0.1 \end{bmatrix}. \tag{52}$$

The SOS conditions presented in Theorem 2 of [10] cannot produce a feasible solution. Using our corollary, we obtain a feasible solution by setting $v_1 = v_2 = 2$ and a polynomial Lyapunov matrix $X(t)$ of fourth order.

$$X(t) = \begin{bmatrix} X_{11}(t) & X_{12}(t) \\ * & X_{22}(t) \end{bmatrix},$$
$$\begin{aligned} X_{11}(t) = 1e^{-8}\Big(&-0.21675x_1^4-0.72733x_1^3x_2 \\ &-0.59966x_1^3+0.58987x_1^2x_2^2+0.33633x_1^2x_2 \\ &-76x_1^2-0.66798x_1x_2^3-0.15007x_1x_2^2 \\ &+94.18x_1x_2+280.11x_1+0.60487x_2^4 \\ &-0.26575x_2^3-19.135x_2^2+14.351x_2+143660\Big), \end{aligned}$$
$$\begin{aligned} X_{12}(t) = 1e^{-8}\Big(&-0.085763x_1^4+0.10917x_1^3x_2-0.023221x_1^3 \\ &-0.26761x_1^2x_2^2-0.5937x_1^2x_2+37.998x_1^2 \\ &+0.013204x_1x_2^3-0.029219x_1x_2^2-21.594x_1x_2 \\ &+374.51x_1-0.17099x_2^4+0.15613x_2^3 \\ &+81x_2^2-338x_2+23832\Big), \end{aligned}$$
$$\begin{aligned} X_{22}(t) = 1e^{-8}\Big(&0.15156x_1^4-0.040076x_1^3x_2-0.83706x_1^3 \\ &+0.76376x_1^2x_2^2+0.35619x_1^2x_2+34201.0x_1^2 \end{aligned}$$

$$+0.10201x_1x_2^3 - 1.2254x_1x_2^2 - 1274x_1x_2 + 4169x_1 + 0.16226x_2^4 - 0.73012x_2^3 + 33953x_2^2 + 2941x_2 + 452370\Big),$$

$$M_1(t) = \begin{bmatrix} M_1^{11}(t) & M_1^{12}(t) \end{bmatrix},$$

$$M_1^{11}(t) = -0.16883e^{-5}x_1^2 - 0.22668e^{-5}x_1x_2 - 0.19061e^{-2}x_1 + 0.1562e^{-5}x_2^2 - 0.14156e^{-2}x_2 - 0.14166e^{-2},$$

$$M_1^{12}(t) = -0.25988e^{-5}x_1^2 - 0.15016e^{-5}x_1x_2 - 0.64972e^{-3}x_1 + 0.14235e^{-4}x_2^2 - 0.29607e^{-3}x_2 - 0.19206e^{-3},$$

$$M_2(t) = \begin{bmatrix} M_2^{11}(t) & M_2^{12}(t) \end{bmatrix},$$

$$M_2^{11}(t) = -0.1622e^{-5}x_1^2 - 0.22268e^{-5}x_1x_2 - 0.19166e^{-2}x_1 + 0.1635e^{-5}x_2^2 - 0.14138e^{-2}x_2 - 0.14214e^{-2},$$

$$M_2^{12}(t) = -0.26041e^{-5}x_1^2 - 0.15019e^{-5}x_1x_2 - 0.67318e^{-3}x_1 + 0.14134e^{-4}x_2^2 - 0.31722e^{-3}x_2 - 0.19275e^{-3},$$

where $1e^{-n} = 10^{-n}, n \geq 0$.

4.2. Example 2

Consider the following polynomial fuzzy model with time delay:

$$\dot{x}(t) = \sum_{i=1}^{2} h_i[A_i(t)x(t) + A_{\tau i}(t-\tau)x(t-\tau) + B_i u(t)], \tag{53}$$

where:

$$A_1(t) = \begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix}, \quad A_2(t) = \begin{bmatrix} -1 & 1 \\ -0.2172 & -1 \end{bmatrix}, \tag{54}$$

$$A_{\tau_1}(t) = A_{\tau_2}(t) = \begin{bmatrix} -0.1 & 0 \\ 0.1 & 0 \end{bmatrix}, \tag{55}$$

$$B_1(t) = B_2(t) = \begin{bmatrix} x_2 \\ 0.1 \end{bmatrix}. \tag{56}$$

For simulation, we consider second-order $X(t)$ and we set $\tau = 0.35$ and $v_1 = v_2 = 4$. By solving the SOS conditions in Theorem 2, we obtain the following feasible solution:

$$X(t) = \begin{bmatrix} X_{11}(t) & X_{12}(t) \\ * & X_{22}(t) \end{bmatrix},$$

$$X_{11}(t) = 1.398e^{-8}x_2^2 - 8.2811e^{-6}x_2 + 0.6014,$$

$$X_{12}(t) = -0.26458e^{-8}x_2^2 + 0.25824e^{-6}x_2 + 2.2547e^{-2},$$

$$X_{22}(t) = 1.2417e^{-8}x_2^2 - 2.5611e^{-6}x_2 + 0.54621,$$

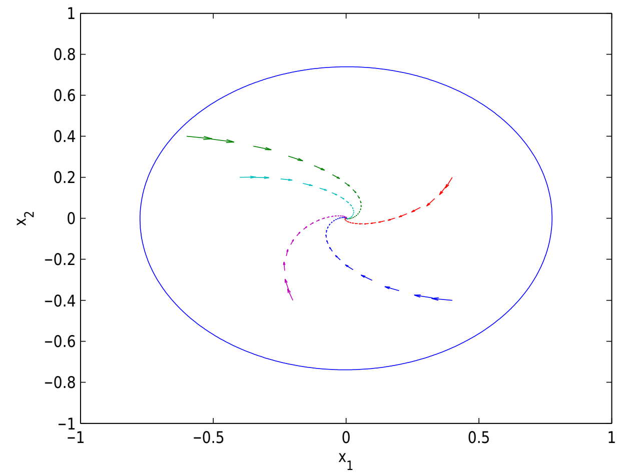

Fig. 1. Invariant set $\varepsilon(X^{-1}(t),1)$ and phase plot of $x_1(t)$ and $x_2(t)$ with several different initial conditions.

$$S = \begin{bmatrix} 0.8805 & 0.04794 \\ * & 0.8140 \end{bmatrix}, \ Z = \begin{bmatrix} 0.6233 & -0.01238 \\ * & 0.6240 \end{bmatrix},$$

$$M_1(t) = \begin{bmatrix} M_1^{11}(t) & M_1^{12}(t) \end{bmatrix},$$

$$M_1^{11}(t) = 0.62312e^{-8}x_2 - 0.47239e^{-5},$$

$$M_1^{12}(t) = -0.10563e^{-7}x_2 - 0.46864e^{-5},$$

$$M_2(t) = \begin{bmatrix} M_2^{11}(t) & M_2^{12}(t) \end{bmatrix},$$

$$M_2^{11}(t) = 0.77e^{-8}x_2 - 0.16935e^{-5},$$

$$M_2^{12}(t) = -0.10802e^{-7}x_2 - 0.33171e^{-5}.$$

Fig. 1 shows the invariant set $\varepsilon(X^{-1}(t),1)$ and the dynamics of the closed-loop system with several different initial conditions. The convergence of state trajectories for different initial conditions shows that the obtained controller gains stabilize the system.

By solving the optimization problem (42) for $\mathcal{X}_R = \varepsilon(I,1)$, we obtain $\mu = 1.01$

$$X(t) = \begin{bmatrix} X_{11}(t) & X_{12}(t) \\ * & X_{22}(t) \end{bmatrix},$$

$$X_{11}(t) = 0.2913e^{-6}x_2^2 + 0.66571e^{-5}x_2 + 0.99032,$$

$$X_{12}(t) = -0.50213e^{-7}x_2^2 - 0.37358e^{-6}x_2 + 0.43092e^{-4},$$

$$X_{22}(t) = 0.28448e^{-6}x_2^2 - 0.2983e^{-5}x_2 + 0.99034,$$

$$S = \begin{bmatrix} 0.6450 & 0.1674 \\ * & 1.5795 \end{bmatrix}, \ Z = \begin{bmatrix} 1.7838 & 0.6625 \\ * & 1.7503 \end{bmatrix},$$

$$M_1(t) = \begin{bmatrix} M_1^{11}(t) & M_1^{12}(t) \end{bmatrix},$$

$$M_1^{11}(t) = 0.85536e^{-7}x_2 - 0.19833e^{-3},$$

$$M_1^{12}(t) = -0.25056e^{-6}x_2 - 0.19915e^{-3},$$

$$M_2(t) = \begin{bmatrix} M_2^{11}(t) & M_2^{12}(t) \end{bmatrix},$$

$$M_2^{11}(t) = 0.12887e^{-6}x_2 - 0.20987e^{-4},$$

$$M_2^{12}(t) = -0.2852e^{-6}x_2 - 0.2219e^{-4}.$$

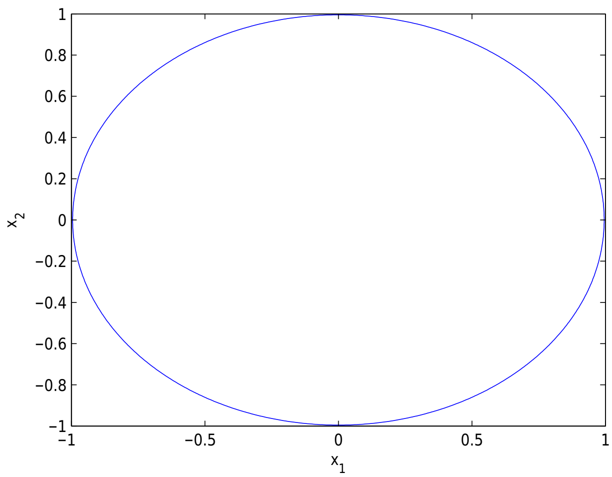

Fig. 2. Invariant set $\varepsilon(X^{-1}(t),1)$ computed by the optimization problem.

Fig. 2 depicts the resulting invariant ellipsoid. The optimization problem maximizes the invariant set $\varepsilon(X^{-1}(t),1)$.

5. CONCLUSION

In this paper, we have studied the delay dependent locally stabilization problem for PFM with time delay. The sufficient conditions are given in terms of SOS which can be symbolically and numerically solved via the SOSOPT and the SeDuMi. The result for delay-free case is easy corollary. The SOS conditions is obtained without imposing any restrictions in the construction of the Lyapunov Krasovskii functional. This improved result is obtained by bounding the variation rates of each state. Several examples have been given to illustrate the effectiveness of the established approach.

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