International Journal of Control, Automation and Systems 14(6) (2016) 1413-1420
http://dx.doi.org/10.1007/s12555-014-0396-6

ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Positive L_1-gain Filter Design for Positive Continuous-time Markovian Jump Systems with Partly Known Transition Rates

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Abstract: The paper is concerned with the problem of positive L_1-gain filter design for positive continuous-time Markovian jump systems with partly known transition rates. Our aim is to design a positive full-order filter such that the corresponding filtering error system is positive and stochastically stable with L_1-gain performance. By applying a linear co-positive Lyapunov function and free-connection weighting vectors, the desired positive L_1-gain filter is provided. The obtained theoretical results are demonstrated by numerical examples.

Keywords: Free-connection weighting vectors, full-order filter, linear programming, partly known transition rates, positive Markovian jump systems.

1. INTRODUCTION

In practical application, there exists a class of special systems whose common property is that the states and outputs are nonnegative whenever initial conditions and inputs are nonnegative. Such systems are denoted as positive systems [1, 2]. For this special class of systems, there are many applications in practice, such as communication networks [3], industrial engineering [4], system control theory [5–12], and other aspects. Therefore, the research on positive switched systems has became a heated topic due to their importance from both theoretical and practical viewpoints. In [13–15], the two basic control problems for stability and stabilization have been considered. Compared with traditional quadratic Lyapunov function, linear co-positive Lyapunov function is more conveniently for solving the corresponding control problems of positive systems.

Markovian jump systems, which consist of the Markovian process (or Markovian chain) and classical differential (or difference) equations, are a special class of hybrid systems and popular in modeling actual control processes that may experience random abrupt changes in their structures or parameters [16–20]. As a key factor, the transition rates determine the system behavior of Markovian jump systems. Until now, most of the analysis and synthesis about Markovian jump systems have been covered under the assumption of a completely accessible knowledge of the transition rates. Actually, the knowledge on the transition rates can be obtained partly due to complex factors. Recently, there are some achievements on the Markovian jump systems with partly known transition rates, such as stability and stabilization [21, 22], H_∞ control [23], saturating actuator [24], finite-time control [25], and fault detection [26]. Very recently, there are only a few papers on the positive Markovian jump systems reported, including stability [27], stabilization [27, 28], and filter design [29].

On the other hand, there are some useful methods on estimation and filtering reported, and H_∞ filtering has been one of the most popular approaches to deal with an external noise [30–32]. Recently, a reduced-order positive H_∞ filter for positive discrete-time linear systems has been designed in [33]. The necessary and sufficient condition is given to obtain the positive filter, which is derived in term of linear matrix inequality. It is noticed that the L_2-form is used to deduce the H_∞ performance index and the traditional quadratic Lyapunov function is used to derive the filter existence conditions in forms of LMI framework. However, due to the positive property, it is natural to apply L_1-form to describe the performance index of positive system. Thus naturally, using the linear co-positive Lyapunov function approach, L_1-gain performance analysis and control have been discussed in [10, 11, 29, 34, 35]. For the positive L_1-gain filter of positive discrete-time Markovian jump systems, the relevant conclusion has been shown in [29]. However, in the published literature, there are no results on the problem of positive L_1-gain filter design for positive continuous-time Markovian jump systems with partly known transition rates, which is the motivation to carry out the challenge and necessary work.

In this paper, we consider the problem of positive L_1-gain filter design for positive continuous-time Markovian

Manuscript received October 4, 2014; revised August 1, 2015; accepted November 21, 2015. Recommended by Associate Editor Soohee Han under the direction of the Editor Duk-Sun Shim. This work was supported by Key Program of National Natural Science Foundation of China under Grant No. 61573088 and 61433004.

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jump systems with partly known transition rates. The main contributions of this paper include: (1) by constructing a linear co-positive Lyapunov function, sufficient condition for stochastic stability is proposed; (2) L_1-gain performance analysis for the considered systems is built based on the stochastic stability; (3) Based on the obtained results, the positive L_1-gain filter is proposed.

Notations: $A \succeq (\preceq 0, \succ, \prec)$ represents that all entries of matrix A are nonnegative (non-positive, positive, negative). $A \succ B$ ($A \succeq B$) means that $A - B \succ 0$ ($A - B \succeq 0$). $\mathbb{R}$ ($\mathbb{R}_+$) is the set of all real (positive real) numbers. $\mathbb{R}^n$ ($\mathbb{R}^n_+$) represents n-dimensional real (positive) vector space. The vector 1-norm is denoted by $||x||_1 = \sum_{k=1}^{n} |x_k|$, where x_k is the kth element of $x \in \mathbb{R}^n$. Given $v : \mathbb{R} \to \mathbb{R}^n$, the L_1-norm is defined by $||v||_{L_1} = \int_0^\infty ||v||_1 dt. L_1[0, +\infty)$ is the space of absolute integrable vector-valued functions on $[0, +\infty)$, i.e., we say $x : [0, +\infty) \to \mathbb{R}^n$ is in $L_1[0, +\infty)$ if $\int_0^\infty ||x(t)||_1 dt < \infty$. Matrix A is said to be a Metzler matrix if its off-diagonal elements are all nonnegative real numbers. $E\{\cdot\}$ represents the mathematical expectation. I_n denotes identity matrix and $\mathbf{1}_n$ stands for the all-ones vector in $\mathbb{R}^n$.

2. PROBLEM STATEMENT AND PRELIMINARIES

Consider the following positive Markovian jump systems:

$$\begin{aligned} \dot{x}(t) &= A(g_t)x(t) + B(g_t)w(t), \\ y(t) &= C(g_t)x(t) + D(g_t)w(t), \\ z(t) &= E(g_t)x(t) + F(g_t)w(t), \\ x(0) &= x_0, \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $w(t) \in \mathbb{R}^w$ is the disturbance input which belongs to $L_1^w[0, +\infty)$; $y(t) \in \mathbb{R}^p$ is the control out; $z(t) \in \mathbb{R}^q$ is the signal to be estimated; $\{g_t, t \geq 0\}$ is a time-homogeneous stochastic Markovian process with right continuous trajectories and takes values in a finite set $S = \{1, 2, \ldots, N\}$ with the transition rate matrix $\Pi = \{\pi_{ij}\}$ ($i, j \in S$) given by:

$$P\{g_{t+\Delta t} = j | g_t = i\} = \begin{cases} \pi_{ij}\Delta t + o(\Delta t), & i \neq j, \\ 1 + \pi_{ij}\Delta t + o(\Delta t), & i = j, \end{cases}$$

where $\Delta t \geq 0$, $\lim\limits_{\Delta t \to 0}(o(\Delta t)/\Delta t) = 0$ and $\pi_{ij} \geq 0$, for $i \neq j$ and $\sum\limits_{j=1, j\neq i}^{N} \pi_{ij} = -\pi_{ii}$.

Throughout the paper, the transition rates are assumed to be partly known, i.e., some elements in matrix $\Pi = \{\pi_{ij}\}$ are unknown. For $\forall i \in S$, the set S^i represents $S^i = S^i_k \bigcup S^i_{uk}$, with

$$\begin{aligned} S^i_k &\triangleq \{j : \pi_{ij} \text{ is known, for } j \in S\}, \\ S^i_{uk} &\triangleq \{j : \pi_{ij} \text{ is unknown, for } j \in S\}. \end{aligned}$$

And if $S^i \neq \varnothing$, it is further given by

$$S^i_k \triangleq \{k^i_1, k^i_2, \ldots, k^i_m\}, 1 \leq m \leq N,$$

where $k^i_m \in S$ represents the mth known transition rate of S^i_k in the ith row of the matrix Π. For simplicity, when $g_t = i$, the system matrices of the ith mode are denoted as A_i, B_i, C_i, D_i, E_i, and F_i.

Definition 1 [2]**:** System (1) is said to be positive if, for any initial condition $x_0 \succeq 0$ and any input $w(t) \succeq 0$, the corresponding trajectories $x(t) \succeq 0$, $y(t) \succeq 0$ and $z(t) \succeq 0$ hold.

Lemma 1 [2]**:** System (1) is said to be positive if and only if A_i are Metzler matrices and $B_i \succeq 0$, $C_i \succeq 0$, $D_i \succeq 0$, $E_i \succeq 0$, $F_i \succeq 0$, $\forall i \in S$.

Lemma 2 [28]**:** Matrix A is a Metzler matrix if and only if there exists a positive constant ε such that $A + \varepsilon I \succeq 0$.

In this paper, the positive full-order linear filter is given as follows:

$$\begin{aligned} \dot{x}_f(t) &= A_{fi}x(t) + B_{fi}y(t), z_f(t) = C_{fi}x(t) + D_{fi}y(t), \\ x_f(0) &= x_{f0}, \end{aligned} \tag{2}$$

where $x_f(t) \in \mathbb{R}^n$ is the filter state; A_{fi}, B_{fi}, C_{fi}, and D_{fi} are the matrices to be determined of compatible dimensions. Then, the resultant filtering error system is given as follows:

$$\begin{aligned} \dot{\bar{x}}_f(t) &= \bar{A}_i x(t) + \bar{B}_i w(t), \quad e(t) = \bar{E}_i x(t) + \bar{F}_i w(t), \\ \bar{x}(0) &= \bar{x}_0 = \begin{bmatrix} x_0^T & x_{f0}^T \end{bmatrix}^T, \end{aligned} \tag{3}$$

where

$$\begin{aligned} \bar{x}(t) &= \begin{bmatrix} x^T(t) & x_f^T(t) - x^T(t) \end{bmatrix}^T, e(t) = z_f(t) - z(t), \\ \bar{A}_i &= \begin{bmatrix} A_i & 0 \\ B_{fi}C_i + A_{fi} - A_i & A_{fi} \end{bmatrix}, \bar{B}_i = \begin{bmatrix} B_i \\ B_{fi}D_i - B_i \end{bmatrix}, \\ \bar{E}_i &= \begin{bmatrix} D_{fi}C_i + C_{fi} - E_i & C_{fi} \end{bmatrix}, \bar{F}_i = D_{fi}D_i - F_i. \end{aligned}$$

Remark 1: The filter (2) is designed to approximate $z(t)$ with $z_f(t)$. Consequently, the estimate $z_f(t)$ is required to be positive, which implies that the filter (2) is supposed to be a positive system. From Lemma 1 , we see that $A_{fi} \succeq 0$, $B_{fi} \succeq 0$, $C_{fi} \succeq 0$, and $D_{fi} \succeq 0$ are needed.

Definition 2 [28]**:** The system (3) ($w(t) = 0$) is said to be stochastically stable if for any initial condition $\bar{x}(0)$ and $g_0 \in S$, the following inequality holds

$$E\left\{\int_0^\infty ||\bar{x}(t)||_1 dt | \bar{x}_0, g_0\right\} < \infty. \tag{4}$$

Definition 3 [28]**:** Given a stable positive system (1), find a positive filter (2) with $\bar{A}_i \succeq 0$, $\bar{B}_i \succeq 0$, $\bar{E}_i \succeq 0$, and $\tilde{F}_i \succeq 0$, such that the filtering error system (3) is positive, stochastically stable and satisfies the performance

$$E\left\{\int_0^\infty ||e(t)||_1 dt\right\} \leq \gamma E\left\{\int_0^\infty ||w(t)||_1 dt\right\} \tag{5}$$

under zero initial conditions.

Definition 4 [20]**:** Considering $V(\bar{x}(t),i)$ as the Lyapunov function for the system (3), we define the weak infinitesimal operator as follows:

$$\begin{aligned}&\Gamma V(\bar{x}(t),i)\\&=\lim_{\Delta t\to 0}\frac{1}{\Delta t}[E\{V(\bar{x}(t+\Delta t),g(t+\Delta t))|\bar{x}(t),g(t)=i)\}\\&\quad -V(\bar{x}(t),g(t)=i)]. \qquad (6)\end{aligned}$$

3. MAIN RESULTS

This section will focus on the problem of stochastic stability analysis, L_1-gain analysis and positive L_1-gain filter design. Firstly, let us consider stochastic stability for system (3) ($w(t)=0$).

Theorem 1: If there exist a set of vectors $\nu_i \in \mathbb{R}^{2n}_+$, $\rho_i \in \mathbb{R}^{2n}$, for $\forall i \in S$, such that

$$\bar{A}_i^T\nu_i+\sum_{j\in S_k^i}\pi_{ij}(\nu_j-\rho_i)\prec 0, \qquad (7)$$

$$\nu_j-\rho_i\preceq 0, j\in S_{uk}^i, j\neq i, \qquad (8)$$

$$\nu_j-\rho_i\succeq 0, j\in S_{uk}^i, j=i, \qquad (9)$$

the filtering error Markovian jump system (3) ($w(t)=0$) with partly known transition rates is stochastically stable.

Proof: For system (3) ($w(t)=0$), choose the copositive type Lyapunov function candidate as

$$V(\bar{x}(t),i)=\bar{x}^T(t)\nu_i. \qquad (10)$$

According to Definition 4, it can be shown that

$$\Gamma V(\bar{x}(t),i)=\bar{x}^T(t)(\bar{A}_i^T\nu_i+\sum_{j=1}^{N}\pi_{ij}\nu_j).$$

Based on $\sum_{j=1}^{N}\pi_{ij}\rho_i=0$ for a set of vectors ρ_i, we have

$$\begin{aligned}\Gamma V(\bar{x}(t),i)&=\bar{x}^T(t)(\bar{A}_i^T\nu_i+\sum_{j=1}^{N}\pi_{ij}\nu_j-\sum_{j=1}^{N}\pi_{ij}\rho_i)\\&=\bar{x}^T(t)(\bar{A}_i^T\nu_i+\sum_{j\in S_k^i}\pi_{ij}(\nu_j-\rho_i)+\sum_{j\in S_{uk}^i}\pi_{ij}(\nu_j-\rho_i)). \qquad (11)\end{aligned}$$

Note that $\pi_{ii}<0(\forall i,j\in S, i=j)$ and $\pi_{ij}\geq 0(\forall i,j\in S, i\neq j)$, therefore, if $\forall j\in S_{uk}^i$, $i\in S_k^i$, inequalities (7)-(8) imply that $\Gamma V(\bar{x}(t),i)<0$. On the other hand, if $\forall j\in S_{uk}^i$, $i\in S_{uk}^i$, we can obtain

$$\begin{aligned}\Gamma V(\bar{x}(t),i)=&\bar{x}^T(t)(\bar{A}_i^T\nu_i+\sum_{j\in S_k^i}\pi_{ij}(\nu_j-\rho_i)\\&+\sum_{j\in S_{uk}^i}\pi_{ij}(\nu_j-\rho_i)+\pi_{ii}(\nu_j-\rho_i)). \qquad (12)\end{aligned}$$

From the fact $\pi_{ii}=-\sum_{j=1,i\neq j}^{N}\pi_{ij}<0$ and inequalities (7)−(9), we can also get $\Gamma V(\bar{x}(t),i)<0$. Following the same line of the proof of [28] leads to

$$E\left\{\int_0^\infty ||\bar{x}(t)||_1dt|\bar{x}_0,g_0\right\}<\infty.$$

The proof is completed. □

Next, we will consider the L_1-gain analysis problem for the filtering error system (3).

Theorem 2: If there exist a set of vectors $\nu_i\in\mathbb{R}^{2n}_+$, $\rho_i\in\mathbb{R}^{2n}$, for $\forall i\in S$, such that the inequalities (8)-(9) and the following inequalities

$$\bar{A}_i^T\nu_i+\sum_{j\in S_k^i}\pi_{ij}(\nu_j-\rho_i)+\bar{E}_i^T\mathbf{1}\prec 0, \qquad (13)$$

$$\bar{B}_i^T\nu_i+\bar{F}_i^T\mathbf{1}-\gamma\mathbf{1}\prec 0, \qquad (14)$$

hold, the filtering error Markovian jump system (3) ($w(t)=0$) with partly known transition rates is stochastically stable with L_1-gain performance.

Proof: It is clear that inequality (7) holds if the inequality (13) is satisfied. According to Theorem 1, we derive that system (3) ($w(t)=0$) is stochastically stable. Choosing a Lyapunov function candidate (10) leads to

$$\begin{aligned}&\Gamma V(\bar{x},i)+\|e(t)\|_{L_1}-\gamma\|w(t)\|_{L_1}\\&=\bar{x}^T(t)(\bar{A}_i^T\nu_i+\sum_{j\in S_k^i}\pi_{ij}(\nu_j-\rho_i)+\sum_{j\in S_{uk}^i}\pi_{ij}(\nu_j-\rho_i))\\&\quad+w^T(t)\bar{B}_i^T\nu_i+\mathbf{1}^T\bar{E}_i\bar{x}(t)+\mathbf{1}^T\bar{F}_iw(t)-\gamma\mathbf{1}^Tw(t)\\&=\bar{x}^T(t)(\bar{A}_i^T\nu_i+\sum_{j\in S_k^i}\pi_{ij}(\nu_j-\rho_i)+\sum_{j\in S_{uk}^i}\pi_{ij}(\nu_j-\rho_i)\\&\quad+\bar{E}_i^T\mathbf{1})+w^T(t)(\bar{B}_i^T\nu_i+\bar{F}_i^T\mathbf{1}-\gamma\mathbf{1}). \qquad (15)\end{aligned}$$

Therefore, if inequalities (8)-(9) and (13)-(14) hold, we have

$$\Gamma V(\bar{x}(t),i)+\|e(t)\|_{L_1}-\gamma\|w(t)\|_{L_1}<0, \qquad (16)$$

which means

$$E\left\{\int_0^\infty ||e(t)||_1dt\right\}\leq\gamma E\left\{\int_0^\infty ||w(t)||_1dt\right\}.$$

The proof is completed. □

By using the L_1-gain performance analysis result in Theorem 2, sufficient conditions are obtained for the existence of the filter (2) as follows:

Theorem 3: Consider the positive system (1) and denote $A_i=[A_{i1}\ A_{i2}\ \cdots\ A_{in}]^T$, $B_i=[B_{i1}\ B_{i2}\ \cdots\ B_{in}]^T$, $C_i=[C_{iq}\ C_{iq}\ \cdots\ C_{iq}]^T$, $D_i=[D_{iq}\ D_{iq}\ \cdots\ D_{iq}]^T$, $e_1=[1\ 0\ \cdots\ 0]^T$, $e_2=[0\ 1\ \cdots\ 0]^T$, $\cdots$, $e_n=[0\ 0\ \cdots\ 1]^T$, with $A_{is}, B_{is}\in\mathbb{R}^r$, C_{it}, $D_{it}\in\mathbb{R}^r$, $s=1,2,\cdots,n$, $t=1,2,\cdots,q$, $i\in S$. For given positive constant γ, the filtering error Markovian jump system (3) with partly known transition rates is positive and stochastically stable with L_1-gain performance, if there exist vectors ν_{1i}, ν_{2i}, ε_i, α_{ais}, β_{bis}, c_{cit}, $d_{dit} \in \mathbb{R}^n_+$, $\rho_{1i}, \rho_{2i} \in \mathbb{R}^n$, $s = 1, 2, \cdots, n$, $t = 1, 2, \cdots, q$, $i \in S$, such that the following linear programming problem

$$A_i^T \nu_{1i} + \sum_{s=1}^{n} \alpha_{ais} + C_i^T \sum_{s=1}^{n} \beta_{bis} - A_i^T \nu_{2i} + \sum_{j\in S_k^i} \pi_{ij}(\nu_{1j} - \rho_{1i}) + C_i^T \sum_{t=1}^{q} d_{dit} + \sum_{t=1}^{q} c_{cit} - E_i^T \mathbf{1} \prec 0, \tag{17}$$

$$\sum_{s=1}^{n} \alpha_{ais} + \sum_{j\in S_k^i} \pi_{ij}(\nu_{2j} - \rho_{2i}) + \sum_{t=1}^{q} c_{cit} \prec 0, \tag{18}$$

$$B_i^T \nu_{1i} + D_i^T \sum_{s=1}^{n} \beta_{bis} - B_i^T \nu_{2i} + D_i^T \sum_{t=1}^{q} d_{dit} - F_i^T \mathbf{1} - \gamma \mathbf{1} \prec 0, \tag{19}$$

$$\nu_{1j} - \rho_{1i} \preceq 0, \nu_{2j} - \rho_{2i} \preceq 0, j \in S_{uk}^i, j \neq i, \tag{20}$$

$$\nu_{1j} - \rho_{1i} \succeq 0, \nu_{2j} - \rho_{2i} \succeq 0, j \in S_{uk}^i, j = i, \tag{21}$$

$$A_{is}\nu_{2is} - \alpha_{ais} - C_i^T \beta_{bis} \preceq 0, B_{is}\nu_{2is} - D_i^T \beta_{bis} \preceq 0, \tag{22}$$

$$E_{it} - c_{cit} - C_i^T d_{dit} \preceq 0, F_{it} - D_i^T d_{dit} \preceq 0,$$
$$\alpha_{ais} + \varepsilon_{is} e_s \succeq 0, \tag{23}$$

is solvable, where $\varepsilon_i = [\varepsilon_{i1} \ \ \varepsilon_{i2} \ \ \cdots \ \ \varepsilon_{in}]^T$, $\nu_{2i} = [\nu_{2i1} \ \nu_{2i2} \ \cdots \ \nu_{2in}]^T$. In this case, the parameters of the L_1-gain filter are given

$$A_{fi} = [\nu_{2i1}^T \alpha_{ai1} \ \nu_{2i2}^T \alpha_{ai2} \ \cdots \ \nu_{2in}^T \alpha_{ain}]^T,$$
$$B_{fi} = [\nu_{2i1}^T \beta_{bi1} \ \nu_{2i2}^T \beta_{bi2} \ \cdots \ \nu_{2in}^T \beta_{bin}]^T,$$
$$C_{fi} = [c_{i1} \ c_{i2} \ \cdots \ c_{iq}]^T, D_{fi} = [d_{i1} \ d_{i2} \ \cdots \ d_{iq}]^T. \tag{24}$$

Proof: Note $\alpha_{ais} + \varepsilon_{is} e_s \succeq 0$, it follows from Lemma 2 that A_{fi} are Metzler matrices. From (24), it is clear that $B_{fi} \succeq 0$, $C_{fi} \succeq 0$, $D_{fi} \succeq 0$. Therefore, the filter (2) is positive.

From (17)-(19) and (22)-(24), we have

$$\sum_{s=1}^{n} \alpha_{ais} = \sum_{s=1}^{n} \alpha_{is}\nu_{2is}^T = A_{fi}^T \nu_{2i}, \sum_{s=1}^{q} c_{cit} = C_{fi}^T \mathbf{1},$$
$$\sum_{s=1}^{n} \beta_{bis} = \sum_{s=1}^{n} \beta_{is}\nu_{2is}^T = B_{fi}^T \nu_{2i}, \sum_{s=1}^{q} d_{dit} = D_{fi}^T \mathbf{1}. \tag{25}$$

Substituting (25) into (17)-(19) and (22)-(23) yields

$$A_i^T \nu_{1i} + A_{fi}^T \nu_{2i} + C_i^T B_{fi}^T \nu_{2i} - A_i^T \nu_{2i} + \sum_{j\in S_k^i} \pi_{ij}(\nu_{1j} - \rho_{1i}) + C_i^T D_{fi}^T \mathbf{1} + C_{fi}^T \mathbf{1} - E_i^T \mathbf{1} \prec 0,$$
$$A_{fi}^T \nu_{2i} + \sum_{j\in S_k^i} \pi_{ij}(\nu_{2j} - \rho_{2i}) + C_{fi}^T \mathbf{1} \prec 0,$$
$$B_i^T \nu_{1i} + D_i^T B_{fi}^T \nu_{2i} - B_i^T \nu_{2i} + D_i^T D_{fi}^T \mathbf{1} - F_i^T \mathbf{1} - \gamma \mathbf{1} \prec 0, \tag{26}$$

and

$$a_{is} + C_i^T b_{is} - A_{is} \succeq 0, D_i^T b_{is} - B_{is} \succeq 0,$$
$$C_i^T d_{dit} + c_{cit} - E_i \succeq 0, D_i^T d_{dit} - F_i \succeq 0. \tag{27}$$

It follows from (27) that

$$B_{fi}C_i + A_{fi} - A_i \succeq 0, B_{fi}D_i - B_i \succeq 0,$$
$$D_{fi}C_i + C_{fi} - E_i \succeq 0, D_{fi}D_i - F_i \succeq 0. \tag{28}$$

Together with Metzler matrices A_i and A_{fi}, it implies that filtering error system (3) is positive. Denote $\nu_i = [\nu_{1i}^T \ \nu_{2i}^T]^T$, $\rho_i = [\rho_{1i}^T \ \rho_{2i}^T]^T$. Together with (26)-(28) and (20)-(21), it implies that (8)-(9) and (13)-(14) hold.

Therefore, the filtering error Markovian jump system (3) with partly known transition rates is positive and stochastically stable with L_1-gain performance.

The proof is completed. □

4. NUMERICAL EXAMPLES

Example 1: Consider two-mode Markovian jump system with parameters:

$$A_1 = \begin{bmatrix} -2.1 & 0.4 \\ 0.5 & -1.6 \end{bmatrix}, \ A_2 = \begin{bmatrix} -1.6 & 0.3 \\ 0.5 & -1.5 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & 0.1 \end{bmatrix}, \ B_2 = \begin{bmatrix} 0.1 & 0.2 \\ 0.1 & 0.1 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 0.1 & 0.1 \\ 0.3 & 0.1 \end{bmatrix}, \ C_2 = \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & 0.1 \end{bmatrix},$$
$$D_1 = \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & 0.2 \end{bmatrix}, \ D_2 = \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 0.1 \end{bmatrix}, \ E_1 = \begin{bmatrix} 0.1 & 0.2 \\ 0.1 & 0.1 \end{bmatrix},$$
$$E_2 = \begin{bmatrix} 0.1 & 0.2 \\ 0.1 & 0.1 \end{bmatrix}, \ F_1 = \begin{bmatrix} 0.1 & 0.1 \\ 0.2 & 0.1 \end{bmatrix}, \ F_2 = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}.$$

The transition rate matrix is given as follows:

$$\begin{bmatrix} ? & ? \\ 0.25 & -0.25 \end{bmatrix},$$

where the unknown element is described by '?'.

For given $\gamma = 0.1$, solving Theorem 3 results in the following full-order filter gain matrices

$$A_{f1} = \begin{bmatrix} -2.4326 & 0.1760 \\ 0.2661 & -1.4453 \end{bmatrix}, B_{f1} = \begin{bmatrix} 0.9361 & 0.8496 \\ 1.1553 & 0.7367 \end{bmatrix},$$
$$C_{f1} = \begin{bmatrix} 3.9404 & 2.7793 \\ 3.9391 & 2.7551 \end{bmatrix}, D_{f1} = \begin{bmatrix} 13.9114 & 11.4075 \\ 13.9136 & 11.4305 \end{bmatrix},$$
$$A_{f2} = \begin{bmatrix} -2.5281 & 0.1632 \\ 0.2778 & -1.5842 \end{bmatrix}, B_{f2} = \begin{bmatrix} 1.3647 & 1.1548 \\ 1.1159 & 0.8565 \end{bmatrix},$$
$$C_{f2} = \begin{bmatrix} 3.8586 & 3.8861 \\ 3.8636 & 3.8637 \end{bmatrix}, D_{f2} = \begin{bmatrix} 15.6492 & 13.7059 \\ 15.5897 & 13.6077 \end{bmatrix}.$$

The simulation results are shown in Figs. 1-5 for the disturbance input $w(t) = \begin{bmatrix} e^{-t}|sin(t)| & e^{-t}|sin(t)| \end{bmatrix}^T$ and initial conditions $g(0) = 1$, $x(0) = x_f(0) = \begin{bmatrix} 5 & 25 \end{bmatrix}^T$.

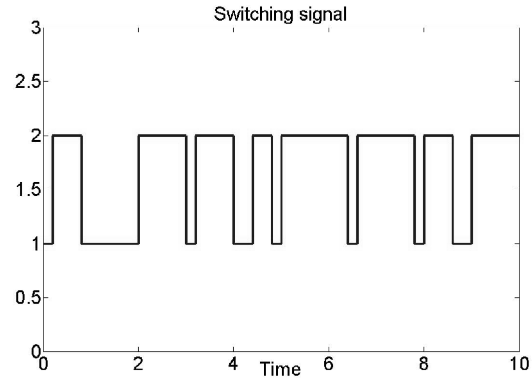

Fig. 1. System modes $g(t)$ of Example 1.

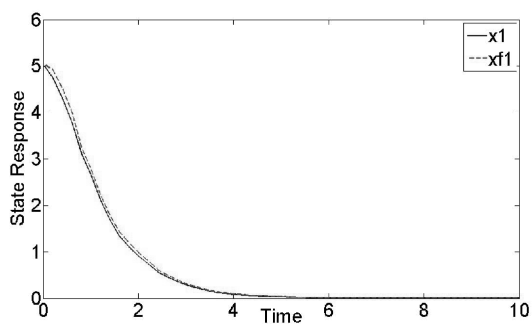

Fig. 2. State responses $x_1(t)$ and $x_{f1}(t)$ of Example 1.

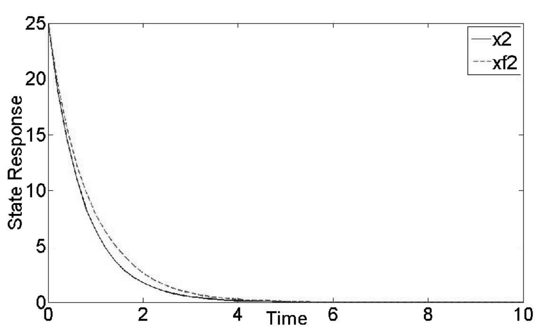

Fig. 3. State responses $x_2(t)$ and $x_{f2}(t)$ of Example 1.

Fig. 1 stands for system modes $g(t)$. Figs. 2-3 show the state responses of real state $x(t)$ and its estimate $x_f(t)$. Figs. 4-5 plot the simulation results of $z(t)$ and its estimate $z_f(t)$. It is evident that the filtering error Markovian jump system (3) with partly known transition rates is positive and stochastically stable with L_1-gain performance.

Remark 2: The completely known transition rate matrix is given as follows:

$$\begin{bmatrix} -0.5 & 0.5 \\ 0.25 & -0.25 \end{bmatrix}.$$

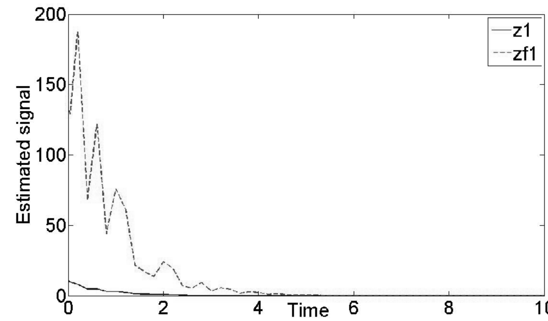

Fig. 4. Estimated signals $z_1(t)$ and $z_{f1}(t)$ of Example 1.

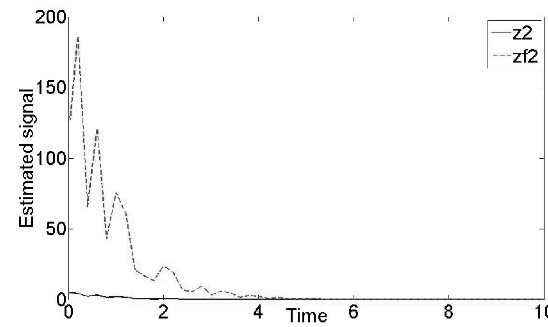

Fig. 5. Estimated signals $z_2(t)$ and $z_{f2}(t)$ of Example 1.

For given $\gamma = 0.1$, solving Theorem 3 results in the following full-order filter gain matrices

$$A_{f1} = \begin{bmatrix} -2.3344 & 0.2034 \\ 0.3490 & -1.6921 \end{bmatrix}, B_{f1} = \begin{bmatrix} 0.7573 & 0.6630 \\ 1.0029 & 0.8134 \end{bmatrix},$$
$$C_{f1} = \begin{bmatrix} 4.1402 & 2.8907 \\ 4.1382 & 2.8632 \end{bmatrix}, D_{f1} = \begin{bmatrix} 12.7812 & 10.5970 \\ 12.7950 & 10.6245 \end{bmatrix},$$
$$A_{f2} = \begin{bmatrix} -2.2231 & 0.0852 \\ 0.1978 & -1.6288 \end{bmatrix}, B_{f2} = \begin{bmatrix} 1.3307 & 0.8970 \\ 1.2924 & 0.7363 \end{bmatrix},$$
$$C_{f2} = \begin{bmatrix} 2.3180 & 1.9658 \\ 2.3232 & 1.9442 \end{bmatrix}, D_{f2} = \begin{bmatrix} 12.8026 & 9.4119 \\ 12.7280 & 9.3255 \end{bmatrix}.$$

It means that the filtering error Markovian jump system (3) with completely known transition rates is positive and stochastically stable with L_1-gain performance.

Example 2: For the mathematical model of virus mutation treatment presented in [12], the dynamic system is described as follows:

$$\begin{aligned} \dot{x}(t) =& (R_i - \delta I + \zeta M)x(t) + B_i w(t), \\ y(t) =& C_i x(t) + D_i w(t), z(t) = E_i x(t) + F_i w(t), \end{aligned} \quad (29)$$

where $x(t) \in \mathbb{R}^2$ indicates two different viral genotypes; $w(t) \in \mathbb{R}^l$ is the disturbance input which belongs to $L_1^n[0, +\infty)$; i indicates a Markovian process with two different states; ζ is a small parameter representing the mutation rate; δ is the death or decay rate; $M = [M_{mn}]$ denotes the system matrices; $M_{mn} \in \{0,1\}$ represents the genetic connections between genotypes, that is, M_{mn}=1 if and only if it is possible for genotype n to mutate into genotype m. The parameter values of the two-mode Markovian jump systems are:

$$R_1 = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.25 \end{bmatrix}, R_2 = \begin{bmatrix} 0.06 & 0 \\ 0 & 0.26 \end{bmatrix}, M = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.1 \end{bmatrix}, B_2 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.2 \end{bmatrix}, C_1 = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.5 \end{bmatrix},$$
$$C_2 = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.9 \end{bmatrix}, D_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, D_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$E_1 = \begin{bmatrix} 0.1 & 0.2 \\ 0.1 & 0.1 \end{bmatrix}, E_2 = \begin{bmatrix} 0.1 & 0.2 \\ 0.1 & 0.1 \end{bmatrix}, F_1 = \begin{bmatrix} 0.1 & 0.1 \\ 0.2 & 0.1 \end{bmatrix},$$
$$F_2 = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}, \delta = 1.3, \zeta = 0.001.$$

The transition rate matrix is given as follows:

$$\begin{bmatrix} ? & ? \\ 0.5 & -0.5 \end{bmatrix},$$

where the unknown element is described by '?'.

For given $\gamma = 0.25$, solving Theorem 3 results in the following full-order filter gain matrices

$$A_{f1} = \begin{bmatrix} -9.1350 & 0.0013 \\ 0.0028 & -2.9332 \end{bmatrix}, B_{f1} = \begin{bmatrix} 5.2624 & 0.0019 \\ 0.0038 & 1.3333 \end{bmatrix},$$
$$C_{f1} = \begin{bmatrix} 2.0045 & 0.5874 \\ 1.9338 & 0.5817 \end{bmatrix}, D_{f1} = \begin{bmatrix} 3.3693 & 1.4517 \\ 4.0862 & 1.4415 \end{bmatrix},$$
$$A_{f2} = \begin{bmatrix} -8.8526 & 0.0132 \\ 0.0132 & -2.8585 \end{bmatrix}, B_{f2} = \begin{bmatrix} 6.2684 & 0.0206 \\ 0.0185 & 2.0297 \end{bmatrix},$$
$$C_{f2} = \begin{bmatrix} 9.7395 & 6.7135 \\ 9.7728 & 6.7010 \end{bmatrix}, D_{f2} = \begin{bmatrix} 14.0180 & 11.0136 \\ 13.7736 & 11.0003 \end{bmatrix}.$$

The simulation results are shown in Figs. 6-7 for the disturbance input $w(t) = \begin{bmatrix} e^{-t}|sin(t)| & e^{-t}|sin(t)| \end{bmatrix}^T$ and initial conditions $g(0) = 2$, $x(0) = \begin{bmatrix} 6 & 10 \end{bmatrix}^T$, $x_f(0) = \begin{bmatrix} 10 & 15 \end{bmatrix}^T$.

Figs. 6-7 stand for system modes $g(t)$, state responses of real state $x(t)$ and its estimate $x_f(t)$. It is shown that the filtering error Markovian jump system (3) with partly known transition rates is positive and stochastically stable with L_1-gain performance.

Remark 3: The completely known transition rate matrix is given as follows:

$$\begin{bmatrix} -0.75 & 0.75 \\ 0.5 & -0.5 \end{bmatrix}.$$

For given $\gamma = 0.25$, solving Theorem 3 results in the following full-order filter gain matrices

$$A_{f1} = \begin{bmatrix} -10.7643 & 0.0593 \\ 0.0551 & -3.7857 \end{bmatrix}, B_{f1} = \begin{bmatrix} 6.4485 & 0.0699 \\ 0.0734 & 2.1127 \end{bmatrix},$$
$$C_{f1} = \begin{bmatrix} 2.4334 & 1.4205 \\ 2.3455 & 1.4141 \end{bmatrix}, D_{f1} = \begin{bmatrix} 3.8417 & 1.9369 \\ 4.4185 & 1.9209 \end{bmatrix},$$
$$A_{f2} = \begin{bmatrix} -10.3443 & 0.0396 \\ 0.0509 & -3.4513 \end{bmatrix}, B_{f2} = \begin{bmatrix} 9.9427 & 0.0742 \\ 0.1584 & 2.7001 \end{bmatrix},$$
$$C_{f2} = \begin{bmatrix} 2.3098 & 0.9578 \\ 2.3236 & 0.9507 \end{bmatrix}, D_{f2} = \begin{bmatrix} 7.6896 & 2.0696 \\ 7.4698 & 2.0431 \end{bmatrix}.$$

It means that the filtering error Markovian jump system (3) with completely known transition rates is positive and stochastically stable with L_1-gain performance.

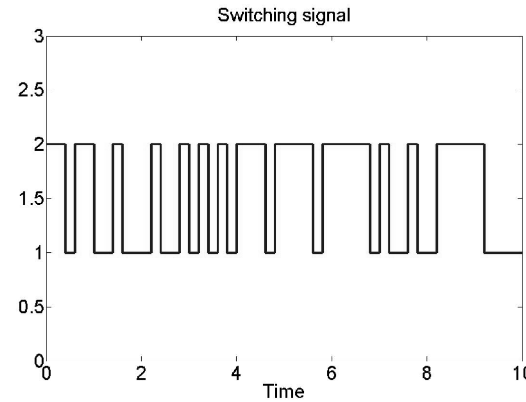

Fig. 6. System modes $g(t)$ of Example 2.

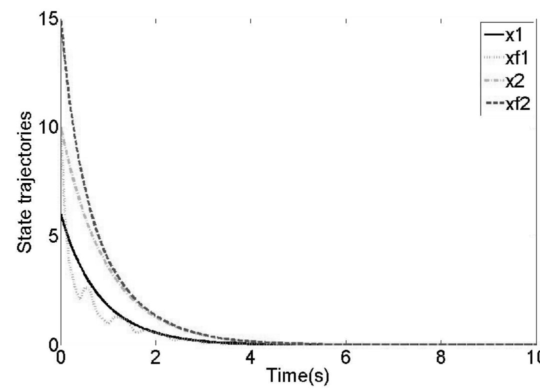

Fig. 7. State responses $x(t)$ and $x_f(t)$ of Example 2.

5. CONCLUSIONS

The problem of positive L_1-gain filter design for positive continuous-time Markovian jump systems with partly known transition rates has been addressed. By applying a linear co-positive Lyapunov function, sufficient conditions, which ensure the filtering error Markovian jump system is positive and stochastically stable with L_1-gain performance, are given in linear programming. Following the approach in the paper, the future work may refer to time delays, output feedback control, observer design, and fault detection.

REFERENCES

[1] L. Farina and S. Rinaldi, *Positive Linear Systems: Theory and Applications*, Wiley, New York, 2000.

[2] T. Kaczorek, *Positive 1D and 2D Systems*, Springer-Verlag, UK, 2002.

[3] R. Shorten, F. Wirth, and D. Leith, "A positive systems model of TCP-like congestion control: asymptotic results," *IEEE/ACM Transactions on Networking*, vol. 14, no. 3, pp. 616-629, June 2006. [click]

[4] L. Caccetta, L. R. Foulds, and V. G. Rumchev, "A positive linear discrete-time model of capacity planning and its controllability properties," *Mathematical and Computer Modelling*, vol. 40, no. 1-2, pp. 217-226, July 2004. [click]

[5] E. Yoshio, P. Dimitri, and A. Denis, "LMI approach to linear positive system analysis and synthesis," *Systems & Control Letters*, vol. 63, pp. 50-56, January 2014. [click]

[6] M. Ait Rami and F. Tadeo, "Controller synthesis for positive linear systems with bounded controls," *IEEE Transactions on Circuits and Systems Part II: Express Briefs*, vol. 54, no. 2, pp. 151-155, March 2007.

[7] J. F. Zhang and Z. Z. Han, "Robust stabilization of switched positive linear systems with uncertainties," *International Journal of Control, Automation, and Systems*, vol. 11, no. 1, pp. 41-47, February 2013.

[8] M. Xiang, Z. R. Xiang, and H. R. Karimi, "Stabilization of positive switched systems with time-varying delays under asynchronous switching," *International Journal of Control, Automation, and Systems*, vol. 12, no. 5, pp. 939-947, October 2014. [click]

[9] S. Li, Z. R. Xiang, and H. R. Karimi, "Mixed l_-/l_1 fault detection observer design for positive switched systems with time-varying delay via delta operator approach," *International Journal of Control, Automation, and Systems*, vol. 12, no. 4, pp. 709-721, August 2014. [click]

[10] X. M. Chen, J. Lam, P. Li, and Z. Shu, "l_1-induced norm and controller synthesis of positive systems," *Automatica*, vol. 49, no. 5, pp. 1377-1385, May 2013. [click]

[11] X. M. Chen, J. Lam, and H. K. Lam, "Positive filtering for positive Takagi-Sugeno fuzzy systems under l_1 performance," *Information Sciences*, vol. 299, pp. 32-41, April 2015. [click]

[12] E. Hernandez-Varga, P. Colaneri, R. Middleton, and F. Blanchini, "Discrete-time control for switched positive systems with application to mitigating viral escape," *International Journal of Robust and Nonlinear Control*, vol. 21, no. 10, pp. 1093-1111, July 2010. [click]

[13] X. W. Liu, "Stability analysis of switched positive systems: a switched linear copositive Lyapunov function method," *IEEE Transactions on Circuits and Systems Part II: Express Briefs*, vol. 56, no. 5, pp. 414-418, May 2009. [click]

[14] E. Fornasini and M. E. Valcher, "Stability and stabilizability criteria for discrete-time positive switched systems," *IEEE Transactions on Automatic Control*, vol. 57, no. 5, pp. 1208-1221, May 2012.

[15] O. Mason and R. Shorten, "On linear copositive Lyapunov functions and the stability of switched positive linear systems," *IEEE Transactions on Automatic Control*, vol. 52, no. 7, pp. 1346-1349, July 2007.

[16] W. H. Chen, J. X. Xu, and Z. H. Guan, "Guaranteed cost control for uncertain Markovian jump systems with mode-dependent time-delays," *IEEE Transactions on Automatic Control*, vol. 48, no. 12, pp. 2270-2277, December 2003.

[17] Y. Song, J. X. Xie, M. R. Fei, and W. Y. Hou, "Mean square exponential stabilization of networked control systems with Markovian packet dropouts," *Transactions of the Institute of Measurement and Control*, vol. 35, no. 1, pp. 75-82, February 2013.

[18] X. H. Ge and Q. L. Han, "Distributed fault detection over sensor networks with Markovian switching topologies," *International Journal of General Systems*, vol. 43, no. 3-4, pp. 305-318, March 2014.

[19] G. L. Wang, "Robust stabilization of singular Markovian jump systems with uncertain switching," *International Journal of Control, Automation, and Systems*, vol. 11, no. 1, pp. 188-193, February 2013. [click]

[20] X. R. Mao, "Stability of stochastic differential equations with Markovian switching," *Stochastic Analysis and Applications*, vol. 79, no. 1, pp. 45-67, January 1999.

[21] L. X. Zhang and E. K. Boukas, "Stability and stabilization for Markovian jump linear systems with partly unknown transition probabilities," *Automatica*, vol. 45, no. 2, pp. 463-468, February 2009. [click]

[22] M. K. Song, J. B. Park, and Y. H. Joo, "Stability and stabilization for discrete-time Markovian jump fuzzy systems with time-varying delays: partially known transition probabilities case," *International Journal of Control, Automation, and Systems*, vol. 11, no. 1, pp. 136-146, February 2013. [click]

[23] X. D, Zhao and Q. S, Zeng, "Delay-dependent H_∞ performance analysis for Markovian jump systems with mode-dependent time varying delays and partially known transition rates," *International Journal of Control, Automation and Systems*, vol. 8, no. 2, pp. 482-489, April 2010. [click]

[24] Y. J. Wang, C. H. Wang, and Z. Q. Zuo, "Controller synthesis for Markovian jump systems with incomplete knowledge of transition probabilities and actuator saturation," *Journal of the Franklin Institute*, vol. 348, no. 9, pp. 2417-2429, November 2011.

[25] X. L. Luan, F. Liu, and P. Shi, "Finite-time filtering for non-linear stochastic systems with partially known transition jump rates," *IET Control Theory and Applications*, vol. 4, no. 5, pp. 735-745, May 2010.

[26] Y. Yin, P. Shi, F. Liu, and J. S. Pan, "Gain-scheduled fault detection on stochastic nonlinear systems with Partially known transition jump rates," *Nonlinear Analysis: Real World Applications*, vol. 13, no. 1, pp. 359-369, February 2012. [click]

[27] P. Bolzern, P. Colaneri, and G. Nicolao, "Stochastic stability of positive Markov jump linear systems," *Automatica*, vol. 50, pp. 4, no. 1181-1187, April 2014. [click]

[28] J. F. Zhang, Z. Z Han, and F. Zhu, "Stochastic stability and stabilization of positive systems with Markovian jump parameters," *Nonlinear Analysis: Hybrid Systems*, vol. 12, no. 147-155, May 2014. [click]

[29] S. Q. Zhu, Q. L. Han, and C. H. Zhang, "l_1-gain performance analysis and positive filter design for positive discrete-time Markov jump linear systems: A linear programming approach," *Automatica*, vol. 50, no. 8, pp. 2098-2107, August 2014. [click]

[30] Y. C. Ding, H. Zhu, and S. M. Zhong, "H_∞ filtering for stochastic systems with Markovian switching and partly unknown transition probabilities," *Circuits, System and Signal Processing*, vol. 32, no. 2, pp. 559-583, April 2013. [click]

[31] J. X. Lin, S. M. Fei, and J. Shen, "Delay-dependent H_∞ filtering for discrete-time singular Markovian jump systems with time-varying delay and partially unknown transition probabilities," *Signal Processing*, vol. 91, no. 2, pp. 277-289, February 2011.

[32] G. L. Wang, "Partially mode-dependent design of H_∞ filter for stochastic Markovian jump systems with mode-dependent time delays," *Journal of Mathematical Analysis and Applications*, vol. 383, no. 2, pp. 573-584, November 2011. [click]

[33] P. Li, J. Lam, and Z. Shu, "H_∞ positive filtering for positive linear discrete-time systems: an augmentation approach," *IEEE Transactions on Automatic Control*, vol. 55, no. 10, pp. 2337-2342, October 2010.

[34] M. Xiang and Z. R Xiang, "Stability, L_1-gain and control synthesis for positive switched systems with time-varying delay," *Nonlinear Analysis: Hybrid Systems*, vol. 9, pp. 9-17, August 2013. [click]

[35] M. Xiang and Z. R Xiang, "Finite-time L_1 control for positive switched linear systems with time-varying delay," *Communications in Nonlinear Science and Numerical Simulation*, vol. 18, no. 11, pp. 3158-3166, November 2013. [click]

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