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Operator-based Robust Control for Nonlinear Uncertain Systems with Unknown Backlash-like Hysteresis

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Abstract: In this paper, operator based robust control for nonlinear uncertain system with unknown backlash-like hysteresis is considered. In detail, a continuous backlash-like hysteresis operator is proved to be corresponding to a one-to-one operator, that is, it is suitable to be used in operator theoretic based control theory. Moreover, an internal model control (IMC) structure with one parallel compensating operator is proposed for nonlinear uncertain system with unknown backlash-like hysteresis. Based on the proposed control scheme, the designed system is robustly stable and the desired output tracking performance can be realized simultaneously. Finally, a simulation example about nonlinear plant preceded by backlash is given to show the design procedure of the proposed method.

Keywords: Nonlinear uncertain systems, operator based robust right coprime factorization, robust nonlinear control, unknown backlash-like hysteresis.

1. INTRODUCTION

Hysteresis phenomenon often occurs in practical systems due to the wide application of the smart material-based actuators and sensors. From control systems point of view, hysteresis is generally non-differentiable, nonlinear, non-smooth, non-symmetric and unknown. As a result, systems preceded by hysteresis usually exhibit undesirable inaccuracies or oscillation, severely degrades system performance and decreases the system response. Therefore, mitigating the effect of hysteresis becomes necessary and important, and it has received increasing attention in recent years [1-12]. Until now, modeling and control issues relating to hysteresis are important topics and still full of challenge due to its physical properties.

It is important to find a model which can describe the nonlinear behavior of the hysteresis process and be utilized for control design. The most common strategy for dealing with hysteresis is the virtual cancelation of this nonlinearity inverting an opportune hysteresis model able to represent the input-output behavior. To ensure an exact cancelation of the hysteresis, the hysteresis model has to precisely represent it. This motivated the introduction of many different models, and a number of mathematical models for hysteresis have been proposed, such as Duhen model [7], Preisach model [8], Krasnosel'skii-Pokrovkii hysteresis model [9], Prandtl-Ishlinskii hysteresis operator [7], Bouc-Wen differential model [11] etc. However, most of the models allow to represent a specific class of hysteresis with given properties. It turns out that in many practical cases those properties do not hold, leading to inaccuracy on hysteresis reconstruction. Therefore, effective control techniques for the case that the hysteresis model is not accurate enough is a typically challenging topic [2-6].

Control of nonlinear system has become mainstream consumer products within the last decades, providing a significant growth opportunity for the mechanical and electrical engineering, computer science and even biological engineering systems etc. So far, there are several significant techniques for analyzing nonlinear systems, for example, Lyapunov function method, sliding mode control method and fuzzy-based method, etc. are proposed [13-16]. In view of the input-output nature of the nonlinear system concept itself, it seems useful to establish computer-oriented approaches to nonlinear control systems analysis and design. Addressing this problem, robust right coprime factorization technique of nonlinear operators, in addition to the above significant techniques which is based on real and complex variable theory, has been a promising technique, where the operators can be either linear or nonlinear, continuous time or discrete time, finite dimensional or infinite dimensional, and can be either in

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the frequency domain or in time domain provided that the formulation is meaningful for the physical problems under investigation [17-24].

Factorization approach was introduced into control system analysis and synthesis by the factorization of transfer matrices since 1980s, which was used for discussing system analysis, design, stabilization, and control. In [18], the authors investigated right coprime factorization of a nonlinear plant in a fairly general operator-theoretic setting. For considering the robustness of nonlinear systems with uncertainties, Deng *et al*. proposed a robust condition on right coprime factorization for nonlinear systems with unknown bounded uncertainty in [19], where, the robustness for the right coprime factorization of a nonlinear plant is guaranteed by one Bezout identity and an inequality, which leads to the robust bounded-input bounded-output (BIBO) stability of the entire feedback control system. The formulated approach is different from traditional approaches in that it links the robustness of the right coprime factorization to the robust stability, which provides new ideas for robust stabilization design of general feedback control systems. In [24], the realization of right factorization problem was considered by using isomorphism, and the existence of two stable controllers was proved satisfying the Bezout identity. Moreover, the output tracking problem was considered in [20], in which, a control system was designed by adding a feedback regulating operator, guaranteeing the robust stability and realizing output tracking performance simultaneously. In summary, operator based robust right coprime factorization approach uses the input-output time function model given by basic physical rules from the real system. In addition, the robustness against significant (constant and time-varying) uncertainties can be guaranteed by one Bezout identity and an inequality. That is, factorization approach provides a relative simple and convenient framework for robust control design of nonlinear uncertain system.

For nonlinear uncertain system with hysteresis, operator based robust right coprime factorization approach has been used to consider nonlinear control for systems with backlash hysteresis (see [22]). However, in operator theory, the hysteresis operator needs to be continuous. Therefore, the following issues will be considered in this paper: i) we will find a continuous backlash-like hysteresis model which is suitable for the application in operator theoretic based control design. ii) The robust control for nonlinear uncertain systems with the proposed backlash-like hysteresis will be considered. In details, for the unknown uncertainties, the sufficient conditions for the system remaining robustly stable and realizing output tracking performance will be discussed.

The remainder of the paper is organized as follows. In Section 2, some relevant notes on operator theory are introduced and problem statement is introduced. In Section 3, the backlash-like hysteresis operator is discussed. Moreover, an operator based internal model like control system design for nonlinear uncertain plant with backlash-like hysteresis is proposed. A numerical example for nonlinear plant with backlash is presented and simulation is done to support the theoretical analysis in Section 4 and conclusion is drawn in Section 5.

2. PRELIMINARIES AND PROBLEM STATEMENT

2.1. Preliminaries

Let X and Y be linear spaces over the field of real numbers, and let X_s and Y_s be normed linear subspaces, called the stable subspaces of X and Y, respectively, defined suitably by two normed linear spaces under certain norm $X_s = \{x \in X : \|x\| < \infty\}$ and$Y_s = \{y \in Y : \|y\| < \infty\}$. Let $Q : X \to Y$ be an operator mapping from X to Y, and denote by $D(Q)$ and $R(Q)$, respectively, the domain and range of Q. Generally, an operator is said to be bounded input bounded output (BIBO) stable or simply, stable if $Q(X_s) \subseteq Y_s$. Otherwise, namely, if Q maps some inputs from X_s to the set $Y \backslash Y_s$ (if not empty), then Q is said to be unstable [18].

Let $S(X,Y)$be the set of stable operators mapping from$X$to$Y$. Then, $S(X,Y)$contains a subset defined by

$$u(X,Y) = \{M : M \in S(X,Y)\}, \tag{1}$$

where, $M$is invertible with$M^{-1} \in S(Y,X)$. Elements of $u(X,Y)$ are called unimodular operators. In this paper, the operators noted are generalized Lipschitz operators defined on extended linear space as ([19])

$$\begin{aligned} \|Q\|_{Lip} &:= \|Q(x_0)\|_Y + \|Q\| \\ &= \|Q(x_0)\|_Y + \sup_{T\in[0,\infty)} \sup_{\substack{x,\tilde{x} \in X \\ x_T \neq \tilde{x}_T}} \frac{\|[Q(x)]_T - [Q(\tilde{x})]_T\|_Y}{\|x_T - \tilde{x}_T\|_X}. \end{aligned} \tag{2}$$

2.2. Problem statement

Backlash is the most familiar and simple hysteresis model which is described by two parallel lines connected via horizontal line segments, the figure can be drawn as Fig. 1 and can be described by

$$\begin{aligned} u^* &= B_a(u)(t) \\ &= \begin{cases} m(u(t)-h) & \text{if } \dot{u} > 0 \text{ and } u^* = m(u(t)-h) \\ m(u(t)+h) & \text{if } \dot{u} < 0 \text{ and } u^* = m(u(t)+h) \\ u(t_i) & \text{otherwise,} \end{cases} \end{aligned} \tag{3}$$

where $m > 0$ is the slopes of lines, $h > 0$ is the threshold value, namely, the backlash distance. $u(t_i)$ means no change occurs in the output $u(t)$. For explaining conveniently, the corner points are marked in the figure. That is,

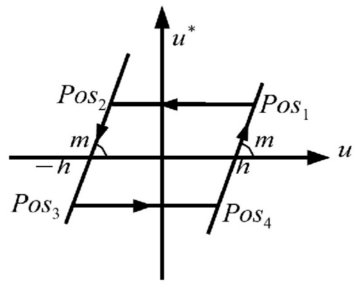

Fig. 1. One backlash hysteresis operator.

when u increases, u^* will vary along Pos3→Pos4→Pos1; when u decreases, u^* will vary along Pos1→Pos2→Pos3. This model requires the input u is piecewise monotone, and itself is discontinuous and may not be amenable to controller design.

Since in operator theory based control design, continuous operator is required. There, a continuous-time dynamic model is given to describe the backlash-like hysteresis [3]

$$\frac{du^*}{dt} = a\left|\frac{du}{dt}\right|(cu-u^*)+b\frac{du}{dt}, \tag{4}$$

where a, c and b are constants, satisfying $c>b$. This model can be solved explicitly for u piecewise monotone

$$u^*(t) = cu(t)+du \tag{5}$$

with

$$\begin{aligned} du =&(u_0^*-cu_0)e^{-a(u-u_0)sgn\dot{u}} \\ &+e^{-ausgn\dot{u}}\int_{u_0}^{u}(b-c)e^{as(sgn\dot{u})}ds \end{aligned} \tag{6}$$

for $\dot{u}$ constant and $u^*(u_0)=u_0^*$. Analyzing (5), we see that it is composed of a line with the slope c, together with a term $d(u)$. It can be rewritten as

$$\begin{aligned} u^* &= B_a(u)(t) \\ &= \begin{cases} cu(t)+(u_0^*-cu_0)e^{-a(u-u_0)} \\ \quad +\dfrac{b-c}{a}(1-e^{-au(u-u_0)}) & \dot{u}>0 \\ cu(t)+(u_0^*-cu_0)e^{a(u-u_0)} \\ \quad -\dfrac{b-c}{a}(1-e^{au(u-u_0)}) & \dot{u}<0. \end{cases} \end{aligned} \tag{7}$$

The model implies that

$$\lim_{u\to\infty}(B_a(u)(t)-cu(t)) = -\frac{c-b}{a}, \tag{8}$$

$$\lim_{u\to-\infty}(B_a(u)(t)-cu(t)) = \frac{c-b}{a}. \tag{9}$$

That is, $B_a(u)$ exponentially converges the output of a play operator with threshold $-\frac{c-b}{a}$ and switches between lines $cu-\frac{c-b}{a}$ and $cu+\frac{c-b}{a}$. Solution (7) and its implied properties (8) and (9) show that $u^*(t)$ eventually satisfies the first and second conditions of (3). Furthermore, setting $\dot{u}=0$ results in $\dot{u}^*=0$, which satisfies the last condition of (3). This implies that the dynamic equation (7) can be used to model a class of backlash-like hysteresis and is an approximation of backlash hysteresis (3).

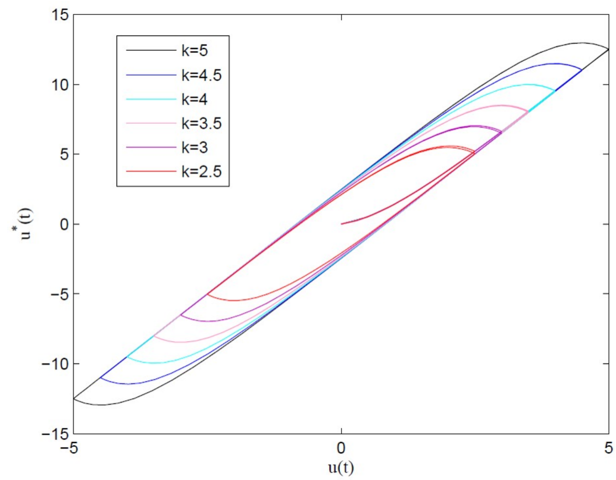

Fig. 2. One backlash hysteresis operator.

Fig. 2 shows that model (4) indeed generates backlash-like hysteresis curves, where $a=1$, $c=3$, $b=0.5$ for $u(t)=k\sin(2t)$ with the initial condition $u(0)=0, u^*(0)=0$, and $k=2.5,3,3.5,4,4.5,5$, respectively. It should be mentioned that the parameter a determines the rate at which $u^*(t)$ switches between the two lines. The larger the parameter a is, the faster the transition in $u^*(t)$ is going to be. Also, the backlash distance is determined by $\frac{c-b}{a}$ and the parameter must satisfy $c>b$.

3. ROBUST CONTROL DESIGN FOR NONLINEAR SYSTEMS WITH BACKLASH-LIKE HYSTERESIS

In this section, robust control for nonlinear systems preceded by backlash-like hysteresis defined as (7) will be considered by using operator based control approach. For considering that, two important issues should be considered. The first one is whether operator based approach can be used to consider the nonlinear control system design preceded by the introduced backlash-like hysteresis. And another is how to design the robust nonlinear control system.

3.1. Property of backlash-like hysteresis

Lemma 1: Let $u(t)\in U_S$, for the different time instances t_1 and t_2 $(t_1\neq t_2)$, but $u(t_1)=u(t_2)$, it leads to $B_a(u)(t_1)\neq B_a(u)(t_2)$ when $c<0$, where $u(t_1)$ and $u(t_2)$ are not the extremums.

Proof: Without losing generality, $B_a(u(0)=0)=0$, (7) becomes

$$B_{ain}(u)(t) = cu(t) + \frac{b-c}{a}(1-e^{-au}), \quad \dot{u} > 0, \tag{10}$$

$$B_{ade}(u)(t) = cu(t) - \frac{b-c}{a}(1-e^{au}), \quad \dot{u} < 0, \tag{11}$$

where, $B_{ain}(u)$ and $B_{ade}(u)$ denote the segment $u(t)$ increases and decreases respectively. When $c < 0$, it has

$$\frac{dB_{ain}(u)}{du} = c + (b-c)e^{-au} < 0, \tag{12}$$

$$\frac{dB_{ade}(u)}{du} = c + (b-c)e^{au} < 0, \tag{13}$$

therefore, B_{ain} and B_{ade} are monotonic. □

Then, based on the analysis, we conclude that for two facultative time instances t_1 and t_2 ($t_1 \neq t_2$), even though $u(t_1) = u(t_2)$, $B_a(u)(t_1) \neq B_a(u)(t_2)$.

Remark 1: Suppose that $H(u)(t)$ is defined as the output of backlash hysteresis. When $B_a(u)(t)$ and $H(u)(t)$ are fed with the same input $u(t)$, the curve of $B_a(u)(t)$ exhibits behavior similar to that of $H(u)(t)$ such as ascending, turning, and descending.

Lemma 2: If there exist two time instances t_1 and t_2, also $t_1 \neq t_2$, such that $B_a(u)(t_1) - B_a(u)(t_2) \to 0$, then $u(t_1) - u(t_2) \to 0$. □

The fact that B_{ain} and B_{ade} are continuous implies the result, the proof is omitted here. Thus, based on the above two lemmas, the following result is derived.

Theorem 1: For the hysteresis operator B_a, there exists a continuous one-to-one mapping Γ, such that $H(u)(t) = \Gamma(u(t), B_a(u)(t))$.

Proof: Firstly, it is proved that Γ is a one-to-one mapping.

Situation 1: Assume that $u(t)$ is not the extremum. In terms of Lemma 1, if there exist two different time instances t_1 and t_2, then

$$(u_1(t), B_a(u)(t_1)) \neq (u_2(t), B_a(u)(t_2)). \tag{14}$$

Therefore, the coordinate $(u(t), B_a(u)(t))$ is uniquely corresponding to hysteresis operator $H(u)(t)$.

Situation 2: Suppose that $u(t)$ is the extremum. In this case, for two different time instances $t_1$and t_2, there will be

$$(u_1(t), B_a(u)(t_1)) = (u_2(t), B_a(u)(t_2)). \tag{15}$$

According to the property of the hysteresis, $H[u(t_1)] = H[u(t_2)]$. Then the coordinate $(u(t),\ B_a(u)(t))$ will be uniquely corresponding to $H(u)(t)$. □

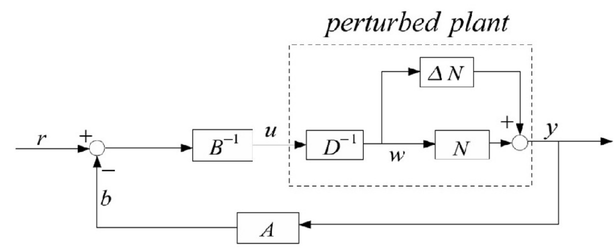

Fig. 3. A perturbed nonlinear feedback system.

Combining the above-mentioned two situations, it is obtained that $\Gamma$is a one-to-one mapping and is continuous.

Remark 2: The properties of the backlash-like hysteresis imply that it is generalized Lipschitz operator, that is, the proposed hysteresis operator is suitable to the application on the robust control by using operator based robust right coprime factorization approach.

3.2. Operator based robust control system design

Before the main results, some basic knowledge on operator theory will be introduced.

A normal operator based nonlinear feedback control for nonlinear uncertain system is shown in Fig. 3, where, the nominal plant and plant perturbation are P and ΔP, respectively, and the overall plant $\tilde{P}$ is $\tilde{P} = P + \Delta P$, $u \in U$ and $y \in V$ are control input and plant output respectively, U and V are used to denote the input and output spaces of a given plant operator P, i.e., $P : U \to V$.

The given plant operator $P : U \to V$ is said to have a *right factorization*, if there exist a linear space W and two stable operators $D : W \to U$ and $N : W \to V$such that D is invertible from U to W, and $P = ND^{-1}$ on U, where, the space W is called a quasi-state space of P. In the right factorization of P, D^{-1} is unstable for the case of P being unstable. That is, N is the stable part of the plant, and the unstable part is included in D^{-1} if the plant is unstable. Moreover, the factorization is said to be *coprime*, or P is said to have a *right coprime factorization,* if there exist two stable operators $A : V \to U$ and $B : U \to U$, satisfying the *Bezout identity*

$$AN + BD = M, \text{ for some } M \in u(W,V), \tag{16}$$

where B is invertible. Usually, P is unstable and (N, D, A, B) are to be determined.

Based on the above definitions, the right factorization of the nominal plant P and the overall plant $\tilde{P}$ are $P = ND^{-1}$, $P + \Delta P = (N + \Delta N)D^{-1}$, where N, ΔN and D are stable operators, D is invertible, $\Delta N$is unknown but the upper and lower bounds are known. Then the stability of the nonlinear feedback control system with perturbation can be guaranteed if the Bezout identity is satisfied and

$$\left\| [A(N + \Delta N) - AN]M^{-1} \right\| < 1. \tag{17}$$

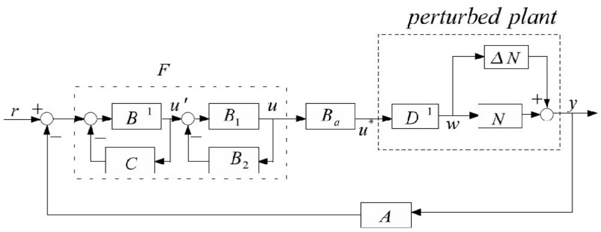

Fig. 4. Proposed control design for nonlinear uncertain system with backlash-like hysteresis.

It's worth to mention that the initial state should also be considered, that is, $AN(w_0,t_0)+BD(w_0,t_0) = M(w_0,t_0)$ should be satisfied. In this paper, we select $t_0 = 0$ and $w_0 = 0$.

Considering the tracking design of the uncertain system shown in Fig. 3, many design schemes were proposed (see [19], [20]). For realizing the perfect tracking performance, nonlinearity, uncertainty and the uncertain hysteresis make it more difficult to design the control system. Therefore, tracking problem is still a challenging problem.

In [23], one regulating operator is designed for compensating the uncertainties and hysteresis, the system is shown as Fig. 4, where, the controller F is constructed by control operator B, regulating operator C, B_1 and B_2 are organized by the known information from backlash-like hysteresis, and are defined as

$$B_1(u) = \frac{u}{c}, \tag{18}$$

$$B_2(u) = d(u). \tag{19}$$

which implies that

$$u'(t) = B_1^{-1}(u)(t) + d(u)(t) = B_a(u)(t) \tag{20}$$

it follows that $u'(t) = u^*(t)$, that is, the effects from hysteresis are compensated, then the other effects from hysteresis is considered as uncertainty and is regulated by C.

As proved in [23], the nonlinear uncertain system with backlash-like hysteresis shown in Fig. 4 is robustly stable and the output tracking performance can be realized under the conditions that

$$N_s + N_u = N, \tag{21}$$

$$\left\|(N_s+\Delta N)N_u^{-1}\right\| < 1, \tag{22}$$

$$A(N)(w) = N(w) + G(w), \tag{23}$$

$$AN_u + (B+C)D = N_u, \tag{24}$$

where N_s is stable operator, N_u is unimodular operator, G is stable and C is regulating operator to be designed.

However, there are two difficult issues existing in the control design of [23]. One is that B_1 and B_2 are obtained by the backlash-like hysteresis, but for modeling error existed in the system, it should be emigrated by the regulating operator C, which induces difficulties into controller design. Another issue is that there is difficulty in designing controller A satisfying (23) in practice, because that the quasi-state w should be predicted, which cause complexity occurs.

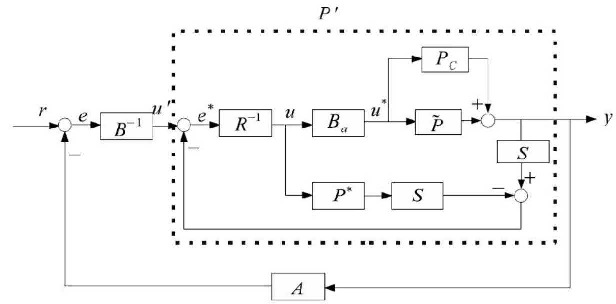

Fig. 5. Proposed control design for nonlinear uncertain system with unknown hysteresis.

Under this account, an internal model like control structure is designed as Fig. 5, where, A, B, R and S are controllers to be designed. Generally, N is non-invertible, however, in this control design, the nominal plant is needed to be invertible. For realizing that, one parallel compensating operator P_C is added in the controlled plant P with $P_C = CD^{-1}$, where, C is stable. Then, the composition plant $P^* = P + P_C = (N+C)D^{-1}$ can be proved having the following properties.

Theorem 2: The composition plant $P^* = P + P_C$ is invertible and its inverse is stable provided that

$$\left\|N_s N_u^{-1}\right\| < 1, \tag{25}$$

where, N_u is unimodular operator and N_s is stable operator such that

$$N + C = N_u + N_s. \tag{26}$$

Proof: According to [20], we can get that $N+C = N_u+N_s$ is unimodular under the condition of (25), then the composition operator $P^* = P+P_C = (N+C)D^{-1} = N^*D^{-1}$is invertible. Based on the definition of unimodular, $(N+C)^{-1}$ is stable, then the inverse of $P^{-1}$is stable. This completes the proof. □

Then, we can prove that the influence of the hysteresis and uncertainty can be rejected by the compensating operator P_C and the controllers R and S.

Theorem 3: As for the proposed control design shown in Fig. 5, the real plant $\tilde{P}$ and the model P have right factorization as $\tilde{P} = \tilde{N}D^{-1} = (N+\Delta N)D^{-1}$ and $P = ND^{-1}$ respectively, P_C is compensating operator such that $P_C = CD^{-1}$, A, B, R and S are operators to be designed. Then, the block diagram P' in the dashed line is equivalent to P^*

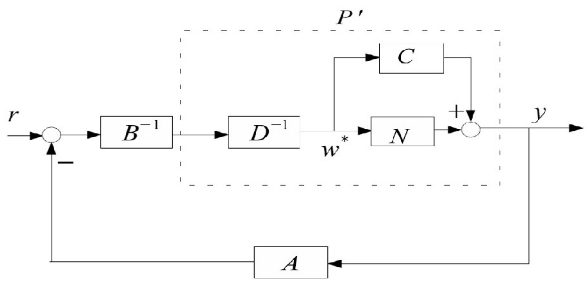

Fig. 6. The equivalent system of Fig. 5.

provided that

$$R = I, \tag{27}$$

$$SP^* = I, \tag{28}$$

where C satisfying the condition (25).

Proof: Based on Theorem 2, P^* is invertible and its inverse is stable under (25). Figure 5 implies that

$$u'(t) = R(u)(t) + S(y)(t) - SP^*(u)(t). \tag{29}$$

Based on (27) and (28), it follows that

$$y(t) = P^*(u')(t).$$

This completes the proof. □

Theorem 3 follows that $P' = P^*$, that is, based on the sufficient conditions, the effects from the uncertainty and hysteresis can be removed.

After that, the whole system can be controlled robustly stable and the output tracking performance can be realized by designing the controllers A and B.

Theorem 4: As for the proposed control design shown in Fig. 5, the system is robustly stable and the output tracking performance can be realized simultaneously if

$$A(N+C) + BD = N + C, \tag{30}$$

where C satisfies (25) and (26).

Proof: Based on Theorem 2, P' in Fig. 5 is equivalent to P^*, then the system can be rewritten as Fig.6. From (29), the system has that

$$\begin{aligned} r(t) &= [A(N+C)+BD](w^*)(t) \\ &= [N+C](w^*)(t), \end{aligned} \tag{31}$$

then the system is stable in virtue of the stability of the operators N and C. Also, the output tracking performance is realized simultaneously because of the fact that $y(t) = [N+C](w^*)(t)$. □

Remark 3: In the proposed design scheme, one parallel compensating operator P_C is added and one IMC-like structure is proposed. That is, the right part of the compensating operator is one regulator, help constructing the unminodular operator. Moreover, the IMC-like structure can reject the effects of the uncertainty and hysteresis, that is, we do not need to know the exact knowledge of the hysteresis to construct the controller. That is one merit of this article. Another merit is that the perfect output tracking performance can be realized simultaneously under the robust conditions.

4. SIMULATION EXAMPLE

The purpose of this section is to demonstrate the effectiveness of the proposed design scheme by considering the robust control of an unstable nonlinear plant preceded by uncertain backlash, which is an elementary unit of nonsymmetric PI hysteresis.

Let $C_{[0,\infty)}$ be the space of continuous functions and $C^1_{[0,\infty)} \subset C_{[0,\infty)}$ consists of all the functions having a continuous first derivative, both defined on $[0,\infty)$. The nominal plant and its right factorization operators are given as follows:

$$\begin{aligned} P(\tilde{u})(t) &= \int_0^t \tilde{u}(\tau)d\tau + e^t\tilde{u}(t), \\ N(w)(t) &= \int_0^t e^{-\tau}w(\tau)d\tau + w(t), \\ D(w)(t) &= e^{-t}w(t). \end{aligned} \tag{32}$$

The plant perturbation and its right factorization operators are given as

$$\begin{aligned} \Delta P(\tilde{u})(t) &= \delta(t)\int_0^t \tilde{u}(\tau)d\tau, \\ \Delta N(w)(t) &= \delta(t)\int_0^t e^{-\tau}w(\tau)d\tau, \\ D(w)(t) &= e^{-t}w(t), \end{aligned} \tag{33}$$

where the input space is $U = C_{[0,\infty)}$ and the output space is $V = \{u + e^t u' | u \in C^1_{[0,\infty)}\} \subset U$, $\tilde{u} \in U$, and so $u(t) := \int_0^t \tilde{u}(\tau)d\tau \in U$ with $P(\tilde{u}) \in V$, $\delta(t)$ is variable such that $\|\delta(t)\| < 1$. In this example, we choose the quasi-state space $W = U$. The norm is defined as usual sup-norm

$$\|u\|_\infty = \sup_{t\in[0,\infty)} |u(t)| \tag{34}$$

and $U_s = \{u(t) : u \in U, \|u\|_\infty < \infty\}$, $U^e = \{u(t) : u \in U, \|u_T\|_\infty < \infty$, for all $T < \infty\}$. It is easy to prove that P is unstable, D is stable.

According to the proposed design scheme, the controllers can be designed as follows:

$$C(w)(t) = -\int_0^t e^{-\tau}w(\tau)d\tau \tag{35}$$

thus, $(N+C)(w)(t) = w(t) + I(w)(t)$, namely, $N+C$ is unimodular operator and $P^* = e^t w(t)$, which is invertible

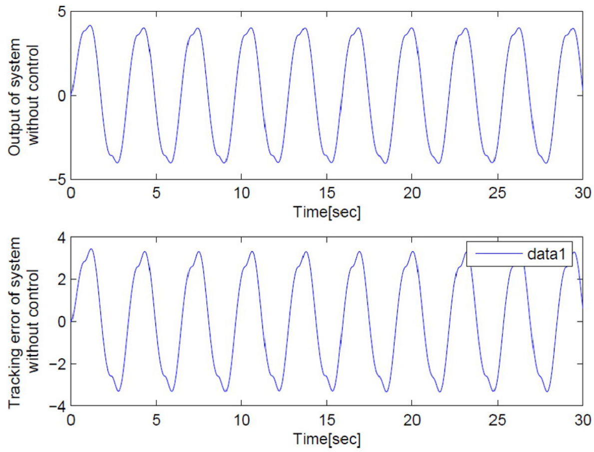

Fig. 7. The plant output and tracking error of system without control.

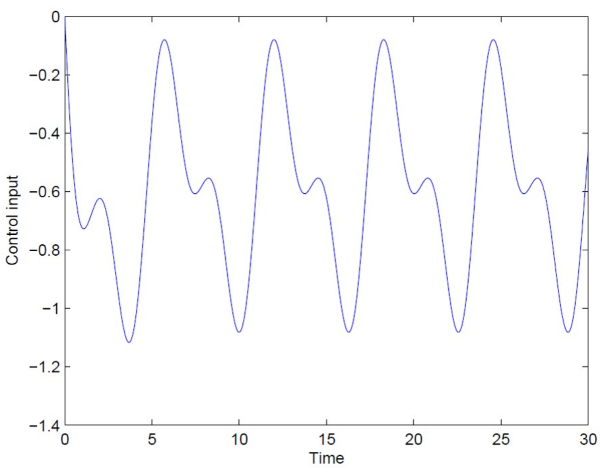

Fig. 8. Control input.

and its inverse is stable. Then,

$$S(y)(t) = P^{*-1} = e^{-t}y(t). \tag{36}$$

And the controllers$A$ and B are designed as

$$A(N+C)(w)(t) = (1-e^{-t})w(t), \tag{37}$$

$$B(u)(t) = I(u)(t). \tag{38}$$

It follows that

$$[A(N+C)+BD](w)(t) = w(t) = (N+C)(w)(t). \tag{39}$$

That is, the required conditions are all satisfied, and the system stability is guaranteed and the output tracking performance can be realized.

In simulation, the actual parameter values are chosen as $a = 1$, $c = 3$, $b = 0.5$, the reference input is chosen as $r(t) = \sin(2t)$ and $\delta(t)$ is assumed to be 0.05. Then the simulation results are given in Figs. 7-9. Fig. 7 shows the

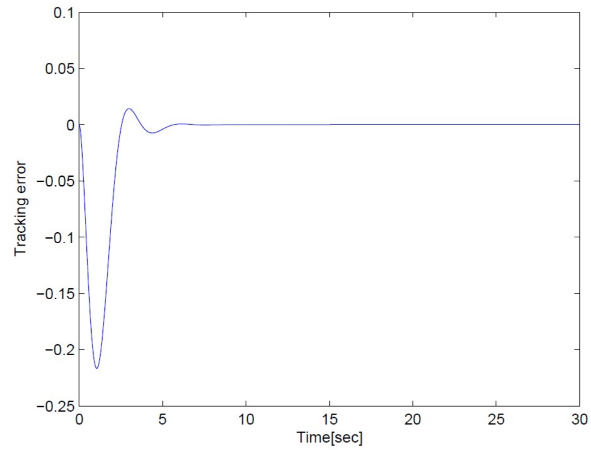

Fig. 9. Tracking error.

plant output and tracking error of system without control. By the proposed control design scheme, the control input $u(t)$ is shown in Fig. 8, and tracking error is drawn by Fig. 9. The simulation results demonstrate the perfect tracking performance of the proposed method.

5. CONCLUSION

Operator based robust control of nonlinear uncertain systems with backlash-like hysteresis is considered in this paper. One class of continuous backlash-like hysteresis operator is discussed and its property implies its applicability in operator based control system design. By using operator based robust right coprime factorization approach, an operator based internal model like control system is designed and sufficient conditions of stability and corresponding controllers design methods are proposed. Based on the proposed design scheme, the designed system is robustly stable and the desired output tracking performance can be realized simultaneously by a feedback regulating operator, helping to construct the needed Bezout identity and to compensate the effect from the hysteresis model uncertainty. Finally, the effectiveness of the proposed method is confirmed through numerical simulations.

With the increasing complexity requirement of the modern technology, multi-input multi-output (MIMO) nonlinear systems have more practical and wider applications than single-input single-output systems. Except that the nonlinearities and uncertainties, the coupling effects existing in the system is still a challenging research topic. On the future work, robust control for MIMO nonlinear uncertain systems proceeded by hysteresis will be considered.

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