International Journal of Control, Automation and Systems 14(2) (2016) 443-451
http://dx.doi.org/10.1007/s12555-014-0533-2
ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Fuzzy Fault Tolerant Predictive Control for a Diesel Engine Air Path

Lamia Ben Hamouda*, Mounir Ayadi, and Nicolas Langlois

Abstract: This paper proposes a Fuzzy Fault Tolerant Predictive Control (FFTPC) with integral action method for a class of nonlinear systems. The Takagi-Sugeno (T-S) fuzzy approach is introduced as a modelling technique in order to consider the active control methods adapted to linear models. The proposed control strategy is based on a combination between Parallel Distributed Compensation (PDC) control law and Model Predictive Control (MPC) where the T-S fuzzy aspect uses the Unmeasurable Premise Variables (UPV). A T-S fuzzy observer provides an L_2 norm estimation of system state vector and faults. The controller and observer gains are obtained by solving Linear Matrix Inequalities (LMIs) derived from the Lyapunov theory. The validity of the proposed Fault Tolerant Control (FTC) strategy is illustrated through an application to a Diesel Engine Air Path (DEAP) system.

Keywords: Diesel engine air path, FTC, LMI, MPC, T-S fuzzy observer.

1. INTRODUCTION

In the last decades, diesel engine has been considered as the most popular thermal engine due to its low fuel consumption and improved reliability. However, it has negative impacts on the environment because of particles and nitrogen oxides (NO_x) emissions. The industries used Diesel particulate filter to remove these polluting emissions from the exhaust gaz of DEAP system. Nevertheless, for economical reasons, it becomes more interesting to reduce the production of these pollutants during the combustion. The air path control problem represents an active research area. The target of most studies in the literature, is to control both of the intake pressure and the air flow using Exhaust Gas Recirculation (EGR) valve and Variable Geometry Turbocharger (VGT).

Firstly in [1], a constructive Lyapunov control design for turbocharged diesel engine based on nonlinear control is studied. In [2], the authors proposed an explicit approach of MPC to take into account DEAP system constraints. Moreover, Layerle et al applied in [3], the design of reconfigurable predictive control to DEAP system. By the same token, in [4], an FTC design scheme based on a T-S fuzzy model of DEAP system has been extended to the state estimation, the leakage identification and the state feedback control law to guaranty the stabilization of the faulty DEAP system. In [5], Abidi et al developed control strategy based on fuzzy logic where fuzzy systems are used to describe different engine speed of DEAP system, which leads to an improvement of the process representation. In [6], authors proposed a nonlinear observer design of diesel engine selective catalytic reduction systems. In this work, the proposed FTC strategy is initiated by a modelling phase. The modelling phase aims to obtain a faithful behaviour description of the healthy nonlinear system. Above all, in [7] and [8], the stability and stabilization of T-S fuzzy model have been widely studied. Among all the proposed approaches, Lyapunov theory and formulation of the stability conditions in terms of LMIs are used to obtain the PDC control gains. In [9], authors investigated H_∞ the filtering problem of discrete-time T-S fuzzy systems in a network environment. In [10] and [11], a fault tolerant Fuzzy-Model-Predictive Control (FMPC) with integral action method for a Simple Input Simple Output (SISO) nonlinear systems is proposed. In [13], a robust FMPC for a SISO nonlinear systems is proposed. In [14] a Fuzzy Predictive Control algorithm (FPCA) for DEAP system subject to a leakage is proposed. In this paper, new sufficient conditions for the existence of the robust FTC are developed in terms of LMIs constraints. A

Manuscript received December 6, 2014; revised April 17, 2015; accepted May 14, 2015. Recommended by Associate Editor Choon Ki Ahn under the direction of Editor Euntai Kim. The authors gratefully thank "Ecole Doctorale Sciences et Techniques de l'Ingénieur de Tunis," MESRST and "Région Haute Normandie," FEDER for financially supporting this work within the framework of the VIRTUOSE project. Likewise, authors are grateful to the associate editor and reviewers for their constructive comments based on which the presentation of this paper has been greatly improved.

Lamia Ben Hamouda is with Université Tunis El Manar, Ecole Nationale d'Ingénieurs de Tunis (ENIT), Laboratoire de Recherche en Automatique, BP 37 le Belvédère, Tunisie, and ESIGELEC-IRSEEM, EA 4353, Avenue Galilée 76899 St Etienne du Rouvray, France (e-mail: lamia.enit.ats@live.fr). Mounir Ayadi is with Université Tunis El Manar, Ecole Nationale d'Ingénieurs de Tunis (ENIT), Laboratoire de Recherche en Automatique, BP 37 le Belvédère, Tunisie (e-mail: mounir.ayadi@enit.rnu.tn). Nicolas Langlois is with Institut de Recherche en Systèmes Electroniques Embarqués (IRSEEM), BP 10024 Avenue Galilée, France (e-mail: nicolas.langlois@esigelec.fr).
* Corresponding author.

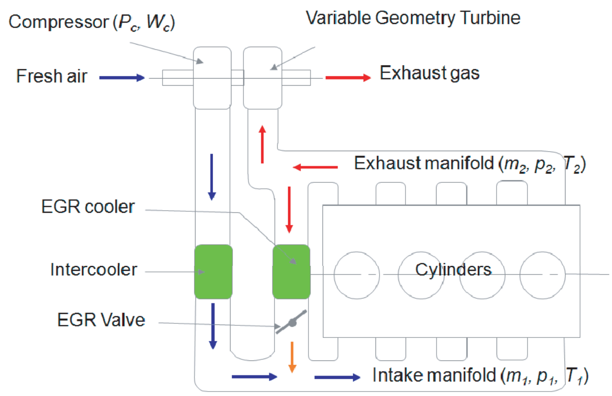


Fig. 1. A schema of a diesel engine airpath system.

T-S fuzzy observer based on UPV is designed for the advanced strategy, in order to estimate DEAP system state variables, the leakage and the sensor fault. When the leakage or the sensor fault occurs, the objective is to conserve the stability and the performances of DEAP system. The layout of this paper is as follows: in Section 2, the T-S fuzzy DEAP system modelling is presented. Then, the FFTPC with integral action is proposed in Section 3. The Section 4 shows the simulation results. Conclusion is presented in the last section.

2. DEAP SYSTEM

2.1. The Jankovic DEAP system modelling

The air goes through the compressor in the intake manifold and then flows into the cylinders where fuel is injected and burned, producing torque on the crank shaft. Part of the hot exhaust gas is pumped out from the exhaust manifold through the turbine. The other part is recirculated back into the intake manifold through the EGR valve. The turbine takes the energy from the exhaust gas to supply power the compressor. To reduce the intake manifold temperature, an intercooler and an EGR cooler are used. There are four sensors in DEAP system measuring both the intake and the exhaust manifold of temperature and pressure. The engine is also equipped with the Air/Fuel Ratio sensor. In the following, the control outputs are the VGT and the EGR valve positions both of them have lower and upper bounds. The considered DEAP system is a two-input-two-output system. The system inputs are $u_1 = W_{egr}$ and $u_2 = W_{vgt}$ as considered in [1]. The following state space model is considered:

$$\begin{cases} \dot{p}_1(t) = -k_1 k_e p_1(t) + \frac{k_1 k_c}{p_1(t)^{\alpha}-1} P_c(t) + k_1 W_{egr}(t), \\ \dot{p}_2(t) = k_2 k_e p_1(t) - k_2 W_{egr}(t) - k_2 W_{vgt}(t), \\ \dot{P}_c(t) = \frac{-P_c(t)}{\tau} + K_0(1 - p_2(t)^{-\alpha}) W_{vgt}(t). \end{cases} \quad (1)$$

A descriptive scheme of DEAP system is shown in Fig. 1 and the nomenclature of variables are given in Table 1.

Table 1. Nomenclature of DEAP system variables.

Variable	Name	Units
p_1	intake manifold pressure	Bar
p_2	exhaust manifold pressure	Bar
P_c	compressor power	Watt
W_c	compressor mass flow	Kg/s
W_{vgt}	turbine mass flow	Kg/s
W_{egr}	gaz flow through the EGR	Kg/s

The compressor mass flow rate is related to the compressor power as follows:

$$W_c(t) = P_c(t) \frac{k_c}{p_1(t)^{\alpha} - 1} \quad (2)$$

with $k_c = \frac{\eta_c}{c_p T_a}$, $k_t = \eta_t c_p T_2$ and $K_0 = \frac{\eta_m}{\tau} k_t$.
The turbine isentropic efficiency η_t, the compressor isentropic efficiency η_c, the time constant τ and the turbocharger mechanical efficiency η_m are assumed to be constant. The model (1) can be expressed under the following control-affine form:

$$\dot{x}(t) = h(x(t)) + g_1(x(t))u_1(t) + g_2(x(t))u_2(t) \quad (3)$$

with $x(t) = (p_1(t), p_2(t), P_c(t))^T$,

$$h(x(t)) = \begin{bmatrix} -k_1 k_e x_1(t) + k_1 k_c \theta_1(x_1(t)) x_3(t) \\ -k_2 k_e x_1(t) \\ \frac{-x_3(t)}{\tau} \end{bmatrix}, \quad (4)$$

$$g_1(x(t)) = \begin{bmatrix} k_1 \\ -k_2 \\ 0 \end{bmatrix}, g_2(x(t)) = \begin{bmatrix} 0 \\ -k_2 \\ K_0 \theta_2(x_2(t)) \end{bmatrix}, \quad (5)$$

where the nonlinear functions are given by:

$$\theta_1(p_1(t)) = \frac{1}{p_1^{\alpha}(t) - 1} \; ; \; \theta_2(p_2(t)) = 1 - p_2^{-\alpha}(t). \quad (6)$$

In [1], DEAP model parameters (k_1, k_2, k_c, k_e, k_t, η_m, τ) are identified under steady state conditions.

$$\left\{ \begin{array}{l} \Omega_{DEAP} = x = (p_1, p_2, P_c)^T : 1 < p_1(t) < p_1^{\max}, \\ 1 < p_2(t) < p_2^{\max}, 0 < P_c(t) < P_c^{\max} \end{array} \right\}$$

where for all conditions $x(t_0) \in \Omega_{DEAP}$, then $x(t) \in \Omega_{DEAP}$, $\forall t \geq t_0$.

2.2. T-S fuzzy model

The T-S fuzzy model is given by the following relation as introduced in [12]:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{N} \mu_i(\theta) \left(A_i x(t) + B_i u(t) \right), \\ y(t) = \sum_{i=1}^{N} \mu_i(\theta) C_i x(t), \end{cases} \quad (7)$$

where $x \in \Re^n$ stands for the state vector, $u \in \Re^m$ denotes the control input vector and $y \in \Re^p$ represents the output vector. $A_i \in \Re^{n\times n}$, $B_i \in \Re^{n\times m}$ and $C_i \in \Re^{p\times n}$ are constant matrices and θ represents the premise variables vector depending on system states and input. $\{A_i, B_i\}$ are the sub-models asymptotically stable matrices and the activation functions μ_i satisfy $\forall\, i \geq 0$ the following:

$$\sum_{i=1}^{N} \mu_i(x(t), u(t)) = 1 \text{ and } 0 \leq \mu_i(x(t), u(t)) \leq 1. \quad (8)$$

Polytope is obtained with $N = 2^r$ peaks, where r is the number of premise variables. In [7], Wang and Tanaka obtain this convex polytopic representation by a direct transform of an affine model in the state called sector nonlinearity approach. This method does not generate an approximation error and has an advantage of reducing the local model number. Infact, the number n of the local models depends on the desired representativeness, the nonlinear system complexity and the choice of the activation functions structure. The fuzzy model obtained is constituted by two sets of sub-linear Time Invariant (LTI) representing the lower and upper bounds $(\underline{\theta}, \bar{\theta})$. The considered model is given by (3). The premise variables are assumed to have lower and upper bounds such that:

$$\forall i \in \{1,2\}, \underline{\theta}_i \leq \theta_i \leq \bar{\theta}_i. \quad (9)$$

The T-S fuzzy model (7) is obtained using the sector nonlinearity approach, where the constant matrices are given below:

$$A_{i=\{1,2\}} = \begin{bmatrix} -k_1 k_e & 0 & k_1 k_c \bar{\theta}_1 \\ k_2 k_e & 0 & 0 \\ 0 & 0 & \frac{-1}{\tau} \end{bmatrix},$$

$$A_{i=\{3,4\}} = \begin{bmatrix} -k_1 k_e & 0 & k_1 k_c \underline{\theta}_1 \\ k_2 k_e & 0 & 0 \\ 0 & 0 & \frac{-1}{\tau} \end{bmatrix},$$

$$B_{i=\{1,3\}} = \begin{bmatrix} k_1 & 0 \\ -k_2 & -k_2 \\ 0 & K_0 \bar{\theta}_2 \end{bmatrix}; B_{i=\{2,4\}} = \begin{bmatrix} k_1 & 0 \\ -k_2 & -k_2 \\ 0 & K_0 \underline{\theta}_2 \end{bmatrix},$$

$$C_{i=\{1,2\}} = \begin{bmatrix} 0 & 0 & k_c \bar{\theta}_1 \\ 0 & 1 & 0 \end{bmatrix}; C_{i=\{3,4\}} = \begin{bmatrix} 0 & 0 & k_c \underline{\theta}_1 \\ 0 & 1 & 0 \end{bmatrix}.$$

In the next section, a T-S fuzzy controller with integral action and based on UPV for DEAP system subject to faults is proposed.

3. PROPOSED FFTPC

3.1. Remind on MPC

In practice all physical systems have some forms of constraints due to physical, economic, safety or performance requirements on control inputs and on system states. The ability to handle input and states constraints systematically in the control algorithm represents one of the primary advantages. Moreover, the MPC structure allows FTC to be embedded: constraints can be redefined, internal model and the control objectives can be changed. In [15], advantages of MPC are exposed. For a dynamic system, the control law based on prediction has two main objectives: the tracking of desired trajectories over time and the stabilization around these trajectories with perturbation rejection. In [16], formulations and experimental evaluations of various constrained MPC schemes applied to a realistic full envelope non-linear model of a fighter aircraft are presented. Investigations are carried out by exploring a variety of scenarios of fault and disturbance combinations. Recently in [18], the problem of predictive output feedback control for networked control systems with random communication delays is studied. The aim of MPC is to minimize the cost function J given in an instant k by:

$$J(k) = \sum_{l=1}^{H_p} \| y(k+l) - y_d(k+l\,|k) \|_Q^2 + \sum_{l=0}^{H_u-1} \| \Delta u(k+l\,|k) \|_R^2 \quad (10)$$

to compute the optimal control for the i^{th} sub-model, subject to the following constraints:

$$\begin{aligned} & x_{\min} \leqslant x_l \leqslant x_{\max}, \text{ where } k+1 \leqslant l \leqslant k+H_p. \\ & u_{\min} \leqslant u_l \leqslant u_{\max}, \ \Delta u(k) = u(k) - u(k-1), \\ & \Delta u_{\min} \leqslant \Delta u_l \leqslant \Delta u_{\max}, \text{ where } k \leqslant l \leqslant k+H_u-1, \end{aligned}$$

where y is the predicted response and y_d is the output desired trajectory. The matrices Q and R are used to weight the corresponding control errors and control actions. The R matrix helps to keep the control inputs within bounds, making sure that smooth control actions result. H_p and H_u are output and control prediction horizons, respectively. The computing time depends on the value chosen of H_p. Hence, the choice of the value of H_p affects the system dynamic. In general, a short control horizon makes the system more robust to uncertainties such as parameter variations. Only the first control increment $\Delta u(k)$ is implemented and the optimization is re-solved at each step. It is also assumed that the dynamic system defined by the model (A_i, B_i) is controllable. The controllability condition is required to ensure that the MPC optimization solved at each step is feasible. This optimization can be formulated as a quadratic programming (QP) problem. In [15], stability proofs of such formulation are given. Predictive control formulations can be designed with nominal asymptotic stability guarantees, provided that the associated optimization problem is feasible at each sampling time. However, model-plant mismatches, external perturbations or faults may cause the optimization to become infeasible. In [17], Afonso and Galvao considered the development of techniques aimed at recovering feasibility without violating hard physical constraints imposed by the nature of the model. The validity of their proposed control strategy is illustrated through an application to an helicopter with three degrees of freedom in the presence of actuator faults.

In the next sub-section, an FTC strategy is proposed to recover feasibility, in the presence of faults, without violating constraints imposed on control inputs and system states.

3.2. Proposed FTC strategy

The proposed FTC schema is given by Fig. 2. The main contribution of this work with respect to other works is about an active FTC based on a combination between a PDC control law and a prediction algorithm where the T-S fuzzy aspect uses the UPV. The method uses a T-S fuzzy observer to estimate DEAP system state variables and faults. The aims of the proposed FFTPC approach with integral action are to maintain system output close to the desired trajectories obtained by the reference model and to preserve stability conditions even when faults occur. For example, in [16], Kale and Chipperfield introduced a straightforward strategy by assuming state feedback as a baseline controller to which predictive control signals are added. Using MPC with the pre-stabilization provides an effective tool to guarantee closed-loop stability in the nominal operation and even in the presence of actuator faults. In previous work [10] and [11], it is shown that the feedback aims to provide further robustness and accuracy. The integral action helps to drive the tracking error to zero. The proposed i^{th} control law signal generated in the nominal operating is given by the following:

$$\begin{cases} u_i(k+l|k) = -k_x^i\,(\hat{x}(k+l|k) - x(k+l|k)) \\ -k_I^i\,x^I(k+l|k) + q_i, \\ l = 0,\dots,H_u - 1 \text{ and } i = 1,\dots,N \end{cases} \tag{11}$$

where $\dot{x}_I = y_d - y$, $k_I^1 = k_I^2 = k_I^3 = k_I^4$ are the integral action gain, $\hat{x}$ represents the estimated state. $k_{u1}, k_{u2}, \dots,$ k_{uN} are the N state feedback gains and q_i the i^{th} predicted control input. Beyond the control horizon, the MPC control q_i is set to zero and the control law becomes when $l \geqslant H_u$:

$$\begin{aligned} u_i(k+l|k) = &-k_x^i\,(\hat{x}(k+l|k) - x(k+l|k)) \\ &-k_I^i\,x^I(k+l|k). \end{aligned} \tag{12}$$

In the faulty case, the nonlinear system described by (7) becomes:

$$\begin{cases} \dot{x}_f(t) = \sum_{i=1}^{N} \mu_i(\theta_f)\left(A_i x_f(t) + B_i u_f(t) + E_a^i f(t)\right), \\ y_f(t) = \sum_{i=1}^{N} \mu_i(\theta_f)\left(C_i x_f(t) + E_s^i f(t)\right), \end{cases} \tag{13}$$

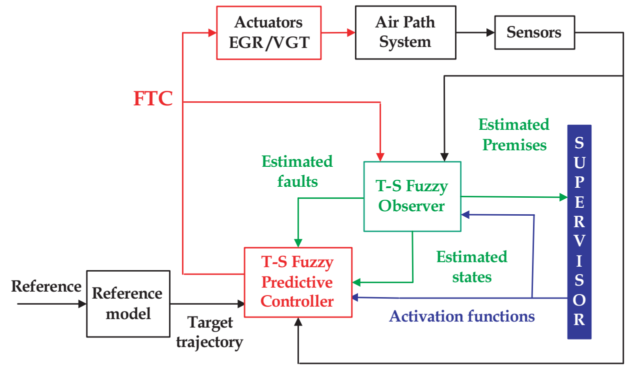


Fig. 2. Proposed structure.

where $f \in \Re^f$ is the fault signal and E_a and E_s represent the fault matrices with appropriate dimensions. The structure given by Fig. 2 is proposed to determine the control inputs $u_f(t)$ such that:

- the closed-loop system is stable,
- $x_f(t)$ converges asymptotically to the reference state vector even in the presence of faults.

In this case, the T-S fuzzy control law is based on the estimated premise variables because UPV depend on the estimated faulty state vector. The following control strategy is then used:

$$u_f(t) = \sum_{i=1}^{N} \mu_i(\hat{\theta}_f)\left(-\hat{f}(t) - k_x^i\,(\hat{x}_f(t) - x(t)) + u(t)\right), \tag{14}$$

where $\hat{f}$ is the fault estimate vector and $u(t)$ is the nominal control input given by (11) and (10):

$$u(t) = \sum_{i=1}^{N} \mu_i(\hat{\theta}(t))u_i(t).$$

The activation functions μ_1 and μ_2 are defined by:

$$\mu_1(\hat{\theta}_f) = \frac{\hat{\theta}_f(\hat{x}_f) - \underline{\theta}}{\bar{\theta} - \underline{\theta}} \quad \text{and} \quad \mu_2(\hat{\theta}_f) = 1 - \mu_1(\hat{\theta}_f). \tag{15}$$

3.3. T-S fuzzy observer

In [19], Bouattour et al proposed the design of a robust fault detection observer for a T-S fuzzy model affected by sensor and actuator faults and unknown bounded disturbances simultaneously with H_∞ performances. The paper [20] represents two approaches of observer for T-S systems with UPV in continuous time case. To illustrate the effectiveness of these proposed approaches simulations results of a two-link robot system are presented and discussed. A recent paper by Sami and Patton [21] described a fault tolerant tracking control strategy. A robust L_2 norm fault estimation and compensation are developed. The advanced strategy is illustrated using a nonlinear inverted pendulum in the presence of simultaneous actuator and sensor faults. In this article, the contribution is about a new FTC structure based on a combination between a PDC control law with the MPC where to estimate simultaneously $x_f(t)$ and $f(t)$, a T-S fuzzy observer is used for system (13):

$$\begin{cases} \dot{\hat{x}}_f(t) = \sum_{i=1}^{N} \mu_i(\hat{\theta}_f) \begin{pmatrix} A_i\hat{x}_f(t) + B_i u_f(t) + E_i \hat{f}(t) \\ +L_i(y_f - \hat{y}_f) \end{pmatrix}, \\ \hat{f}(t) = \sum_{i=1}^{N} \mu_i(\hat{\theta}_f)\left(G_i C_i (x_f(t) - \hat{x}_f(t))\right). \end{cases} \tag{16}$$

The extended error system, containing the two error dynamics $\dot{x}_f(t) - \dot{\hat{x}}_f(t)$ and $\dot{f}(t) - \dot{\hat{f}}(t)$ is given by:

$$\begin{pmatrix} \dot{x}_f(t) - \dot{\hat{x}}_f(t) \\ \dot{f}(t) - \dot{\hat{f}}(t) \end{pmatrix} = \sum_{i=1}^{N} \mu_i(\hat{\theta}_f) \begin{pmatrix} A_i - L_iC_i & E_a^i - L_iE_s^i \\ -G_iC_i & -G_iE_s^i \end{pmatrix} \begin{pmatrix} x_f(t) - \hat{x}_f(t) \\ f(t) - \hat{f}(t) \end{pmatrix}. \tag{17}$$

The tracking error $e(t) = x(t) - x_f(t)$ is given by:

$$\dot{e}(t) = \sum_{i=1}^{N}\sum_{j=1}^{N} \mu_i(\theta)\mu_j(\theta_f) \begin{pmatrix} (A_i - B_iK_x^j)e(t) \\ -E_a^i(f(t) - \hat{f}(t)) \\ -B_iK_x^j(x_f(t) - \hat{x}_f(t)) \end{pmatrix} + I_{n\times n}\Delta_1(t), \tag{18}$$

where $\Delta_1(t) = \sum_{i=1}^{N} (\mu_i(\theta) - \mu_i(\theta_f))(A_i x(t) + B_i u(t))$.

An extended error system $\tilde{e}(t)$, containing the tracking error $e(t)$, the state estimation error $x_f(t) - \hat{x}_f(t)$ and the fault estimation error $f(t) - \hat{f}(t)$, can be expressed as:

$$\dot{\tilde{e}}(t) = \sum_{i=1}^{N}\sum_{j=1}^{N} \mu_i(\hat{\theta}_f)\mu_j(\theta_f)\tilde{A}_{ij}\tilde{e}(t) + \Gamma\Delta(t), \tag{19}$$

where

$$\tilde{e}(t) = \begin{pmatrix} x(t) - x_f(t) \\ x_f(t) - \hat{x}_f(t) \\ f(t) - \hat{f}(t) \end{pmatrix}, \Gamma = \begin{pmatrix} I_{n\times n} & 0 \\ 0 & I_{n\times n} \\ 0 & 0 \end{pmatrix},$$

$$\Delta = \begin{pmatrix} \Delta_1(t) \\ \Delta_2(t) \end{pmatrix},$$

$$\tilde{A}_{ij} = \begin{pmatrix} A_i - B_iK_x^j & -B_iK_x^j & -E_a^i \\ 0 & A_i - L_iC_j & E_a^i - L_iE_s^j \\ 0 & -G_iC_j & -G_iE_s^j \end{pmatrix},$$

$$\Delta_2(t) = \sum_{i=1}^{N} \left(\mu_i(\theta_f) - \mu_i(\hat{\theta}_f)\right) \begin{pmatrix} A_i x_f(t) + B_i u_f(t) \\ +E_a^i f(t) \end{pmatrix}.$$

Hypothesis 1: It is assumed that the following conditions are satisfied:

- The term $\Delta(t)$ have lower and upper bounds.
- The open-loop system is stable.

The stability analysis of system (20), guarantying the tracking performance under the L_2-gain, allows to introduce the Theorem 1.

Theorem 1: The tracking error $e(t)$, the state estimation error $x_f(t) - \hat{x}_f(t)$ and the fault estimation error $f(t) - \hat{f}(t)$ converge asymptotically to zero, if there exists symmetric positive definite matrices X_1 and P_2, $P_3 = I$, gain matrices K_x^j, $\bar{L}_i$ and G_i and a positive scalar $\bar{\gamma}$ solutions of the following optimization problem:

$$\min_{X1, P_2, K_x^j, \bar{L}_i, G_i} \bar{\gamma},$$

such that the following LMIs are verified:

$$\begin{pmatrix} \Omega_i & -B_iK_x^j & -E_a^i & -B_iK_x^j & X1 & X1 & 0 \\ * & \Xi_{ij} & \Psi_{ij} & 0 & 0 & 0 & P_2 \\ * & * & Z_{ij} & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & -\bar{\gamma}I & 0 \\ * & * & * & * & * & * & -\bar{\gamma}I \end{pmatrix} < 0, \tag{20}$$

$$\begin{aligned} &\Omega_i = A_iX1 + X1A_i^T \\ &\Xi_{ij} = P_2A_i + A_i^TP_2 - \bar{L}_iC_j - C_j^T\bar{L}_i^T \\ &\Psi_{ij} = P_2E_a^i - \bar{L}_iE_s^j - C_j^T\bar{G}_i^T \\ &Z_{ij} = -\bar{G}_iE_s^j - E_s^{j\,T}G_i^T \\ &i, j = 1, \ldots, N. \end{aligned}$$

The gains of the controller are K_x^j and the gains of the observer are given by $L_i = P_2^{-1}\bar{L}_i$ and G_i. The attenuation rate is obtained by $\gamma = \sqrt{\bar{\gamma}}$.

Proof: The proof is given in the appendix.

4. SIMULATIONS RESULTS

The diesel engine is assumed to run at 1800 rpm. Considered DEAP system parameters in the numerical simulations are given by Table 2. These numerical values are taken from identification results exposed in [1] and [5]. For this engine, $1.2 \leq x_1(t) \leq 2$ and $1.3 \leq x_2(t) \leq 4$. There are two premise variables, so there are four sub-models. The tuning parameters used in the MPC are given in Table 3. The performance of the proposed FFTPC strategy has been evaluated by numerical simulations considering two fault scenarios. The first fault scenario is given by Fig. 3, which represents the leakage mass flow as considered in [4]. In the presence of the air leakage, DEAP model described by (1) becomes:

$$\begin{cases} \dot{x}_1(t) = -k_1k_ex_1(t) + \frac{k_1k_c}{x_1^{\alpha}(t)-1}x_3(t) + k_1u_1(t) - \beta f(t), \\ \dot{x}_2(t) = k_2k_ex_1(t) - k_2u_1(t) - k_2u_2(t), \\ \dot{x}_3(t) = \frac{-x_3(t)}{\tau} + K_0(1 - x_2^{-\alpha}(t))u_2(t). \end{cases} \tag{21}$$

The intake manifold leakage signal is given by:

$$f(t) = \begin{cases} 0, \; t < 25 \; s \\ 0.05 \text{ kg/s}, \; t \geq 25 \; s \end{cases}$$

Table 2. Numerical values of DEAP system parameters.

Parameter	Value
k_1	31.2500
k_2	333.2000
k_c	0.0026
k_t	388.9474
k_e	0.0945
τ	0.300
η_m	0.95
α	0.2850
K_0	$1.2317\ e^{+03}$

Table 3. MPC tuning parameters.

Prediction horizon H_p	4
Control horizon H_u	3
Input constraints	$-8 \leqslant u_1(k) \leqslant 8$ $-8 \leqslant u_2(k) \leqslant 8$
Output constraints	$-0.05 \leqslant y_1(k) \leqslant 0.25$ $1.3 \leqslant y_2(k) \leqslant 4$
Input weights R	$\begin{bmatrix} 1 & 1 \end{bmatrix}$
Output weights Q	$\begin{bmatrix} 10 & 10 \end{bmatrix}$

with $\dot{f}(t) = 0$, where fault matrices $E_a^i = \begin{bmatrix} -\beta & 0 & 0 \end{bmatrix}^T$ for $i = 1,\ldots,4$. The second scenario given by figure 6 represents a sensor fault, where fault matrices $E_s^i = \begin{bmatrix} 0 & \alpha \end{bmatrix}$ for $i = 1,\ldots,4$. Solutions satisfying stability conditions under LMIs in Theorem 1 are found with the attenuation rate value: $\gamma = 0.861$. The designed controller and observer gains are:

$$K_1 = K_2 = 10^{-4}\begin{bmatrix} -0.0040 & 0.0000 & 0.1271 \\ -0.0013 & -0.0007 & -0.1271 \end{bmatrix};$$

$$K_3 = K_4 = 10^{-4}\begin{bmatrix} -0.0040 & 0.0000 & 0.4071 \\ -0.0013 & -0.0007 & -0.4071 \end{bmatrix};$$

$$L_1 = \begin{bmatrix} 0.3216 & 6.6288 \\ 0.1880 & 34.9487 \\ -0.2026 & 0.5442 \end{bmatrix};$$

$$L_2 = \begin{bmatrix} 0.3216 & 6.6288 \\ 0.1880 & 34.9485 \\ -0.2027 & 0.5442 \end{bmatrix};$$

$$L_3 = \begin{bmatrix} -0.0298 & 6.6439 \\ -0.0437 & 34.8189 \\ -0.0271 & -0.3532 \end{bmatrix};$$

$$L_4 = \begin{bmatrix} -0.0298 & 6.6440 \\ -0.0437 & 34.8186 \\ -0.0271 & -0.3533 \end{bmatrix}.$$

The system responses obtained by FTC strategy and by a classical predictive control from initial conditions $x_0 = (1.3229, 1.3596, 5.6095)^T$ are shown in Figs. 4 and 7. The obtained performances in terms of tracking, confirm the effectiveness of the introduced control strategy. Faults are estimated with a high accuracy and are given by Figs. 3 and 6. Fig. 5 shows the FTC signals in the presence of the leakage. Fig. 8 illustrates the evolution of the activation functions in the faulty cases. When the leakage occurs, the compressor mass flow W_c decreases. As a result, the pressure is reduced in the intake manifold and the pressure p_2 also goes down. Consequently, the solution is to close partially the EGR valve (W_{egr} decreases) and the VGT valve (W_{vgt} increases). From the simulation results, it is concluded that the performances of the FFTPC strategy are very satisfactory. Furthermore, the advanced idea to combine a PDC control law with the MPC represents an interesting approach. The proposed controller accommodates faults properly and ensures the stability of DEAP system. In this respect, the chosen Lyapunov polytopic function is set out as an optimisation convex problem in terms of LMIs to reduce the pessimism of the method, considering that this approach heed informations contained in the activations functions.

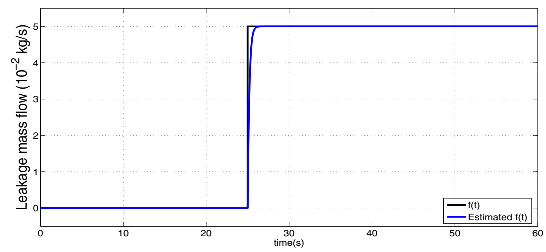


Fig. 3. Intake manifold leakage signal with its estimation vs. time.

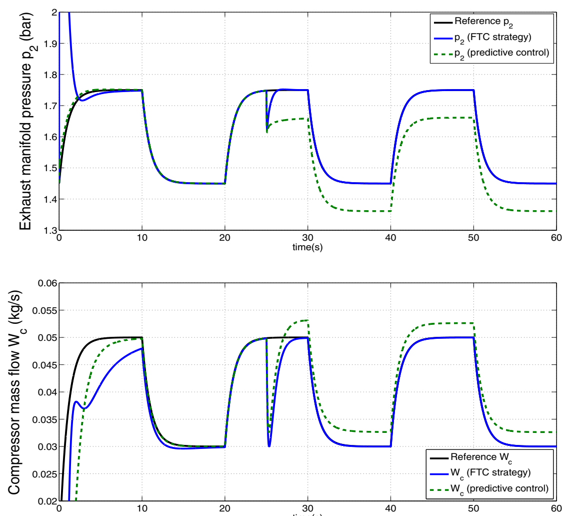


Fig. 4. Output signals vs. time in the presence of the leakage.

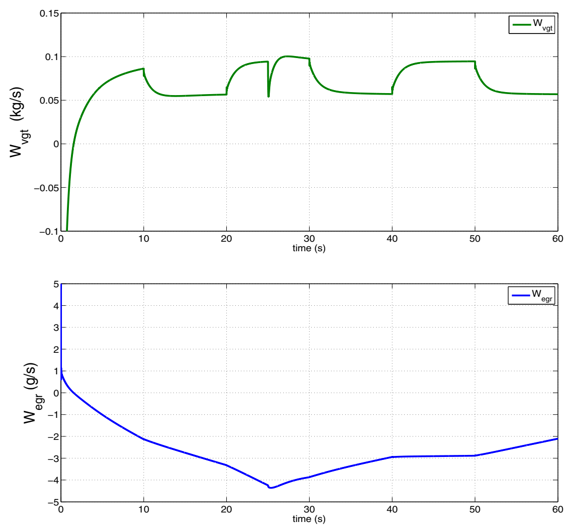


Fig. 5. Control signals vs. time in the faulty case.

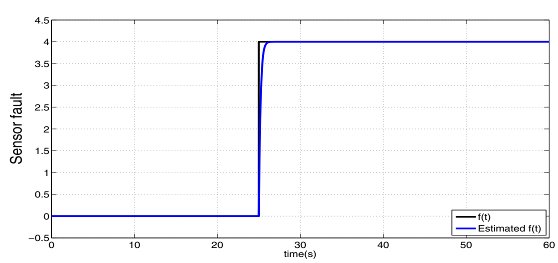


Fig. 6. Sensor Fault with its estimate signals vs. time.

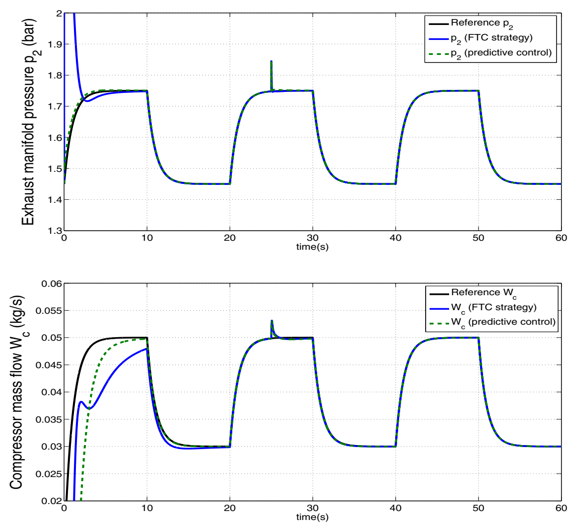


Fig. 7. Output signals vs. time in the sensor faulty case.

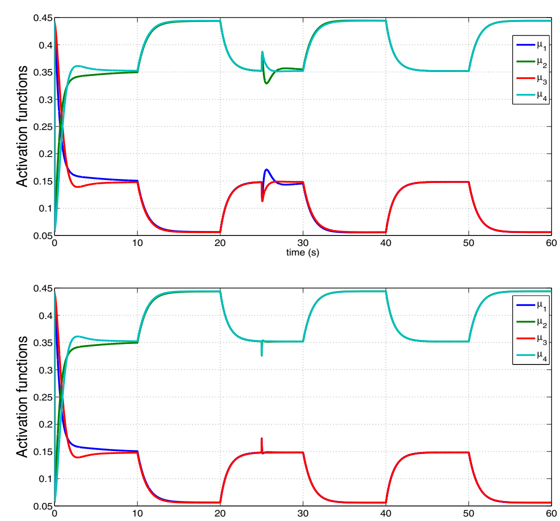


Fig. 8. Activation functions with UPV signals vs. time in the presence of the leakage (top) and in the sensor faulty case (bottom).

5. CONCLUSION

The development of a new FTC strategy called FFTPC with integral action for DEAP system is proposed in this paper. The aims are to tolerate faults and to allow the system to operate properly. The contributions and novelties with respect to other works are to combine a PDC control law with the MPC where the T-S fuzzy aspect is considered. Actually, the use of the sector nonlinearity approach has reduced the conservatism related to the number of LMIs to solve. On top of that, the chosen form of the function $V(x(t))$ and the T-S fuzzy structure have significantly decreased the pessimism of sufficient stability conditions derived from Lyapunov theories.

APPENDIX A

Lemma 1: Let us consider two matrices X and Y of appropriate dimensions. The following inequality is verified for each matrix Q:

$$X^T Y + XY^T \leq X^T Q^{-1} X + YQY^T.$$

Lemma 2 (Schur complement)**:** The following two inequalities are equivalent:

1) $\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} > 0$ where $Q = Q^T$ and $R = R^T$

2) $R > 0$, $Q - SR^{-1}S^T > 0$.

Proof: The proof of the Theorem 1 is established using the following Lyapunov's function:

$$V(\tilde{e}(t)) = \tilde{e}(t)^T P \tilde{e}(t),\ P = P^T > 0, \tag{A.1}$$

where the matrix P is defined as follows:

$$\begin{pmatrix} P_1 & 0 & 0 \\ 0 & P_2 & 0 \\ 0 & 0 & P_3 \end{pmatrix}.$$

The derivative of $V(\tilde{e}(t))$ is written as:

$$\dot{V}(\tilde{e}(t)) = \sum_{i=1}^{N}\sum_{j=1}^{N} \mu_i(\hat{\theta}_f)\mu_j(\theta_f)\left(\tilde{e}(t)^T\, \Upsilon_{ij}\, \tilde{e}(t)\right) \quad (A.2)$$

with

$$\Upsilon_{ij} = \Lambda\left(\left(\begin{array}{ccc} P_1A_i - P_1B_iK_x^j & -P_1B_iK_x^j & -P_1E_a^i \\ 0 & P_2A_i - P_2L_iC_j & P_2E_a^i - P_2L_iE_s^j \\ 0 & -P_3G_iC_j & -P_3G_iE_s^j \end{array}\right)\right),$$

where $\Lambda(X)$ denote the Hermitian of the matrix X:

$$\Lambda(X) = X^T + X.$$

The derivative of the Lyapunov function is negative if the following inequalities are satisfied

$$\Upsilon_{ij} < 0,\ i,j = 1,\ldots,N \quad (A.3)$$

using the lemma of congruence as follows:

$$\Upsilon_{ij} < 0 \Leftrightarrow \left(\begin{array}{ccc} P_1^{-1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{array}\right)\Upsilon_{ij}\left(\begin{array}{ccc} P_1^{-1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{array}\right). \quad (A.4)$$

The following inequalities are obtained:

$$\left(\begin{array}{ccc} \xi_{ij}^1 & -B_iK_x^j & -E_a^i \\ * & \xi_{ij}^2 & P_2E_a^i - P_2L_iE_s^j - C_j^TG_i^TP_3 \\ * & * & -P_3G_iE_s^j - E_s^{j^T}G_i^TP_3 \end{array}\right) < 0, \quad (A.5)$$

where

$$\xi_{ij}^1 = A_iX1 + X1A_i^T - B_iK_x^jX1 - X1K_x^{j^T}B_i^T.$$
$$\xi_{ij}^2 = P_2A_i + A_i^TP_2 - \bar{L}_iC_j - C_j^T\bar{L}_i^T$$

with $X1 = P_1^{-1}$. The inequalities (A.5) can be written as:

$$\left(\begin{array}{ccc} A_iX1 + X1A_i^T & -B_iK_x^j & -E_a^i \\ * & \xi_{ij}^2 & P_2E_a^i - P_2L_iE_s^j - C_j^TG_i^TP_3 \\ * & * & -P_3G_iE_s^j - E_s^{j^T}G_i^TP_3 \end{array}\right)$$
$$+\left(\begin{array}{c} -B_iK_x^j \\ 0 \\ 0 \end{array}\right)\left(\begin{array}{c} X1 \\ 0 \\ 0 \end{array}\right)^T + \left(\begin{array}{c} X1 \\ 0 \\ 0 \end{array}\right)\left(\begin{array}{c} -B_iK_x^j \\ 0 \\ 0 \end{array}\right)^T < 0. \quad (A.6)$$

Using Lemma 1, the inequalities (A.6) becomes:

$$\left(\begin{array}{ccc} A_iX1 + X1A_i^T & -B_iK_x^j & -E_a^i \\ * & \xi_{ij}^2 & P_2E_a^i - P_2L_iE_s^j - C_j^TG_i^TP_3 \\ * & * & -P_3G_iE_s^j - E_s^{j^T}G_i^TP_3 \end{array}\right) +$$
$$\left(\begin{array}{c} -B_iK_x^j \\ 0 \\ 0 \end{array}\right)\Theta^{-1}\left(\begin{array}{c} -B_iK_x^j \\ 0 \\ 0 \end{array}\right)^T + \left(\begin{array}{c} X1 \\ 0 \\ 0 \end{array}\right)\Theta\left(\begin{array}{c} X1 \\ 0 \\ 0 \end{array}\right)^T < 0, \quad (A.7)$$

where Θ is a symmetric definite positive matrix. Using Lemma 2, we obtain the LMIs of Theorem 1, with $\bar{L}_i = P_2L_i$, $\bar{G}_i = P_3G_i$ and $\Theta = I$.

The objective is to minimize the L_2-gain of the perturbation transfer from $\Delta(t)$ to the errors $\tilde{e}(t)$, this is formulated by:

$$\frac{\|\tilde{e}(t)\|_2}{\|\Delta(t)\|_2} < \gamma, \quad \|\Delta(t)\|_2 \neq 0. \quad (A.8)$$

Then, the problem can be formulated as follows:

$$\dot{V}(\tilde{e}(t)) + \tilde{e}(t)^T\tilde{e}(t) - \gamma\Delta(t)^T\Delta(t) < 0. \quad (A.9)$$

□

REFERENCES

[1] M. Jankovic and I. Kolmanovsky, "Constructive Lyapunov control design for turbocharger diesel engine," *IEEE Trans. on Control Systems Technology,* vol. 8, no. 3, pp. 288-299, 2000. [click]

[2] P. Langthaler and L. del Re, "Robust model predictive control of a diesel engine airpath," *Proc. of the 17th IFAC World congress*, pp. 9485-9490, Seoul, 2008. [click]

[3] K. Layerle, N. Langlois, H. Chafouk, "Design of reconfigurable predictive control applied to the air path of a diesel engine," *United Kingdom Automatic Control Conference,* Manchester, 2008.

[4] I. Djemili, A. Aitouche, and V. Cocquempot, "Fault tolerant control of internal combustion Engine subject to intake Manifold leakage," *Proc. of IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes,* Mexico City, pp. 600-605, 2012. [click]

[5] I. Abidi, J. Bosche, and A. El Hajjaji, "Control of a turbocharged diesel engine with EGR system using Takagi-Sugeno approach," *Proc. of Mediterranean Conference on Control and Automation,* Barcelona, 2012. [click]

[6] H. Zhang, J. Wang, and Y. Wang, "Nonlinear observer design of diesel engine selective catalytic reduction systems with NO_x sensor measurements," *IEEE/ASME Transactions on Mechatronics,* vol. PP, no. 99, pp. 01-10, 2014. [click]

[7] K. Tanaka, T. Hori, and H.O. Wang, "A fuzzy Lyapunov approach to fuzzy control and design," *Proc. of Amerian Control Conference,* Arlington, pp. 4790-4795, 2001. [click]

[8] K. Tanaka, T. Hori, and H.O. Wang, "A multiple Lyapunov function approach to stabilization of fuzzy control systems," *IEEE Trans. on Fuzzy Systems,* vol. 4, no. 11, pp. 582-589, 2003. [click]

[9] H. Zhang and J. Wang, "State estimation of discrete-time Takagi-Sugeno fuzzy systems in a network environment," *IEEE Transactions on Cybernetics,* vol. PP, no. 99, pp. 1, 2014. [click]

[10] L. Ben Hamouda, O. Bennouna, M. Ayadi, and N. Langlois, "Quadratic stability and LMIs for tolerance to faults: fuzzy model predictive control," *Proc. of International Conference on System Theory, Control and Computing,* Sinaia, pp. 387-392, October 2014. [click]

[11] L. Ben Hamouda, O. Bennouna, M. Ayadi, and N. Langlois, "Fuzzy fault tolerant control based on unmeasurable premise variables: quadratic stability and LMIs," *XVI Congreso Latinoamericano de Control Automático,* Cancún, pp. 319-324, October 2014.

[12] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its application to modeling and control," *IEEE Trans. on Systems, Man and Cybernetics,* vol. 15, pp. 116-132, 1985. [click]

[13] L. Ben Hamouda, M. Ayadi, and N. Langlois, "Fuzzy fault tolerant predictive control for a class of nonlinear uncertain systems," *Systems Science and Control Engineering tSSC*, 2015. [click]

[14] L. Ben Hamouda, M. Ayadi, and N. Langlois, "Takagi-Sugeno fuzzy observer and predictive control for a diesel engine air path," *International Journal of Automation and Control IJAAC*, 2015. [click]

[15] J. M. Maciejowski, *Predictive Control with Constraints*, Prentice Hall, Englewood Cliffs, NJ, USA, 2002.

[16] M. M. Kale and A. J. Chipperfield, "Stabilized MPC formulations for robust reconfigurable flight control," *Control Engineering Practice J.,* vol. 13, no. 6, pp. 771-788, 2005. [click]

[17] R. J. M. Afonso and R. K. H. Galvao, "Predictive control of a helicopter model with tolerance to actuator faults," *Proc. of Conference on Control and Fault-Tolerant Systems,* Nice, pp. 744-751, 2010. [click]

[18] Y. Rongni, L. Guo-Ping, S. Peng, C. Thomas and M.V. Basin, "Predictive output feedback control for networked control systems," *IEEE Transactions on Industrial Electronics,* vol. 61, no. 1, pp. 512-520, 2014. [click]

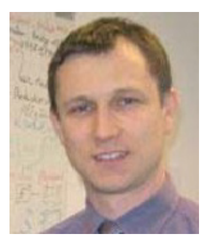
[19] M. Bouattour, M. Chadli, M. Chaabane and A. El Hajjaji, "Design of robust fault detection observer for Takagi-Sugeno models using the descriptor approach," *International Journal of Control, Automation and Systems,*vol. 9, no. 5, pp. 973-979, 2011. [click]

[20] A. Salem, Z. Kardous, N. B. Braiek, and J. Ragot, "On the state observer based stabilization of Takagi-Sugeno systems with immeasurable premise variables," *International Journal of Control, Automation and Systems,*vol. 10, no. 6, pp. 1153-1163, 2012. [click]

[21] M. Sami and R. J. Patton, "Active fault tolerant control for nonlinear systems with simultaneous actuator and sensor faults," *International Journal of Control, Automation and Systems,*vol. 11, no. 6, pp. 1149-1161, 2013. [click]

Lamia Ben Hamouda received a Ph.D. degree in Electrical Engineering and Physics in 2015 from the National School of Engineers of Tunis (ENIT), LA.R.A, Tunisia and from University of Rouen, IRSEEM, FRANCE. She received her Master degree in Automatic and Signal Processing in 2011 from ENIT. She is currently Research professor in Cnam of Paris, France. Her research is related to predictive control, Fault-tolerant control, stability and stabilization of non linear systems described by T-S fuzzy model, diagnostic and prognostic.

Mounir Ayadi graduated from Ecole Nationale d'Ingénieurs de Tunis in 1998 and received his Ph.D. degree in Automatic Control from the Institut National Polytechnique de Toulouse in 2002. He was a post-doctoral fellow at the Ecole Supérieure d'Ingéni-eurs en Génie Electrique de Rouen in 2003. He is currently Maître de Conférences at the Ecole Nationale d'Ingénieurs de Tunis and the head of Electrical Engineering Department in ENIT. His research interests are in the area of control system theory, predictive and adaptive control and flat systems.



Nicolas Langlois received the Ph.D. degree in Automatic Control and signal processing from the University of Rouen, Rouen, France in 2001. In 2000, he joined the graduate school of electrical engineering ESIGELEC, Rouen, France. At present, he is the head in charge of skills acquisition through research of ESIGELEC where he currently teaches control systems and digital signal processing. He is also the head of the "Automatic Control and Systems" research team at the research institute IRSEEM since 2008. His research interests include fault tolerant control. He is the co-author of over 90 publications in the field of control systems theory during these 10 last years.