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A Coverage Algorithm for Multiple Autonomous Surface Vehicles in Flowing Environments

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Abstract: This paper is concerned with the coverage problem with multiple autonomous surface vehicles (ASVs) in time-varying flowing environment, where the interest information distribution is unknown to the coverage networks. While taking the model parameter uncertainty into consideration, a decentralized, adaptive control law is proposed such that the coverage network will converge to the optimal assigned region from arbitrary positions. For ease of exploration, we first investigate the static coverage problem of two-agent systems in flowing environment and present an example by extending the two-agent systems into the general case. In addition, Gaussian Estimation is introduced to predict the value of the sensory function through the sampled measurements. By using the static coverage partition as theoretical foundation, we transform the optimal coverage control into the moving target tracking problems, where the target is the centroid of the assigned region for each ASV. Based on these techniques, a decentralized kinematic control algorithm is developed to navigate the multi-ASV systems. Furthermore, the adaptive back-stepping techniques are employed to extend the kinematic controller into dynamic case with uncertain model parameters. Finally, simulation studies are provided to demonstrate the feasibility and effectiveness of the proposed approaches.

Keywords: Adaptive backstepping techniques, coverage control, Gaussian functions, multi-ASV systems.

1. INTRODUCTION

Cooperative control of multiple agents have the efficiency and robustness merits compared with single agent [1–6]. One of the typical problems in multi-agent cooperations is Voronoi diagram based coverage control, which has been intensively studied in recent years [7, 8]. These coverage algorithms have been viewed as important tools in different engineering fields, including targets search [9], environment monitoring [10], spatial estimation [11], etc. In these applications, the multi-agent networks can be treated as the resources and the target areas are consumers. The purpose of coverage control is to assign the multiple agents optimally to the consumers such that the proposed objective function would reach the minimum. For instance, a coverage algorithm is proposed for multi-robot systems in [12] by using the gradient decent methods, which reveals the characteristics of Voronoi cell in coverage control. In [13], the Gaussian functions are employed to estimate the sensory function over the target region such that the multi-agent systems can be assigned optimally based on the density of local information. Furthermore, distributed trajectories for multi-agent systems are proposed in [14] to monitor the target area periodically, where the decentralized pathes are constructed by adding virtual agents into the coverage networks.

In this work, we focus on the optimal Voronoi Partition of multi-ASV systems in time-varying flowing environment. As the coverage problem is region assignment strategy of multiple agents, it is important to figure out the sensor region for each agent in flowing environment. There are some coverage algorithms in special environment: A coverage algorithm in river environment is presented in [15], of which the metric in river is defined by the sum of reachable sets of each agent. In [16], a coverage control algorithm in constant flowing environment is proposed, which takes both energy consumption and traveling time as the Voronoi metric and presents the Voronoi cell though geometry approximation. For indoor environments, a Voronoi diagram based configuration of multi-agent systems is proposed with the consideration of the energy constraints in [17].

Furthermore, we address a particular system model based coverage problem in which the Euler-Lagrange equations with uncertain model parameters are introduced to make this paper practically. The ideal of employing

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special system model have appeared in some articles. For example, a coverage algorithm of wheeled dynamic is proposed in [18], where constant and variable forward speed are taken into consideration, respectively. In [19], a coverage control law for multi-robot networks is provided to monitor the environment, where the anisotropic of the robot is limited in a sector region.

The main contribution of this paper is that a coverage algorithm for multi-ASV systems is proposed in flowing environment, where the sensory function over the objective region is unknown to the coverage network. Comparing with the existing results, the advantages of this paper are that: (i) In many engineering field, the operators in monitoring tasks are complicated. For most of the existing linear coverage algorithms, the under-actuated system model is used in this paper such that the proposed algorithms are close to the engineering applications. (ii) A redeveloped Voronoi Partition is provided while the objective flied is influenced by the time-varying currents. (iii) The adaptive backstepping technique is employed to design the dynamic controller while the ASV model parameters are uncertain. Therefore, the proposed coverage algorithms for ASV model in flowing environment is significant and practical to the engineering applications.

The remainder of the paper is organized as follows: The formulation of coverage problem in time-varying flowing environment is presented in Section 2. In Section 3, the static coverage control strategy with time-varying currents is provided. Furthermore, the kinematic and dynamic coverage algorithms for Euler-Lagrange equations are proposed to deal with model parameters uncertainty in Section 4. In Section 5, simulation studies are provided to demonstrate the feasibility and effectiveness of the proposed approaches. In final, concluding remarks are drawn in Section 6.

2. PROBLEM FORMULATION

Let $Q \in \mathbb{R}^2$ be a time-varying flowing environment and q be an arbitrary point in Q. Denote V_c as the time-varying currents, which is given by [20]

$$\dot{V}_c + gV_c = \omega, \tag{1}$$

where g are constant and ω are white noise, ϕ_c is the direction of the currents.

Consider n ASVs coverage the horizontal plane Q, of which the sensory function is described by $\phi(q)$. The kinematic and dynamic motion of ASV are respectively shown as follows [21]:

$$\begin{cases} \dot{x} = u\cos\psi - v\sin\psi + V_c\cos\phi_c, \\ \dot{y} = u\sin\psi + v\cos\psi + V_c\sin\phi_c, \\ \dot{\psi} = r, \end{cases} \tag{2}$$

where u (surge speed), v (sway speed) and r (angular speed) are the velocity of ASV in body-fixed coordinate frame $\{B\}$ with the respect to water. x, y and ψ are the posture of ASV in global coordinate frame $\{U\}$.

$$\begin{cases} m_u\dot{u} - m_v vr + d_u u = \tau_u, \\ m_v\dot{v} + m_u ur + d_v v = 0, \\ m_r\dot{r} - m_{uv}uv + d_r r = \tau_r, \end{cases} \tag{3}$$

where m_u, m_v, m_r and m_{uv} are the mass and hydrodynamic added mass terms, d_u, d_v, d_r capture hydrodynamic damping effects. τ_u, τ_r denote the control force in surge and torque of ASV, respectively. These model parameters can be found in [20] for details.

By using Euler-Lagrange (E-L) equations as ASV model in (3), the under-actuation of ASV can be presented in three component equations, which is practical and meaningful for engineering applications. In addition, the problem of model parameter uncertainty can be formulated and solved conveniently because the weight or the inertia of ASV are taken into account in E-L model.

In order to explicit the coverage strategy of multi-ASV networks, a cost function over the region Q is defined as

$$\mathcal{H}(P) = \sum_{i=1}^{n}\int_{W_i} f(\|q-p_i\|)\phi(q)dq, \tag{4}$$

where W_i is the assigned region for ASV i, $\phi(q)$ is the sensory function over Q, $P = [p_1, \ldots, p_n]^T$ are the positions of the multiple ASVs and $f(\|q-p_i\|) = \|q-p_i\|^2$ means the cost for ASV i traveling from p_i to q.

The sensory function $\phi(q)$ respects the distribution of the interest information such as temperature, salinity and the probability of events occur over the task region. By taking this sensory function $\phi(q)$ into $\mathcal{H}(P)$, the weigh of every point in region Q is taken into consideration. In this way, the coverage strategy will pay more attention on the point with high sensory values. The method of getting the sensory function will be introduced in section 4 for detail.

According to the mathematic form of cost function in (4), the value of $\mathcal{H}(P)$ is not only associate with P, but the assignment region to W_i. Therefore, the optimization of coverage strategy is to be performed with respect to the positions of ASV and the partition of the space. Furthermore, by using the integration of the sensory function $\phi(q)$, the weigh at every point in the region is taken into consideration. In this way, the coverage network will have an optimal configuration if the proposed cost function $\mathcal{H}(P)$ reaches the minimum.

Therefore, the coverage problems of multiple ASVs in time-varying flowing environment is formulated as follows:

Problem 1: Consider n ASVs in a time-varying flowing region Q, of which the sensory function $\phi(q)$ is unknown to the coverage networks. The kinematic and dynamic motion of ASV are described by (2) and (3), respectively. With the uncertain model parameters, the purpose of this paper is to design an adaptive control law to deploy the multiple ASVs into the optimal coverage cells from arbitrary initial positions such that the cost function $\mathcal{H}(P)$ would reach the minimum.

3. STATIC VORONOI PARTITION WITH FLOWING CURRENTS

In this section, we formulate a static coverage framework for multi-ASV systems in time-varying flowing environment. Before we start the region assignment, a brief proposition in [22] would be used to deal with the optimal Voronoi cells.

Lemma 1: The optimal configuration of multi-agent system in a sensory area is the centroidal Voronoi Partition.

The Standard Voronoi region of ASV i is given by

$$V_i = \{q \in Q | \|q - p_i\| \le \|q - p_j\|, \forall i, j \in n, i \neq j\}, \quad (5)$$

where the mass M_{V_i} and the centroid C_{V_i} are shown as follows

$$\begin{cases} M_{V_i} = \int_{V_i} \phi(q) dq, \\ C_{V_i} = \frac{1}{M_{V_i}} \int_{V_i} q\phi(q) dq. \end{cases} \quad (6)$$

Consider two distinct neighbor points (p_1, p_2) in Euclidean plane. Denote $V(p_1)$ and $V(p_2)$ as the Voronoi Region generated by p_1 and p_2, respectively. The boundary between $V(p_1)$ and $V(p_2)$ is described by

$$\begin{aligned} \zeta(p_1, p_2) &\triangleq \{q \in Q: \ V(p_1) \cap V(p_2)\} \\ &= \{q \in Q: \ c(p_1, q) = c(p_2, q)\}, \end{aligned} \quad (7)$$

where $c(p_1, q)$ and $c(p_2, q)$ are the traveling cost from p_1, p_2 to q, respectively.

According to the geometric property of standard Voronoi Partition, $\zeta(p_1, p_2)$ is the bisector of p_1 and p_2, which satisfies that $\|p_1 - q\| = \|p_2 - q\|, q \in \zeta(p_1, p_2)$. However, the boundary will be warped by the time-varying currents, which is denoted by $\zeta'(p_1, p_2)$. Fig. 1 illustrates the relationship between $\zeta(p_1, p_2)$ and $\zeta'(p_1, p_2)$.

For the standard Voronoi Partition, we usually define that $f(\|p_i - q\|) = \|p_i - q\|^2$ [12]. In flowing environment, however, the motion of ASVs is either assisted or impeded by V_c. We define a novel criticism to take the currents V_c into account, which is shown by

$$c'(p_1, q) = \left\| p_1 - q + \int_{p_1}^{q} V_c(\tau) d\tau \right\|. \quad (8)$$

By defining the cost function $f(\|p_1 - q\|) = c'(p_1, q)$, it is practical to show the time cost form p_1 to q when the region is influenced by currents V_c.

Hence, the boundary curve $\zeta'(p_1, p_2)$ of two-agent systems in flowing environment is shown as follows:

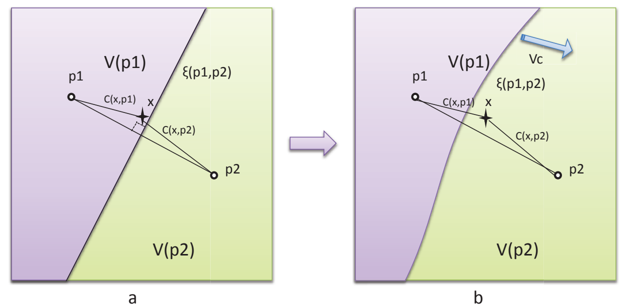

Fig. 1. Voronoi partition of two-agent systems.

Proposition 1: Consider two-ASV systems (p_1, p_2) in time-varying flowing environment Q, of which the time-varying currents V_c is described by (1). The boundary curve between $V(p_1)$ and $V(p_2)$ is provided as

$$\zeta(p_1, p_2) = \{q \in Q \mid c'(p_1, q) = c'(p_2, q)\}. \quad (9)$$

where c' is shown by (8).

Therefore, the Voronoi cell of p_1 in flowing environment is presented by

$$V(p_1) = \{q \in Q \mid c'(p_1, q) \le c'(p_2, q)\}. \quad (10)$$

For the general case in time-varying flowing environment, the Voronoi cells of multiple ASVs are geometric polygon associated with neighbors. It is necessary to find out the neighbor sets $\mathcal{N}_i, i = 1, \dots, n$ such that each ASV can calculate its Voronoi cell in time-varying flowing environment.

There are two kinds of relationship between ASV i and the others: adjacent and separated. For instance, p_1 and p_2 are adjacent if and only if $V(p_1)$ is not contained by any other Voronoi cell $V'(p_1)$, which is generated by p_1 and any other ASVs.

By comparing the ASV i with the other ASVs, it is easy to find out the neighbor sets $\mathcal{N}_i$. Depending on the two-agent case, the example of static Vornoi Partition for multi-ASV networks in time-varying flowing environment is presented in Fig 2, where the time-varying currents $V_c = [-0.6e^{-t} + 0.3t, -0.4e^{-t} + 0.6t]^T$.

In Fig. 2, the 'o' points are the positions of multi-ASV network. The solid lines represent the Standard Voronoi Partition and the dotted lines are the warped Voronoi Partition. The simulation result shows us that the standard Voronoi cells of multi-ASV systems are warped by V_c, which means the monitoring region for each ASV should be reassigned related to V_c. Therefore, it is significant to focus on the region assignment of coverage control in flowing environment.

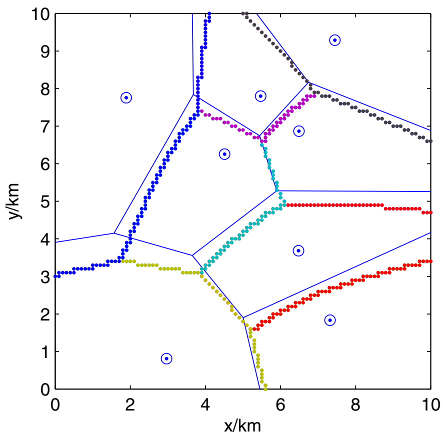

Fig. 2. Static Voronoi partition in flowing environments.

4. COVERAGE CONTROL FOR MULTI-ASV SYSTEMS WITH TIME-VARYING CURRENTS

4.1. Estimation of the sensory information

Generally, the sensory function $\phi(q)$ of information over region Q is unknown to the multi-ASV systems. In order to assign the multi-ASV networks based on local information, the Gaussian functions are introduced to estimate the sensory information.

We assume that the collected data from the sensor is accurate and the sensory function $\phi(q)$ can be parameterized as an unknown linear combination of a set of Gaussian function $\mathcal{K}(q) = [K_1(q), \ldots, K_m(q)]$, which is denoted by

$$\phi(q) = \mathcal{K}(q)^T \mathbf{a}, \ \ \exists \mathbf{a} \in \mathbb{R}^m_+. \tag{11}$$

Let $\hat{\mathbf{a}}_i$ be the approximation of parameter vector for ASV i and we obtain

$$\hat{\phi}(q)_i = \mathcal{K}^T \hat{\mathbf{a}}_i, \ \ i = 1, \ldots, n. \tag{12}$$

Define the errors of parameter vectors and sensory function as

$$\begin{aligned} &\tilde{\mathbf{a}}_i = \hat{\mathbf{a}}_i - \mathbf{a}, \\ &\tilde{\phi}_i = \hat{\phi}_i - \phi. \end{aligned} \tag{13}$$

where $i = 1, \ldots, n$.

The estimation methods in [23] are employed to approximate the sensory function in region Q, which can be described as follows:

Lemma 2: Consider a distributed estimation systems. The adaptive estimation law of parameter vector $\hat{\mathbf{a}}_i$ are proposed by

$$\begin{aligned} &\dot{\hat{\mathbf{a}}}_{pre_i} = -F_i \hat{\mathbf{a}}_i - \gamma J_i, \\ &\dot{\hat{\mathbf{a}}}_i = \Xi(\dot{\hat{\mathbf{a}}}_{pre_i} - I_{proj_i} \dot{\hat{\mathbf{a}}}_{pre_i}), \end{aligned} \tag{14}$$

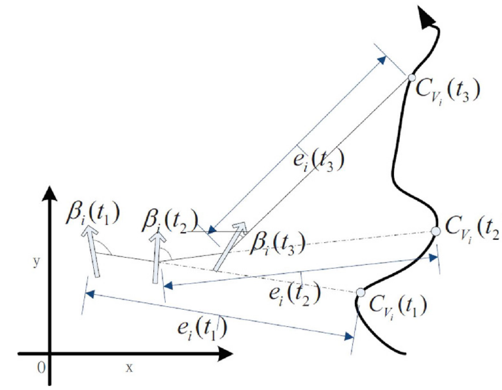

Fig. 3. Tracking proceeding of ASV i.

where $\Xi \in \mathbb{R}^{m\times m}$ is a diagonal, positive definite adaptation gain matrix. $\gamma \in \mathbb{R}_+$ is an adaptation gain scalar. F_i and J_i are described by

$$\begin{cases} F_i = \frac{\int_{V_i} \mathcal{K}(q)(q-p_i)^T dq \int_{V_i} (q-p_i)\mathcal{K}(q)^T dq}{\int_{V_i} \hat{\phi} dq}, \\ J_i = \omega(\tau)\mathcal{K}(\tau)\tilde{\phi}_i, \ \ q \in V_i, \end{cases} \tag{15}$$

where $\omega(t)$ is positive and determines the sampling weight.

The diagonal matrix I_{proj_i} is defined element-wise as

$$I_{proj_i} = \begin{cases} 0 \ \text{ for } \hat{\mathbf{a}}_i(j) > 0, \\ 0 \ \text{ for } \hat{\mathbf{a}}_i(j) = 0 \text{ and } \dot{\hat{\mathbf{a}}}_{pre_i}(j) \geq 0, \\ 1 \ \text{ otherwise}, \end{cases} \tag{16}$$

where j denotes the jth element for vector $\mathbf{a}_i$.

According to the adaptive law in (14), the estimated parameter vector $\hat{\mathbf{a}}_i$ is only related to p_i and V_i. Hence, the approximation of sensory information ϕ_i is applicable in V_i and the mathematical form of $\hat{M}_{V_i}$ and $\hat{C}_{V_i}$ are re-provided by

$$\begin{cases} \hat{M}_{V_i} &= \int_{V_i} \hat{\phi}_i(q) dq, \\ \hat{C}_{V_i} &= \frac{1}{\hat{M}_{V_i}} \int_{V_i} q \hat{\phi}_i(q) dq. \end{cases} \tag{17}$$

4.2. Kinematic controller for multi-ASVs systems

According to Lemma 1, the optimal position for ASV i is the centroid of its Voronoi cell, which can be denoted as the pursue target of ASV i. Fig. 3 illustrates the relationship of ASV i and its target along time. Denote $\beta_i \triangleq \beta_i(t)$ as the angle measured between the heading of ASV i and its target $\hat{C}_{V_i}$. $e_i \triangleq e_i(t)$ is the distance from ASV i to $\hat{C}_{V_i}$ in $\{U\}$. Therefore, the coverage problem in region Q with time-varying flowing currents can be transformed into the tracking problem with moving target $\hat{C}_{V_i}$.

The mathematical forms of e_i and β_i are

$$\begin{aligned} e_i &= \sqrt{(x_i - x_i^c)^2 + (y_i - y_i^c)^2}, \\ \beta_i &= \tan^{-1}\left(\frac{(y_i - y_i^c)}{(x_i - x_i^c)}\right), \end{aligned} \tag{18}$$

where $[x_i, y_i]^T \triangleq p_i$, $[x_i^c, y_i^c]^T \triangleq \hat{C}_{V_i}$.

It is verified that the possible strategy for the controller has to satisfy the following requirements: (1). Manipulate r_i to make that $\lim_{t\to\infty} \beta_i = 0$. This will align the direction of attitude and speed of ASV i. (2). Actuate u_i to force the position of ASV i such that $\lim_{t\to\infty} e_i = 0$. In order to satisfy these requirements, we get the following theorem:

Theorem 1: Let a coverage network consisting of n ASVs monitor the objective region Q, where the motion of ASV is described by (2) and (3). The warped partition and the sensory function are calculated by the redeveloped Voronoi partition in (10) and the Gaussian estimation in (14), respectively. Based on these two techniques, the decentralized kinematic control laws in closed-loop are proposed as

$$\begin{aligned} u_i =& k_1 e_i - V_c \cos(\psi_i - \varphi_c) + k_3 \tilde{\phi}_i, \quad i = 1, \ldots, n \\ r_i =& k_1 \sin\beta_i + \frac{V_c}{e_i}\sin(\psi_i - \varphi_c)\cos\beta_i \\ & - \frac{v_i}{e_i}\cos\beta_i + k_2\beta + \frac{\sin\beta_i}{e_i}k_3\tilde{\phi}_i, \end{aligned} \tag{19}$$

where e_i and β_i are defined in (18) and k_1, k_2, k_3 are positive constants.

Proof: Firstly, the derivative of β_i and e_i of ASV i is provided by

$$\begin{aligned} \dot{e}_i =& - u_i \cos\beta_i - v_i \sin\beta_i - V_c\cos(\beta_i - \psi_i - \varphi_c), \\ \dot{\beta}_i =& \frac{\sin\beta_i}{e_i}u_i - \frac{\cos\beta_i}{e_i}v_i - r_i + \frac{V_c}{e_i}\sin(\beta_i + \psi_i - \varphi_c). \end{aligned} \tag{20}$$

Define the Lyapunov function candidate as

$$V_k = \sum_{i=1}^{n}(\frac{1}{2}\beta_i^2 + \frac{1}{2}e_i^2) = \sum_{i=1}^{n}(V_1 + V_2), \tag{21}$$

where $V_1 = \frac{1}{2}\beta_i^2$, $V_2 = \frac{1}{2}e_i^2$.

Consider the positive definite function

$$V_1 = \frac{1}{2}\beta_i^2. \tag{22}$$

while substituting (20) into V_1, the time derivative of V_1 is given by

$$\begin{aligned} \dot{V}_1 =& \beta_i[\frac{\sin\beta_i}{e_i}u_i - \frac{\cos\beta_i}{e_i}v_i - r_i \\ & + \frac{V_c}{e_i}\sin(\beta_i + \psi_i - \varphi_c)]. \end{aligned} \tag{23}$$

According to the control law in (19), we obtain

$$\dot{V}_1 = -k_2\beta_i^2. \tag{24}$$

Thus, we conclude that β_i is bounded and converges exponentially to zero as $t \to \infty$.

Then, consider another positive definite function

$$V_{ass} = \frac{1}{2}e_i^2 + \frac{1}{2}{v_i}^2. \tag{25}$$

Take the derivative of V_{ass} and obtain

$$\dot{V}_{ass} = e_i\dot{e}_i + v_i\dot{v}_i. \tag{26}$$

Rewrite $\dot{e}_i$ and $\dot{v}_i$ in dynamics though (19)

$$\begin{aligned} \dot{e}_i &= -k_1\cos\beta_i e_i + f_i^e - v_i\sin\beta_i, \\ \dot{v}_i^r &= -\frac{1}{m_v}(m_u u_i r_i + d_i^v v_i) = -d_i v_i + g_i e_i + f_i^v, \end{aligned} \tag{27}$$

where

$$\begin{aligned} f_i^e =& V_c[\cos(\psi_i - \varphi_c)\cos\beta_i - \cos(\beta_i + \psi_i - \varphi_c)] \\ & + k_3\tilde{\phi}\cos\beta_i, \\ d_i =& \frac{d_i^v}{m_v} - \frac{V_c}{e_i}\cos(\psi_i - \varphi_c)\cos\beta_i \\ & - \frac{m_u}{m_v}[k_1\cos\beta_i - k_3\tilde{\phi}\cos\beta_i], \\ g_i =& -\frac{m_u}{m_v}(k_1^2\sin\beta_i + k_1k_2\beta_i), \\ f_i^v =& -\frac{m_u}{m_v}[k_1V_c\sin(\psi_i - \beta_i - \varphi_c) + k_2k_3\beta_i\tilde{\phi} \\ & - \frac{V_c^2}{e_i}\cos(\psi_i - \varphi_c)\sin(\psi_i - \varphi_c)\cos\beta_i \\ & - k_2V_c\cos(\psi_i - \varphi_c)\beta_i + k_1k_3\tilde{\phi}\sin\beta_i \\ & + \frac{V_c}{e_i}k_3\tilde{\phi}\sin(\psi_i - \varphi_c)\cos\beta_i]. \end{aligned} \tag{28}$$

Thus, $\dot{V}_{ass}$ can be simplified as

$$\begin{aligned} \dot{V}_{ass} =& - k_1\cos\beta_i e_i^2 + f_i^e e_i - e_i v_i \sin\beta_i \\ & - d_i{v_i}^2 + g_i v_i e_i f_i^v v_i. \end{aligned} \tag{29}$$

Since β_i is bounded, there exist $\bar{d}_T \triangleq \max_{t_0\le t\le T}|d_i(t)|$, $\bar{g}_T \triangleq \max_{t_0\le t\le T}|g_i(t)|$, $\bar{f}_T^e \triangleq \max_{t_0\le t\le T}|f_i^e(t)|$, $\bar{f}_T^v \triangleq \max_{t_0\le t\le T}|f_i^v(t)|$.

Hence, we imply that

$$\dot{V}_{ass} \le k_1e_i^2 + \bar{f}_T^e e_i + e_i|v_i| + \bar{d}_T{v_i}^2\bar{g}_T e_i|v_i| + \bar{f}_T^v|v_i|. \tag{30}$$

According to the inequality theorem $ab \le \frac{\gamma}{2}a^2 + \frac{1}{2\gamma}b^2$, we have

$$\begin{aligned} \dot{V}_{ass} \le& (1 + k_1 + \frac{\bar{g}_T}{2} + \frac{\bar{f}_T^e}{2})e_i^2 + \frac{\bar{f}_T^e}{2} + \frac{\bar{f}_T^v}{2} \\ & + (1 + \bar{d}_T + \frac{\bar{g}_T}{2} + \frac{\bar{f}_T^v}{2}){v_i}^2, \\ \le& \lambda V_{ass} + \frac{\bar{f}_T^e}{2} + \frac{\bar{f}_T^v}{2}, \end{aligned} \tag{31}$$

where $\lambda \triangleq 2\max\{1+k_1+\frac{\bar{g}_T}{2}+\frac{\bar{f}_T^e}{2}, 1+\bar{d}_T+\frac{\bar{g}_T}{2}+\frac{\bar{f}_T^v}{2}\}$.

Therefore, we conclude that e_i and v_i are bounded in finite time.

According to (27), the dynamics of e_i in closed-loop satisfies

$$\dot{e}_i = -k_1 \cos\beta e_i + \varepsilon_i, \tag{32}$$

where

$$\begin{aligned}\varepsilon_i =& V_c[\cos(\psi_i - \varphi_c)\cos\beta_i - \cos(\beta_i + \psi_i - \varphi_c)] \\ &+ k_3\tilde{\phi}\cos\beta_i - v_i \sin\beta_i.\end{aligned} \tag{33}$$

Since $\lim_{t\to\infty}\beta_i = 0$ and v_i^r is bounded, we imply that $\lim_{t\to\infty}\varepsilon_i = 0$.

Take the derivative of V_2 and obtain

$$\begin{aligned}\dot{V}_2 &\le -k_1 e_i^2 + e_i\varepsilon_i, \\ &\le -k_1(1-\delta)e_i^2 - \delta e_i^2 + e_i\varepsilon_i, \\ &\le -k_1(1-\delta)e_i^2, \ \ \forall |e_i| \ge \frac{|\varepsilon_i|}{k_1\delta},\end{aligned} \tag{34}$$

where $k_1 > 0$, $0 < \delta < 1$.

Thus, e_i is input-to-stable(ISS) system with ε_i as input [24]. Moreover, since $\lim_{t\to\infty}\varepsilon_i = 0$, we have that $\lim_{t\to\infty} e_i = 0$, which shows us $\lim_{t\to\infty}\|p_i - C_{V_i}\| = 0$.

By using these proposed results, we conclude that the multi-ASV coverage networks will converge to the optimization configuration in finite time from arbitrary initial positions. □

4.3. Adaptive dynamic controller for multi-ASVs systems

Due to the uncertainty of ASV model parameters in time-varying following environment, the adaptive backstepping techniques are employed to ensure the robustness and controllability of the multi-ASV systems.

Let α_i^1 and α_i^2 be the ideal control inputs for ASV i, which can be viewed by (19). Define the auxiliary variables z_{1i} and z_{2i} as

$$\begin{aligned}z_{1i} &= u_i - \alpha_i^1, \\ z_{2i} &= r_i - \alpha_i^2,\end{aligned} \tag{35}$$

where u_i and r_i are the virtual control inputs.

Denote the model parameters set of ASV i as

$$\begin{aligned}\Theta_i \triangleq& [m_u, m_v, m_{uv}, m_r, d_u, d_r, \frac{m_u}{m_v}m_r, \frac{d_v}{m_v}m_r]^T \\ \triangleq& [\theta_i^1, \dots, \theta_i^8]^T.\end{aligned} \tag{36}$$

Let $\hat{\Theta}_i$ be the approximation of model parameters and define that

$$\tilde{\Theta}_i \triangleq \Theta_i - \hat{\Theta}_i, \tag{37}$$

where $\tilde{\Theta}_i$ are the estimation errors.

Define the Lyapunov function candidate by

$$\begin{aligned}V_{adp} =& \sum_{i=1}^{n}(\frac{1}{2}\beta_i^2 + \frac{1}{2}z_{1i}^2 + \frac{1}{2}z_{2i}^2 + \frac{1}{2}\tilde{\Theta}_i^T\Gamma_i^{-1}\tilde{\Theta}_i) \\ =& \sum_{i=1}^{n} V_{adp_i}.\end{aligned} \tag{38}$$

Based on the dynamic motion of ASV in (3), the time derivative of V_{adp_i} is given by

$$\begin{aligned}\dot{V}_{adp_i} =& z_{1i}[\tau_i^u + m_v v_i r_i - d_u u - m_u\dot{\alpha}_i^1 + \frac{\sin\beta_i}{e_i}\beta_i] \\ &+ z_{2i}[\tau_i^r + m_{uv}u_i v_i - d_r r_i - m_r\dot{\alpha}_i^2 - \beta_i] \\ &- k_2\beta_i^2 - \tilde{\Theta}_i^T\Gamma_i^{-1}\dot{\hat{\Theta}}_i.\end{aligned} \tag{39}$$

where τ_i^u and τ_i^r are the dynamic control inputs of ASV i.

In order to make the indefinite terms in (39) cancel out, we provide the dynamic control laws by

$$\begin{aligned}\tau_i^u =& -\hat{\theta}_i^2 v_i r_i + \hat{\theta}_i^5 u_i + \hat{\theta}_i^1\dot{\alpha}_i^1 - \frac{\sin\beta_i}{e_i}\beta_i - k_4 z_{1i}, \\ \tau_i^r =& -\hat{\theta}_i^3 u_i v_i + \hat{\theta}_i^6 r_i + \hat{\theta}_i^4\dot{\alpha}_i^{2a} + \hat{\theta}_i^7 u_i r_i\frac{\cos\beta_i}{e_i} \\ &+ \hat{\theta}_i^4(\frac{v_i\dot{\beta}_i}{\sin}\beta_i e_i + \frac{v_i\dot{e}_i\cos\beta_i}{e_i^2}) + \beta_i \\ &+ \hat{\theta}_i^8 v_i\frac{\cos\beta_i}{e_i} - k_5 z_{2i},\end{aligned} \tag{40}$$

where

$$\begin{aligned}\alpha_i^{2a} =& k_1\sin\beta_i + \frac{V_c}{E_i}\sin(\psi_i - \varphi_c)\cos\beta_i \\ &+ k_2\beta + \frac{\sin\beta_i}{e_i}k_3\tilde{\phi}.\end{aligned} \tag{41}$$

Thus, we get the mathematical form of V_{adp_i} as follows

$$\dot{V}_{adp_i} = -k_2\beta_i^2 - k_4 z_{1i}^2 - k_5 z_{2i}^2 + \tilde{\Theta}_i[\Lambda_i - \Gamma^{-1}\dot{\hat{\Theta}}_i], \tag{42}$$

where Λ_i is shown by

$$\begin{aligned}\Lambda_i \triangleq \mathrm{diag}\Big\{& -z_{1i}, z_{1i}v_i r_i, z_{2i}u_i v_i, b_i, -z_{1i}u_i, -z_{2i}r_i, \\ & -z_{2i}u_i r_i\frac{\cos\beta_i}{e_i}, -z_{2i}v_i\frac{\cos\beta_i}{e_i}\Big\}.\end{aligned} \tag{43}$$

where $b_i = -(\dot{\alpha}_i^{2a} + \frac{v_i\sin\beta_i\dot{\beta}_i}{e_i} + \frac{v_i\cos\beta_i\dot{e}_i}{e_i^2})$.

Based on the simplified equation in (42), the adaptive control laws for the model parameters of ASV i are proposed by

$$\dot{\hat{\Theta}}_i = \Gamma_i\Lambda_i. \tag{44}$$

Therefore, we obtain

$$\dot{V}_{adp_i} = -k_2\beta_i^2 - k_4 z_{1i}^2 - k_5 z_{2i}^2 \le 0. \tag{45}$$

It can be concluded that $(\beta_i, z_{1i}, z_{2i})$ will converge to zero as $t \to \infty$, which can be used to proof the following theorem.

Theorem 2: Let a coverage network consisting of n ASVs monitor the objective region Q, where the motion of ASV is described by (2) and (3). The warped partition and the sensory function are calculated by the redeveloped Voronoi partition in(10) and the Gaussian estimation in (14), respectively. To deal with the model parameters uncertainty, an adaptive update law in (44) is provided to estimate the system model parameters. Based these techniques, the dynamic controllers of the multi-ASV systems are proposed in (40) such that the coverage network will converge to the optimal partition from arbitrary initial positions in finite times.

Proof: Define the Lyapunov function candidate V_{adp} as (38). By resorting to (45), we can easily obtain that

$$\dot{V}_{adp} \leq 0. \tag{46}$$

Based on the LaSalle's invariance principle, the multi-ASV coverage networks will converge to the optimization positions in region Q with time-varying currents and model parameters uncertainty. □

5. SIMULATIONS

Simulation studies are displayed to illustrate the performance of the multi-ASV coverage networks in flowing environment. Consider 8 ASVs in 10km × 10km region, where the time-varying currents $V_c = [(\exp^{0.6t} + 0.2)\exp^{-0.6t}, (\exp^{0.4t} + 0.3)\exp^{-0.4t}]$. The kinematic and dynamic motion of ASV are briefly described by (2) and (3), respectively.

According to the estimation laws in **Lemma** 2, region Q is divided into 3×3 unit square and 9 Gaussian functions $\mathcal{K} = [\mathcal{K}(1), \dots, \mathcal{K}(9)]$ are chosen to parameterized the sensory function. Each component of Gaussian functions is implemented by

$$\mathcal{K}(j) = \frac{1}{\sigma_j\sqrt{2\pi}} \exp\left\{-\frac{(q-\mu_j)^2}{2\sigma_j^2}\right\}, \quad j = 1, \dots, 9,$$

where μ_j is the center of each grid square and $\sigma(j) = [2.0, 2.3, 2.4, 2.6, 3.6, 2.9, 2.6, 1.5, 3.0]^T$.

Then, the initial value of parameters in the adaptive control laws are: $J(0) = [0,0,0,0,0,0,0,0,0]^T$, $\hat{\Theta}(0) = \text{diag}\{0,0,0,0,0,0,0,0\}$; The control gain parameters for each ASV are selected as follows: $k_1 = 0.4$, $k_2 = 0.8$,$k_3 = 1.3$,$\gamma = 3000$, $k_4 = 1000$, $k_5 = 500$, $\Gamma = \text{diag}\{10, 20, 30, 1.3, 3.5, 3.6, 5.2, 4.9\} \times 10^3$.

Simulation results of the multi-ASV coverage networks in time-varying flowing environment are shown in Fig. 4 and Fig.5.Fig.4 displays the approximation of the sensory information over region Q, which shows us the distribution of the probability where event occurs. Fig. 5 illustrates us that the errors between the estimated and accurate Voronoi area of multi-ASV systems is limited in the range $[0, 100]$ and converge to zeros, which means that the estimated sensory function for each vehicle will converge to the accurate value of the interest information in finite time.

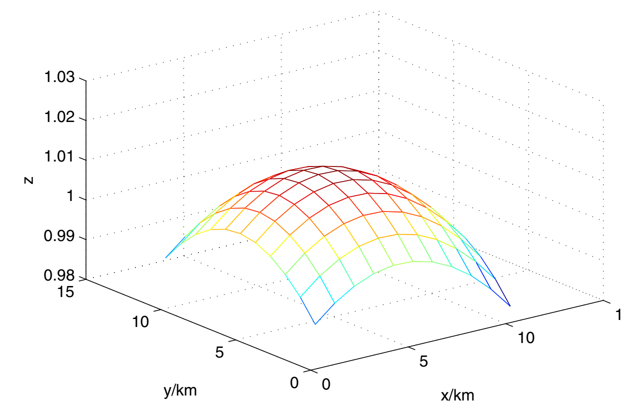

Fig. 4. Estimated sensory function.

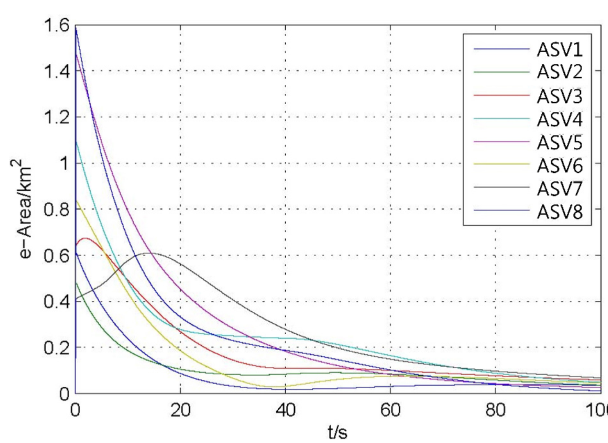

Fig. 5. Estimated errors of Voronoi cells.

The proceeding of multi-ASV coverage control strategy from arbitrary positions is illustrated in Fig. 6. In Fig. 6, the 'o' points denote the initial positions of multi-ASV systems and '+' points are the final optimal positions. The moving trajectories of multi-ASV systems are presented by the dash lines. The solid partition lines represent the standard Voronoi Partition of the coverage network and the dot lines are redeveloped Voronoi boundary in time-varying flowing environment.

Note that the sensory function is estimated through sampled data online, the partition of coverage network is time-varying based on the moving position of the multi-ASV systems. Therefore, the presented partition of coverage networks in Fig. 6 is temporary, which is the optimization for coverage network at final time. The simulation result in Fig 6 shows us that even though the Voronoi cells of multi-ASV systems in region Q are actually influenced by the currents, the multi-ASV coverage networks

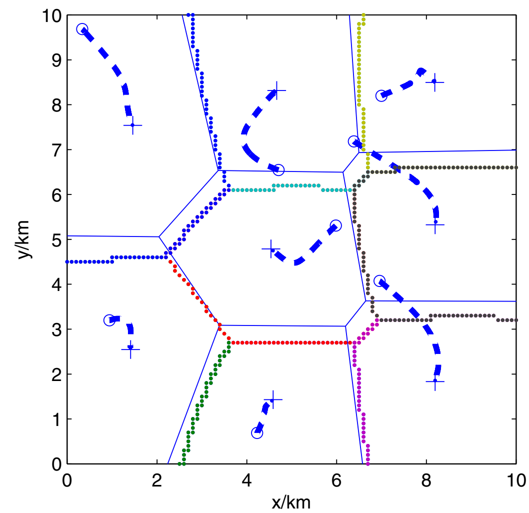

Fig. 6. Proceeding of multi-ASV coverage control systems in flowing environment.

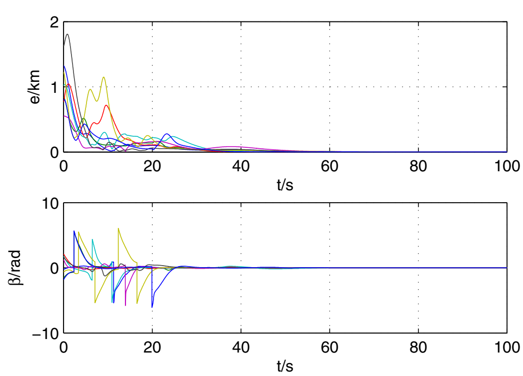

Fig. 7. The time evolution of tracking errors (e and β).

can still converge to the optimization configuration.

The time evolution of the tracking errors (e, β) and the control inputs (τ_u, τ_r) are shown in Fig. 7 and Fig. 8, respectively. In Fig. 7, we observe that the e and β of the multi-ASV systems converge to zeros in finite time, which means that the multiple ASVs reach the centroid of its Voronoi cell. Furthermore, the control inputs of the multi-ASV systems in Fig 8 present the corresponding action tendency with the tracking errors and show us the stabilities of the closed-loop coverage systems.

6. CONCLUSION

This paper has addressed the coverage problems of multi-ASV systems in time-varying flowing environment. A decentralized coverage algorithm was proposed such that the multiple ASVs could be navigated to the assigned cells optimally. In order to find out the influence of the time-varying currents to the coverage networks, we presented the static Voronoi boundaries for multi-ASV networks to reconstruct the Voronoi cells by extending the two-ASV case. In addition, Gaussian functions were employed to estimate the sensory function such that the multi-ASV networks could spread over the region based on the interest information. By transforming the coverage problems into the pursue ones, a kinematic control law was provided to drive the ASVs to the optimal positions. Moreover, we proposed an adaptive dynamic control algorithm to deal with the uncertainty of the model parameters by using the backstepping techniques. In this framework, simulation results were provided to show the feasibility and stability of the proposed approaches.

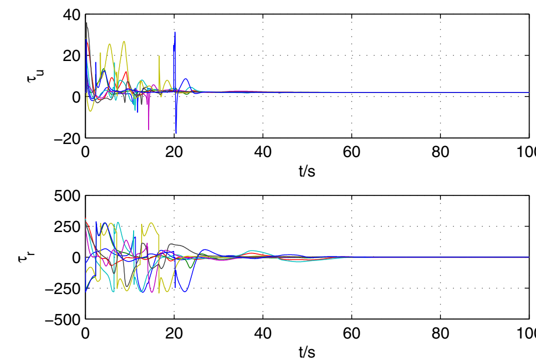

Fig. 8. The time evolution of control inputs (τ_u and τ_r).

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