International Journal of Control, Automation and Systems 14(5) (2016) 1254-1261
http://dx.doi.org/10.1007/s12555-014-0541-2
ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Observer-based H_∞ Guaranteed Cost Control for Uncertain Singular Time-delay Systems with Input Saturation

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Abstract: In this paper, the problem of observer-based H_∞ guaranteed cost control for uncertain singular time-delay systems with actuator saturation is concerned. A delay-dependent sufficient condition is proposed, which guarantees that the closed-loop system is admissible via Lyapunov theory and linear matrix inequality (LMI) approach. Then, with this condition, the estimation of stability region, the upper bound of cost function and the design method of observer-based H_∞ guaranteed cost controller are given by solving linear matrix inequalities and convex optimization problems. Finally, numerical examples are provided to demonstrate the effectiveness of the proposed method.

Keywords: Free-weighting matrix, guaranteed cost, input saturation, linear matrix inequality, observer-based H_∞ control, singular time-delay systems.

1. INTRODUCTION

It has been recognized that the state variables are often not measurable in most practical situations. Many observer design problems, which are concerned with using the available information on inputs and outputs to reconstruct the unmeasured states, have been widely investigated for many practical applications in [1, 2], whereas only a few works have been carried out on singular systems [3, 4]. Singular systems (also known as descriptor systems, generalized state-space systems, implicit systems, differential-algebraic systems) can describe many practical systems more reasonably than regular ones [5,6]. It should be pointed out that the stability, stabilization and H_∞ control problem for singular systems are much more complicated than the regular ones. During the past decades, singular systems have received considerable interest and many fundamental system theories have been established, see [7, 8] and the references therein.

During the recent years, much attention has been devoted to the study of singular systems with time-delay. The methods may be classified into two categories: delay-independent cases [9] and delay-dependent cases [10]. Generally speaking, delay-independent cases are likely to be conservative, especially when the delay is comparatively small. Therefore, more attention has been paid to the study on delay-dependent stability of singular time-delay system and several results are obtained, see [11, 12]. At present, H_∞ control problem and robust control problem have made great progress [9–12]. Most aforementioned references are concerned with asymptotic stability, but the system can converge quickly is desired in practice. Moreover, when controlling a real plant, it is also desirable to design a control system which is not only asymptotically stable but also guarantees an adequate level of performance. Therefore, the problems of guaranteed cost control for the singular systems have received considerable attention in recent years [13–15].

In addition, nearly all physical systems are subject to saturation constraints. If the controller is designed without considering this kind of nonlinearity, the presence of actuator saturation can lead to the performance degradation. Hence, additional constraints should be imposed on the analysis of systems [16–19]. The control synthesis problem for a class of linear time-delay systems with actuator saturation is investigated in [16]. Certainly, the problems of stability analysis and controller design for singular linear systems subject to actuator saturation are more complex than those for normal systems. There have a few but not many works dealing with the problem for singular systems [17–19]. It established a set of conditions under which an ellipsoid is contractively invariant with respect to a singular linear system under a saturated linear feedback in [18, 19].

The novelty of our research is to design an observer-based guaranteed cost controller such that the uncertain

Manuscript received December 8, 2014; revised March 19, 2015, May 5, 2015, and September 5, 2015; accepted October 17, 2015. Recommended by Associate Editor Jiuxiang Dong under the direction of Editor Yoshito Ohta. This work is supported by the National Natural Science Foundation of China (No. 61273004) and the Natural Science Foundation of Hebei province (No. F2014203085).

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singular time-delay system is not only robustly stable, but also satisfies a prescribed H_∞ performance level. First, a new delay-dependent LMI condition which guarantees that the closed-loop systems are admissible with H_∞ performance γ is derived. Adding a free-weighting matrix will play a significant role in the derivation of a less conservative delay-dependent result. Then, using the conditions, the estimation of stability region, the observer and the guaranteed cost controller are given by solving a convex optimization problem. Finally, numerical examples show the less conservativeness of the results and demonstrate the validity and merit of the proposed approach.

Notations: X^{T} denotes the transpose of X. X^{+} denotes the pseudo-inverse matrix of X. Symmetric elements in the matrix are denoted by $*$. $sym(X)$ denotes $X+X^{\mathrm{T}}$. For a matrix X, x_j denotes j-th row of X.

2. PROBLEM DESCRIPTION AND PRELIMINARIES

Consider the following uncertain singular time-delay systems

$$\begin{cases} E\dot{x}(t) = (A+\Delta A)x(t) + (A_d+\Delta A_d)x(t-d(t)) \\ \qquad + (B+\Delta B)\,\mathrm{sat}(u(t)) + (B_\omega+\Delta B_\omega)\,\omega(t), \\ y(t) = C_1 x(t), \\ z(t) = (C+\Delta C)x(t), \\ x(t) = \phi(t), t\in[-d,0], \end{cases} \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state, $u(t)\in\mathbb{R}^l$ is the control input, $\omega(t)\in\mathbb{R}^q$ is the disturbance input which belongs to $\omega(t)\in\mathcal{L}_2[0,\infty)$, $z(t)\in\mathbb{R}^m$ is the control output, $y(t)\in\mathbb{R}^p$ is the measured output, $\phi(t)$ is the initial condition of the system, $d(t)$ is the time-varying delay satisfying $0<d(t)\le d$ and $\dot{d}(t)\le\mu$. The saturating term $\mathrm{sat}(u(t))$ in (1) is a vector-valued function defined as follows:

$$\mathrm{sat}(u(t)) = [\mathrm{sat}(u_1(t)),\cdots,\mathrm{sat}(u_l(t))]^{\mathrm{T}}.$$

The matrix $E\in\mathbb{R}^{n\times n}$ may be singular, and we assume that $\mathrm{rank}(E)=r\le n$. A, A_d, B, B_ω, C and C_1 are known real constant matrices of appropriate dimensions, $\triangle A$, $\triangle A_d$, $\triangle B$, $\triangle B_\omega$ and $\triangle C$ are unknown matrices representing norm-bounded parametric uncertainties, and are assumed to be of the following form

$$\begin{aligned} &\Delta A = E_1F_1(t)H_1,\ \ \Delta A_d = E_2F_2(t)H_2, \\ &\Delta B = E_3F_3(t)H_3,\ \ \Delta B_\omega = E_4F_4(t)H_4, \\ &\Delta C = E_5F_5(t)H_5, \end{aligned} \tag{2}$$

where $E_i, H_i\,(i=1,\cdots,5)$ are known real constant matrices with appropriate dimensions, and $F_i(t)\,(i=1,\cdots,5)$ are unknown time-varying matrices satisfying

$$F_i^{\mathrm{T}}(t)F_i(t)\le I,\ (i=1,\cdots,5), \tag{3}$$

The parametric uncertainties $\triangle A$, $\triangle A_d$, $\triangle B$, $\triangle B_\omega$ and $\triangle C$ are said to be admissible if both (2) and (3) hold.

Remark 1: We assume that C_1 is a known real constant matrix with appropriate dimensions. However, C_1 is assumed to be a matrix of full-row rank in [11]. And, in this paper, there is no restriction on the rank of C_1. Therefore, the condition that C_1 is a non-singular matrix or a matrix of full-row rank can be seen as a special case of our article. Therefore, the method proposed by us has a wider application.

Associated with the system (1) is the following cost function

$$J = \int_0^\infty \left[x^{\mathrm{T}}(t)Qx(t) + u^{\mathrm{T}}(t)Ru(t)\right]dt, \tag{4}$$

where Q and R are given positive-definite symmetric matrices of appropriate dimensions.

Now, we construct the following observer-based controller

$$\begin{cases} E\dot{\bar{x}}(t) = A\bar{x}(t) + A_d\bar{x}(t-d(t)) \\ \qquad +Bu(t) + L(y(t) - C_1\bar{x}(t)), \\ u(t) = K\bar{x}(t), \end{cases} \tag{5}$$

where $\bar{x}(t)$ is the estimated state, K and L are controller gain and observer gain to be designed, respectively. Define the state estimated error $e(t)=x(t)-\bar{x}(t)$.

Definition 1 [5]**:** 1) The singular time-delay system

$$\begin{cases} E\dot{x}(t) = Ax(t) + A_dx(t-d(t)) \\ x(t) = \phi(t), t\in[-d,0] \end{cases} \tag{6}$$

is said to be regular and impulse free, if the pair (E,A) is regular and impulse free.

2) The system (6) is said to be asymptotically stable, if for any $\varepsilon>0$, there exists a scalar $\delta(\varepsilon)>0$, such that for any compatible initial condition $\phi(t)$ with $\sup\limits_{-d<t\le 0}\|\phi(t)\|<\delta(t)$, the solution $x(t)$ of (6) satisfies $\|x(t)\|<\varepsilon$ for $t>0$ and $\lim\limits_{t\to 0}x(t)=0$.

3) The system (6) is said to be admissible, if it is regular, impulse free and asymptotically stable.

Definition 2 [20]**:** For a matrix $H\in\mathbb{R}^{l\times n}$, let h_i be the i-th row of the matrix H and $\mathcal{L}(H)$ is defined as

$$\mathcal{L}(H) = \{x(t)\in\mathbb{R}^n : |h_ix(t)|\le 1,\ i\in[1,l]\}.$$

Definition 3 [20]**:** Let $P\in\mathbb{R}^{n\times n}$ be a symmetric matrix and satisfies $E^{\mathrm{T}}PE\ge 0$, for a scalar $\eta>0$, $\mathcal{E}\left(E^{\mathrm{T}}PE,\eta\right)$ is denoted as

$$\mathcal{E}\left(E^{\mathrm{T}}PE,\eta\right) = \left\{x(t)\in\mathbb{R}^n : x(t)^{\mathrm{T}}E^{\mathrm{T}}PEx(t)\le\eta\right\}.$$

Lemma 1 [20]**:** Let $K,H\in\mathbb{R}^{l\times n}$, then for any $x(t)\in\mathcal{L}(H)$, we have

$$\mathrm{sat}(Kx(t))\in co\left\{D_jKx(t) + D_j^{\,-}Hx(t),\ j=1,2,\cdots,2^l\right\},$$

or equivalently, $\mathrm{sat}(Kx(t)) = \sum_{j=1}^{2^l} \alpha_j \left(D_j K + D_j^- H \right) x(t)$, where co stands for the convex hull, α_j for $j = 1, 2, \cdots, 2^l$ are some scalars which satisfy $0 \le \alpha_j \le 1$ and $\sum_{j=1}^{2^l} \alpha_j = 1$.

Let $\mathcal{D}$ be the set of $l \times l$ diagonal matrices whose diagonal elements are either I or 0. Suppose each element of $\mathcal{D}$ is labelled as $D_j,\ j = 1, 2, \cdots, 2^l$, and denote $D_j^- = I - D_j$. Clearly, if $D_j \in \mathcal{D}$, we have $D_j^- \in \mathcal{D}$.

From Lemma 1, for any $\tilde{x}(t) \in \mathcal{L}\left(\begin{bmatrix} H & -H \end{bmatrix}\right), \bar{x}(t) \in \mathcal{L}(H)$, we have

$$\begin{aligned} &\mathrm{sat}(K\bar{x}(t)) = \mathrm{sat}([K \ \ -K]\tilde{x}(t)) \\ &= \sum_{j=1}^{2^l} \alpha_j \left(\left(D_j [K \ \ -K] + D_j^- [H \ \ -H] \right) \tilde{x}(t) \right), \end{aligned} \tag{7}$$

where $\tilde{x}(t) = \begin{bmatrix} x^{\mathrm{T}}(t) & e^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}$.

Consider the equation (7), we obtain the closed-loop system as follows:

$$\begin{cases} \tilde{E}\dot{\tilde{x}}(t) = \sum_{j=1}^{2^l} \alpha_j \tilde{A}_j \tilde{x}(t) + \tilde{A}_d \tilde{x}(t - d(t)) + \tilde{B}_\omega \omega(t), \\ z(t) = (C + \Delta C) x(t), \end{cases} \tag{8}$$

where $\bar{A}_d = A_d + \Delta A_d$, $\bar{B}_\omega = B_\omega + \Delta B_\omega$,

$$\tilde{E} = \begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix}, \tilde{A}_d = \begin{bmatrix} \bar{A}_d & 0 \\ \Delta A_d & A_d \end{bmatrix}, \tilde{B}_\omega = \begin{bmatrix} \bar{B}_\omega \\ \bar{B}_\omega \end{bmatrix},$$

$$\tilde{A}_j = \begin{bmatrix} \bar{A}_j & -\bar{B}_j \\ \Delta A - BK + \bar{B}_j & A - LC_1 + BK - \bar{B}_j \end{bmatrix},$$

$$\bar{A}_j = A + \Delta A + (B + \Delta B) D_j K + (B + \Delta B) D_j^- H,$$

$$\bar{B}_j = (B + \Delta B) D_j K + (B + \Delta B) D_j^- H.$$

The corresponding closed-loop cost function of subsystem (8) is

$$J = \int_0^\infty \tilde{x}^{\mathrm{T}}(t) Q^* \tilde{x}(t)\, dt, \tag{9}$$

where $Q^* = \begin{bmatrix} Q + K^{\mathrm{T}}RK & -K^{\mathrm{T}}RK \\ -K^{\mathrm{T}}RK & K^{\mathrm{T}}RK \end{bmatrix}$ is a positive-definite symmetric matrix of appropriate dimensions.

The set $\mathcal{E}\left(\tilde{E}^{\mathrm{T}}\tilde{P}\tilde{E}, \gamma^2\eta\right)$ is defined as follows:

$$\mathcal{E}\left(\tilde{E}^{\mathrm{T}}\tilde{P}\tilde{E}, \gamma^2\eta\right) = \left\{ \tilde{x}(t) \in \mathbb{R}^{2n} : \tilde{x}(t)^{\mathrm{T}} \tilde{E}^{\mathrm{T}}\tilde{P}\tilde{E}\tilde{x}(t) \le \gamma^2\eta \right\}.$$

The aim of this paper is to design an observer-based guaranteed cost controller such that for any time-vary delay $0 < d(t) \le d$, satisfies the following conditions:

1) The closed-loop system (8) is admissible.
2) The closed-loop value of the cost function (9), under the condition of $\omega(t) = 0$, satisfies $J \le J^*$, where J^* is some specified constant.
3) The closed-loop system (8) satisfies H_∞ performance γ, which means, under the zero initial condition, system (8) satisfies

$$J_\omega = \int_0^\infty \left(z^{\mathrm{T}}(t) z(t) - \gamma^2 \omega^{\mathrm{T}}(t) \omega(t) \right) dt < 0, \tag{10}$$

for any nonzero $\omega(t) \in \mathcal{L}_2[0, \infty)$ and all admissible uncertainties.

Lemma 2 [21]**:** Given a set of suited dimension real matrices T_1, T_2 and $F(t)$ is a time-varying matrix with $F(t)^{\mathrm{T}} F(t) \le I$. Then, we have

1) For any scalar $\varepsilon > 0$,

$$T_1 F(t) T_2 + T_2{}^{\mathrm{T}} F(t)^{\mathrm{T}} T_1{}^{\mathrm{T}} \le \varepsilon T_1 T_1{}^{\mathrm{T}} + \varepsilon^{-1} T_2{}^{\mathrm{T}} T_2,$$

2) For any positive-definite matrix G,

$$T_1 T_2 + T_2{}^{\mathrm{T}} T_1{}^{\mathrm{T}} \le T_1 G T_1{}^{\mathrm{T}} + T_2{}^{\mathrm{T}} G^{-1} T_2.$$

3. MAIN RESULTS

3.1. Delay-dependent stability analysis for singular systems

In this section, we concentrate our attention on the problems of stability and H_∞ performance analysis for system (1) with cost function (4).

Theorem 1: For prescribed scalar $d > 0$, $0 \le \mu < 1$, the system (8) with cost function (9) is admissible with H_∞ performance γ within the set $\mathcal{E}\left(\tilde{E}^{\mathrm{T}}\tilde{P}\tilde{E}, \gamma^2\eta\right)$, if there exist matrices $\bar{U}, N_i, M_j, (i, j = 1, \cdots, 5)$, invertible matrix W and positive-definite symmetric matrices $\tilde{P}$, Q_1, R_1, S, such that the following matrix inequality holds

$$\Sigma = \begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} & \tilde{E}^{\mathrm{T}} M_4^{\mathrm{T}} & \theta_{15} & dM_1 & -\tilde{A}_j^{\mathrm{T}} W & 0 \\ * & -(1-\mu) Q_1 & -M_2\tilde{E} & N_2 & 0 & dM_2 & -\tilde{A}_d^{\mathrm{T}} W & N_2 \\ * & * & \theta_{33} & \theta_{34} & -\tilde{E}^{\mathrm{T}} M_5^{\mathrm{T}} & dM_3 & 0 & N_3 \\ * & * & * & \theta_{44} & N_5^{\mathrm{T}} & dM_4 & 0 & N_4 \\ * & * & * & * & -\gamma^2 I & dM_5 & -\tilde{B}_\omega^{\mathrm{T}} W & N_5 \\ * & * & * & * & * & -dS & 0 & 0 \\ * & * & * & * & * & * & -W & 0 \\ * & * & * & * & * & * & * & -W \end{bmatrix} < 0, \tag{11}$$

where

$$\theta_{11} = sym\left(\tilde{A}_j^{\mathrm{T}} W + M_1\tilde{E}\right) + Q_1 + R_1 + Q^* + \bar{C}^{\mathrm{T}}\bar{C},$$

$$\theta_{12} = W^{\mathrm{T}}\tilde{A}_d + \tilde{E}^{\mathrm{T}} M_2^{\mathrm{T}},\ \ \theta_{13} = -M_1\tilde{E} + \tilde{E}^{\mathrm{T}} M_3^{\mathrm{T}},$$

$$\theta_{15} = W^{\mathrm{T}}\tilde{B}_\omega + \tilde{E}^{\mathrm{T}} M_5^{\mathrm{T}},\ \ \theta_{33} = -R_1 - sym\left(M_3\tilde{E}\right),$$

$$\theta_{34} = N_3 - \tilde{E}^{\mathrm{T}} M_4^{\mathrm{T}},\ \ \theta_{44} = dS + sym(N_4),$$
$$\tilde{H} = \begin{bmatrix} H & -H \end{bmatrix},\ \bar{C} = \begin{bmatrix} C+\Delta C & 0 \end{bmatrix},$$
$$W = \tilde{P}\tilde{E} + \bar{R}\bar{U}^{\mathrm{T}} = \mathrm{diag}\{W_1, W_1\},$$

$\bar{R} \in \mathbb{R}^{2n\times 2(n-r)}$ is any full-column rank matrix satisfying $\tilde{E}^{\mathrm{T}}\bar{R} = 0$, $\mathrm{rank}(\bar{R}) = 2(n-r)$. Moreover $\mathcal{E}\left(\tilde{E}^{\mathrm{T}}\tilde{P}\tilde{E}, \gamma^2\eta\right) \subset \mathcal{L}\left(\tilde{H}\right)$ and the corresponding cost function satisfies

$$\begin{aligned} J \le J^* =& \tilde{x}^{\mathrm{T}}(0)\tilde{E}^{\mathrm{T}}\tilde{P}\tilde{E}\tilde{x}(0) \\ &+ \int_{0-d(0)}^{0} \tilde{x}^{\mathrm{T}}(s) Q_1 \tilde{x}(s)\,ds \\ &+ \int_{0-d}^{0} \tilde{x}^{\mathrm{T}}(s) R_1 \tilde{x}(s)\,ds \\ &+ \int_{-d}^{0}\int_{0+\theta}^{0} \dot{\tilde{x}}^{\mathrm{T}}(s)\tilde{E}^{\mathrm{T}} S\tilde{E}\dot{\tilde{x}}(s)\,ds\,d\theta. \end{aligned}$$

Proof: From (11), it follows that $\theta_{11} < 0$. We choose two nonsingular matrices M and N, such that $M\tilde{E}N = \begin{bmatrix} I_{2r} & 0 \\ 0 & 0 \end{bmatrix}$, $M\tilde{A}_j N = \begin{bmatrix} \tilde{A}_{j11} & \tilde{A}_{j12} \\ \tilde{A}_{j21} & \tilde{A}_{j22} \end{bmatrix}$, $M^{-\mathrm{T}}\bar{R} = \begin{bmatrix} 0 \\ \bar{R}_2 \end{bmatrix}$, $N^{\mathrm{T}}\bar{U} = \begin{bmatrix} \bar{U}_1^{\mathrm{T}} & \bar{U}_2^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$. Then, pre- and post-multiplying $\theta_{11} < 0$ by N^{T} and N, respectively, it is obtained that $\bar{U}_2\bar{R}_2^{\mathrm{T}}\tilde{A}_{j22} + \tilde{A}_{j22}^{\mathrm{T}}\bar{R}_2\bar{U}_2^{\mathrm{T}} < 0$, which means $\tilde{A}_{j22}$ is nonsingular and the pair $\left(\tilde{E}, \tilde{A}_j\right)$ is regular and impulse free according to [20]. By Definition 1, the closed-loop system (8) is regular and impulse free within $\mathcal{E}\left(\tilde{E}^{\mathrm{T}}\tilde{P}\tilde{E}, \gamma^2\eta\right)$.

Next, we show the stability of system (8). Choose a Lyapunov-Krasovskii function as follows:

$$\begin{aligned} V(\tilde{x}_t) =& \tilde{x}^{\mathrm{T}}(t)\tilde{E}^{\mathrm{T}}\tilde{P}\tilde{E}\tilde{x}(t) \\ &+ \int_{t-d(t)}^{t} \tilde{x}^{\mathrm{T}}(s) Q_1\tilde{x}(s)\,ds + \int_{t-d}^{t} \tilde{x}^{\mathrm{T}}(s) R_1\tilde{x}(s)\,ds \\ &+ \int_{-d}^{0}\int_{t+\theta}^{t} \dot{\tilde{x}}^{\mathrm{T}}(s)\tilde{E}^{\mathrm{T}} S\tilde{E}\dot{\tilde{x}}(s)\,ds\,d\theta, \end{aligned}$$

where $\tilde{x}_t = \tilde{x}(t+\alpha)$, $-d \le \alpha \le 0$. Taking the time derivative of $V(\tilde{x}_t)$ along the trajectory of the system (8) yields

$$\begin{aligned} \dot{V}(\tilde{x}_t) \le& 2\tilde{x}^{\mathrm{T}}(t)\tilde{E}^{\mathrm{T}}\tilde{P}\tilde{E}\dot{\tilde{x}}(t) + \dot{\tilde{x}}^{\mathrm{T}}(t)\tilde{E}^{\mathrm{T}}\bar{R}\bar{U}^{\mathrm{T}}\tilde{x}(t) \\ &+ \tilde{x}^{\mathrm{T}}(t)\bar{U}\bar{R}^{\mathrm{T}}\tilde{E}\dot{\tilde{x}}(t) + \tilde{x}^{\mathrm{T}}(t) Q_1\tilde{x}(t) \\ &- (1-\mu)\tilde{x}^{\mathrm{T}}(t-d(t)) Q_1\tilde{x}(t-d(t)) \\ &+ d\dot{\tilde{x}}^{\mathrm{T}}(t)\tilde{E}^{\mathrm{T}} S\tilde{E}\dot{\tilde{x}}(t) - \tilde{x}^{\mathrm{T}}(t-d) R_1\tilde{x}(t-d) \\ &+ \tilde{x}^{\mathrm{T}}(t) R_1\tilde{x}(t) - \int_{t-d}^{t} \dot{\tilde{x}}^{\mathrm{T}}(s)\tilde{E}^{\mathrm{T}} S\tilde{E}\dot{\tilde{x}}(s)\,ds. \end{aligned} \quad (12)$$

For any matrices N_i and M_j of appropriate dimensions, we can get that the following equations hold

$$2\varsigma^{\mathrm{T}}(t)\tilde{N}\left(\tilde{E}\dot{\tilde{x}}(t) - \tilde{A}_j\tilde{x}(t) - \tilde{A}_d\tilde{x}(t-d(t))\right) = 0, \quad (13)$$

$$2\varsigma^{\mathrm{T}}(t)\tilde{M}\left(\tilde{E}\tilde{x}(t) - \tilde{E}\tilde{x}(t-d) - \int_{t-d}^{t}\tilde{E}\dot{\tilde{x}}(s)\,ds\right) = 0, \quad (14)$$

where

$$\varsigma^{\mathrm{T}}(t) = \begin{bmatrix} \tilde{x}^{\mathrm{T}}(t) & \tilde{x}^{\mathrm{T}}(t-d(t)) & \tilde{x}^{\mathrm{T}}(t-d) & \left(\tilde{E}\dot{\tilde{x}}(t)\right)^{\mathrm{T}} \end{bmatrix},$$
$$\tilde{M} = \begin{bmatrix} M_1^{\mathrm{T}} & M_2^{\mathrm{T}} & M_3^{\mathrm{T}} & M_4^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}},$$
$$\tilde{N} = \begin{bmatrix} 0 & N_2^{\mathrm{T}} & N_3^{\mathrm{T}} & N_4^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}},$$

when $\omega(t) = 0$, substituting the left side of (13) and (14) into (12), by using the Schur complement and resulting from the matrix inequality (11), it is easy to see that the following inequality holds

$$\dot{V}(\tilde{x}_t) \le -\tilde{x}^{\mathrm{T}}(t) Q^*\tilde{x}(t) \le -\lambda_{\min}(Q^*)\|\tilde{x}(t)\|^2 < 0,$$

where $\lambda_{\min}$ denotes the minimum eigenvalue of matrix $(\cdot)$. According to Lyapunov's stability theory, the closed-loop system (8) is asymptotically stable. And the result from the above inequality is that $-\dot{V}(\tilde{x}_t) \ge \tilde{x}^{\mathrm{T}}(t) Q^*\tilde{x}(t)$, integrating both sides of the inequality from 0 to ∞, and exploit the stabilization of the systems show that

$$\begin{aligned} J \le J^* =& \tilde{x}^{\mathrm{T}}(0)\tilde{E}^{\mathrm{T}}\tilde{P}\tilde{E}\tilde{x}(0) \\ &+ \int_{0-d(0)}^{0} \tilde{x}^{\mathrm{T}}(s) Q_1 \tilde{x}(s)\,ds \\ &+ \int_{0-d}^{0} \tilde{x}^{\mathrm{T}}(s) R_1 \tilde{x}(s)\,ds \\ &+ \int_{-d}^{0}\int_{0+\theta}^{0} \dot{\tilde{x}}^{\mathrm{T}}(s)\tilde{E}^{\mathrm{T}} S\tilde{E}\dot{\tilde{x}}(s)\,ds\,d\theta. \end{aligned}$$

When $\omega(t) \ne 0$, we show that for any nonzero $\omega(t) \in \mathcal{L}_2[0,\infty)$, the system (8) has H_∞ performance γ. We consider the index $J_\omega = \int_0^\infty \left(z^{\mathrm{T}}(t) z(t) - \gamma^2\omega^{\mathrm{T}}(t)\omega(t)\right)dt$.

Under the zero initial condition, it is obtained that

$$J_\omega \le \int_0^\infty \left[\dot{V}(x_t) + z^{\mathrm{T}}(t) z(t) - \gamma^2\omega^{\mathrm{T}}(t)\omega(t)\right] dt.$$

It is easy to see that (11) is the sufficient condition to ensure $J_\omega < 0$ for any nonzero $\omega(t) \in \mathcal{L}_2[0,\infty)$. □

Remark 2: By adding some free-weighting matrices N_i, M_j and using Lyapunov-Krasovskii function, while not relying on the restriction of $E^{\mathrm{T}}P = P^{\mathrm{T}}E$, a delay-dependent sufficient condition is derived in Theorem 1, which guarantees that system (8) is admissible. We consider Example 1 to compare our delay-dependent stability condition. Example 1 shows that the obtained admissible upper bound of time-delay using Theorem 1 in this paper is better than some previous articles. Hence, the condition of Theorem 1 is less conservative.

3.2. Observer-based H_∞ guaranteed cost controller design

In the following, the problem that we are dealing with is observer-based H_∞ guaranteed cost controller design for singular system (1). We shall give sufficient condition for the existence of controller and present the corresponding observer-based guaranteed cost controller design method. Now, we give the following theorem.

Theorem 2: For prescribed scalars $d>0$, $0\le\mu<1$ and the cost function (9), the considered observer-based H_∞ guaranteed cost controlling problem of the closed-loop system (8) within $\mathcal{E}\left(\tilde{E}^{\mathrm{T}}\tilde{P}\tilde{E},\gamma^2\eta\right)$ is solvable for all permissible parametric uncertainties, if there exist positive-definite symmetric matrices $\tilde{Q}_1$, $\tilde{R}_1$, $\tilde{S}$, invertible matrix X, matrices Y, $\bar{H}$, Y_L, $\bar{N}_i\,(i=2,\cdots,5)$, $\bar{M}_j\,(j=1,\cdots,5)$ and scalar $\varepsilon>0$, such that the following LMIs hold

$$\begin{bmatrix} -\frac{1}{\gamma^2\eta} & \bar{h}_j & -\bar{h}_j \\ * & -EX & 0 \\ * & * & -EX \end{bmatrix}\le 0, \tag{15}$$

$$\begin{bmatrix} \Psi_{18\times18} & \bar{O}_1 & \bar{O}_3^{\mathrm{T}} \\ * & -\varepsilon^{-1}I & 0 \\ * & * & -\varepsilon I \end{bmatrix}<0, \tag{16}$$

where

$$\bar{\theta}_{11}=sym\left(A_j\tilde{X}+\bar{M}_1\tilde{E}^{\mathrm{T}}\right)+\tilde{Q}_1+\tilde{R}_1,$$

$$\bar{\theta}_{12}=A_D\tilde{X}+\tilde{E}\bar{M}_2^{\mathrm{T}},\ \bar{\theta}_{13}=-\bar{M}_1\tilde{E}^{\mathrm{T}}+\tilde{E}\bar{M}_3^{\mathrm{T}},$$

$$\bar{\theta}_{15}=B_W+\tilde{E}\bar{M}_5^{\mathrm{T}},\ \bar{\theta}_{33}=-\tilde{R}_1-sym\left(\bar{M}_3\tilde{E}^{\mathrm{T}}\right),$$

$$\bar{\theta}_{34}=\bar{N}_3-\tilde{E}\bar{M}_4^{\mathrm{T}},\ \bar{\theta}_{44}=d\tilde{S}+sym\left(\bar{N}_4\right),$$

$$g_{21}=H_3\left(D_jY+D_j^-\bar{H}\right),\ A_D=\mathrm{diag}\left\{\begin{array}{cc} A_d & A_d\end{array}\right\},$$

$$A_j=\begin{bmatrix} A+B\left(D_jK+D_j^-H\right) & -B\left(D_jK+D_j^-H\right) \\ -BK+ & A-LC_1+BK- \\ B\left(D_jK+D_j^-H\right) & B\left(D_jK+D_j^-H\right)\end{bmatrix},$$

$$A_j\tilde{X}=\begin{bmatrix} AX+B\left(D_jY+D_j^-\bar{H}\right) & -B\left(D_jY+D_j^-\bar{H}\right) \\ -BY+ & AX-Y_L+BY- \\ B\left(D_jY+D_j^-\bar{H}\right) & B\left(D_jY+D_j^-\bar{H}\right)\end{bmatrix},$$

$$B_W=\begin{bmatrix} B_\omega \\ B_\omega\end{bmatrix},\ \bar{\Lambda}=\begin{bmatrix} QX & 0 \\ RY & -RY\end{bmatrix},\ \Lambda=\begin{bmatrix} Q & 0 \\ 0 & R\end{bmatrix},$$

$$\Psi_{18\times18}=\left[\begin{array}{cccccccccc}
\bar{\theta}_{11} & \bar{\theta}_{12} & \bar{\theta}_{13} & \tilde{E}\bar{M}_4^{\mathrm{T}} & \bar{\theta}_{15} & d\bar{M}_1 & -\tilde{X}^{\mathrm{T}}A_j^{\mathrm{T}} & 0 & \bar{\Lambda}^{\mathrm{T}} & \left[\begin{array}{cc} C & 0\end{array}\right]^{\mathrm{T}} \\
* & (\mu-1)\tilde{Q}_1 & -\bar{M}_2\tilde{E}^{\mathrm{T}} & \bar{N}_2 & 0 & d\bar{M}_2 & -\tilde{X}^{\mathrm{T}}A_D^{\mathrm{T}} & \bar{N}_2 & 0 & 0 \\
* & * & \bar{\theta}_{33} & \bar{\theta}_{34} & -\tilde{E}\bar{M}_5^{\mathrm{T}} & d\bar{M}_3 & 0 & \bar{N}_3 & 0 & 0 \\
* & * & * & \bar{\theta}_{44} & \bar{N}_5^{\mathrm{T}} & d\bar{M}_4 & 0 & \bar{N}_4 & 0 & 0 \\
* & * & * & * & -\gamma^2I & d\bar{M}_5 & -B_W^{\mathrm{T}} & \bar{N}_5 & 0 & 0 \\
* & * & * & * & * & -d\tilde{S} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -\tilde{X}^{\mathrm{T}} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -\tilde{X}^{\mathrm{T}} & 0 & 0 \\
* & * & * & * & * & * & * & * & -\Lambda & 0 \\
* & * & * & * & * & * & * & * & * & -I
\end{array}\right]<0,$$

$$\bar{O}_1=\begin{bmatrix}
E_1 & E_3 & E_2 & 0_{1\times5} & E_4 & 0_{1\times8} & 0 \\
E_1 & E_3 & E_2 & 0_{1\times5} & E_4 & 0_{1\times8} & 0 \\
0_{9\times1} & 0_{9\times1} & 0_{9\times1} & 0_{9\times5} & 0_{9\times1} & 0_{9\times8} & 0_{9\times1} \\
-E_1 & -E_3 & -E_2 & 0_{1\times5} & -E_4 & 0_{1\times8} & 0 \\
-E_1 & -E_3 & -E_2 & 0_{1\times5} & -E_4 & 0_{1\times8} & 0 \\
0_{4\times1} & 0_{4\times1} & 0_{4\times1} & 0_{4\times5} & 0_{4\times1} & 0_{4\times8} & 0_{4\times1} \\
0 & 0 & 0 & 0_{1\times5} & 0 & 0_{1\times8} & E_5
\end{bmatrix},$$

$$\bar{O}_3=\begin{bmatrix}
H_1X & 0 & 0 & 0_{1\times5} & 0 & 0_{1\times9} \\
g_{21} & -g_{21} & 0 & 0_{1\times5} & 0 & 0_{1\times9} \\
0 & 0 & H_2X & 0_{1\times5} & 0 & 0_{1\times9} \\
0_{5\times1} & 0_{5\times1} & 0_{5\times1} & 0_{5\times5} & 0_{5\times1} & 0_{5\times9} \\
0 & 0 & 0 & 0_{1\times5} & H_4 & 0_{1\times9} \\
0_{8\times1} & 0_{8\times1} & 0_{8\times1} & 0_{8\times5} & 0_{8\times1} & 0_{8\times9} \\
H_5 & 0 & 0 & 0_{1\times5} & 0 & 0_{1\times9}
\end{bmatrix}.$$

In addition, the feedback controller gain and the observer gain in (8) are given by $K=YX^{-1}$ and $L=Y_LX^{-1}C_1^+$.

Proof: From $\mathcal{E}\left(\tilde{E}^{\mathrm{T}}\tilde{P}\tilde{E},\gamma^2\eta\right)\subset\mathcal{L}\left(\tilde{H}\right)$ and since the system (8) is regular and impulse free, there exist two other nonsingular matrices $\bar{M}$ and $\bar{N}$ such that

$$\bar{M}\tilde{E}\bar{N}=\begin{bmatrix} I_{2r} & 0 \\ 0 & 0\end{bmatrix},\ \bar{M}^{-\mathrm{T}}\tilde{P}\bar{M}^{-1}=\begin{bmatrix} P_{11} & P_{12} \\ * & P_{22}\end{bmatrix},$$

with $\tilde{H}\bar{N}=\left[\begin{array}{cc}\tilde{H}_1 & \tilde{H}_2\end{array}\right]$ and $\bar{N}^{-1}\tilde{x}(t)=\begin{bmatrix}\tilde{x}_1(t) \\ \tilde{x}_2(t)\end{bmatrix}$, it follows that $\tilde{H}_2=0$. Otherwise, let $\tilde{x}_1(t)=0$ and $\left|\tilde{h}_{2j}\tilde{x}_2(t)\right|>\gamma\eta^{1/2}$, then $\tilde{x}^{\mathrm{T}}(t)\tilde{E}^{\mathrm{T}}\tilde{P}\tilde{E}\tilde{x}(t)=0$, $\left|\tilde{h}_{2j}\tilde{x}_2(t)\right|>\gamma\eta^{1/2}$, it contradicts that $\mathcal{E}\left(\tilde{E}^{\mathrm{T}}\tilde{P}\tilde{E},\gamma^2\eta\right)\subset\mathcal{L}\left(\tilde{H}\right)$. Then the condition $\mathcal{E}\left(\tilde{E}^{\mathrm{T}}\tilde{P}\tilde{E},\gamma^2\eta\right)\subset\mathcal{L}\left(\tilde{H}\right)$ is equivalent to $\tilde{h}_{1j}P_{11}^{-1}\tilde{h}_{1j}^{\mathrm{T}}\le\frac{1}{\gamma^2\eta}$, $(j=1,2,\cdots,l)$, which, by Schur complement, is equivalent to

$$\begin{bmatrix} -\frac{1}{\gamma^2\eta} & \left[\begin{array}{cc}\tilde{h}_{1j} & 0\end{array}\right] \\ \left[\begin{array}{cc}\tilde{h}_{1j} & 0\end{array}\right]^{\mathrm{T}} & -\Gamma\end{bmatrix}\le 0,\ j=1,2,\cdots,l, \tag{17}$$

where

$$\Gamma=\begin{bmatrix} I_{2r} & 0 \\ 0 & 0\end{bmatrix}\begin{bmatrix} P_{11} & P_{12} \\ * & P_{22}\end{bmatrix}\begin{bmatrix} I_{2r} & 0 \\ 0 & 0\end{bmatrix}.$$

Setting $\tilde{X}=W^{-1}=\left(\tilde{P}\tilde{E}+\bar{R}\bar{U}^{\mathrm{T}}\right)^{-1}$, and together with $\tilde{E}^{\mathrm{T}}\left(\tilde{P}\tilde{E}+\bar{R}\bar{U}^{\mathrm{T}}\right)=\left(\tilde{P}\tilde{E}+\bar{R}\bar{U}^{\mathrm{T}}\right)^{\mathrm{T}}\tilde{E}$, we have

$$\tilde{X}^{\mathrm{T}}\tilde{E}^{\mathrm{T}}\left(\tilde{P}\tilde{E}+\bar{R}\bar{U}^{\mathrm{T}}\right)\tilde{X}=\tilde{X}^{\mathrm{T}}\left(\tilde{P}\tilde{E}+\bar{R}\bar{U}^{\mathrm{T}}\right)^{\mathrm{T}}\tilde{E}\tilde{X},$$

that is to say $\tilde{X}^{\mathrm{T}}\tilde{E}^{\mathrm{T}}=\tilde{E}\tilde{X}$.

Pre- and post-multiplying (17) by $\mathrm{diag}\left\{1,\bar{N}^{-\mathrm{T}}\tilde{X}^{\mathrm{T}}\right\}$ and its transpose, respectively, we obtain

$$\begin{bmatrix} -\frac{1}{\gamma^2\eta} & \left[\begin{array}{cc}\bar{h}_j & -\bar{h}_j\end{array}\right] \\ * & -\tilde{E}\tilde{X}\end{bmatrix}\le 0.$$

According to Theorem 1 and Schur complement, pre- and post-multiplying $\mathrm{diag}\left\{\underbrace{\tilde{X}^{\mathrm{T}}\cdots\tilde{X}^{\mathrm{T}}}_{4},I,\underbrace{\tilde{X}^{\mathrm{T}}\cdots\tilde{X}^{\mathrm{T}}}_{3},\underbrace{I\cdots I}_{3}\right\}$

and its transpose. Setting $\tilde{Q}_1 = \tilde{X}^{\mathrm{T}} Q_1 \tilde{X}$, $\tilde{R}_1 = \tilde{X}^{\mathrm{T}} R_1 \tilde{X}$, $\tilde{S} = \tilde{X}^{\mathrm{T}} S \tilde{X}$, $\bar{N}_i = \tilde{X}^{\mathrm{T}} N_i \tilde{X}$, $(i = 2,3,4)$, $\bar{N}_5 = N_5 \tilde{X}$, $\bar{M}_j = \tilde{X}^{\mathrm{T}} M_j \tilde{X}$, $(j = 1,2,3,4)$, $\bar{M}_5 = M_5 \tilde{X}$, $Y = KX$, $Y_L = LC_1 X$, Theorem 2 holds. □

Remark 3: In this paper, C_1 is a general matrix which can be non-singular matrix or full-row rank matrix. Therefore, when we deal with the problem of observer design, we need to guarantee the condition that the matrix equation $Y_L = LC_1 X$ is solvable. Then, we get the observer gain L which is the LN (least-norm) solution of the matrix equation. That is to say, the method of an observer design in Theorem 2 is reasonable.

The guaranteed costs in Theorem 2 depend on the choice of guaranteed cost controllers. The following convex optimization problem will select the guaranteed cost controller which minimizes the upper bound of the closed-loop cost function.

Corollary 1: Consider the system (8) with cost function (9). If the optimization problem

$$\begin{aligned} &\min\ J^* \\ &s.t.\ \ \text{inequality}\,(15)\,\text{and}\,(16) \end{aligned}$$

has solution, then there exists an observer-based controller which minimizes the upper bound of the closed-loop cost function such that system (8) is admissible.

Now, we will give a LMI-based optimization algorithm to obtain the largest invariant ellipsoid for the system (8). With Theorem 2, an exact invariant set with least degree of conservativeness can be formulated as

$$\begin{aligned} &\max\ \ v_1 \\ &s.t. \begin{cases} (a)\ \ v_1 \tilde{x}_0 \in \mathcal{E}\left(\tilde{E}^{\mathrm{T}} \tilde{P} \tilde{E}, \gamma^2 \eta\right), \\ (b)\ \ \text{inequality}\,(15), \\ (c)\ \ \text{inequality}\,(16), \end{cases} \end{aligned} \tag{18}$$

where $\tilde{x}_0 = \left[\begin{array}{cc} x_0^{\mathrm{T}} & e_0^{\mathrm{T}} \end{array}\right]^{\mathrm{T}}$. Applying Lemma 2 and Schur complement, constraint (a) is equivalent to

$$\left[\begin{array}{ccc} -v_2 & \tilde{x}_0^{\mathrm{T}} \tilde{E}^{\mathrm{T}} & \tilde{x}_0^{\mathrm{T}} \\ * & -2I & 0 \\ * & * & \Upsilon \end{array}\right] \leq 0, \tag{19}$$

where $v_2 = \frac{\gamma^2 \eta}{v_1^2}$, $\Upsilon = -\tilde{X}^{\mathrm{T}} - \tilde{X} + \frac{1}{2} I$.

From the above discussion, (18) can be transformed to the following LMI optimization problem

$$\begin{aligned} &\min\ v_2 \\ &s.t.\ \ \text{inequality}\ (15), (16)\ \text{and}\ (19). \end{aligned} \tag{20}$$

4. NUMERICAL EXAMPLE AND SIMULATION

In this section, we give the examples to illustrate the effectiveness of the proposed conditions.

Table 1. Comparisons of the allowed upper bound $d(t)$.

Methods	[22]	[23]	[24]	Theorem 1
$d(t)_{\max}$	1.1372	1.9841	2.4865	4.5027

Table 2. Comparisons of maximum value of $d(t)$.

Methods	[25]	[26]	Theorem 2
$d_{\max}$	1.854	1.955	3.572

Example 1: Consider a singular time-varying delay system (6) with

$$E = \left[\begin{array}{cc} 9 & 3 \\ 6 & 2 \end{array}\right],\ A_d = \left[\begin{array}{cc} -18.6 & -10.4 \\ -25.2 & -16.8 \end{array}\right],$$
$$A = \left[\begin{array}{cc} -13.1 & -13.7 \\ -15.4 & -23.8 \end{array}\right].$$

In this example, we choose $\mu = 0.5$. By comparing the stability criterion of Theorem 1 with those of [22–24], we have Table 1. Hence, for this example, the stability criterion we derived is less conservative than those reported in the above-mentioned papers.

Example 2: Consider the linear system with actuator saturation in [26] described by

$$\begin{aligned} \dot{x}(t) = &\left[\begin{array}{cc} 0.6 & 0.4 \\ 0 & -0.5 \end{array}\right] x(t - d(t)) \\ &+ \left[\begin{array}{cc} 0.5 & -1 \\ 0.5 & -0.5 \end{array}\right] x(t) + \left[\begin{array}{c} 1 \\ 1 \end{array}\right] \mathrm{sat}(u(t)). \end{aligned}$$

Table 2 shows the maximum value of d with $\mu = 0$ which guarantees stability of the system by applying Theorem 2 in this work. Hence, example 2 shows that the stabilization criterion for time-delay systems with saturating actuators obtained by the method in this paper is less conservative than those [25, 26].

Example 3: Consider the uncertain singular time-delay system (1) with

$$A = \left[\begin{array}{ccc} -1 & 0.5 & 0 \\ 0 & -1 & 0.5 \\ 0 & -0.8 & -1 \end{array}\right],\ B = \left[\begin{array}{ccc} -5 & -1 & 0 \\ 0 & -5 & -1 \\ 0 & -10 & 5 \end{array}\right],$$
$$E = \mathrm{diag}\left\{\ 1\ \ 1\ \ 0\ \right\},\ C = \mathrm{diag}\left\{\ 0.1\ \ 0.1\ \ 0.1\ \right\},$$
$$A_d = \mathrm{diag}\left\{\ -0.5\ \ -0.5\ \ -0.3\ \right\},$$
$$B_\omega = \mathrm{diag}\left\{\ 0.1\ \ 0.1\ \ 0.5\ \right\}, C_1 = \left[\begin{array}{ccc} 2 & 0 & 1 \\ 0 & -1 & 0 \end{array}\right],$$
$$E_i = \mathrm{diag}\left\{\ 0.1\ \ 0.1\ \ 0.1\ \right\},\ i = 1, \cdots, 5,$$
$$H_i = \mathrm{diag}\left\{\ 0.1\ \ 0.1\ \ 0.1\ \right\},\ i = 1, \cdots, 5.$$

Let $\varepsilon = 0.5$, $\gamma = 3$, $\mu = 0.01$, $d = 1.9$. Solving the LMIs got in Theorem 2 by using Toolbox in Matlab, the controller gain matrix and observer gain matrix are obtained as

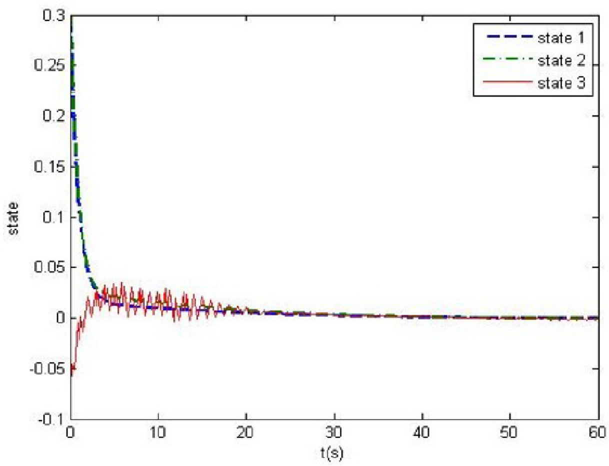

Fig. 1. Trajectories of states.

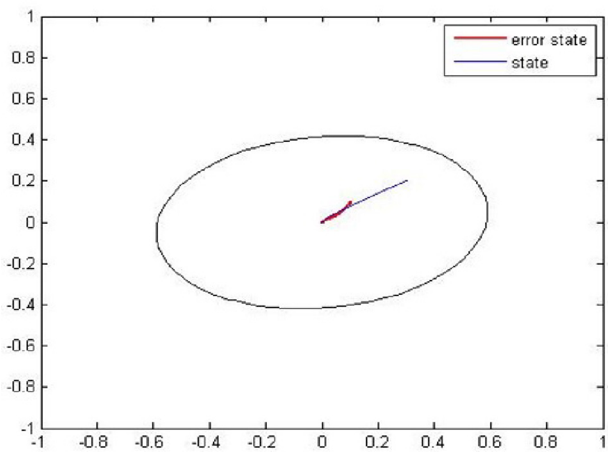

Fig. 2. The invariant ellipsoid and the trajectories of states and error states.

$$K=\begin{bmatrix} -0.0018 & 0.0575 & -0.0039 \\ 0.0009 & -0.025 & 0.0011 \\ -0.0074 & 0.0585 & -0.0013 \end{bmatrix},$$
$$L=\begin{bmatrix} 0.0038 & -0.3324 \\ 0.0911 & 0.1214 \\ 0.0236 & 0.7003 \end{bmatrix}.$$

Choosing the function $F(t) = 0.8+0.2\sin(t)$, and $\omega(t) = 0.1e^{0.051t}\cos(0.03t)$. The initial state is assumed to be $x_0 = \begin{bmatrix} 0.3 & 0.3 & 0 \end{bmatrix}^{\mathrm{T}}$, $e_0 = \begin{bmatrix} 0.1 & 0.1 & 0 \end{bmatrix}^{\mathrm{T}}$. The state dynamic system is shown in Fig. 1. From the figure, our proposed method can guarantee that the singular system (8) is well stabilized.

Solving the LMI optimization problem (20), it is obtained that $v_2^{\min} = 0.3$, then the corresponding upper bound of the cost function is $J^* = 5.704$. The larger invariant ellipsoid is shown in Fig. 2. Fig. 2 shows the states and the error states trajectories of the system (8) which start from the invariant ellipsoid will remain inside it.

5. CONCLUSIONS

In this paper, the problem of delay-dependent robust H_∞ guaranteed cost stability and observer-based guaranteed cost stabilization of uncertain singular time-delay systems with saturating actuators is concerned. By choosing an appropriate Lyapunov function and using free-weighting matrix approach, a new criterion which guarantees that the closed-loop systems are admissible with H_∞ performance is derived. Then, delay-dependent sufficient conditions for the existence of the observer and the guaranteed cost controller are derived. Besides, convex optimization problems have been formulated to choose a controller minimizing the upper bound of the guaranteed cost and the larger invariant ellipsoid. Finally, numerical examples show the reduced conservativeness of the obtained stability and demonstrate the validity of the proposed approach.

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