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Mean-square Exponential Stability of Impulsive Stochastic Time-delay Systems with Delayed Impulse Effects

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Abstract: This paper is concerned with the mean-square exponential stability problem for a class of impulsive stochastic systems with delayed impulses. The delays exhibit in both continuous subsystem and discrete subsystem. By constructing piecewise time-varying Lyapunov functions and Razumikhin technique, sufficient conditions are derived which guarantee the mean-square exponential stability for impulsive stochastic delay system. It is shown that the obtained stability conditions depend both on the lower bound and the upper bound of impulsive intervals, and the stability of system is robust with regard to sufficiently small impulse input delays. Finally, two examples are proposed to verify the efficiency of the proposed results.

Keywords: Delayed impulse, impulsive systems, LMIs, Ruzumikhin-type method, stochastic time-delay systems.

1. INTRODUCTION

Being typically composed of reference input, plant output, control input, networked control systems (NCSs) whose components information are exchanged via communication networks have attracted a great of attention in recent studies [1–5]. It is known that in the transmission of the impulse information, input delays are usually encountered named as delayed impulses. For example, in the application of networked control systems (see section 2: model for networked control systems, for some details). In addition to modeling impulsive control, delayed impulses are also used to model abrupt changes in the state variables. These changes may be related to such phenomena as shocks, harvesting or other faults. In many cases, time delays are assumed to have a bound and can be dealt with by impulses [6–14]. In Peng, and Wang [11], the mean-square exponential stabilization property for a class of stochastic systems with time delay was investigated via impulsive control, in which the time varying delays are assumed having a bound in order to analyze the impact of delays.

Stochastic functional differential systems can be applied to modeling many real world phenomenons, such as science and engineering (see Mao, [15]). Consequently, stability analysis issue has received a lot of attention (see, e.g., [16, 17]). Among the concepts of stability in existing references about stochastic systems, mean square exponentially stability has been well studied. It is a consensus that noise can stabilize an unstable system even make a stable system more stable. In the years pass by, a large number of literatures on the stabilization problems have been made [8, 16, 18, 19].

As we all know that impulse effects exist in practical systems, due widely to state changing abruptly at certain moments of time, especially when impulsive control permit the discontinuous inputs and stochastic interferences. And in the transmission of the impulse information, input delays are often encountered.

Recently, a number of papers investigated the stability and control problems for stochastic impulsive systems, see [20–22], for more details. In Wu, and Sun [21], some criteria for $p-$moment stability of stochastic differential equations with impulsive jump and Markovian switching was obtained by using Lyapunov function method. Based on Lyapunov-Razumikhin technique, the mean square exponential stability of uncertain linear impulsive stochastic systems with Markovian switching was established in [17].

Moreover, Rakkiyappan, and Balasubramaniam studied the mean square asymptotic stability for a class of Markovian jumping impulsive stochastic Cohen-Grossberg neural networks, and some delay-interval dependent stability criteria are obtained by the Lyapunov-Krasovskii functional technique. Recently, in [23], linear impulsive stochastic delay system has been studied and the mean

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square exponential stability conditions are construed based on a set of LMIs. Unfortunately, many parameters are included in LMIs in [22, 23] which making the cost expensive. Therefore, it is necessary to conduct our work to reduce the complexity and the cost.

However, it should be pointed out that the linear impulsive stochastic systems with delayed impulses have not been fully investigated. The stability analysis in these literatures using time-invariant Lyapunov functions, are likely to neglect some helpful information regarding to state jumps that happening at impulse instant. For example, the results in [11] may be conservative when both the upper bound and the lower bound of impulse intervals are known for the stability conditions. The aforementioned discussions motivate an investigation into the current research. The main contributions are highlighted as follows: 1) A novel time-varying Lyapunov function is constructed and some new stability criteria are obtained to guarantee the mean-square exponential stability for impulsive stochastic delay systems; 2) The results depend both on the lower bound and the upper bound of impulsive intervals. Therefore, the results that we derived relax the constraints of impulsive intervals and reduce the conservatism of existing results; 3) In line with the theorems, an impulsive controller with delayed impulses is designed via solving a set of LMIs.

2. MODEL FOR NETWORKED CONTROL SYSTEM AND PRELIMINARIES

In this section, an example of NCS is given to illustrate the delayed impulses may exist in actual background.

The structure of the NCS is shown in Fig. 1. It is consisted of a continuous time plant and a discrete time controller, which only act at sampling instant and change state. The continuous-time plant model of the NCSs is supposed to take the form :

$$\dot{x} = Ax(t) + Bu^*(t) + C\omega(t), u^*(t) = u_k, \ \ t > 0, \tag{1}$$

where $x(t), u^*(t), \omega(t), u_k = u(t_k)$ are the state vector, control input, exogenous signal and delayed discrete time input. The matrices A, B, C are constant matrices of appropriate dimensions. As shown in Fig. 1, the random delays exist in the sensor-to-controller (S-C) and controller-to-actuator (C-A) sides. Here τ_k represents the S-C delay, and r_k stands for the C-A delay.

In the NCSs, the delay information is important for the controller design. Through utilizing the embedded processor and time-stamping technique [24], the information of $r_{k-1-\tau_k}$ at time instant s_k is known at the controller node if the time delay τ_k exits. By considering the effect of random delays, the mode-dependent state-feedback controller is designed as

$$u_k = Ky_k,$$

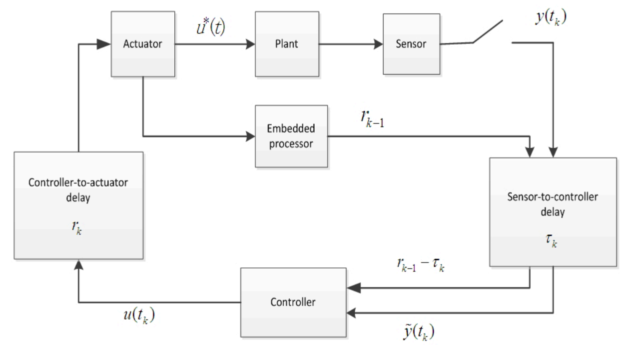

Fig. 1. Schematic overview of the networked control system.

and we adopt the more general impulsive control law

$$\Delta x(t_k) = B_k u_k.$$

to stabilize the plant. Then the impulses can be modified as

$$\Delta x(t_k) = B_k x((t-d_k)^-), t = t_k. \tag{2}$$

Thus, the impulses are obtained as the following form

$$x(t_k) = B_{1k}x(t^-) + B_{2k}x((t-d_k)^-), t = t_k. \tag{3}$$

Throughout this paper, we assume that matrices have compatible dimensions, if not particular statement. Let $\mathbb{R} = (-\infty, +\infty)$, $\mathbb{R}^+ = [0, +\infty)$, $N = \{1, 2, \ldots\}$. The nation $M > 0 (< 0)$ is used to denote a symmetric positive (negative) definite matrix. $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ denote the maximum and minimum eigenvalues of the corresponding matrix, respectively. $|\cdot|$ represents the Euclidean norm for vectors or the spectral norm for matrices. For $\tau > 0$, let $PC([-\tau, 0], \mathbb{R}^n)$ denote the set of piecewise right continuous function $\phi : [-\tau, 0] \to \mathbb{R}^n$ with the norm defined by $\|\phi\| = \sup_{-\tau \le \theta \le 0} \|\phi(\theta)\|$. If $x \in PC([t_0 - \tau, +\infty], \mathbb{R}^n)$, then for each $t \ge t_0$, we define $x_t \in PC([-\tau, 0], \mathbb{R}^n)$ by $x_t(s) = x(t+s)$ for $-\tau \le s \le 0$. Let $B(t)$ is standard one dimensional Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, P)$ with a natural filtration $\{\mathcal{F}_t\}_{t \ge 0}$ generated by $\{\omega(s; 0 \le s \le t)\}$. $E\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure P.

In this paper, we can obtain the following linear impulsive stochastic delay closed-loop system

$$\begin{aligned} dx(t) &= [A_0 x(t) + A_1 x(t-\tau)]dt + [D_0 x(t) \\ &\quad + D_1 x(t-\tau)]dB(t), \ t \ge t_0, t \ne t_k, \\ x(t_k) &= C_0 x(t_k^-) - C_1 x((t_k - d_k)^-), \ k \in N, \end{aligned} \tag{4}$$

where $x(t) \in \mathbb{R}^n$ is the system state. $A_0, A_1, D_0, D_1, C_0, C_1$, are the corresponding dimension matrices. $\{t_k\}$ is a strictly increasing sequence of impulse times in $[0, \infty)$, denote $\lim_{h \to 0^+} x(t_k - h) = x(t_k^-), \lim_{h \to 0^+} x(t_k + h) = x(t_k^+)$.

d_k denotes the impulse input delay at impulse time $t_k, k \in N$. We assume that $t_0 = 0, \lim_{k\to\infty} t_k = \infty$ and $d_k \in [0,d]$. Denote $\mathbf{S}(\delta_1,\delta_2) = \{\{t_k\}; \delta_1 \le t_k - t_{k-1} \le \delta_2, k \in N\}$ for some positive scalars δ_1 and δ_2 satisfying $\delta_1 \le \delta_2$. The aim of control is to find out an appropriate gain C_0 and C_1 such that for any $\{t_k\} \in \mathbf{S}(\delta_1,\delta_2)$ and $d_k \in [0,d]$ system (4) is mean square exponentially stable. For a prescribed scalar $\tau > 0$, suppose that there exist any positive scalars $c_i, i = 1,2,...,n$ such that $0 \le \frac{x_i(t-\tau)}{x_i(t)} \le c_i$, thus for any positive scalars $k_i, i = 1,2,...,n$, the following holds

$$\begin{aligned} 0 &\le \textstyle\sum_{i=1}^n k_i x(t-\tau)[c_i x(t) + x_i(t-\tau)] \\ &= x^T(t)\Lambda S x(t-\tau) - x^T(t-\tau(t))\Lambda x(t-\tau), \end{aligned} \tag{5}$$

where $\Lambda = diag\{k_1,...,k_n\}$, $S = diag\{c_1,...,c_n\}$. Moreover, $|D_1^2||P_i| < 2|\Lambda_{ij}|, i,j = 1,2$, where P_i satisfying the following conditions in Theorem 1. Set

$$\kappa_1 = \left|A_0 + A_1 e^{\frac{\varepsilon_0}{2}d} + D_0 + D_1 e^{\frac{\varepsilon_0}{2}d}\right|, \kappa_2 = |C_0 - C_1|.$$

Definition 1: For given class $\mathcal{S}$ of admissible impulse time sequences, the system (1) is said to be mean-square exponentially stable over $\mathcal{S}$ if there exist a pair of positive scalars ρ and ν such that

$$E \parallel x(t,t_0,x_0) \parallel^2 \le \rho \exp(-\nu(t-t_0)) E\{\parallel x_0 \parallel^2\},\ t \ge t_0,$$

for all impulse time sequence $\{t_k\} \in \mathcal{S}$.

For given impulsive time sequence $\{t_k \in \mathbf{S}(\delta_1,\delta_2)\}$, we introduce the following two piecewise linear functions ρ, $\rho_1 \in [t_0,\infty) \to \mathbb{R}^+$ for $t \in [t_{m-1},t_m), k \in N$

$$\rho(t) = \frac{t_m - t}{t_m - t_{m-1}},\quad \rho_1(t) = \frac{1}{t_m - t_{m-1}}. \tag{6}$$

It is easy to show that there exists $\rho_2(t) \in [0,1]$, such that

$$\rho_1(t) = \frac{1-\rho_2(t)}{\delta_1} + \frac{\rho_2(t)}{\delta_2}. \tag{7}$$

Notice that $\rho(t) \in [0,1)$, for $t \ge t_0, \rho(t_m^-) = 0$ and $\rho(t_m) = \rho(t_m^+) = 1, k \in N$.

3. MAIN RESULTS

In this section, we are aiming at establishing two sufficient criteria for linear impulsive stochastic delay system (4) with known gain C_0 and C_1, by utilizing LMIs and Razumikhin method. For this purpose, we firstly give two lemmas, which will be applied to prove our theorems.

Lemma 1 [8]: For any vectors $x,y \in \mathbb{R}^n$, matrices $A,P,D,E,N,F \in \mathbb{R}^{n\times n}$, with $\|F\| \le 1$, and scalars $\varepsilon > 0$, the following holds

$$DFN + N^T F^T D^T \le \varepsilon N N^T + \varepsilon^{-1} N^T N. \tag{8}$$

Lemma 2 [11]: Let $C^{2,1}(\mathbb{R}^+ \times \mathbb{R}^n, \mathbb{R}^+)$ denote a family of all nonnegative functions $V(t,x)$ on $\mathbb{R}^+ \times \mathbb{R}^n$ which are continuously twice differentiable in x and once differentiable in t. If $V \in C^{2,1}(\mathbb{R}^+ \times \mathbb{R}^n, \mathbb{R}^+)$, then for any stopping times $0 \le t_1 \le t_2$,

$$EV(t_2,x(t_2)) = EV(t_1,x(t_1)) + E\int_{t_1}^{t_2} \mathcal{L}V(t,x(t))dt, \tag{9}$$

as long as the integrations involved exist and are finite.

Theorem 1: Assume that the impulsive time sequence $\{t_k\} \in \mathbf{S}(\delta_1,\delta_2)$ and impulse input delays $d_k \in (0,d)$, where $\delta_1 \le \delta_2$, $l\delta_1 \le d \le (l+1)\delta_1$ for some nonnegative integer l. System (4) can be mean square exponentially stable if for positive scalars $\varepsilon_2, \varepsilon_3$ $\mu \in (0,1)$, there exist $n\times n$ matrices $P_1 > 0, P_2 > 0$, and positive definite diagonal matrices $\Lambda_{ij} \in R^n \times R^n, i,j = 1,2$, such that the following matrix inequalities hold:

$$\begin{gathered} \begin{bmatrix} \Omega_{ij} & P_iA_1 + D_0^T P_i D_1 + \Lambda_{ij} S \\ * & D_1^T P_i D_1 - 2\Lambda_{ij} \end{bmatrix} < 0,\ i,j = 1,2 \\ \begin{bmatrix} -(\mu - \varepsilon\frac{\kappa^2}{\lambda_0})P_1 & (C_0 - C_1)^T P_2 & 0 \\ * & -P_2 & P_2C_1 \\ 0 & * & -\varepsilon I \end{bmatrix} < 0, \end{gathered} \tag{10}$$

where $\Omega_{ij} = A_0^T P_i + P_i A_0 + D_0^T P_i D_0 + \frac{\ln\mu}{\delta_2} P_i + \frac{1}{\delta_j}(P_1 - P_2)$, $\kappa = d\kappa_1 + l\kappa_2$, then system (4) is mean square exponentially stable over $\mathbf{S}(\delta_1,\delta_2)$.

Proof: For $\phi \in \mathcal{C}^b_{\mathcal{F}_0}([-\tau,0],R^n)$, we denote the solution $x(t,t_0,\phi)$ of (4) by $x(t)$. From (10), there exist small enough scalars ε_0, and $\varepsilon_1 \in (0, 1-\mu)$, such that

$$\begin{aligned} \Xi_{ij} &= \begin{bmatrix} \tilde{\Omega}_{ij} & P_iA_1 + D_0^T P_i D_1 + \Lambda_{ij} S \\ * & D_1^T P_i D_1 - 2\Lambda_{ij} \end{bmatrix} \\ &< 0,\quad i,j = 1,2 \\ \Xi_1 &= \begin{bmatrix} -(\mu - \varepsilon\frac{\tilde{\kappa}^2}{\lambda_0})P_1 & (C_0 - C_1)^T P_2 & 0 \\ * & -P_2 & P_2C_1 \\ 0 & * & -\varepsilon I \end{bmatrix} \\ &< 0, \end{aligned} \tag{11}$$

where $\tilde{\Omega}_{ij} = A_0^T P_i + P_i A_0 + D_0^T P_i D_0 + (\varepsilon_0 - \mu_1)P_i + \frac{1}{\delta_j}(P_1 - P_2)$, $\mu_1 = -\frac{\ln(\mu+\varepsilon_1)}{\delta_2}$, $\tilde{\kappa} = (d\kappa_1 + l\kappa_2)e^{\varepsilon_0 d}$. For any given scalar $\varepsilon > 0$, choose $\delta > 0$, such that $\lambda_1\delta < \mu\lambda_0\varepsilon$, $\lambda_1 = \max\{\lambda_{\max}(P_1), \lambda_{\max}(P_2)\}, \lambda_0 = \min\{\lambda_{\min}(P_1), \lambda_{\min}(P_2)\}$. We assume that $\phi \in \mathcal{C}^b_{\mathcal{F}_0}([-\tau,0],R^n)$ satisfies $E\|\phi\|^2 < \delta$. Choose a time dependent Lyapunov function candidate for system (4) as

$$V(t) = x^T(t)P(t)x(t),$$

where $P(t) = (1-\rho(t))P_1 + \rho(t)P_2$. We will prove that

$$V(t) \le \lambda_0 \varepsilon e^{-\varepsilon_0(t-t_0-d)},\ t \in [t_0 - \tau, +\infty). \tag{12}$$

We assume that the impulsive time sequence on $t \in [t_0 - d, +\infty)$ is $\{t_k\}$. For any given $t \in [t_k, t_{k+1})$, set $W(t) = e^{-\varepsilon_0(t-t_0-d)}V(t)$. In the following, we will prove that

$$EW(t) < \lambda_0\varepsilon,\ t \in [t_0 - \tau, +\infty). \tag{13}$$

Firstly, we will prove that

$$EW(t) < \lambda_0\varepsilon,\ t \in [t_0-\tau, t_1). \tag{14}$$

Noting that

$$EW(t+\theta) < \lambda_1 E\|\phi\|^2 < \lambda_1\delta < \mu\lambda_0\varepsilon < \lambda_0\varepsilon,\quad \theta \in [-\tau, 0). \tag{15}$$

Next, one only needs to prove that

$$EW(t) < \lambda_0\varepsilon, \tag{16}$$

for $t \in [t_0, t_1)$. On the contrary, there exist some $t \in (t_0, t_1)$ such that $EW(t) \geq \lambda_0\varepsilon$. Set $\tilde{t} = \inf\{t \in (t_0, t_1) : EW(t) \geq \lambda_0\varepsilon\}$. After that we have $\tilde{t} \in (t_0, t_1)$ and $EW(\tilde{t}) \geq \lambda_0\varepsilon$. Set $\tilde{\tilde{t}} = \sup\{t \in (t_0+\tau, \tilde{t}) : EW(t) \leq \mu\lambda_0\varepsilon\}$. Then $\tilde{\tilde{t}} \in (t_0+\tau, \tilde{t})$ and $EW(\tilde{\tilde{t}}) = \mu\lambda_0\varepsilon$. So for $t \in [\tilde{\tilde{t}}, \tilde{t})$, taking the derivative of the Lyapunov function along the trajectories of (4) and using the inequalities (10), we have

$$\begin{aligned}
&D^+W(t) \\
&\leq e^{\varepsilon_0(t-t_0-\tau)}\varepsilon_0 EV(t) + D^+EW(t) - \mu_1 EW(t) \\
&\quad + \mu_1 EW(t) \\
&\leq e^{\varepsilon_0(t-t_0-\tau)}\{(\varepsilon_0-\mu_1)x^T P(t)x \\
&\quad + 2x^T P(t)(A_0x + A_1\eta) \quad + trace[D_0x + D_1\eta]^T P(t) \\
&\quad [D_0x + D_1\eta] + \rho_1(t)x^T(P_1 - P_2)x\} \quad + \mu_1 EW(t) \\
&\quad + \sum_{i=1}^{2}\sum_{j=1}^{2} 2\rho_{ij}(t)[x^T\Lambda_{ij}S\eta - \eta^T\Lambda_{ij}\eta] \\
&\leq e^{\varepsilon_0(t-t_0-\tau)}\sum_{i=1}^{2}\sum_{j=1}^{2}\rho_{ij}(t)\xi^T\Xi_{ij}\xi + \mu_1 EW(t)
\end{aligned} \tag{17}$$

where $\xi^T = (x^T, \eta^T), x = x(t), \eta(t) = x(t-\tau(t))$. It follows from (11) and (17)

$$D^+W(t) < \mu_1 EW(t),\ t \in [\tilde{\tilde{t}}, \tilde{t}]. \tag{18}$$

This leads to

$$EW(\tilde{t}) < EW(\tilde{\tilde{t}})e^{\mu_1\delta_2} \leq \mu\lambda_0\varepsilon e^{\mu_1\delta_2} < \lambda_0\varepsilon. \tag{19}$$

This is a contradiction, so (16) holds.

Now we assume that for some $m \in N$,

$$EW(t) < \lambda_0\varepsilon,\ t \in [t_0-\tau, t_m]. \tag{20}$$

We shall prove that

$$EW(t) < \lambda_0\varepsilon,\ t \in [t_m, t_{m+1}). \tag{21}$$

First, we will give an estimate of $W(t_m)$. From (20), we have

$$|x(s)|^2 \leq \varepsilon e^{-\varepsilon_0(t_m-t_0-\tau)},\ s \in [t_0-\tau, t_m). \tag{22}$$

We notice that $\{t_k\} \in \mathbf{S}(\delta_1, \delta_2)$ and $l\delta_1 \leq d \leq (l+1)d$. So there are at most l impulse times on $[t_m-d_m, t_m)$. We assume that impulsive instants in $[t_m-d_m, t_m)$ are $t_{m_j}, j = 1, 2, ..., l_0,\ l_0 \leq l$. By (22), we get

$$\begin{aligned}
&\left|x(t_m^-) - x((t_m-d_m)^-)\right| \\
&\leq \int_{t_m-d_m}^{t_m}|\dot{x}(s)|\,ds + \sum_{j=1}^{l_0}|\triangle x(t(m_j))| \\
&\leq \int_{t_m-d_m}^{t_m}\kappa_1|x(s)|\,ds + \sum_{j=1}^{l_0}\kappa_2\left|x((t_{m_j}-d_{m_j})^-)\right| \\
&\leq (d\kappa_1 e^{\varepsilon_0 d} + l_0\kappa_2 e^{\varepsilon_0 d})\varepsilon e^{-\frac{\varepsilon_0}{2}(t_m-t_0-d)} \\
&\leq \tilde{\kappa}\varepsilon^{\frac{1}{2}}e^{-\frac{\varepsilon_0}{2}(t_m-t_0-d)}.
\end{aligned} \tag{23}$$

Denotes $\triangle\tilde{x}_m = x((t_m-d_m)^-) - x(t_m^-)$. Pre- and post-multiplying (11) by $\mathrm{diag}\{e^T(t_m^-), I, I\}$ and its transpose, respectively, via using the fact that $V(t_m^-) = x^T(t_m^-)P_1x(t_m^-)$, we have

$$\begin{bmatrix} -\tilde{V}(t_m) & x^T(t_m^-)(C_0-C_1)^T P_2 & 0 \\ * & -P_2 & P_2C_1 \\ 0 & * & -\varepsilon I \end{bmatrix} < 0, \tag{24}$$

where $\tilde{V}(t_m) = (\mu - \varepsilon\frac{\tilde{\kappa}^2}{\lambda_0})V(t_m^-)$. It follows from (20) that

$$\begin{bmatrix} (\mu\lambda_0 - \varepsilon\tilde{\kappa}^2)\tilde{\varepsilon} & x^T(t_m^-)(C_0-C_1)^T P_2 & 0 \\ * & -P_2 & P_2C_1 \\ 0 & * & -\varepsilon I \end{bmatrix} < 0, \tag{25}$$

where $\tilde{\varepsilon} = -\varepsilon e^{-\varepsilon_0(t_m-t_0-d))}$. Then by (23) and Schur complement, we further obtain

$$\begin{aligned}
&\begin{bmatrix} -\mu\lambda_0\varepsilon e^{-\varepsilon_0(t_m-t_0-d))} & x^T(t_m^-)(C_0-C_1)^T P_2 \\ * & -P_2 \end{bmatrix} \\
&\quad + \varepsilon\begin{bmatrix} \triangle\tilde{x}^T(t_m^-) \\ 0 \end{bmatrix}\begin{bmatrix} \triangle\tilde{x}(t_m^-) & 0 \end{bmatrix} \\
&\quad + \varepsilon^{-1}\begin{bmatrix} 0 \\ P_2C_1 \end{bmatrix}\begin{bmatrix} 0 & C_1^T P_2 \end{bmatrix} \\
&< 0.
\end{aligned} \tag{26}$$

By Lemma 1, for any scalar $\varepsilon > 0$, combining (26) and using $x(t_m) = C_0x(t_m^-) - C_1x((t_m-d_m)^-)$ yields

$$\begin{bmatrix} -\mu\lambda_0\varepsilon e^{-\varepsilon_0(t_m-t_0-d))} & x^T(t_m)P_2 \\ * & -P_2 \end{bmatrix} < 0. \tag{27}$$

Then by Schur complement, we have

$$x^T(t_m)P_2x(t_m) < \mu\lambda_0\varepsilon e^{-\varepsilon_0(t_m-t_0-d))}. \tag{28}$$

Observing that $P(t_m) = P_2$, it follows that $V(t_m) \leq \mu\lambda_0\varepsilon$ $e^{-\varepsilon_0(t_m-t_0-d))}$. Thus we obtain $W(t_m) < \mu\lambda_0\varepsilon < \lambda_0\varepsilon$. Hence, $EW(t) \leq \lambda_0\varepsilon, t \in [t_m, t_{m+1})$. So if that is not true, there exists $t \in [t_m, t_{m+1})$ such that $EW(t) \geq \lambda_0\varepsilon$. Set $t^* = \inf\{t \in [t_m, t_{m+1}) : EW(t_m^-) \geq \lambda_0\varepsilon\}$. Thus, we have that $t^* \in (t_m, t_{m+1})$ and $EW(t^*) = \lambda_0\varepsilon$. Set $\bar{t} = \sup\{t \in [t_m, t_*) : EW(t_m^-) \le \mu\lambda_0\varepsilon\}$. Then $\bar{t} \in (t_m, t^*)$ and $EW(\bar{t}) = \mu\lambda_0\varepsilon$. Applying the similar argument used in the proof of (19), we have $EW(t^*) \le EW(\bar{t})e^{\mu_1\delta_2} < \lambda_0\varepsilon$, which yields a contradiction. Therefore, (21) holds, which in turn implies that (12) holds by the induction method. So system is globally mean square exponentially stable.

Remark 1: Stability conditions in Theorem 1 imply that the upper bound of impulsive input delays can be determined by a set of LMIs as well as the upper bound of impulse input delays. The assumption $\mu \in (0,1)$ and LMI conditions mean that system (4) is stable and the impulses are stabilizing. Therefore, Theorem 1 proves that the stability of system (4) is robust to quite small impulse input delays.

Remark 2: It is noted that conditions $\{t_k\} \in \mathbf{S}(\delta_1, \delta_2)$ and $l\delta_1 \le d \le (l+1)\delta_1$ inditcate that there are at most l impulse times on $[t_m - d_m, t_m)$, i.e., the impulse times is finite on any interval $[t_m - d_m, t_m)$. This constraint on δ_1 grantees the implementation of (23).

Next, let us generalize the results to stochastic impulsive switched system

$$\begin{aligned} dx(t) =&[A_0^\iota x(t) + A_1^\iota x(t-\tau(t))]dt \\ &+ [D_0^\iota x(t) + D_1^\iota x(t-\tau(t))]dB(t), \ \ t \ge t_0, t \ne t_k, \\ x(t_k) =&C_0^\iota x(t_k^-) - C_1^\iota x(t_k^- - d_k)), \ k \in N, \end{aligned} \tag{29}$$

where ι is a switching signal. Naturally, we can obtain a sufficient condition for system (29).

Theorem 2: Suppose that the impulsive time sequence $\{t_k\} \in \mathbf{S}(\delta_1, \delta_2)$ and impulse input delays $d_k \in (0,d)$, where $\delta_1 \le \delta_2$, $l\delta_1 \le d \le (l+1)\delta_1$ for some nonnegative integer l. System (29) can be mean square exponentially stable if for positive scalars $\varepsilon_2, \varepsilon_3$ $\mu \in (0,1)$, there exist $n\times n$ matrices $P_1^\iota > 0, P_2^\iota > 0$, and positive definite diagonal matrices $\Lambda_{ij} \in R^n \times R^n, i,j = 1,2$, such that the following matrix inequalities hold:

$$\begin{aligned} &\begin{bmatrix} \Omega_{ij} & P_i^\iota A_1 + (D_0^\iota)^T P_i^\iota D_1^\iota + \Lambda_{ij} S \\ * & (D_1^\iota)^T P_i^\iota D_1^\iota - 2\Lambda_{ij} \end{bmatrix} < 0, \ i,j = 1,2 \\ &\begin{bmatrix} -(\mu - \varepsilon\frac{\kappa^2}{\lambda_0})P_1^\iota & (C_0^{\tilde\iota} - C_1^{\tilde\iota})^T P_2^{\tilde\iota} & 0 \\ * & -P_2^{\tilde\iota} & P_2^{\tilde\iota} C_1^{\tilde\iota} \\ 0 & * & -\varepsilon I \end{bmatrix} < 0, \end{aligned} \tag{30}$$

where $\Omega_{ij} = (A^\iota)_0^T P_i^\iota + P_i^\iota A_0^\iota + (D_0^\iota)^T P_i^\iota D_0^\iota + \frac{\ln\mu}{\delta_2}P_i^\iota + \frac{1}{\delta_j}(P_1^\iota - P_2^\iota), \kappa = d\kappa_1 + l\kappa_2$, then system (29) is mean square exponentially stable over $\mathbf{S}(\delta_1, \delta_2)$.

Proof: Choose a time-dependent Lyapunov function candidate for system (29) as $V^\iota(t) = x^T(t)P^\iota(t)x(t)$, where $P^\iota(t) = (1-\rho(t))P_1^\iota + \rho(t)P_2^\iota$. We will prove that

$$V(t) \le \lambda_0\varepsilon e^{-\varepsilon_0(t-t_0-d)}, \ t \in [t_0 - \tau, +\infty). \tag{31}$$

Denotes $\triangle\tilde{x}_m = x((t_m - d_m)^-) - x(t_m^-)$. Pre- and post-multiplying (11) by $\mathrm{diag}\{e^T(t_m^-), I, I\}$ and its transpose, respectively, via using the fact that $V^\iota(t_m^-) = x^T(t_m^-)P_1^\iota x(t_m^-)$, we have

$$\begin{bmatrix} -\tilde{V}(t_m) & x^T(t_m^-)(C_0^{\tilde\iota} - C_1^{\tilde\iota})^T P_2^{\tilde\iota} & 0 \\ * & -P_2^{\tilde\iota} & P_2^{\tilde\iota} C_1^{\tilde\iota} \\ 0 & * & -\varepsilon I \end{bmatrix} < 0, \tag{32}$$

where $\tilde{V}(t_m) = (\mu - \varepsilon\frac{\kappa^2}{\lambda_0})V^\iota(t_m^-)$.

By Lemma 1, for any scalar $\varepsilon > 0$, combining (26) and using $x(t_m) = C_0^{\tilde\iota} x(t_m^-) - C_1^{\tilde\iota} x((t_m - d_m)^-)$ yield

$$\begin{bmatrix} -\mu\lambda_0\varepsilon e^{-\varepsilon_0(t_m - t_0 - d))} & x^T(t_m)P_2^{\tilde\iota} \\ * & -P_2^{\tilde\iota} \end{bmatrix} < 0. \tag{33}$$

The rest of the proof is very similar to Theorem 1, thus we omit it here.

Remark 3: In this paper, we use $V^{\tilde\iota}(t_k)$ to define the Lyapunov function which reflects the impulse effects at constant t_k in mode $\tilde\iota$ and $V^\iota(t_k^-)$ to define the Lyapunov function which is different to impulses at constant t_k in mode ι.

When applying Theorem 1 to system (4), impulsive controller are obtained as follows:

Corollary 1: Consider system (4). Assume that the impulsive time sequence $\{t_k\} \in \mathbf{S}(\delta, \delta)$ and impulse input delays $d_k \in [0,d]$, where $d \le \delta_1 \le \delta_2$. If for prescribed positive scalars $\mu \in (0,1), \lambda_0, \varepsilon$, there exist $n\times n$ matrices $P_1 > 0, P_2 > 0$, and positive definite diagonal matrix $\Lambda_{ij} \in \mathbb{R}^{n\times n}, i,j = 1,2$, and matrix $Y \in \mathbb{R}^{n\times n}$, such that (10) and the following LMI hold:

$$\begin{bmatrix} -(\mu - \varepsilon\frac{(d\kappa_1)^2}{\lambda_0})P_1 & Y^T P_2 & 0 \\ * & -P_2 & P_2 C_1 \\ 0 & * & -\varepsilon I \end{bmatrix} < 0, \tag{34}$$

then system (4) is mean square exponentially stable. Moreover, the impulsive controller gain of (4) is given by $C_0 - C_1 = Y$. It is easy to show according to Theorem 1, thus we omit it here.

Remark 4: Corollary 1 gives the way to design the controller gain C_0, C_1. The main procedure can be summarized as follows:

Step 1. Suppose that the impulses are equidistant, i.e., $\{t_k\} \in \mathbf{S}(\delta, \delta)$, where δ is known and $\delta > d$.

Step 2. For given constant $\mu, \varepsilon, \lambda_0 \in (0,1)$, we can obtain d. Further, choosing $P_1 > 0, P_2 > 0$, we have C_0, C_1 by solving LMI (10), and $C_i > 0, i = 1,2$.

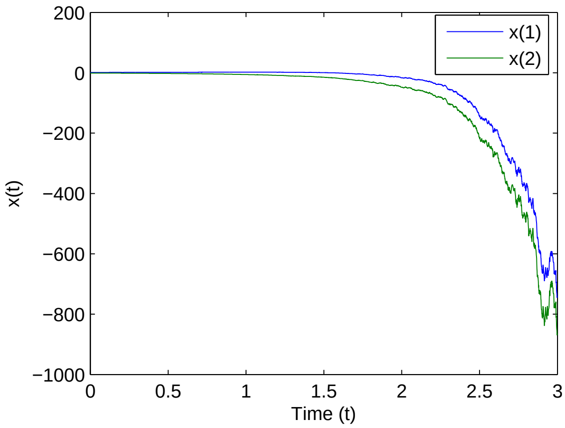

Fig. 2. State response of Example 1 without impulses.

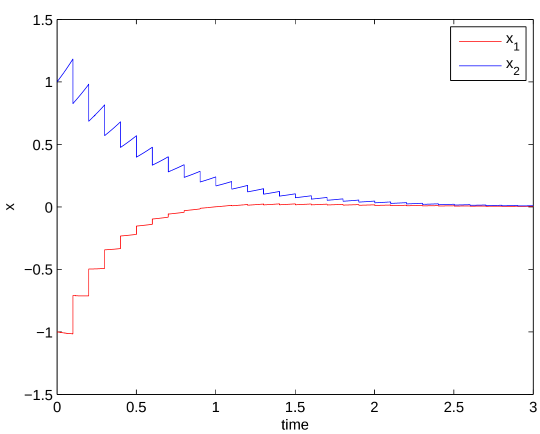

Fig. 3. State response of Example 1 under impulses with $t_k - t_{k-1} = 0.1001$.

4. NUMERICAL EXAMPLE

Example 1: Consider the following stochastic delay system

$$\begin{aligned} dx(t) =& [A_0x(t) + A_1x(t-\tau)]dt \\ &+ [D_0x(t) + D_1x(t-\tau)]dB(t), \ \ t \geq t_0, \end{aligned} \tag{35}$$

where $A_0 = \begin{bmatrix} 0.5 & 0.3 \\ 0 & 0.4 \end{bmatrix}, A_1 = \begin{bmatrix} 1.3 & 0.3 \\ 0.23 & 1.5 \end{bmatrix}, D_0 = \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & 0.1 \end{bmatrix}$, and $D_1 = 0, 1I$.

From Fig. 2, we know that system (35) is not stable with $\tau = 0.002$, initial data $\phi(\theta) = [1, -1]$ and $\theta \in [-\tau, 0]$.

When the delayed impulses are involved in the system, one can use Corollary 1 to stabilize it. Now, without lose of generality, we assume the impulses are equidistant, i.e., $\{t_k\} \in \mathbf{S}(\delta, \delta), \delta = 0.1$. Noticing that $d < \delta$, from $(l-1)\delta \leq d \leq l\delta$, it implies $l = 0$. According to Corollary 1, choose $\varepsilon = 0.5, \mu = 0.5, \lambda_0 = 0.2$, we have $d < 0.1400$ and

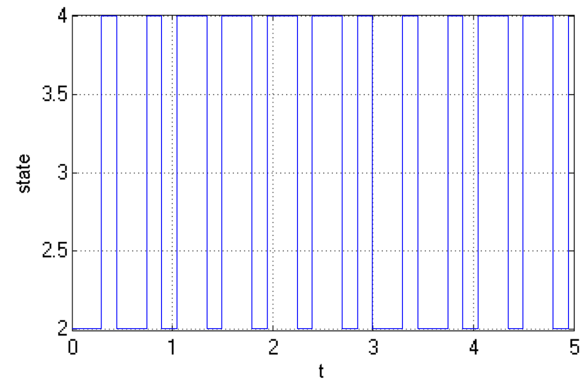

Fig. 4. The switching signal.

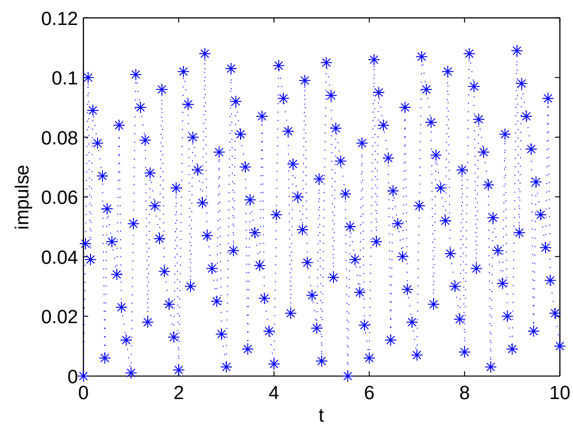

Fig. 5. The impulsive signal with $t_k - t_{k-1} = 0.1110$.

$k_1 \geq 3.1952$. The corresponding impulsive gain matrixes are $C_0 = 0.8089I, C_1 = 0.1110I$. Therefore, we choose $t_k - t_{k-1} = 0.1001$, the simulation results are presented in Fig. 3.

Remark 5: From Fig. 3, we can clearly see that system (35) is stable. This shows that our results hold. What's more, the results are better than [6]. In [6], with the same impulsive control system and the initial data, our stability conditions have wider adaptive scope.

Remark 6: Peng [3] calculate $t_k - t_{k-1} < 0.011$, while using our condition, we get $t_k - t_{k-1} < 0.13$. It can enlarge the maximum limitation of impulsive interval, which, in practice, shows that the robustness of system (35) about impulsive delay is amplified comparing to [3]. Therefore, our results are more superior.

Example 2: Consider the following stochastic switching delay system

$$\begin{aligned} dx(t) = & [A_0^\iota x(t) + A_1^\iota x(t-\tau)]dt \\ &+ [D_0^\iota x(t) + D_1^\iota x(t-\tau)]dB(t), \ t \geq t_0, \end{aligned} \tag{36}$$

where $A_0^1 = \begin{bmatrix} 0.5 & 0.3 \\ 0 & 0.4 \end{bmatrix}, A_1^1 = \begin{bmatrix} -1.3 & 0.3 \\ 0.23 & -1.5 \end{bmatrix}, D_0^1 = \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & 0.1 \end{bmatrix}$, and $D_1^1 = 0.1I$; $A_0^2 = \begin{bmatrix} -1 & 3 \\ 0.1 & -2 \end{bmatrix}, A_1^2 =$

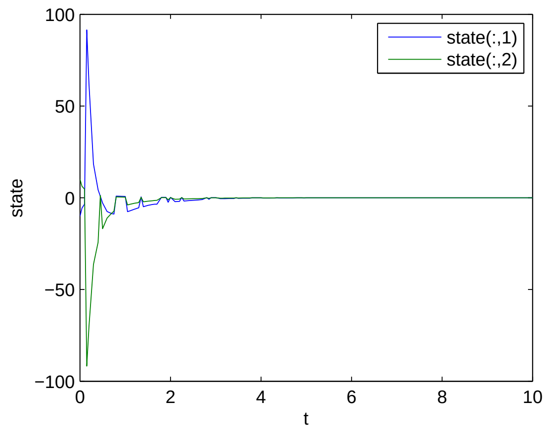

Fig. 6. State response of Example 2 under any switching signal and the impulsive period=0.1110.

$\begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}, D_0^1 = \begin{bmatrix} -0.7 & 0.8 \\ 0.4 & -0.6 \end{bmatrix}$, and $D_1^2 = -0.2I$.

Here, we will demonstrate that the stability property of system can be applied to any switching signal. See the following simulation results.

The corresponding impulsive gain matrixes are $C_0 = 0.8089I, C_1 = 0.1110I$, and we choose $t_k - t_{k-1} = 0.1110$. Set the switching period is arbitrarily number.

5. CONCLUSIONS

In this paper, we have dealt with the problem of exponential stability for impulsive stochastic system with delayed impulses. And the impulsive controller have been designed. In order to derive a less conservative upper bound of impulsive intervals and stability condition, we have considered the time-varying Lyapunov function into LMIs. By employing the Lyapunov-Ruzumikhin method, a sufficient condition has been established to ensure linear impulsive stochastic delay system mean square exponentially stable. Further, we generate the result to impulsive switched stochastic delay system. At last, the effectiveness of the proposed result has been demonstrated by two examples.

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