

# Robust Delay Dependent Fault Estimation for a Class of Interconnected Nonlinear Time Delay Systems

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**Abstract:** This paper focuses on the problem of fault estimation for a class of interconnected nonlinear systems with time varying delays. In contrast to the common assumption imposed on the problem in most literature, here, there is no need for the delay rate to be less than one. Both actuator and component faults are considered within the general fault model invoked as multiplicative faults in this study. Robust adaptive observers are used to detect and estimate simultaneously the states and the parameter faults in each subsystem. The designed observers ensure a prescribed  $H_\infty$  performance level for the fault estimation error, irrespective of the uncertainties which are assumed here to be the unknown interconnections between the subsystems. With the aid of  $H_\infty$  performance index, the common assumption regarding the observer matching condition is no longer required. Sufficient conditions for asymptotic stability of the observers are derived via a matrix inequality approach with the aid of Lyapunov-Krasovskii function. Finally, a simulation example is presented to show the validity and feasibility of the proposed method.

**Keywords:** Fault estimation, interconnected nonlinear systems, Lyapunov-Krasovskii approach, time delay.

## 1. INTRODUCTION

The increase in the demand for safety and reliability of dynamical systems invokes further development of fault detection and isolation techniques. The faults may have harmful effect on systems if they are not detected in time. This is why, the fault detection and isolation (FDI) techniques are of practical significance [1]. The main task of the FDI techniques is to detect the abnormality in the process and to determine which subsystem or component has encountered with a fault [2]. Then, the magnitude of the fault can be determined via some on-line fault estimation approaches. Finally, the calculated fault information can be exploited to compensate the effect of the fault.

Generally, fault detection and isolation (FDI) can be organized as model based or signal processing based approaches [3]. The model based FDI approaches have been found quite effective for FDI from both theory and practical point of views [4].

Among different model-based techniques such as parity space, observer-based, and parameter estimation-based approaches [5], the most common technique is the observer based approach. In the fault diagnosis scheme based on an adaptive observer, the faults can be detected and approximated, and the estimated faults may be further used for fault-tolerant control.

Time delay is a common phenomenon in chemical pro-

cesses, biological reactors, rolling mills, communication networks, etc. Despite the massive research on fault diagnosis techniques for nonlinear uncertain systems and the valuable results provided [6-9], the works on fault diagnosis for time-delay systems are few.

In general, faults can be classified into additive and multiplicative based on their effects on the system dynamics and outputs [10]. Some of the component faults including actuator faults may appear in the form of multiplicative faults. Nevertheless most of the fault estimation techniques focus on the effects of additive faults. On the other hand, some studies on multiplicative fault estimation are investigated in [10, 11]. Fault estimation techniques for multiplicative faults are often more complicated.

To design of an asymptotic observer, disturbance decoupling should be feasible [12]. This is a necessary existence condition which is commonly referred to as the observer matching condition (i.e.,  $\text{rank}(CH) = \text{rank}(H)$  where  $C$  and  $H$  are the output matrix and the disturbance input matrix, respectively). When the unknown disturbances are considered as completely unknown and unbounded observer, matching condition should be satisfied [13]. However, such a necessary existence condition is very restrictive and there are many practical systems that do not satisfy this condition. In this paper, with aid of  $H_\infty$  performance index, matching condition is simply ignored.

In the following, the most relevant works on this topic

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are reviewed.

A fault detection, estimation, and accommodation problem for a class of nonlinear time delay systems is investigated by [14], where an iterative learning observer (ILO) for fault detection, estimation, and compensation is utilized. The considered system is represented in a centralized form. Furthermore, in the proposed approach in [14], the disturbance and its derivative should be bounded which limit the applications.

The problem of robust reliable  $H_\infty$  control for a class of uncertain Takagi-Sugeno fuzzy systems with actuator failures and time-varying delay is investigated in [15]. A state feedback reliable  $H_\infty$  controller is designed such that, the resulting closed loop system is robustly asymptotically stable with a prescribed  $H_\infty$  performance level. There, the considered system is a linear uncertain time delay system and only actuator fault is considered

A robust fault detection filter design problem is investigated for nonlinear time-delay systems with unknown inputs in [16]. In that work, by applying robust  $H_\infty$  optimization control technique, the existence conditions of the robust fault detection filter for the nonlinear time-delay systems with unknown inputs are presented in terms of a linear matrix inequality (LMI) formulation, with no dependency on the time delay. Moreover, in the mentioned paper, constant time delay is considered and the faults are represented in an additive form.

An actuator fault diagnosis method is investigated for a class of time-delayed nonlinear systems in [17]. There, the considered system is represented by a dynamic state space model where the time delays are embedded into the state vector. Then, an adaptive fault diagnosis observer is designed and the Lyapunov stability theory is used to derive the required adaptive tuning rule for the estimation of the nonlinear actuator fault. The problem of  $H_\infty$  fault detection for a kind of linear singular systems with time-varying delay is investigated in [18]. There, the residual is generated by a generalized form of observer-based fault detection filter. Besides, delay-dependent conditions on the existence of the  $H_\infty$  fault detection filter are derived by applying Lyapunov-Krasovskii function approach. In [28], an Adaptive observer for a class of nonlinear systems with time-varying delays is proposed which estimates both states and unknown parameters simultaneously. The sufficient conditions for existence of the observer are derived using the linear matrix inequality approach. There, it is needed that the rate of time delay to be less than one and no uncertainties are considered in the system description.

The discussed papers consider fault detection and estimation both for time delay nonlinear or linear systems in centralized forms.

A decentralized fault detection design for a class of distributed large-scale nonlinear uncertain systems is developed in [19], where a fault detection estimator is designed

by utilizing local measurements and certain communicated information from the interconnected subsystems.

It is well-known that time delays are inherent in many real physical systems. On the other hand, existence of faults in interconnected systems is more probable due to their wide distribution in space and the interdependencies among the subsystems. Therefore, the study of fault detection for interconnected system is quite important. This motivates the present fault detection and estimation research for interconnected nonlinear systems with time delays without satisfying the matching condition. The states are assumed to be unavailable while the outputs are measurable. In this paper, the robust adaptive observers are investigated for a class of interconnected nonlinear systems with time varying delays.

In our research, faults are modeled in multiplicative form which are presented by unknown parameters and known functions and can represent both actuator and process faults. The known functions should satisfy Lipschitz condition. While in many papers, actuator faults are only considered [15]. In each subsystem after detecting faults, the proposed robust adaptive observers estimate simultaneously the system's states and the parameter faults. By incorporating the appropriate Lyapunov-Krasovskii function, some sufficient conditions in term of matrix inequalities which depend on time delays for stability of the proposed observers are derived. The maximum rates of delays are obtained where the matrix inequalities are feasible. Moreover, any restrictive assumption is not necessary to be imposed on the uncertainties except to be bounded. Furthermore, with the help of  $H_2$  performance, the common assumption regarding the observer matching condition is no longer required. In comparison to [20], it is shown that the observation error converges to zero rather than being bounded.

The rest of the paper is arranged as follows. Section 2 explains preliminaries. In Section 3, problems formulation and some assumptions are presented for a class of decentralized nonlinear time-delay systems. The proposed fault detection observer design method is given in Section 4. Section 5 concentrates on fault estimation approach. The validity and feasibility of the proposed method are illustrated in Section 6. Finally, conclusions are provided in Section 7.

## 2. PRELIMINARIES

Following [21], some propositions are given which will be used in the later analysis and focuses on observer matching condition.

Consider a nonlinear system as

$$\dot{x} = Ax + \varphi(x, u, t), \quad (1)$$

$$y = Cx, \quad (2)$$

where  $x \in \mathbb{R}^n$ ,  $u \in \mathbb{R}^m$ ,  $y \in \mathbb{R}$  are the system states inputs and outputs respectively. The term  $\varphi(x, u, t) \in \mathbb{R}^n$  is continuous in its arguments.

**Definition 1:** For the matrix pair  $(A, C)$ , and the assumption that for any  $Q > 0$  there exists a matrix  $L$  with appropriate dimension such that the Lyapunov equation

$$(A - LC)^T P + P(A - LC) = -Q \quad (3)$$

has a unique solution  $P > 0$ . Then, the term  $(\varphi(\cdot))$  is said to be matched with respect to the pair  $(A, C)$  if the following decomposition

$$\varphi(x, u, t) = P^{-1} C^T \psi(x, u, t) \quad (4)$$

holds for some continuous function  $\psi(x, u, t)$ . The triple  $(A, C, \varphi(\cdot))$  is then said to satisfy the observer matching condition.

**Proposition 1:** Consider the system presented by (1)-(2). The triple  $(A, C, \varphi(\cdot))$  satisfies the observer matching condition if and only if the term  $\varphi(x, u, t)$  has decomposition

$$\varphi(x, u, t) = D\psi(x, u, t) \quad (5)$$

with some constant matrix  $D$  such that the matrix equation

$$D^T P = TC \quad (6)$$

is solvable with some matrix  $T$ . Here,  $P$  satisfies equation (3). See [21], for more details.

### 3. PROBLEM FORMULATION

Consider a system consisted of  $n$  interconnected subsystems with time varying delays, which may be subjected to multiple faults occurring at unknown times. The  $i$  th subsystem,  $i = 1, 2, \dots, n$ , is described by:

$$\begin{aligned} \dot{x}_i &= A_i x_i + f_i(x_i, x_i(t - d_i(t))) + E_i H_i(u_i, y_i, x_i) \theta_i(t) \\ &\quad + B_i u_i + G_i \Delta_i(x, x(t - d_i(t))), \\ y_i &= C_i x_i, \\ x_i &= g_i(t), \quad -d_i(t) \leq t \leq 0, \end{aligned} \quad (7)$$

where  $x_i \in \mathbb{R}^{n_i}$ ,  $u_i \in \mathbb{R}^{m_i}$ ,  $y_i \in \mathbb{R}$  are the states, input, and output of the  $i$  th subsystem respectively.  $f_i(x_i, x_i(t - d_i(t)))$  is a nonlinear function and  $d_i(t)$  is a time varying delay and satisfies  $0 < d_i(t) \leq h_i < \infty$ ,  $\dot{d}_i(t) \leq \tau_i$  where  $h_i$  and  $\tau_i$  are scalar constants.  $\Delta_i(\cdot)$  represents the unknown interconnection effects between the  $i$  th subsystem and the remaining subsystems.  $A_i$ ,  $B_i$ ,  $C_i$ ,  $E_i$  and  $G_i$  are the system matrices of appropriate dimensions.  $g_i(t)$  is a continuous function on the interval  $[-d_i(t), 0]$  which indicates the initial states.

The term  $H_i(u_i, y_i, x_i) \in \mathbb{R}^{n_i \times p_i}$  is a known nonlinear matrix function of  $u_i, y_i, x_i$ . The  $\theta_i(t) \in \mathbb{R}^{p_i}$  is a vector of fault parameters which can change unexpectedly when a fault

occurs.  $\theta_i(t) = \begin{cases} = \theta_{iH}(t) & t < T_i \\ \neq \theta_{iH}(t) & t \geq T_i \end{cases}$ , where  $T_i$  is the unknown fault occurrence time, and  $\theta_{iH}$  is a piecewise constant.

The following assumptions are necessary for our proof.

**Assumption 1:** There exist known positive constants  $\gamma_{i1}, \gamma_{i2}, \gamma_{i3}$  ( $i = 1, \dots, n$ ), such that the following Lipschitz inequalities hold:

$$\begin{aligned} \|f_i(x_i, x_i(t - d_i(t))) - f_i(\hat{x}_i, \hat{x}_i(t - d_i(t)))\| &\leq \\ \gamma_{i1} \|x_i - \hat{x}_i\| + \gamma_{i2} \|x_i(t - d_i(t)) - \hat{x}_i(t - d_i(t))\|, \end{aligned} \quad (8)$$

$$\begin{aligned} \|H_i(u_i, y_i, x_i) - H_i(u_i, y_i, \hat{x}_i)\| &\leq \gamma_{i3} \|x_i - \hat{x}_i\|, \\ \forall u_i, y_i \in \mathbb{R}, \end{aligned} \quad (9)$$

**Assumption 2:** It is assumed that  $\|\theta_i(t)\| \leq \theta_{i0}$  where  $\theta_{i0}$  is known.

The following lemmas will be required in the proof of the main result of the paper:

**Lemma 1** [21]: Assume that  $X$  and  $Y$  are vectors or matrices with appropriate dimension, then a constant  $\alpha > 0$  can be chosen, such that the following inequality always holds:

$$X^T Y + Y^T X \leq \alpha X^T X + \alpha^{-1} Y^T Y. \quad (10)$$

**Lemma 2** [22]: For any constant matrix  $M$ ,  $M = M^T > 0$ , and a positive scalar  $\kappa > 0$  such that the integrations in (11) are well defined, the following inequality holds:

$$\frac{1}{\kappa} \left[ \int_0^\kappa w(s) ds \right]^T M \left[ \int_0^\kappa w(s) ds \right] \leq \int_0^\kappa w^T(s) M w(s) ds. \quad (11)$$

**Lemma 3** (Barbalat's Lemma [10]): If  $\lim_{t \rightarrow \infty} \int_0^t f(\tau) d\tau$  exists and is finite, and  $f(t)$  is a uniformly continuous function, then  $\lim_{t \rightarrow \infty} f(t) = 0$ .

**Remark 1:** Assumption 1 is widely considered in the literature to design observers for Lipschitz kinds of nonlinear systems [24, 25].

**Remark 2:** Assumption 2 is a common assumption in representing multiplicative faults [10].

The following section focuses on the proposed fault detection observer design.

### 4. THE PROPOSED FAULT DETECTION OBSERVER DESIGN APPROACH

To detect faults when  $\theta_i(t)$  changes suddenly, fault detection observers for each subsystems are designed by the following:

$$\begin{aligned} \dot{\hat{x}}_i &= A_i \hat{x}_i + f_i(\hat{x}_i, \hat{x}_i(t - d_i(t))) + B_i u_i \\ &\quad + E_i H_i(u_i, y_i, \hat{x}_i) \theta_{iH}(t) + L_i (y_i - \hat{y}_i), \end{aligned} \quad (12)$$

$$\begin{aligned} \hat{x}_i &= j_i(t), \quad -d_i(t) \leq t \leq 0, \\ \hat{y}_i &= C_i \hat{x}_i, \end{aligned} \quad (13)$$

$$\begin{aligned}
\dot{V}_i &\leq \dot{e}_i^T P_{1i} e_i + e_i^T P_{1i} \dot{e}_i + e_i^T P_{2i} e_i - (1 - \tau_i) e_i^T (t - d_i(t)) P_{2i} e_i (t - d_i(t)) + h_i \dot{e}_i^T P_{3i} \dot{e}_i - \int_{t-h_i}^t \dot{e}_i^T (s) P_{3i} \dot{e}_i (s) ds \\
\dot{V}_i &\leq ((A_i - L_i C_i) e_i + f_i(x_i, x_i(t - d_i(t))) - f_i(\hat{x}_i, \hat{x}_i(t - d_i(t)))) + E_i (H_i(u_i, y_i, x_i) - H_i(u_i, y_i, \hat{x}_i)) \theta_{iH}(t) \\
&\quad + G_i \Delta_i(x, x(t - d_i(t)))^T P_{1i} e_i + e_i^T P_{1i} ((A_i - L_i C_i) e_i + f_i(x_i, x_i(t - d_i(t))) - f_i(\hat{x}_i, \hat{x}_i(t - d_i(t)))) \\
&\quad + E_i (H_i(u_i, y_i, x_i) - H_i(u_i, y_i, \hat{x}_i)) \theta_{iH}(t) + G_i \Delta_i(x, x(t - d_i(t))) + e_i^T P_{2i} e_i - (1 - \tau_i) e_i^T (t - d_i(t)) P_{2i} e_i (t - d_i(t)) \\
&\quad + h_i \dot{e}_i^T P_{3i} \dot{e}_i - \int_{t-h_i}^t \dot{e}_i^T (s) P_{3i} \dot{e}_i (s) ds
\end{aligned} \tag{19}$$

where  $\hat{x}_i$  and  $\hat{y}_i$  represent the state and output estimation vectors,  $L_i$  is the observer gain matrix which is determined by the matrix inequality approach.  $j_i(t)$  is a continuous function on the interval  $[-d_i(t), 0]$  which indicates the initial states. The estimation error ( $e_i = x_i - \hat{x}_i$ ) dynamic and observation error ( $e_{yi}$ ) for each subsystem are given by

$$\begin{aligned}
\dot{e}_i &= (A_i - L_i C_i) e_i \\
&\quad + f_i(x_i, x_i(t - d_i(t))) - f_i(\hat{x}_i, \hat{x}_i(t - d_i(t))) \\
&\quad + E_i (H_i(u_i, y_i, x_i) - H_i(u_i, y_i, \hat{x}_i)) \theta_{iH}(t) \\
&\quad + G_i \Delta_i(x, x(t - d_i(t))),
\end{aligned} \tag{14}$$

$$e_{yi} = y_i - \hat{y}_i. \tag{15}$$

Consider there exists a positive constant  $\gamma_i$  such that the following condition is satisfied:

$$\int_0^\infty \|e_i\|^2 dt < \gamma_i \int_0^\infty \|\Delta_i\|^2 dt, \forall \Delta_i \in L_2[0, \infty), \Delta_i \neq 0. \tag{16}$$

The following theorem provides sufficient condition of convergence to zero of estimation error  $e_i$ .

**Theorem 1.** If there are positive definite matrices  $P_{1i} = P_{1i}^T > 0$ , and  $P_{2i} = P_{2i}^T > 0$ , and  $P_{3i} = P_{3i}^T > 0$ , and vectors  $L_i$  and  $Y_{1i}$ , and positive scalars  $\gamma_i$ ,  $\varepsilon_i$ ,  $\mu_i$  such that the following matrix inequalities hold

$$\begin{aligned}
\bar{\Lambda}_i &= \\
&\begin{bmatrix} \bar{\Phi}_{11} & \varepsilon_i \gamma_{i2} \gamma_{i1} I & P_{1i} & P_{1i} E_i & P_{1i} G_i & h_i (A_i^T P_{1i} - C_i^T Y_{1i}) \\ * & \Phi_{22} & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon_i I & 0 & 0 & h_i P_{1i} \\ * & * & * & -\mu_i I & 0 & h_i E_i^T P_{1i} \\ * & * & * & * & -\gamma_i I & h_i G_i^T P_{1i} \\ * & * & * & * & * & -h_i P_{1i} P_{3i}^{-1} P_{1i} \end{bmatrix} \\
&< 0,
\end{aligned} \tag{17}$$

where

$$\begin{aligned}
\bar{\Phi}_{11} &= A_i^T P_{1i} - C_i^T Y_{1i}^T + P_{1i} A_i - Y_{1i}^T C_i + P_{2i} \\
&\quad + \varepsilon_i \gamma_{i1}^2 I + \mu_i \theta_{i0}^2 \gamma_{i3}^2 I - \frac{P_{3i}}{h_i} + I, \\
\Phi_{22} &= \& - (1 - \tau_i) P_{2i} + \varepsilon_i \gamma_{i2}^2 I - \frac{P_{3i}}{h_i}, \\
Y_{1i} &= P_{1i} L_i,
\end{aligned}$$

then, it can be concluded that  $e_i$  is bounded, furthermore if  $\Delta_i(\cdot) \in L_\infty$ , as inequality (16) indicates  $e_i \in L_2$ . Because  $e_i \in L_\infty$ ,  $\dot{e}_i$  is uniformly bounded. Based on Barbalat lemma,  $e_i \rightarrow 0$ .

“\*” and “I” denote, respectively, the symmetric elements in a symmetric matrix and identity matrix with appropriate dimensions.

**Proof:** Consider the following Lyapunov-Krasovskii function:

$$\begin{aligned}
V_i &= e_i^T P_{1i} e_i + \int_{t-d_i(t)}^t e_i^T (s) P_{2i} e_i (s) ds. \\
&\quad + \int_{-h_i}^0 \int_{t+z}^t \dot{e}_i^T (s) P_{3i} \dot{e}_i (s) ds dz.
\end{aligned} \tag{18}$$

The time derivative of  $V_i$  along the trajectories of error dynamic (14) is given by (19). Denote

$$\begin{aligned}
F_i &= f_i(x_i, x_i(t - d_i(t))) - f_i(\hat{x}_i, \hat{x}_i(t - d_i(t))), \\
\Psi_i &= H_i(u_i, y_i, x_i) - H_i(u_i, y_i, \hat{x}_i).
\end{aligned}$$

According to Assumption 1, for any scalars  $\varepsilon_i$ ,  $\mu_i > 0$ , the following inequalities are achieved:

$$\begin{aligned}
\varepsilon_i F_i^T F_i &\leq \chi_i, \\
\chi_i &= \varepsilon_i (\gamma_{i1}^2 e_i^T e_i + \gamma_{i2}^2 e_i^T (t - d_i) e_i (t - d_i) \\
&\quad + \gamma_{i2} \gamma_{i1} e_i^T e_i (t - d_i) + \gamma_{i2} \gamma_{i1} e_i e_i^T (t - d_i)),
\end{aligned} \tag{20}$$

$$\begin{aligned}
\|H_i(u_i, y_i, x_i) \theta_{iH}(t) - H_i(u_i, y_i, \hat{x}_i) \theta_{iH}(t)\| \\
\leq \|H_i(u_i, y_i, x_i) - H_i(u_i, y_i, \hat{x}_i)\| \theta_{i0} \leq \theta_{i0} \gamma_{i3} \varepsilon_i,
\end{aligned} \tag{21}$$

$$\mu_i (\Psi_i \theta_{iH}(t))^T \Psi_i \theta_{iH}(t) \leq \mu_i (\theta_{i0}^2 \gamma_{i3}^2 e_i^T e_i). \tag{22}$$

By using Lemma 2, the following inequality is obtained.

$$\begin{aligned}
-\int_{t-h_i}^t \dot{e}_i^T (s) P_{3i} \dot{e}_i (s) ds &\leq - \left[ \int_{t-h_i}^t \dot{e}_i (s) ds \right]^T \frac{P_{3i}}{h_i} \left[ \int_{t-h_i}^t \dot{e}_i (s) ds \right] \\
&\leq - \left[ \int_{t-d_i(t)}^t \dot{e}_i (s) ds \right]^T \frac{P_{3i}}{h_i} \left[ \int_{t-d_i(t)}^t \dot{e}_i (s) ds \right] \\
&\leq -(e_i(t) - e_i(t - d_i(t)))^T \frac{P_{3i}}{h_i} (e_i(t) - e_i(t - d_i(t)))
\end{aligned}$$

And moving the left hand sides of (20) and (22) to the right hand side and then adding these positives terms to the Lyapunov equation (19), it yields (23). The term  $h_i \dot{e}_i^T P_{3i} \dot{e}_i$  is simplified as follows:

$$\begin{aligned}
\dot{V}_i \leq & e_i^T (A_i - L_i C_i)^T P_{1i} e_i + e_i^T P_{1i} (A_i - L_i C_i) e_i + e_i^T P_{1i} F_i + e_i P_{1i} F_i^T + 2(\Delta_i(x, x(t-d_i(t))))^T G_i^T P_{1i} e_i \\
& - \varepsilon_i F_i^T F_i + \varepsilon_i (\gamma_{i1}^2 e_i^T e_i + \gamma_{i2}^2 e_i^T (t-d_i(t)) e_i (t-d_i(t)) + \gamma_{i2} \gamma_{i1} e_i^T e_i (t-d_i(t)) + \gamma_{i2} \gamma_{i1} e_i e_i^T (t-d_i(t))) \\
& + e_i^T P_{1i} E_i \psi_i \theta_{iH}(t) + (\psi_i \theta_{iH}(t))^T E_i^T P_{1i} e_i - \mu_i (\psi_i \theta_{iH}(t))^T \psi_i \theta_{iH}(t) + \mu_i (\theta_{i0}^2 \gamma_{i3}^2 e_i^T e_i) \\
& + e_i^T P_{2i} e_i - (1 - \tau_i) e_i^T (t-d_i(t)) P_{2i} e_i (t-d_i(t)) + h_i \dot{e}_i^T P_{3i} \dot{e}_i \\
& - (e_i(t) - e_i(t-d_i(t)))^T \frac{P_{3i}}{h_i} (e_i(t) - e_i(t-d_i(t)))
\end{aligned} \tag{23}$$

$$h_i \dot{e}_i^T P_{3i} \dot{e}_i = h_i \bar{\zeta}_i^T \psi_i^T P_{3i} \psi_i \bar{\zeta}_i, \tag{24}$$

where

$$\bar{\zeta}_i(t) = \begin{bmatrix} e_i \\ F_i \\ \psi_i \theta_{iH}(t) \\ \Delta_i \end{bmatrix}, \quad \psi_i^T = [ (A_i - L_i C_i) \quad I \quad E_i \quad G_i ].$$

After some manipulations, one can get:

$$\dot{V}_i \leq \zeta_i^T(t) \Upsilon_i \zeta_i(t) + h_i \bar{\zeta}_i^T(t) \psi_i^T P_{3i} \psi_i \bar{\zeta}_i(t), \tag{25}$$

where

$$\Upsilon_i = \begin{bmatrix} \varphi_{11} & \varepsilon_i \gamma_{i2} \gamma_{i1} I & P_{1i} & P_{1i} E_i & G_i^T P_{1i} \\ * & \varphi_{22} & 0 & 0 & 0 \\ * & * & -\varepsilon_i I & 0 & 0 \\ * & * & * & -\mu_i I & 0 \\ * & * & * & * & 0 \end{bmatrix}, \tag{26}$$

where

$$\varphi_{11} = A_i^T P_{1i} - C_i^T Y_{1i}^T + P_{1i} A_i - Y_{1i}^T C_i + P_{2i} + \varepsilon_i \gamma_{i1}^2 I + \mu_i \theta_{i0}^2 \gamma_{i3}^2 I - \frac{P_{3i}}{h_i},$$

$$\varphi_{22} = -(1 - \tau_i) P_{2i} + \varepsilon_i \gamma_{i2}^2 I - \frac{P_{3i}}{h_i},$$

$$Y_{1i} = P_{1i} L_i, \quad \zeta_i(t) = \begin{bmatrix} e_i \\ e_i(t-d_i(t)) \\ F_i \\ \psi_i \theta_{iH}(t) \\ \Delta_i \end{bmatrix}.$$

By absorbing the second term in (25) into the first and using Schur Lemma, (25) can be written as

$$\dot{V}_i \leq \zeta_i^T \Lambda_i \zeta_i,$$

where matrixes  $\Lambda_i$  is as follows:

$$\Lambda_i = \begin{bmatrix} \varphi_{11} & \varepsilon_i \gamma_{i2} \gamma_{i1} I & P_{1i} & P_{1i} E_i & P_{1i} G_i & h_i (A_i^T P_{1i} - C_i^T Y_{1i}) \\ * & \varphi_{22} & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon_i I & 0 & 0 & h_i P_{1i} \\ * & * & * & -\mu_i I & 0 & h_i E_i^T P_{1i} \\ * & * & * & * & 0 & h_i G_i^T P_{1i} \\ * & * & * & * & * & -h_i P_{1i} P_{3i}^{-1} P_{1i} \end{bmatrix}. \tag{27}$$

It follows from (16) that

$$e_i^T e_i - \gamma_i \Delta_i^T \Delta_i + \dot{V}_i < \zeta_i^T \bar{\Lambda}_i \zeta_i, \tag{28}$$

where

$$\bar{\Lambda}_i = \begin{bmatrix} \bar{\varphi}_{11} & \varepsilon_i \gamma_{i2} \gamma_{i1} I & P_{1i} & P_{1i} E_i & P_{1i} G_i & h_i (A_i^T P_{1i} - C_i^T Y_{1i}) \\ * & \varphi_{22} & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon_i I & 0 & 0 & h_i P_{1i} \\ * & * & * & -\mu_i I & 0 & h_i E_i^T P_{1i} \\ * & * & * & * & -\gamma_i I & h_i G_i^T P_{1i} \\ * & * & * & * & * & -h_i P_{1i} P_{3i}^{-1} P_{1i} \end{bmatrix}, \tag{29}$$

$$\begin{aligned} \bar{\varphi}_{11} = & A_i^T P_{1i} - C_i^T Y_{1i}^T + P_{1i} A_i - Y_{1i}^T C_i + P_{2i} + \varepsilon_i \gamma_{i1}^2 I \\ & + \mu_i \theta_{i0}^2 \gamma_{i3}^2 I - \frac{P_{3i}}{h_i} + I. \end{aligned}$$

The time derivative of the Lyapunov function of the overall system satisfies

$$\sum_{i=1}^n (e_i^T e_i - \gamma_i \Delta_i^T \Delta_i + \dot{V}_i) < \sum_{i=1}^n \zeta_i^T \bar{\Lambda}_i \zeta_i. \tag{30}$$

It is clear that if  $\bar{\Lambda}_i < 0$  ( $i = 1, 2, \dots, n$ ), then  $\dot{V} < 0$ . Therefore, it can be concluded that  $e_i$  will converges to zero. Integrating both sides of (28) yields

$$\begin{aligned} & \int_0^\infty (e_i^T e_i - \gamma_i^2 \Delta_i^T \Delta_i) dt + V_i(\cdot)|_{t=\infty} - V_i(\cdot)|_{t=0} \\ & < \int_0^\infty \zeta_i^T \bar{\Lambda}_i \zeta_i dt. \end{aligned}$$

Using the fact that  $V_i > 0$  for all  $t \neq 0$ ; one can get

$$\int_0^\infty e_i^T e_i dt \leq \int_0^\infty \gamma_i \Delta_i^T \Delta_i dt.$$

Hence, the inequality (16) is guaranteed, and the proof is completed. Based on Theorem 1, it can be concluded that as far as the system is at its normal condition and there is no fault the observation error ( $e_{yi}$ ) is zero, but when a fault occur the observation error ( $e_{yi}$ ) deviate from zero and the fault can be detected. The next step is to estimate the faults. This is investigated in the following section.

## 5. THE PROPOSED FAULT ESTIMATION DESIGN APPROACH

To estimate the fault after an alarm has been generated, the following observer is constructed for each subsystem which has the same structure as the fault detector design except for  $\theta_{iH}(t)$  which is substituted by  $\hat{\theta}_i(t)$ :

$$\begin{aligned}\dot{\hat{x}}_i &= A_i \hat{x}_i + f_i(\hat{x}_i, \hat{x}_i(t-d_i(t))) + B_i u_i \\ &\quad + E_i H_i(u_i, y_i, \hat{x}_i) \hat{\theta}_i(t) + L_i(y_i - \hat{y}_i), \\ \hat{x}_i &= j_i(t), \quad -d_i(t) \leq t \leq 0,\end{aligned}\quad (31)$$

where  $\hat{x}_i$  is the state vector observer and  $\hat{\theta}_i(t)$  is an estimate of  $\theta_i(t)$ . It is assumed that after the occurrence of fault,  $\theta_i(t) \neq \theta_{iH}(t)$ .

The state estimation error dynamic of each subsystem can be obtained as:

$$\begin{aligned}\dot{e}_i &= A_i e_i + f_i(x_i, x_i(t-d_i(t))) - f_i(\hat{x}_i, \hat{x}_i(t-d_i(t))) \\ &\quad + E_i(H_i(u_i, y_i, x_i)\theta_i(t) - H_i(u_i, y_i, \hat{x}_i)\hat{\theta}_i(t)) \\ &\quad + G_i \Delta_i(x, x(t-d_i(t))) - L_i(y_i - \hat{y}_i).\end{aligned}\quad (32)$$

The next theorem specifies sufficient conditions for the stability of the estimators in the time varying delay cases.

**Theorem 2:** If there are positive definite matrixes  $P_{1i} = P_{1i}^T > 0$  and  $P_{2i} = P_{2i}^T > 0$  and  $P_{3i} = P_{3i}^T > 0$ , and matrixes  $S_i$ , and vectors  $L_i$  and  $Y_{1i}$ , and positive scalars  $\gamma_i, \varepsilon_i, \mu_i$ , the matrix inequality (17) holds and

$$E_i^T P_{1i} = S_i C_i \quad (33)$$

and the fault parameter error ( $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ ) update law is selected as

$$\dot{\tilde{\theta}}_i = -\Gamma_i^{-1} H_i(u_i, y_i, \hat{x}_i)^T E_i^T P_{1i} e_i = -\Gamma_i^{-1} H_i(u_i, y_i, \hat{x}_i)^T S_i e_{y_i}. \quad (34)$$

Then it can be concluded that the estimators are stable and the fault parameter error  $\tilde{\theta}_i$  remains bounded.

**Proof:** Consider the following Lyapunov-Krasovskii function

$$\begin{aligned}V_i &= e_i^T P_{1i} e_i + \int_{t-d_i(t)}^t e_i^T(s) P_{2i} e_i(s) ds \\ &\quad + \int_{-h_i}^0 \int_{t+z}^t \dot{e}_i^T(s) P_{3i} \dot{e}_i(s) ds dz + \tilde{\theta}_i^T \Gamma_i \tilde{\theta}_i.\end{aligned}\quad (35)$$

$$\begin{aligned}\dot{V}_i &\leq e_i^T (A_i - L_i C_i)^T P_{1i} e_i + e_i^T P_{1i} (A_i - L_i C_i) e_i + e_i^T P_{1i} F_i + e_i P_{1i} F_i^T + 2(\Delta_i(x, x(t-d_i(t))))^T G_i^T P_{1i} e_i \\ &\quad - \varepsilon_i F_i^T F_i + \varepsilon_i (\gamma_{1i}^2 e_i^T e_i + \gamma_{12}^2 e_i^T(t-d_i(t)) e_i(t-d_i(t)) + \gamma_{12} \gamma_{11} e_i^T e_i(t-d_i(t)) + \gamma_{12} \gamma_{11} e_i e_i^T(t-d_i(t))) \\ &\quad + e_i^T P_{1i} E_i \Psi_i \theta_i(t) + (\Psi_i \theta_i(t))^T E_i^T P_{1i} e_i - \mu_i (\Psi_i \theta_i(t))^T \Psi_i \theta_i(t) + \mu_i (\theta_{10}^2 \gamma_{13}^2 e_i^T e_i) \\ &\quad + e_i^T P_{2i} e_i - (1 - \tau_i) e_i^T(t-d_i(t)) P_{2i} e_i(t-d_i(t)) + h_i \dot{e}_i^T P_{3i} \dot{e}_i \\ &\quad - (e_i(t) - e_i(t-d_i(t)))^T \frac{P_{3i}}{h_i} (e_i(t) - e_i(t-d_i(t)))\end{aligned}\quad (39)$$

Its derivative with respect to time is

$$\begin{aligned}\dot{V}_i &\leq \dot{e}_i^T P_{1i} e_i + e_i^T P_{1i} \dot{e}_i + e_i^T P_{2i} e_i \\ &\quad - (1 - \tau_i) e_i^T(t-d_i(t)) P_{2i} e_i(t-d_i(t)) \\ &\quad + h_i \dot{e}_i^T P_{3i} \dot{e}_i - \int_{t-h_i}^t \dot{e}_i^T(s) P_{3i} \dot{e}_i(s) ds + 2\tilde{\theta}_i^T \Gamma_i \dot{\tilde{\theta}}_i.\end{aligned}\quad (36)$$

From assumption 1, for any scalars  $\mu_i > 0$ , the following inequalities are obtained:

$$\begin{aligned}\|H_i(u_i, y_i, x_i)\theta_i(t) - H_i(u_i, y_i, \hat{x}_i)\theta_i(t)\| \\ \leq \|H_i(u_i, y_i, x_i) - H_i(u_i, y_i, \hat{x}_i)\| \theta_{i0} \leq \theta_{i0} \gamma_{13} e_i,\end{aligned}\quad (37)$$

$$\mu_i (\Psi_i \theta_i(t))^T \Psi_i \theta_i(t) \leq \mu_i (\theta_{10}^2 \gamma_{13}^2 e_i^T e_i). \quad (38)$$

By considering (20) and (38), and choosing the fault parameter error ( $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ ) as (34) and after some manipulations and similar to the previous section, one can get (39).

The time derivative of the Lyapunov function can be written as:

$$e_i^T e_i - \gamma_i \Delta_i^T \Delta_i + \dot{V}_i < \zeta_i^T \bar{\Lambda}_i \zeta_i, \quad (40)$$

$$\text{where } \zeta_i(t) = \begin{bmatrix} e_i \\ e_i(t-d_i(t)) \\ F_i \\ \Psi_i \theta_i(t) \\ \Delta_i \end{bmatrix} \text{ and } \bar{\Lambda}_i \text{ is given in (29).}$$

Therefore, the time derivative of the Lyapunov function of the overall system satisfies

$$\sum_{i=1}^n e_i^T e_i - \gamma_i \Delta_i^T \Delta_i + \dot{V}_i < \sum_{i=1}^n \zeta_i^T \bar{\Lambda}_i \zeta_i. \quad (41)$$

It is evident that if  $\bar{\Lambda}_i < 0$  ( $i = 1, 2, \dots, n$ ), then  $\dot{V} = \sum_{i=1}^n \dot{V}_i \leq 0$ . This implies that the estimators are stable, and the fault parameter error  $\tilde{\theta}_i$  remains bounded. The proof of this theorem is similar to Theorem 1, thus the detail is omitted.

## 6. SIMULATION RESULTS

In this section an example is given to verify the effectiveness of the proposed method. Consider the equation of an interconnected two pendulum systems as (42). The

figure of two inverted pendulums connected by a spring is shown in Fig. 1.

$$\begin{aligned}
\dot{x}_{11} &= x_{12}, \\
\dot{x}_{12} &= \left( \frac{m_1 g r}{J_1} - \frac{kr^2}{4J_1} \right) \sin(x_{11}(t - d_1(t))) \\
&\quad + \frac{kr(l-b)}{2J_1} + \frac{u_1}{J_1} + \frac{kr^2}{4J_1} \sin(x_{22}), \\
x_1 &= [x_{11}, x_{12}]^T, \\
y_1 &= C_1 x_1, \\
\dot{x}_{21} &= x_{22}, \\
\dot{x}_{22} &= \left( \frac{m_2 g r}{J_2} - \frac{kr^2}{4J_2} \right) \sin(x_{21}(t - d_2(t))) \\
&\quad + \frac{kr(l-b)}{2J_2} + \frac{u_2}{J_2} + \frac{kr^2}{4J_2} \sin(x_{11}), \\
y_2 &= C_2 x_2, \\
x_2 &= [x_{21}, x_{22}]^T,
\end{aligned} \tag{42}$$

where  $x_{ij}$ ,  $i, j = 1, 2$  are the states of each subsystem and  $y_i$ ,  $i = 1, 2$  depict the output of each subsystem.  $m_1 = 2$  kg and  $m_2 = 2.5$  kg,  $J_1 = 0.5 \frac{N \cdot S}{m^2}$  and  $J_2 = 0.625 \frac{N \cdot S}{m^2}$ ,  $k = 100$  N/m,  $r = 0.5$  m,  $l = 0.5$  m, and  $g = 9.81 \frac{m}{s^2}$ , and  $b = 0.4$  m.

We consider a simple multiplicative actuator fault in subsystem 1 and 2, respectively. Specifically, for  $i = 1, 2$ , we let  $u_i = \bar{u}_i + \theta_i \bar{u}_i$ , where  $\bar{u}_i$  is the nominal control input in the non-fault case ( $\bar{u}_i = -20y_i$ ), and  $\theta_i \in [-1, 0]$  is the parameter characterizing the magnitude of the fault.

To simulate a fault, we set  $\theta_1 = -0.5$ , for subsystem 1 at  $T_0 = 5$  sec and  $\theta_2 = -0.75$ , for subsystem 2 at  $T_0 = 7$  sec. Besides, it is assumed  $\theta_{iH}$  ( $i = 1, 2$ ) is zero. The outputs of the system are defined such that  $C_i = [1 \ 0.5]$ ,  $i = 1, 2$ .  $S_i$  ( $i = 1, 2$ ) are obtained to be 0.05 which satisfy the assumption in Theorem 1. The optimization problem which is formulated by matrix inequality form is solved by CVX software [23].

**Remark 3:** Equation (17) is a kind of nonlinear matrix inequality. The problem is solved by the iterative linear

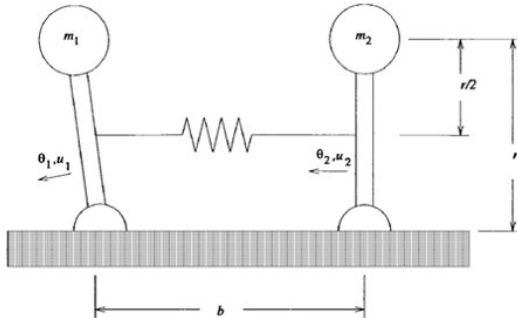


Fig. 1. Two inverted pendulums connected by a spring [26].

matrix inequality approach which splits the problem into two simpler optimization problems, where each is linear in the decision variables and then solves the problem iteratively. Actually, this approach changes the problem from optimal to a suboptimal one. So each of the simpler optimization problems can be solved by a linear matrix inequality (LMI) or CVX toolbox. For more information please refer to [27].

### 6.1. Simulation results-slow time varying delay case

In this case, it is assumed that the rate of delay ( $\tau_i$ ,  $i = 1, 2$ ) is less than one. The considered delays are  $d_1(t) = 0.3 + 0.3 \sin(t)$  and  $d_2(t) = 0.4 + 0.2 \sin(t)$  respectively. Observation error of the first and the second subsystem in a time varying delay case are shown in Fig. 2 and Fig. 3 respectively. After the fault occurrences, the observation error ( $e_{yi}$ ) deviates from zero, and the fault is detected. Next, the fault estimator is activated and the parameter fault changes according the adaptive rule.

It is obvious that when the faults are estimated, observation error converges to zero. The fault magnitude of the first and the second subsystems are estimated with satisfactory accuracy as shown in Fig. 4 and Fig. 5 respectively. The fault magnitude of each subsystem converges approximately to their true values ( $\theta_1 = -0.5$ , for subsystem 1) and ( $\theta_2 = -0.75$ , for subsystem 2). From the above simulation results, it is seen that the proposed observer and estimator have acceptable performances.

In this case, it is assumed that the rate of delay ( $\tau_i$ ,  $i = 1, 2$ ) is less than one. The considered delays are  $d_1(t) = 0.3 + 0.3 \sin(t)$  and  $d_2(t) = 0.4 + 0.2 \sin(t)$  respectively. Observation error of the first and the second subsystem in a time varying delay case are shown in Fig. 2 and Fig. 3 respectively. After the fault occurrences, the observation error ( $e_{yi}$ ) deviates from zero, and the fault is detected. Next, the fault estimator is activated and the parameter fault changes according the adaptive rule. It is obvious that when the faults are estimated, observation error converges to zero. The fault magnitude of the first and the second subsystems are estimated with satisfactory accuracy as shown in Fig. 4 and Fig. 5 respectively. The estimation of fault magnitude of each subsystem converges approximately to their true values ( $\theta_1 = -0.5$ , for subsystem 1) and ( $\theta_2 = -0.75$ , for subsystem 2). From the above simulation results, it is seen that the proposed observer and estimator have acceptable performances.

### 6.2. Simulation results- fast time varying delay case

In this case, a fast time-varying delay is considered. It is assumed that the time-varying delay of the first and the second subsystem are  $d_1(t) = 0.3 + 0.3 \sin(8t)$ ,  $d_2(t) = 0.4 + 0.2 \sin(8t)$  respectively. The rate of time delay ( $\tau_2$ ) is greater than one. Observation error of the first and the second subsystem for time varying delay case are shown in Fig. 6 and Fig. 7, respectively. The fault magnitude

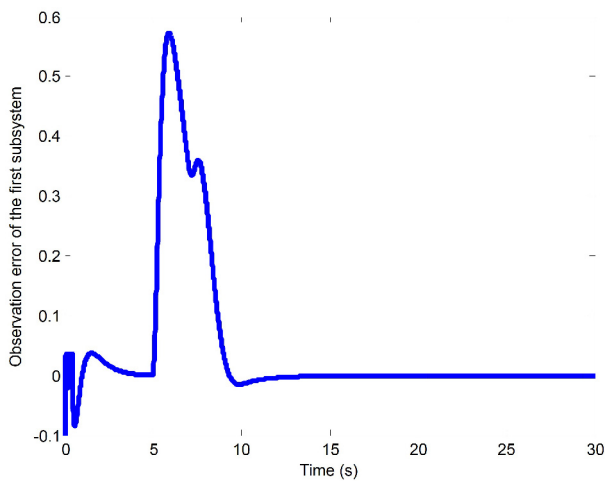


Fig. 2. Observation error of the first subsystem- the slow time varying delay.

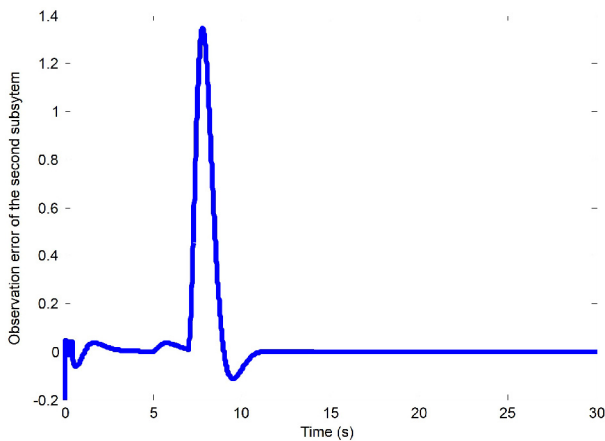


Fig. 3. Observation error of the second subsystem- the slow time varying delay.

of the first and the second subsystems are estimated with satisfactory accuracy as shown in Fig. 8 and Fig. 9, respectively.

## 7. CONCLUSIONS

In this paper, a fault detection and estimation scheme for a class of interconnected nonlinear systems with time varying delays was considered. The rate of the delay can be greater than one. The proposed robust adaptive observers estimate the system's states and the parameter faults at the same time for each subsystem. The proposed fault estimator has been shown to be capable of ensuring a prescribed  $H_\infty$  performance level for the fault estimation error, irrespective of the uncertainties. The estimated fault can be further used in a fault-tolerant control design stage. The simulation results show that the proposed method works reasonably well. Furthermore, the

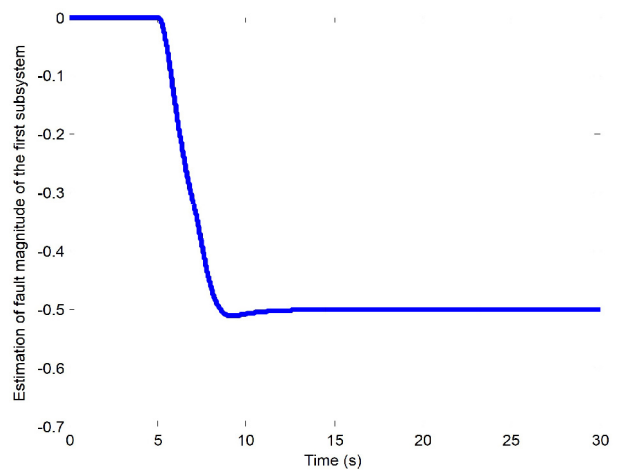


Fig. 4. Estimation of fault magnitude of the first subsystem-the slow time varying delay case.

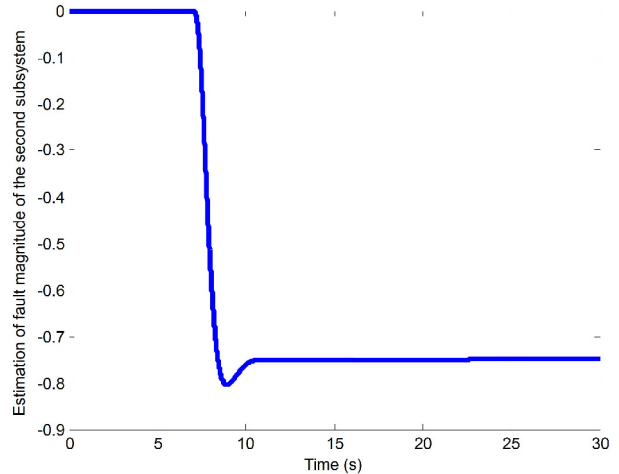


Fig. 5. Estimation of fault magnitude of the second subsystem-the slow time varying delay case.

maximum rate of the time varying delays can be obtained from the feasible matrix inequalities.

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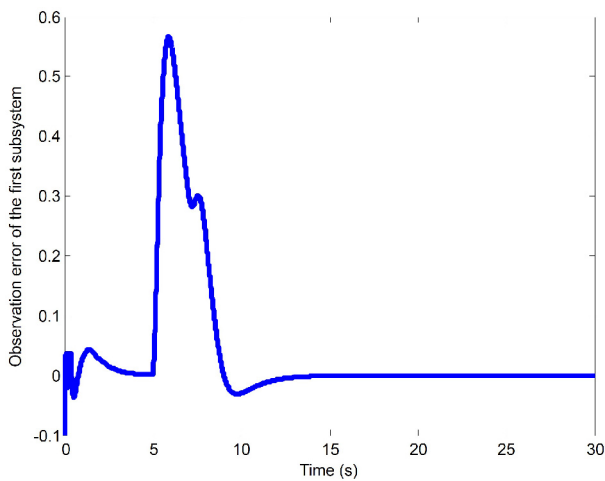


Fig. 6. Observation error of the second subsystem- the fast time varying delay case.

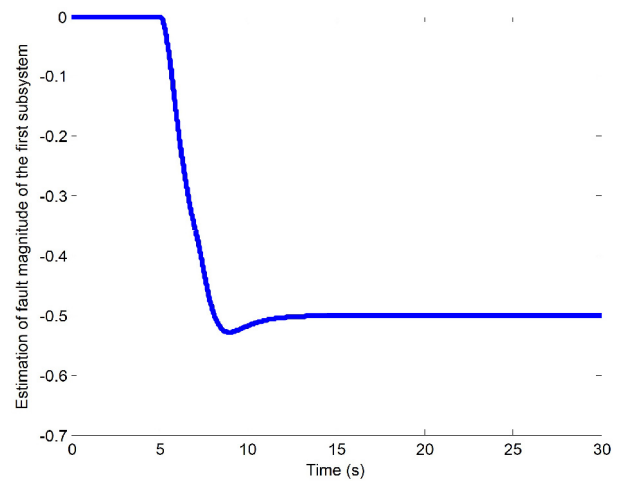


Fig. 8. Estimation of fault magnitude of the first subsystem- the fast time varying delay.

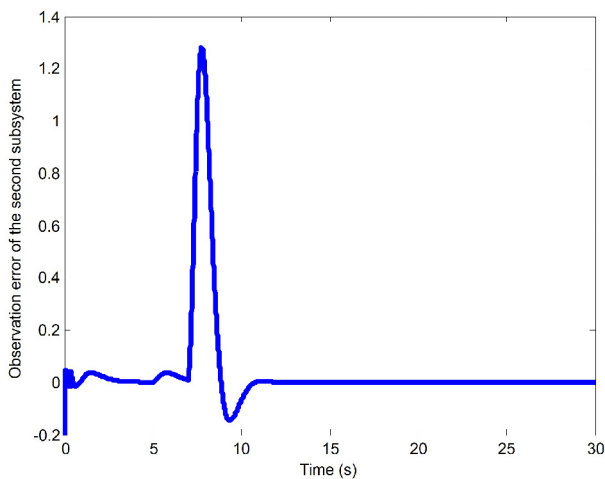


Fig. 7. Observation error of the second subsystem- the fast time varying delay case.

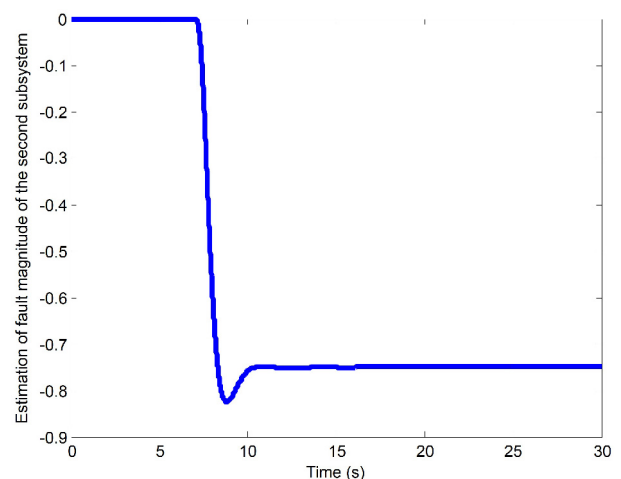


Fig. 9. Estimation of fault magnitude of the second subsystem- the fast time varying delay.

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