

l_∞ Fuzzy Filter Design for Nonlinear Systems with Missing Measurements: Fuzzy Basis-dependent Lyapunov Function Approach

Sun Young Noh, Geun Bum Koo, Jin Bae Park*, and Young Hoon Joo

Abstract: In this paper, l_∞ fuzzy filtering problem is dealt for nonlinear systems with both persistent bounded disturbances and missing probabilistic sensor information. The Takagi–Sugeno (T–S) fuzzy model is adopted to represent a nonlinear dynamic system. The measurement output is assumed to contain randomly missing data, which is modeled by a Bernoulli distributed with a known conditional probability. To design the l_∞ fuzzy filter and guarantee tracking performance, the effect of the perturbation against persistent bounded disturbances is reduced by using the minimum l_∞ performance. By using the fuzzy basis-dependent Lyapunov function approach, a sufficient condition is established that ensure the mean square exponential stability of the filtering error. The proposed sufficient condition is represented as some linear matrix inequalities (LMIs), and the filter gain is obtained by the solution to a set of LMIs. Finally, the effectiveness of the proposed design method is shown via an example.

Keywords: l_∞ fuzzy filter, missing measurements, Takagi–Sugeno fuzzy model, fuzzy basis-dependent Lyapunov function, linear matrix inequalities.

1. INTRODUCTION

The $l_2 - l_\infty$ filtering has received considerable attention for nonlinear dynamic systems, because the $l_2 - l_\infty$ filtering is more suitable than the traditional Kalman filtering for such systems as hybrid systems, time-delay systems, uncertain systems, and so on [1]. Especially, some literatures show that the $l_2 - l_\infty$ filtering problem of persistent bounded disturbances can be formulated as a minimax optimization problem, which is to minimize the maximum peak value amplitude of the estimation error for all possible bounded energy disturbances [2–5]. However, $l_2 - l_\infty$ filtering is very complicated and an inefficient algorithm to solve the l_∞ filtering problem for the nonlinear systems [6, 7]. To conquer the limitation of the previous $l_2 - l_\infty$ filtering approaches, fuzzy estimation approaches [8, 9] have motivated to robust H_∞ fuzzy filtering approaches [9–14] based on the Takagi–Sugeno (T–S) fuzzy system [15–21]. However, these algorithms are involved to eliminate the effect of the external disturbance but they do not consider to eliminate the persistent bounded disturbances. Unlike the H_∞ approach, the l_∞ approach reduces the influence of the energy of an external disturbance with persistent bounded disturbance on the energy of the output signal as small as possible [22–25].

On the other hand, above all studies are based on the implicit assumption that the communication between the physical plant and filter is perfect. However, the signals transmitted from the plant to the filter can not be arrived at the filter simultaneously and perfectly in real-world applications. Because of clear engineering insights, [28] and [29] considered the filtering problem for stochastic systems with missing measurements, and [30] investigated the performance problem of the Kalman filtering with intermittent observations, while [14], [31], and [13] discussed it for stochastic systems with time delays. In [9], only the persistent bounded disturbances was considered without missing measurements. Up until now, to the best of the author's knowledge, there has not been investigated the l_∞ filter design for the T–S fuzzy systems in the presence of intermittent measurements and persistent bounded disturbances simultaneously yet, which still remains open and challenging.

In this paper, we study the problem of l_∞ fuzzy filtering for nonlinear systems with both persistent bounded disturbances and missing probabilistic sensor information. The purpose of this paper is to design the optimal l_∞ fuzzy filter for missing measurements, which is to attenuate the peak of the estimation error of persistent bounded disturbance. To design the fuzzy filter, the nonlinear plant system is

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represented by the T–S fuzzy model. The measurement output is assumed to contain randomly missing data, which is modeled by a Bernoulli distributed with a known conditional probability. In order to design the l_∞ fuzzy filter and guarantee the tracking performance of the fuzzy system, the effect of the perturbation against persistent bounded disturbances is reduced by using the minimum l_∞ performance, its stability condition is established by using the fuzzy basis-dependent Lyapunov function (FB-DLF) [33, 37]. By using some lemmas, sufficient conditions are established to ensure the exponential meansquare stability of the filtering error in the LMI format. Based on the proposed LMIs [40–46], the filter l_∞ gain is obtained by the solution to a set of LMIs. Finally, the effectiveness of the proposed design methods on the FBDLF are shown via an example.

2. PROBLEM STATEMENT

A target dynamics can be modeled based on nonlinear systems. It can be approximated as locally linear systems in much the same way that a fuzzy linear dynamic models have been proposed by the T–S fuzzy model to represent local linear systems of nonlinear systems. This is described by the following IF-THEN rules and will be employed here to deal with a filter error system. The i th rule of the fuzzy linear model for the nonlinear systems is of the following form:

Plant Rule R_i :

IF $z_1(k)$ is Γ_{i1} and \dots and $z_p(k)$ is Γ_{ip} ,

$$\text{THEN } \begin{cases} x(k+1) = A_i x(k) + B_i w(k) \\ y(k) = \gamma(k) C_i x(k) + D_i v(k), \end{cases} \quad (1)$$

where $A_i \in \mathbb{R}^{n \times n}$, $B_i \in \mathbb{R}^{n \times l}$, $C_i \in \mathbb{R}^{m \times n}$, $D_i \in \mathbb{R}^{m \times s}$ are assumed known for $i \in \mathcal{S}$, R_i denotes the i th fuzzy rule; $z_p(k)$ is the p th premise variable for $p \in \mathcal{S}_q$, Γ_{ip} is the fuzzy set of $z_p(k)$ in R_i , $x(k) \in \mathbb{R}^n$ denotes the vector of the state, $y(k) \in \mathbb{R}^m$ denotes the vector of the output, $w(k) \in \mathbb{R}^l$ denotes the vector of the bounded external disturbance, $v(k) \in \mathbb{R}^s$ denotes the vector of the bounded measurement disturbance, and $\gamma(k)$ denotes data loss phenomenon, which is assumed to satisfy the Bernoulli distributed white sequence taking values on 0 and 1. Also, the stochastic variable $\gamma(k)$ has the following probability.

$$\mathbf{Prob} \{ \gamma(k) = 1 \} = \mathbb{E} \{ \gamma(k) \} := \bar{\gamma}$$

where $\bar{\gamma}$ is a known positive constant, and $\gamma(k)$ is assumed to be independent of $w(k)$, $v(k)$, and $x(k)$. Therefore, we

$$\sigma^2 = \mathbb{E} \left\{ (\gamma(k) - \bar{\gamma})^2 \right\} = (1 - \bar{\gamma}) \bar{\gamma}$$

where σ is a scalar zero mean stochastic sequence with variance.

Using the center-average defuzzifier, product inference, and singleton fuzzifier, the fuzzy rule (1) is inferred as follows:

$$\begin{aligned} x(k+1) &= \sum_{i=1}^r h_i(z(k)) (A_i x(k) + B_i w(k)), \\ y(k) &= \sum_{i=1}^r h_i(z(k)) (\gamma(k) C_i x(k) + D_i v(k)), \end{aligned}$$

where

$$\begin{aligned} h_i(z(k)) &= \mu_i(z(k)) / \sum_{i=1}^r \mu_i(z(k)), \\ \mu_i(z(k)) &= \prod_{p=1}^q \Gamma_{ip}(z_p(k)) \end{aligned}$$

in which $\Gamma_{ip}(z_p(k))$ is the fuzzy membership grade of $z_p(k)$ in Γ_{ip} . Because $0 \leq \mu_i(z(k)) \leq 1$ for all k , we get

$$h_i(z(k)) \geq 0, \quad \sum_{i=1}^r h_i(z(k)) = 1 \quad (2)$$

for $i = 1, 2, \dots, r$.

For notational convenience in the following discussions, we will denote that

$$\begin{aligned} \tilde{A}(k) &= \sum_{i=1}^r h_i(z(k)) A_i, & \tilde{B}(k) &= \sum_{i=1}^r h_i(z(k)) B_i, \\ \tilde{C}(k) &= \sum_{i=1}^r h_i(z(k)) C_i, & \tilde{D}(k) &= \sum_{i=1}^r h_i(z(k)) D_i. \end{aligned}$$

Then fuzzy system model can be rewritten as the following form:

$$\begin{aligned} x(k+1) &= \tilde{A}(k)x(k) + \tilde{B}(k)w(k), \\ y(k) &= \gamma(k)\tilde{C}(k)x(k) + \tilde{D}(k)v(k). \end{aligned} \quad (3)$$

Based on the fuzzy system model (3), the following l_∞ fuzzy filter with missing measurements is proposed to deal with the state estimation error:

Filter Rule R_i :

IF $z_1(k)$ is Γ_{i1} and \dots and $z_p(k)$ is Γ_{ip} ,

$$\text{THEN } \hat{x}(k+1) = A_{fi} \hat{x}(k) + L_{fi} (y_f(k) - \hat{y}(k)) \quad (4)$$

where $\hat{x}(k)$ is an estimate of $x(k)$, A_{fi} is a filter parameter to be determined as A_i , L_{fi} is the filter gain with i th rule, and $\hat{y}(k) = \sum_{i=1}^r h_i(z(k)) C_{fi}(k) \hat{x}(k)$. Using the center-average defuzzification, product inference, and singleton fuzzifier, the defuzzified output is given by

$$\begin{aligned} \hat{x}(k+1) &= \sum_{i=1}^r \sum_{j=1}^r h_i(z(k)) h_j(z(k)) (\gamma(k) L_{fi} C_j x(k) \\ &\quad + (A_{fi} - L_{fi} C_{fj}) \hat{x}(k) + L_{fi} D_j v(k)). \end{aligned} \quad (5)$$

For notational convenience in the following discussions, we will denote that

$$\begin{aligned}\tilde{A}_f(k) &= \sum_{i=1}^r h_i(z(k))A_{fi}, & \tilde{C}_f(k) &= \sum_{i=1}^r h_i(z(k))C_{fi}, \\ \tilde{L}_f(k) &= \sum_{i=1}^r h_i(z(k))L_{fi}.\end{aligned}$$

In this paper, we will parameterize all desired l_∞ filter gain to reduce the influence of the peak of external disturbance on the peak of the estimated error signal as small as possible.

3. ROBUST FUZZY FILTER DESIGN USING FBDLF

In this section, we will parameterize all desired l_∞ filter gain to reduce the influence of the peak of external disturbance on the peak of the estimated error signal as small as possible. The following closed-loop system with fuzzy filter containing missing measurements will be proposed to deal with the state estimation error.

$$\begin{aligned}\eta(k+1) &= A_m(k)\eta(k) + B_m(k)d(k) \\ &= \begin{bmatrix} \tilde{A}(k) & 0 \\ \gamma(k)\tilde{L}_f(k)\tilde{C}(k) & \tilde{A}_f(k) - \tilde{L}_f(k)\tilde{C}_f(k) \end{bmatrix} \begin{bmatrix} x(k) \\ \hat{x}(k) \end{bmatrix} \\ &+ \begin{bmatrix} \tilde{B}(k) & 0 \\ 0 & \tilde{L}_f(k)\tilde{D}(k) \end{bmatrix} \begin{bmatrix} w(k) \\ v(k) \end{bmatrix},\end{aligned}\quad (6)$$

where

$$\begin{aligned}\eta(k+1) &:= \begin{bmatrix} x(k+1)^T & \hat{x}(k+1)^T \end{bmatrix}^T, \\ d(k) &:= \begin{bmatrix} w(k)^T & v(k)^T \end{bmatrix}^T, \\ A_m &:= \begin{bmatrix} \tilde{A}(k) & 0 \\ \gamma(k)\tilde{L}_f(k)\tilde{C}(k) & \tilde{A}_f(k) - \tilde{L}_f(k)\tilde{C}_f(k) \end{bmatrix}, \\ B_m &:= \begin{bmatrix} \tilde{B}(k) & 0 \\ 0 & \tilde{L}_f(k)\tilde{D}(k) \end{bmatrix}.\end{aligned}$$

The augmented system (6) can be expressed as the fuzzy filter error system:

$$\eta(k+1) = A_{m1}(k)\eta(k) + \tilde{\gamma}(k)A_{m2}(k)\eta(k) + B_m(k)d(k), \quad (7)$$

where

$$\begin{aligned}A_{m1}(k) &:= \begin{bmatrix} \tilde{A}(k) & 0 \\ \tilde{\gamma}\tilde{L}_f(k)\tilde{C}(k) & \tilde{A}_f(k) - \tilde{L}_f(k)\tilde{C}_f(k) \end{bmatrix}, \\ A_{m2}(k) &:= \begin{bmatrix} 0 & 0 \\ \tilde{L}_f(k)\tilde{C}(k) & 0 \end{bmatrix},\end{aligned}$$

and $\tilde{\gamma}(k) = \gamma(k) - \tilde{\gamma}$.

The estimation error is defined as

$$e(k) := x(k) - \hat{x}(k) = \begin{bmatrix} I & -I \end{bmatrix} \begin{bmatrix} x(k) \\ \hat{x}(k) \end{bmatrix} = I_m \eta(k), \quad (8)$$

where $I_m = \begin{bmatrix} I & -I \end{bmatrix}$.

The objective of this paper is to design an optimal l_∞ fuzzy filter for all possible missing measurements. First,

we consider the stability properties by using the FBDLF. Let P_i be a symmetric positive definite matrix for $1 \leq i \leq r$ and be defined as

$$\tilde{P}(k) := \sum_{i=1}^r h_i(z(k))P_i = \sum_{i=1}^r h_i(z(k)) \begin{bmatrix} P_{1i} & 0 \\ 0 & P_{2i} \end{bmatrix},$$

$$\begin{aligned}\tilde{P}^+(k) &:= \sum_{j=1}^r h_j(z(k+1))P_j \\ &= \sum_{j=1}^r h_j(z(k+1)) \begin{bmatrix} P_{1j} & 0 \\ 0 & P_{2j} \end{bmatrix}\end{aligned}\quad (9)$$

for all $k \geq 0$.

The following Lemmas 1 and 2 are used to construct the LMI condition in proof of Theorem 2 and to define a parameter-dependent Lyapunov function in the stability analysis of a T-S fuzzy system, respectively.

Lemma 1 [32]: Given any matrices X and $P = P^T \succ 0$, we have

$$-X^T P^{-1} X \leq P - X^T - X.$$

Lemma 2: Let P_{1i} and P_{2i} are symmetric positive definite matrices, if there exist the matrices P_{1i} , P_{2i} , Ω_1 , Ω_2 , M_i , such that the following LMIs hold

$$\begin{bmatrix} -\lambda P_i & * & * \\ \Phi_{ij} & P_g - \Omega - \Omega^T & * \\ \sigma \Psi_{ij} & 0 & P_g - \Omega - \Omega^T \end{bmatrix} < 0, \quad (10)$$

where

$$\begin{aligned}\Phi_{ij} &= \begin{bmatrix} \Omega_1 A_i & 0 \\ \tilde{\gamma} M_i C_j & \Omega_2 A_{fi} - M_i C_j \end{bmatrix}, \\ \Psi_{ij} &= \begin{bmatrix} 0 & 0 \\ M_i C_j & 0 \end{bmatrix}, \quad \Omega = \begin{bmatrix} \Omega_1 & 0 \\ 0 & \Omega_2 \end{bmatrix}, \\ M_i &= \Omega_2 L_{fi}\end{aligned}$$

and $*$ is the transposed element in symmetric positions for $1 \leq g, i, j \leq r$, then so does the following inequality:

$$\begin{aligned}A_{m1}(k)^T \tilde{P}^+(k) A_{m1}(k) + \\ A_{m2}(k)^T \sigma^2 \tilde{P}^+(k) A_{m2}(k) - \lambda \tilde{P}(k) < 0.\end{aligned}$$

for all $k \geq \mathbb{Z}_{\geq 0}$, where λ is any positive real number.

Proof: Lemma 2 can be easily proved by [33], [34], and [35], hence the proof is omitted. \square

We will study the condition under which the filter error system (7) is stochastically stable in the mean square with a given l_∞ performance α . The following theorems show that the l_∞ performance of the filter error system can be guaranteed if there exist some fuzzy basis dependent matrices satisfying the LMIs.

Theorem 1: If there exist symmetric positive definite matrices P_{1i} and P_{2i} , and some matrices Ω_1 , Ω_2 and M_i ,

such that the LMI (10) holds for $1 \leq i \leq r$ and some λ with $0 < \lambda < 1$, then the filter error system (7) with $d(k) = 0$ is mean square exponential stable with

$$\mathbb{E} \{ \|\eta(k)^2\| \} \leq \frac{\lambda_{\max}(\tilde{P}(k))}{\lambda_{\min}(\tilde{P}(k))} \lambda^{k-k_0} \mathbb{E} \{ \|\eta(0)^2\| \} \quad (11)$$

for all $k \geq \mathbb{Z}_{\geq 0}$, where k_0 is an arbitrary initial time, $\eta(k_0)$ is an arbitrary initial condition, and the positive constants $\lambda_{\min}(\tilde{P}(k))$ and $\lambda_{\max}(\tilde{P}(k))$ are defined as $\lambda_{\min}(\tilde{P}(k)) = \min_{1 \leq i \leq r} (\lambda_{\min}(P_i))$ and $\lambda_{\max}(\tilde{P}(k)) = \max_{1 \leq i \leq r} (\lambda_{\max}(P_i))$, respectively.

Proof: Define a parameter-dependent Lyapunov function as

$$\begin{aligned} V(\eta(k), z(k)) &\triangleq \eta(k)^T \left\{ \sum_{i=1}^r h_i(z(k)) P_i \right\} \eta(k) \\ &= \eta(k)^T \tilde{P}(k) \eta(k), \end{aligned} \quad (12)$$

where the matrix P_i satisfies a positive-definite for $1 \leq i \leq r$. By using the properties of the membership function in (2), it is obvious that $V(\eta(k), z(k))$ is positive-definite. Also, from (12), it can be shown that

$$\begin{aligned} \lambda_{\min}(\tilde{P}(k)) \|\eta(k)\|^2 &\leq V(\eta(k), z(k)) \\ &\leq \lambda_{\max}(\tilde{P}(k)) \|\eta(k)\|^2. \end{aligned} \quad (13)$$

With the definition of $V(\eta(k), z(k))$ it follows that

$$\begin{aligned} &V(\eta(k+1), z(k+1)) \\ &= \eta(k)^T (A_{m1}(k) + \tilde{\gamma}(k)A_{m1}(k))^T \tilde{P}^+(k) \\ &\quad \times (A_{m1}(k) + \tilde{\gamma}(k)A_{m2}(k)) \eta(k) \end{aligned} \quad (14)$$

We apply the conditional mean operator $\mathbb{E}\{\cdot|F_{k-1}\}$ to the both side of (14).

$$\begin{aligned} &\mathbb{E} \{ V(\eta(k+1), z(k+1)) | F_k \} \\ &= \mathbb{E} \left\{ \eta(k)^T (A_{m1}(k) + \tilde{\gamma}(k)A_{m2}(k))^T \tilde{P}^+(k) \right. \\ &\quad \times (A_{m1}(k) + \tilde{\gamma}(k)A_{m2}(k)) \eta(k) \} \\ &= \eta(k)^T A_{m1}(k)^T \tilde{P}^+(k) A_{m1}(k) \eta(k) \\ &\quad + \eta(k)^T A_{m2}(k)^T \sigma^2 \tilde{P}^+(k) A_{m2}(k) \eta(k). \end{aligned}$$

Thus, if the LMI (10) is satisfied, then the following inequality is satisfied by Lemma 2.

$$\begin{aligned} \mathbb{E} \{ V(\eta(k+1), z(k+1)) | F_k \} &\leq \lambda \eta(k)^T \tilde{P}(k) \eta(k) \\ &= V(\eta(k), z(k)) \end{aligned} \quad (15)$$

Using the smoothing property of the conditional mean [36], $\{\eta(k), z(k)\}$ are F_{k-1} -measurable and $\{\eta(k+1), z(k+1)\}$ are also F_k -measurable. Tacking the conditional expectation operator $\mathbb{E}\{\cdot|F_{k-1}\}$ again to the both sides of (15), we have

$$\begin{aligned} &\mathbb{E} \{ V(\eta(k+1), z(k+1)) | F_{k-1} \} \\ &\leq \lambda^2 V(\eta(k-1), z(k-1)). \end{aligned}$$

By continuing this procedure by sequentially applying $\mathbb{E}\{\cdot|F_{k-2}\}$, $\mathbb{E}\{\cdot|F_{k-3}\}$, \dots , $\mathbb{E}\{\cdot|F_{k_0}\}$, we can obtain almost surely.

$$\begin{aligned} &\mathbb{E} \{ V(\eta(k+1), z(k+1)) | F_{k-1} \} \\ &\leq \lambda^{k+1-k_0} V(\eta(k_0), z(k_0)). \end{aligned} \quad (16)$$

And then, taking the expectation of the last inequality, we can rearrange (16) as follows:

$$\mathbb{E} \{ V(\eta(k), z(k)) \} \leq \lambda^{k-k_0} \mathbb{E} \{ V(\eta(k_0), z(k_0)) \}.$$

Finally, using the fact of (13), inequality (11) is obtained. The proof is completed.

Second, the optimal l_∞ filtering problem is almost converged to find a filter gain L_{fi} in (4) and to minimize the peak value $\|e(k)\|_\infty$ of the estimation error $e(k)$ over all bounded energy disturbances $d(k)$, that is

$$\min_{L_{fi}} \sup_{d(k) \in l_\infty} \frac{\|e(k)\|_\infty}{\|d(k)\|_\infty} \leq \alpha.$$

Remark 1: There is $\|e(k)\|_\infty \triangleq \sup_k |e(k)|$ for $e(k) \in \mathbb{R}^n$, where $|e(k)| \triangleq \sqrt{e(k)^T e(k)}$ and $e(k) \in l_\infty$ if $\|e(k)\|_\infty < \infty$.

The optimal l_∞ filter minimizes the energy-to-peak gain of the system from the disturbance $d(k)$ to the estimation error [26, 39]. However, it is very difficult to solve the minimax problem for nonlinear systems directly. So, the α -suboptimal L_{fi} filtering problem is considered to minimize the upper bound of the l_∞ norm. Given a disturbance attenuation level α the l_∞ fuzzy filter is said to be solvable if there exists the filter gain L_{fi} with zero initial conditions.

$$\sup_{d(k) \in l_\infty} \mathbb{E} \{ \|e(k)\|_\infty \} \leq \alpha \mathbb{E} \{ \|d(k)\|_\infty \}$$

for all $d(k) \in l_\infty$, where we define

$$\begin{aligned} \mathbb{E} \{ \|e(k)\|_\infty \} &\triangleq \mathbb{E} \left\{ \sqrt{e(k)^T e(k)} \right\}, \\ \mathbb{E} \{ \|d(k)\|_\infty \} &\triangleq \mathbb{E} \left\{ \sqrt{d(k)^T d(k)} \right\}. \end{aligned}$$

Then, under the effect of the persistently bounded disturbance signal $d(k)$, the peak of the estimation error $e(k)$ can be attenuated by a level α .

Next, the l_∞ performance criteria for the filter error system (7) will be established. The following theorem shows that the l_∞ performance of the filter error system can be guaranteed if there exists the attenuation level α which can be minimized to satisfy the certain LMIs.

Theorem 2: If there exist a symmetric matrix P_{1i} and P_{2i} , some matrices Ω_1 , Ω_2 and M_i , and some scalar τ , such that the LMI (10) and the following optimization problem

are satisfied, then the filter error system (7) is stochastically stable with a given minimum l_∞ performance α .

$$\begin{aligned} \min \quad & \tau \\ \text{subject to} \quad & \begin{bmatrix} -\tilde{P}_{2i} & * & * & * & * \\ 0 & -\rho^{-1}\tau I & * & * & * \\ \tilde{\Phi}_{ij} & \Lambda_{ij} & P_g & * & * \\ \sigma\tilde{\Psi}_{ij} & 0 & 0 & P_g & * \\ I_m & 0 & 0 & 0 & -\rho I \end{bmatrix} \leq 0 \end{aligned} \quad (17)$$

where

$$\begin{aligned} P_g &= P_{2g} - \Omega_2 - \Omega_2^T, & \tilde{P}_{2i} &= \begin{bmatrix} P_{2i} & -P_{2i} \\ -P_{2i} & P_{2i} \end{bmatrix}, \\ \tilde{\Phi}_{ij} &= \begin{bmatrix} \Omega_2 A_i - \tilde{\gamma} M_i C_j & -\Omega_2 A_{fi} + M_i C_j \end{bmatrix}, \\ \Lambda_{ij} &= \begin{bmatrix} \Omega_2 B_i & -M_i D_j \end{bmatrix}, & \tilde{\Psi}_{ij} &= \begin{bmatrix} -M_i C_j & 0 \end{bmatrix} \end{aligned}$$

for $1 \leq g, i, j \leq r$. Here, the minimum l_∞ performance α and the fuzzy gain L_{fi} are obtained by $\sqrt{\tau}$ and $\Omega_2^{-1} M_i$, respectively.

Proof: We first establish the l_∞ performance criteria for the filter error systems (7).

$$\begin{aligned} J &= \mathbb{E}\{V(k+1)\} - \mathbb{E}\{V(k)\} + \mathbb{E}\{\|e(k)\|^2\} \\ &\quad - \alpha^2 \mathbb{E}\{\|d(k)\|^2\}, \end{aligned}$$

where $\alpha > 0$, and $V(k) \geq 0$ is a family of positive real valued function. If $J \leq 0$, then the filter error system has l_∞ -gain. Choosing a Lyapunov function as $V(k) = e(k)^T \sum_{i=1}^r h_i(z(k)) \rho P_{2i} e(k)$ where ρ is a given constant, the l_∞ performance criteria J can be rewritten as

$$J = \Delta V(k) + e(k)^T e(k) - \alpha^2 d(k)^T d(k). \quad (18)$$

First, we have

$$\begin{aligned} \Delta V(k) &= \mathbb{E}\{V(k+1)|F_k\} - V(k) \\ &= \mathbb{E}\left\{ (I_m \eta(k+1))^T \rho \tilde{P}_2^+(k) (I_m \eta(k+1)) | F_k \right\} \\ &\quad - \eta(k)^T I_m^T \rho \tilde{P}_2(k) I_m \eta(k) \\ &= \mathbb{E}\left\{ ((A_{m1}(k) + \tilde{\gamma}(k)A_{m2}(k))\eta(k) + B_m(k)d(k))^T I_m^T \right. \\ &\quad \times \rho \tilde{P}_2(k) I_m ((A_{m1}(k) + \tilde{\gamma}(k)A_{m2}(k))\eta(k) \\ &\quad \left. + B_m(k)d(k)) | F_k \right\} \\ &\quad - \eta(k)^T I_m^T \rho \tilde{P}_2(k) I_m \eta(k) \\ &= \mathbb{E}\left\{ \xi(k)^T \begin{bmatrix} (A_{m1}(k)^T + \tilde{\gamma}(k)A_{m2}(k)^T) I_m^T \\ B_m(k)^T I_m^T \end{bmatrix} \rho \tilde{P}_2^+(k) \right. \\ &\quad \times \left. \begin{bmatrix} I_m(A_{m1}(k) + \tilde{\gamma}(k)A_{m2}(k)) & I_m B_m(k) \end{bmatrix} \xi(k) \right\} \\ &\quad - \eta(k)^T I_m^T \rho \tilde{P}_2(k) I_m \eta(k) \\ &= \xi(k)^T \begin{bmatrix} A_{m1}(k)^T I_m^T \\ B_m(k)^T I_m^T \end{bmatrix} \rho \tilde{P}_2^+(k) \begin{bmatrix} I_m A_{m1}(k) & I_m B_m(k) \end{bmatrix} \xi(k) \\ &\quad + \xi(k)^T \sigma^2 \begin{bmatrix} A_{m2}(k)^T I_m^T \\ 0 \end{bmatrix} \rho \tilde{P}_2^+(k) \begin{bmatrix} I_m A_{m1}(k) & 0 \end{bmatrix} \xi(k) \end{aligned}$$

$$- \eta(k)^T I_m^T \rho \tilde{P}_1(k) I_m \eta(k), \quad (19)$$

where $\tilde{P}_2(k) = \sum_{i=1}^r h_i(z(k)) P_{2i}$, $\tilde{P}_2^+(k) = \sum_{i=1}^r h_i(z(k+1)) P_{2i}$ and $\xi(k) = [\eta(k)^T \quad d(k)^T]^T$. Considering the persistent bounded disturbances, the following holds:

$$\begin{aligned} e(k)^T e(k) - \alpha^2 d(k)^T d(k) \\ = \xi(k)^T \begin{bmatrix} I_m^T I_m & 0 \\ 0 & -\alpha^2 I \end{bmatrix} \xi(k). \end{aligned} \quad (20)$$

From (19), (20) and Schur complement, if the following inequality is satisfied

$$\begin{aligned} \begin{bmatrix} A_{m1}(k)^T I_m^T & \sigma A_{m2}(k)^T I_m^T \\ B_m(k)^T I_m^T & 0 \end{bmatrix} \begin{bmatrix} \tilde{P}_2^+(k) & 0 \\ 0 & \tilde{P}_2^+(k) \end{bmatrix} \\ \times \begin{bmatrix} I_m A_{m1}(k) & I_m B_m(k) \\ \sigma I_m A_{m2}(k) & 0 \end{bmatrix} \\ - \begin{bmatrix} I_m^T P_2(k) I_m - \rho^{-1} I_m^T I_m & 0 \\ 0 & \rho^{-1} \alpha^2 I \end{bmatrix} \leq 0 \end{aligned} \quad (21)$$

then $J \leq 0$. Also, the inequality (21) is rearranged as the follows:

$$\begin{aligned} \begin{bmatrix} A_{m1}(k)^T I_m^T & \sigma A_{m2}(k)^T I_m^T & I_m^T \\ B_m(k)^T I_m^T & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{P}_2^+(k) & 0 & 0 \\ 0 & \tilde{P}_2^+(k) & 0 \\ 0 & 0 & \rho^{-1} \end{bmatrix} \\ \times \begin{bmatrix} I_m A_{m1}(k) & I_m B_m(k) \\ \sigma I_m A_{m2}(k) & 0 \\ I_m & 0 \end{bmatrix} - \begin{bmatrix} I_m^T P_2(k) I_m & 0 \\ 0 & \alpha^2 \rho^{-1} I \end{bmatrix} \leq 0. \end{aligned} \quad (22)$$

By using the Schur complement to (22), we obtain the following inequality:

$$\begin{bmatrix} -I_m^T P_2(k) I_m & * & * & * & * \\ 0 & -\alpha^2 \rho^{-1} I & * & * & * \\ I_m A_{m1}(k) & I_m B_m(k) & -Q^+(k) & * & * \\ \sigma I_m A_{m2}(k) & 0 & 0 & -Q^+(k) & * \\ I_m & 0 & 0 & 0 & \rho I \end{bmatrix} \leq 0, \quad (23)$$

where $Q^+(k) = (\tilde{P}_2^+(k))^{-1}$.

Then, employing the congruence transformation with $\text{diag}\{I, I, \Omega_2, \Omega_2, I\}$ to (23), applying Lemma 1, and using the fuzzy property, the LMI (17) can be obtained. The proof is completed.

Remark 2: Theorem 1 and 2 are based on the fuzzy system (3) with data loss phenomenon of the measurement output. If the measurement output of the fuzzy system has not data loss phenomenon or is perfectly measured, the LMIs in Theorem 1 and 2 are simplified as follows:

$$(10) \Rightarrow \begin{bmatrix} -\lambda P_i & * \\ \tilde{\Phi}_{ij} & P_g - \Omega - \Omega^T \end{bmatrix} \prec 0 \quad (24)$$

$$(17) \Rightarrow \begin{bmatrix} -\tilde{P}_{2i} & * & * & * \\ 0 & -\rho^{-1}\tau I & * & * \\ \tilde{\Phi}_{ij} & \Lambda_{ij} & P_{2g} - \Omega_2 - \Omega_2^T & * \\ I_m & 0 & 0 & -\rho I \end{bmatrix} \prec 0 \quad (25)$$

where

$$\begin{aligned} \hat{\Phi}_{ij} &= \begin{bmatrix} \Omega_1 A_i & 0 \\ M_i C_j & \Omega_1 A_{fi} - M_i C_j \end{bmatrix}, \\ \tilde{\Phi}_{ij} &= \begin{bmatrix} \Omega_2 A_i - M_i C_j & -\Omega_2 A_{fi} + M_i C_j \end{bmatrix}. \end{aligned}$$

4. SIMULATION EXAMPLE

In this section, we demonstrate a discretized chaotic Lorenz system with both persistent bounded disturbances and missing measurements, which can be represented a T-S fuzzy system [27] as follows:

$$\begin{aligned} x(k+1) &= \sum_{i=1}^2 h_i(x_1(k)) (A_i x(k) + B_i u(k) + G_i w(k)), \\ y(k) &= \sum_{i=1}^2 h_i(x_1(k)) (C_i x(k) + v(k)), \end{aligned}$$

where

$$\begin{aligned} A_1 &= \begin{bmatrix} 1 - \sigma T_s & \sigma T_s & 0 \\ c T_s & 1 - T_s & -M_1 T_s \\ 0 & M_1 T_s & 1 - b T_s \end{bmatrix}, \\ A_2 &= \begin{bmatrix} 1 - \sigma T_s & \sigma T_s & 0 \\ c T_s & 1 - T_s & -M_2 T_s \\ 0 & M_2 T_s & 1 - b T_s \end{bmatrix}, \\ B_1 = B_2 &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \\ C_1 = C_2 &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \quad G_1 = G_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \\ h_1(x_1(k)) &= \frac{-x_1(k) + M_2}{M_2 - M_1}, \quad h_2(x_1(k)) = \frac{x_1(k) - M_1}{M_2 - M_1}. \end{aligned}$$

To simulate the above system, we use the following system parameters:

$$\begin{aligned} \sigma &= 10, & b &= 28, & c &= 8/3, \\ T_s &= 0.1, & M_1 &= -20, & M_2 &= 30 \end{aligned}$$

and an exogenous disturbance $w(k)$ and the measurement disturbance $v(k)$ are assumed as $5 \cos(0.5k)$ and $0.5 \sin(0.5k)$, which are the persistent and bounded sinusoidal functions.

Now, by following the design procedure in the previous section, the discretized chaotic Lorenz system has to be stochastically stable with a guaranteed l_∞ norm bound α . To stabilization of the discretized chaotic Lorenz system, the control input $u(k)$ is obtained by using the fuzzy

control technique described in [38]. Supposing $\rho = 1.8$ and solving LMIs in (17), we obtain the filter gains L_{fi} for $\bar{\gamma} = 0.8$ and $\bar{\gamma} = 0.5$ cases respectively as follows:

$$\begin{aligned} \bar{\gamma} = 0.8: \quad L_{f1} &= \begin{bmatrix} -0.1371 \\ 0.1342 \\ -0.0095 \end{bmatrix}, \quad L_{f2} = \begin{bmatrix} -0.1312 \\ 0.1332 \\ -0.0115 \end{bmatrix}, \\ \bar{\gamma} = 0.5: \quad L_{f1} &= \begin{bmatrix} -0.1374 \\ 0.1344 \\ -0.0095 \end{bmatrix}, \quad L_{f2} = \begin{bmatrix} -0.1310 \\ 0.1331 \\ -0.0114 \end{bmatrix}. \end{aligned}$$

In the simulation, the data losses are generated randomly according to $\bar{\gamma} = 0.8$ and $\bar{\gamma} = 0.5$, and Figs. 1 and 2 shows when the data loss phenomenon is occurred for each case. With zero initial condition, Figs. 3, 4 and 5 show each state variable for both $\bar{\gamma} = 0.8$ and $\bar{\gamma} = 0.5$ cases. As shown the graphs, the filtering performance of the state variable $x_1(k)$ is insufficient, but the outstanding filtering performances are represented in the case of the state variable $x_2(k)$ and $x_3(k)$. In Figs. 6, 7 and 8, the estimation errors of each state variable are respectively shown for both $\bar{\gamma} = 0.8$ and $\bar{\gamma} = 0.5$ cases, and we show that the case of $\bar{\gamma} = 0.8$ has better performance than the case of $\bar{\gamma} = 0.5$ in the proposed fuzzy filter.

5. CONCLUSIONS

In this paper, we have investigated the problem of l_∞ fuzzy filtering with both the persistent bounded disturbances and missing probabilistic sensor information. The T-S fuzzy model was adopted to represent a nonlinear system. The measurement output is assumed to contain randomly missing data that has been modeled by the Bernoulli distributed with a known conditional probability. We have reduced the effect of the perturbation against persistent bounded disturbances by using the minimum l_∞ performance based on the LMI. The LMI was obtained based on a relaxed approach in the FBDLF concept. By using FBDLF, the sufficient conditions have been established that ensure the exponential mean square stability of the filtering error, and the filter gain was obtained by the solution to a set of LMI. Finally, the effectiveness of the proposed design methods on the FBDLF have been shown via an example.

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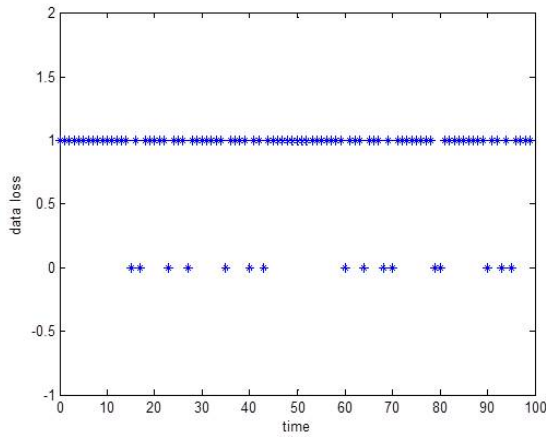


Fig. 1. The data loss phenomenon with $\bar{\gamma} = 0.8$.

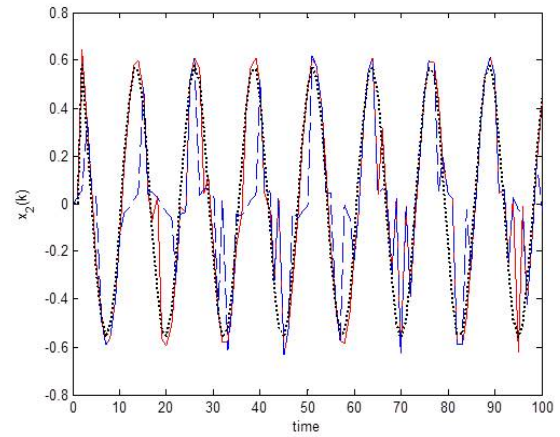


Fig. 4. Time responses: the state $x_2(k)$ (dotted), the estimations $\hat{x}_2(k)$ for $\bar{\gamma} = 0.8$ (solid) and $\bar{\gamma} = 0.5$ (dashed).

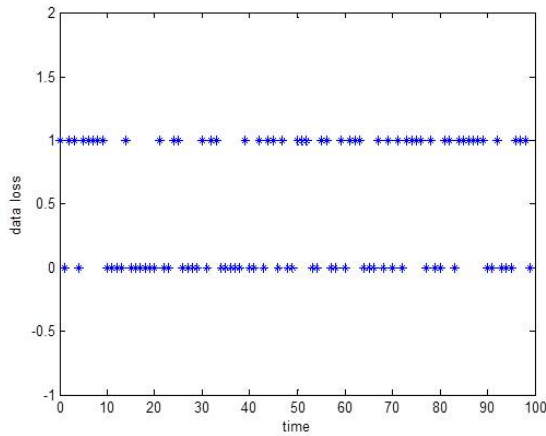


Fig. 2. The data loss phenomenon with $\bar{\gamma} = 0.5$.

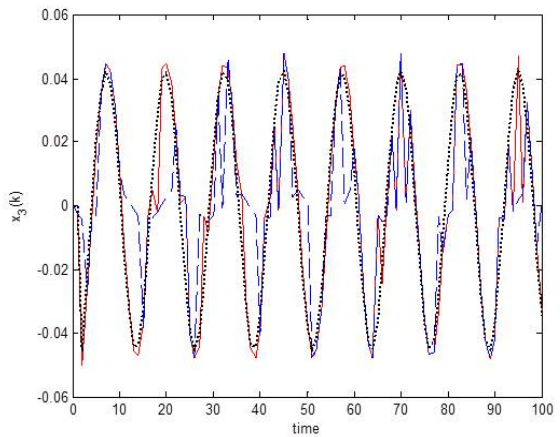


Fig. 5. Time responses: the state $x_3(k)$ (dotted), the estimations $\hat{x}_3(k)$ for $\bar{\gamma} = 0.8$ (solid) and $\bar{\gamma} = 0.5$ (dashed).

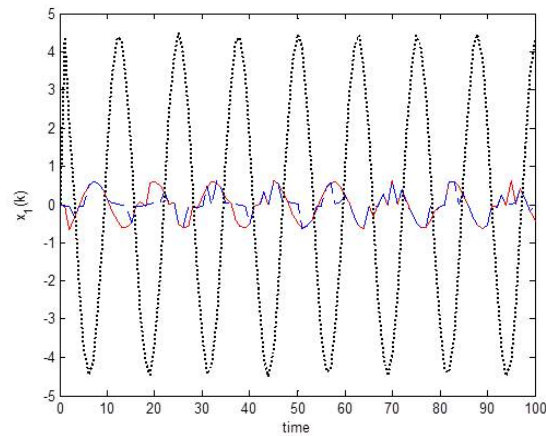


Fig. 3. Time responses: the state $x_1(k)$ (dotted), the estimations $\hat{x}_1(k)$ for $\bar{\gamma} = 0.8$ (solid) and $\bar{\gamma} = 0.5$ (dashed).

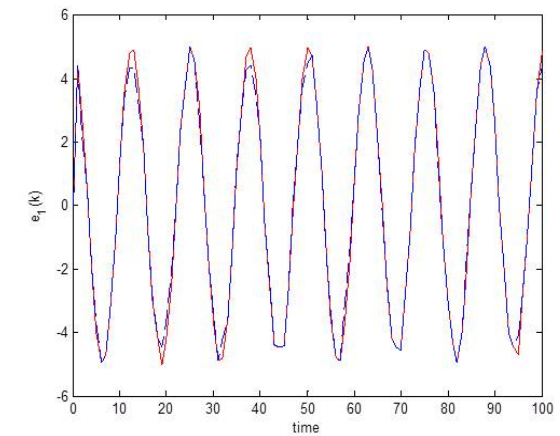


Fig. 6. Time responses: the estimation error $e_1(k)$ for $\bar{\gamma} = 0.8$ (solid) and $\bar{\gamma} = 0.5$ (dashed).

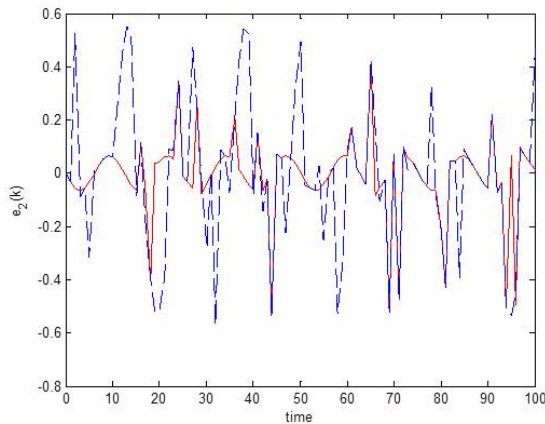


Fig. 7. Time responses: the estimation error $e_2(k)$ for $\bar{\gamma} = 0.8$ (solid) and $\bar{\gamma} = 0.5$ (dashed).

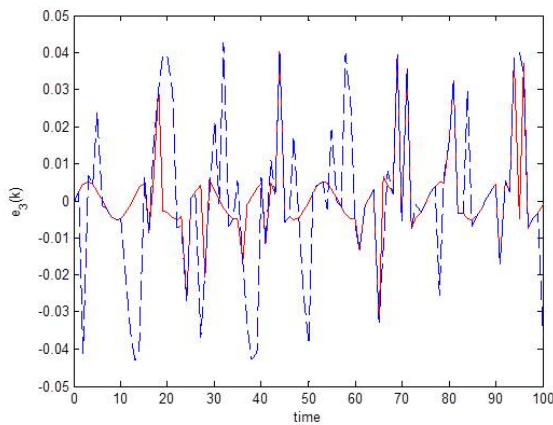


Fig. 8. Time responses: the estimation error $e_3(k)$ for $\bar{\gamma} = 0.8$ (solid) and $\bar{\gamma} = 0.5$ (dashed).

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