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Improved Stability Criteria for T-S fuzzy Systems with Time-varying Delay by Delay-partitioning Approach

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Abstract: This paper focuses on the robust stability criteria of uncertain T-S fuzzy systems with time-varying delay by delay-partitioning approach. An appropriate Lyapunov-Krasovskii functional is established in the framework of state vector augmentation. Then, on the basis of the Finsler's lemma, some tighter bounding inequalities (Seuret-Wirtinger's integral inequality and Peng-Park's integral inequality) are employed to deal with (time-varying) delay-dependent integral items. Therefore, less conservative delay-dependent stability criteria are obtained in terms of linear matrix inequalities (LMIs), which can be solved efficiently with the Matlab LMI toolbox. Finally, two numerical examples are provided to show that the proposed conditions are less conservative than existing ones.

Keywords: Delay-partitioning approach, Linear matrix inequalities (LMIs), Lyapunov-Krasovskii functional (LKF), Stability, Time-varying delay, Takagi-Sugeno (T-S) fuzzy systems.

1. INTRODUCTION

Since Takagi-Sugeno (T-S) fuzzy model [1] was first introduced, much effort has been made in the stability analysis and control synthesis of such a model during the past two decades, due to the fact that it can combine the flexibility of fuzzy logic theory and fruitful linear system theory into a unified framework to approximate complex nonlinear systems [2,3]. On the other hand, as a source of instability and deteriorated performance, time-delay often occurs in many dynamic systems such as biological systems, chemical processes, communication networks and so on. Therefore, stability analysis for T-S fuzzy systems with time-delay has received more interest and achieved fruitful results, see, e.g., [4-10] and references therein. These stability criteria can be classified into two types: one is delay-dependent and the other is delay-independent. The delay-dependent criteria are less conservative than delay-independent ones since they consider the length information of the delay [5,7,11].

Among the recent techniques adopted in the stability analysis of T-S fuzzy systems with time-varying delay, the most noteworthy is the delay-partitioning approach: the delay interval is divided into multiple uniform/non-uniform segments [11-15]. It has been proved that less conservative results may be expected with the increasing delay-partitioning segments [11,14]. On the other hand, to the system with time-varying delays, it is seen from [15] that the results based on reciprocally convex technique [16] generally have the less conservativeness than those based on Jensen inequality, since none of any useful integral items are arbitrarily ignored in the proof [14]. Recently, by dividing the delay interval into two uniform segments, [15] obtained the less conservative results than those in [5,17] for time-varying delay T-S fuzzy systems. More recently, on the basis of delay-partitioning approach and Peng-Park's integral inequality established by reciprocally convex approach, [14] has developed less conservative stability criteria than those in [5,13,15] for the uncertain T-S fuzzy systems with interval time-varying delay. Most recently, via the idea of combining delay-decomposition with state vector augmentation, a novel LKF is established, then by employing the reciprocally convex approach, [11] has achieved less conservative results than those in [5,14,18-22] for the uncertain T-S fuzzy systems with time-varying delay. However, when revisiting this problem, we find that the aforementioned works still leave plenty of room for improvement.

This paper will develop less conservative stability criteria of uncertain T-S fuzzy systems with time-varying delay by means of delay-partitioning approach and

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Finsler's lemma. An appropriate augmented LKF is established in the framework of state vector augmentation. Then, some improved stability criteria are obtained by employing Seuret-Wirtinger's integral inequality and Peng-Park's integral inequality to deal with (time-varying) delay-dependent integral items. Finally, two numerical examples are provided to show that the proposed criteria are less conservative than existing ones.

The rest of this paper is organized as follows. The main problem is formulated in Section 2 and improved stability criteria for the uncertain T-S fuzzy systems with time-varying delay are derived in Section 3. In Section 4, two numerical examples are provided; and a concluding remark is given in Section 5.

Notations: Through this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n\times m$ real matrices; the notation $A>(\geq)B$ means that $A-B$ is positive (semi-positive) definite; I (0) is the identity (zero) matrix with appropriate dimension; A^{T} denotes the transpose; $\mathrm{He}(A)$ represents the sum of A and A^{T}; $\|\bullet\|$ denotes the Euclidean norm in $\mathbb{R}^n$; "*" denotes the elements below the main diagonal of a symmetric block matrix; $C([-\tau,0],\mathbb{R}^n)$ is the family of continuous functions ϕ from interval $[-\tau,0]$ to $\mathbb{R}^n$ with the norm $\|\phi\|_\tau=\sup_{-\tau\leq\theta\leq 0}\|\phi(\theta)\|$; let $x_t(\theta)=x(t+\theta)$, $\theta\in[-\tau,0]$.

2. PROBLEM FORMULATION

In this section, a class of uncertain T-S fuzzy systems with time-varying delay is concerned. For each $i=1,2,\cdots,r$ (r is the number of plant rules), the ith rule of this T-S fuzzy model is represented as follows:

Plant Rule i**: IF** $\theta_1(t)$ **is** M_{i1}, $\theta_2(t)$ **is** M_{i2}, $\cdots$, $\theta_p(t)$ **is** M_{ip}, **THEN**

$$\begin{cases}\dot{x}(t)=[A_i+\Delta A_i(t)]x(t)+[A_{di}+\Delta A_{di}(t)]x(t-\tau(t)),\ t\geq 0\\ x(t)=\phi(t),\quad t\in[-\tau,0],\end{cases}\tag{1}$$

where $\theta_1(t),\ \theta_2(t),\ \cdots,\theta_p(t)$ are the premise variables, and each $M_{il}(i=1,2,\cdots,r;l=1,2,\cdots,p)$ is a fuzzy set; $x(t)\in\mathbb{R}^n$ is the state vector; $\phi(t)\in C([-\tau,0],\mathbb{R}^n)$ is the initial function; A_i and A_{di} are constant real matrices with appropriate dimensions; the delay $\tau(t)$ is a time-varying functional satisfying

$$0\leq\tau(t)\leq\tau,\tag{2}$$

$$\dot{\tau}(t)<\mu,\tag{3}$$

where τ and μ are constants assumed to exist; The matrices $\Delta A_i(t)$ and $\Delta A_{di}(t)$ denote the uncertainties in the system and are defined as

$$[\Delta A_i(t),\Delta A_{di}(t)]=HF(t)[E_i,E_{di}],\tag{4}$$

where H, E_i and E_{di} are known constant matrices and $F(t)$ is an unknown matrix function satisfying

$$F^{\mathrm{T}}(t)F(t)\leq I.\tag{5}$$

By a center-average defuzzier, product inference and singleton fuzzifier, the dynamic fuzzy model in (1) can be represented by

$$\begin{cases}\dot{x}(t)=\sum_{i=1}^{r}h_i(\theta(t))\{A_i(t)x(t)+A_{di}(t)x(t-\tau(t))\},\\ x(t)=\phi(t),\quad t\in[-\tau,0],\end{cases}\tag{6}$$

where $A_i(t)=A_i+\Delta A_i(t)$, $A_{di}(t)=A_{di}+\Delta A_{di}(t)$ and

$$h_i(\theta(t))=\frac{\prod_{l=1}^{p}M_{il}(\theta_l(t))}{\sum_{i=1}^{r}\prod_{l=1}^{p}M_{il}(\theta_l(t))},\quad i=1,\cdots,r,\tag{7}$$

in which $M_{il}(\theta_l(t))$ is the grade of membership of $\theta_l(t)$ in M_{il}, and $\theta(t)=(\theta_1(t),\cdots,\theta_r(t))$; By definition, the fuzzy weighting functions $h_i(\theta(t))$ satisfy $h_i(\theta(t))\geq 0$ and $\sum_{i=1}^{r}h_i(\theta(t))=1$. For notational simplicity, h_i is used to represent $h_i(\theta(t))$ in the following description.

Before proceeding, recall the following lemmas which will be used throughout the proofs.

Lemma 1 (Finsler's lemma) [23]**:** Let $\zeta\in\mathbb{R}^n$, $\Phi=\Phi^{\mathrm{T}}\in\mathbb{R}^{n\times n}$, and $B\in\mathbb{R}^{m\times n}$ such that $rank(B)<n$. Then the following statements are equivalent:

(i) $\zeta^{\mathrm{T}}\Phi\zeta<0,\ \forall\ B\zeta=0,\ \zeta\neq 0;$

(ii) ${B^{\perp}}^{\mathrm{T}}\Phi B^{\perp}<0;$

(iii) $\exists\, Y\in\mathbb{R}^{n\times m}:\Phi+\mathrm{He}(YB)<0,$

where $B^{\perp}\in\mathbb{R}^{n\times(n-rank(B))}$ is the right orthogonal complement of B.

Lemma 2 (Peng-Park's integral inequality) [14,16]**:** For any matrix $\begin{bmatrix}Z & S\\ * & Z\end{bmatrix}\geq 0$, positive scalars τ and $\tau(t)$ satisfying $0<\tau(t)<\tau$, vector function $\dot{x}:[-\tau,0]\to\mathbb{R}^n$ such that the concerned integrations are well defined, then

$$-\tau\int_{t-\tau}^{t}\dot{x}^{\mathrm{T}}(s)Z\dot{x}(s)ds\leq\varpi^{\mathrm{T}}(t)\Omega\varpi(t),$$

where

$$\varpi(t)=[x^{\mathrm{T}}(t),x^{\mathrm{T}}(t-\tau(t)),x^{\mathrm{T}}(t-\tau)]^{\mathrm{T}},$$

$$\Omega=\begin{bmatrix}-Z & Z-S & S\\ * & -2Z+\mathrm{He}(S) & -S+Z\\ * & * & -Z\end{bmatrix}.$$

Lemma 3 (Seuret-Wirtinger's integral inequality) [24]**:** For any matrix $Z>0$, the following inequality holds for all continuously differentiable function $x:[\alpha,\beta]\to\mathbb{R}^n$:

$$\int_{\alpha}^{\beta}\dot{x}^{\mathrm{T}}(s)Z\dot{x}(s)ds\geq\frac{1}{\beta-\alpha}\nu^{\mathrm{T}}(t)\Theta\nu(t),$$

where

$$\nu(t)=\begin{bmatrix}x(\beta)\\ x(\alpha)\\ \frac{1}{\beta-\alpha}\int_{\alpha}^{\beta}x(s)ds\end{bmatrix},\quad\Theta=\begin{bmatrix}4Z & 2Z & -6Z\\ * & 4Z & -6Z\\ * & * & 12Z\end{bmatrix}.$$

Lemma 4 [25]**:** Let $Q=Q^{\mathrm{T}}$, H, E and $F(t)$ satisfying $F^{\mathrm{T}}(t)F(t)\le I$ are appropriately dimensional matrices, then the following inequality

$$Q+\mathrm{He}\{HF(t)E\}<0$$

is true, if and only if the following inequality holds for any $\varepsilon>0$,

$$Q+\varepsilon^{-1}HH^{\mathrm{T}}+\varepsilon E^{\mathrm{T}}E<0.$$

3. MAIN RESULTS

This section aims to develop a novel robust stability criteria for uncertain fuzzy system (6) with time-varying delay by delay-partitioning approach.

For any integer $m\ge 1$, define $\delta=\frac{\tau}{m}$, then $[0,\tau]$ can be divided into m segments, i.e.,

$$[0,\tau]=\bigcup_{j=1}^{m}[(j-1)\delta, j\delta]. \tag{8}$$

For notational simplification, motivated by [14], let

$$\begin{cases} e_s=[\underbrace{0,\cdots,0}_{s-1},I,\underbrace{0,\cdots,0}_{m-s+4}]^{\mathrm{T}}, \quad s=1,2,\cdots,m+4 \\ \zeta(t)=[x^{\mathrm{T}}(t-\tau(t)),\zeta_1^{\mathrm{T}}(t),x^{\mathrm{T}}(t-m\delta), \\ \qquad \frac{1}{\delta}\int_{t-\delta}^{t}x^{\mathrm{T}}(s)ds, \dot{x}(t)]^{\mathrm{T}}, \end{cases} \tag{9}$$

where

$$\zeta_1(t)=[x^{\mathrm{T}}(t),x^{\mathrm{T}}(t-\delta),\cdots,x^{\mathrm{T}}(t-(m-1)\delta)]^{\mathrm{T}}.$$

Based on Lyapunov-Krasovskii stability theorem [26], we firstly state the following stability criterion for the nominal system (6), i.e., system (6) without parameter uncertainties.

Theorem 1: Given a positive integer m, scalars $\tau\ge 0$, μ, and $\delta=\frac{\tau}{m}$, then the nominal system (6) with a time-delay $\tau(t)$ satisfying (2) and (3) is asymptotically stable if there exist symmetric positive matrices

$$P=\begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix},\quad R_l=\begin{bmatrix} R_{1l} & R_{2l} \\ * & R_{3l} \end{bmatrix},$$

$$X=[X_{ij}]_{m\times m}\triangleq\begin{bmatrix} X_{11} & \cdots & X_{1m} \\ \vdots & \ddots & \vdots \\ * & \cdots & X_{mm} \end{bmatrix},$$

Q_j, Z_0, Z_j, and any matrices Y and S_{ij} $(i=1,2,\cdots,r;$ $j=1,2,\cdots,m;$ $l=1,2,\cdots,m-1)$ with appropriate dimensions, such that the following LMIs hold for $i=1,2,\cdots,r$ and $k=1,2,\cdots,m$:

$$\Xi(i,k)+\mathrm{He}(Y\Gamma_i)<0, \tag{10}$$

$$\Upsilon(i,k)=\begin{bmatrix} Z_k & S_{ik} \\ * & Z_k \end{bmatrix}\ge 0, \tag{11}$$

where

$$\Xi(i,k)=\sum_{j=0}^{3}\Xi_j+\Xi_4(k)+\Xi_5(i,k) + e_{m+4}\left(\delta^2\sum_{j=0}^{m}Z_j\right)e_{m+4}^{\mathrm{T}},$$

$$\Gamma_i=A_ie_2^{\mathrm{T}}+A_{di}e_1^{\mathrm{T}}-e_{m+4}^{\mathrm{T}},$$

$$\Xi_0=\begin{bmatrix} e_2^{\mathrm{T}} \\ e_3^{\mathrm{T}} \\ e_{m+3}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} -4Z_0 & -2Z_0 & 6Z_0 \\ * & -4Z_0 & 6Z_0 \\ * & * & -12Z_0 \end{bmatrix}\begin{bmatrix} e_2^{\mathrm{T}} \\ e_3^{\mathrm{T}} \\ e_{m+3}^{\mathrm{T}} \end{bmatrix},$$

$$\Xi_1=\mathrm{He}\left\{\begin{bmatrix} e_2^{\mathrm{T}} \\ \delta e_{m+3}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix}\begin{bmatrix} e_{m+4}^{\mathrm{T}} \\ e_2^{\mathrm{T}}-e_3^{\mathrm{T}} \end{bmatrix}\right\},$$

$$\Xi_2=\begin{bmatrix} e_2^{\mathrm{T}} \\ e_3^{\mathrm{T}} \\ \vdots \\ e_{m+1}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}X\begin{bmatrix} e_2^{\mathrm{T}} \\ e_3^{\mathrm{T}} \\ \vdots \\ e_{m+1}^{\mathrm{T}} \end{bmatrix}-\begin{bmatrix} e_3^{\mathrm{T}} \\ e_4^{\mathrm{T}} \\ \vdots \\ e_{m+2}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}X\begin{bmatrix} e_3^{\mathrm{T}} \\ e_4^{\mathrm{T}} \\ \vdots \\ e_{m+2}^{\mathrm{T}} \end{bmatrix},$$

$$\Xi_3=\sum_{j=1}^{m-1}\left(\begin{bmatrix} e_{j+1}^{\mathrm{T}} \\ e_{j+2}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}R_j\begin{bmatrix} e_{j+1}^{\mathrm{T}} \\ e_{j+2}^{\mathrm{T}} \end{bmatrix}-\begin{bmatrix} e_{j+2}^{\mathrm{T}} \\ e_{j+3}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}R_j\begin{bmatrix} e_{j+2}^{\mathrm{T}} \\ e_{j+3}^{\mathrm{T}} \end{bmatrix}\right),$$

$$\Xi_4(k)=\sum_{j=1}^{k-1}\left[e_{j+1}Q_je_{j+1}^{\mathrm{T}}-e_{j+2}Q_je_{j+2}^{\mathrm{T}}\right] + e_{k+1}Q_ke_{k+1}^{\mathrm{T}}-(1-\mu)e_1Q_ke_1^{\mathrm{T}},$$

$$\Xi_5(i,k)=\sum_{j=1,j\neq k}^{m}\begin{bmatrix} e_{j+1}^{\mathrm{T}} \\ e_{j+2}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} -Z_j & Z_j \\ * & -Z_j \end{bmatrix}\begin{bmatrix} e_{j+1}^{\mathrm{T}} \\ e_{j+2}^{\mathrm{T}} \end{bmatrix} + \begin{bmatrix} e_{k+1}^{\mathrm{T}} \\ e_1^{\mathrm{T}} \\ e_{k+2}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} -Z_k & Z_k-S_{ik} & S_{ik} \\ * & -2Z_k+\mathrm{He}(S_{ik}) & Z_k-S_{ik} \\ * & * & -Z_k \end{bmatrix}\begin{bmatrix} e_{k+1}^{\mathrm{T}} \\ e_1^{\mathrm{T}} \\ e_{k+2}^{\mathrm{T}} \end{bmatrix}.$$

Proof: For any $t\ge 0$, there should exist an integer $k\in\{1,2,\cdots,m\}$, such that $\tau(t)\in[(k-1)\delta, k\delta]$. Then, choose the following augmented LKF candidate:

$$V(t,x_t)\big|_{\{\tau(t)\in[(k-1)\delta,k\delta]\}}=\sum_{i=1}^{5}V_i(x_t), \tag{12}$$

where

$$V_1(x_t)=\eta_0^{\mathrm{T}}(t)P\eta_0(t),$$

$$V_2(x_t)=\int_{t-\delta}^{t}\zeta_1^{\mathrm{T}}(s)X\zeta_1(s)ds,$$

$$V_3(x_t)=\sum_{j=1}^{m-1}\int_{t-\delta}^{t}\eta_j^{\mathrm{T}}(s)R_j\eta_j(s)ds,$$

$$V_4(x_t)=\sum_{j=1}^{k-1}\int_{t-j\delta}^{t-(j-1)\delta}x^{\mathrm{T}}(s)Q_jx(s)ds + \int_{t-\tau(t)}^{t-(k-1)\delta}x^{\mathrm{T}}(s)Q_kx(s)ds,$$

$$V_5(x_t) = \sum_{j=1}^{m} \delta \int_{-j\delta}^{-(j-1)\delta} \int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s) Z_j \dot{x}(s) ds d\theta + \delta \int_{-\delta}^{0} \int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s) Z_0 \dot{x}(s) ds d\theta,$$

with $\eta_0(t) = [x^{\mathrm{T}}(t), \int_{t-\delta}^{t} x^{\mathrm{T}}(s)ds]^{\mathrm{T}}$ and $\eta_j(s) = [x^{\mathrm{T}}(s-(j-1)\delta), x^{\mathrm{T}}(s-j\delta)]^{\mathrm{T}}$, $j = 1,2,\cdots,m-1$.

Taking derivative of $V(t,x_t)|_{\{\tau(t)\in[(k-1)\delta,k\delta]\}}$ along the trajectory of the nominal system (6) yields:

$$\dot{V}(t,x_t)|_{\{\tau(t)\in[(k-1)\delta,k\delta]\}} = \sum_{i=1}^{5} \dot{V}_i(x_t), \tag{13}$$

where

$$\dot{V}_1(x_t) = 2\eta_0^{\mathrm{T}}(t) P \dot{\eta}_0(t) = \zeta^{\mathrm{T}}(t) \Xi_1 \zeta(t), \tag{14}$$

$$\begin{aligned}\dot{V}_2 &= \zeta_1^{\mathrm{T}}(t) X \zeta_1(t) - \zeta_1^{\mathrm{T}}(t-\delta) X \zeta_1(t-\delta) \\ &= \zeta^{\mathrm{T}}(t) \Xi_2 \zeta(t),\end{aligned} \tag{15}$$

$$\begin{aligned}\dot{V}_3(x_t) &= \sum_{j=1}^{m-1} [\eta_j^{\mathrm{T}}(t) R_j \eta_j(t) - \eta_j^{\mathrm{T}}(t-\delta) R_j \eta_j(t-\delta)] \\ &= \zeta^{\mathrm{T}}(t) \Xi_3 \zeta(t),\end{aligned} \tag{16}$$

$$\begin{aligned}\dot{V}_4(x_t) \le& \sum_{j=1}^{k-1} [x^{\mathrm{T}}(t-(j-1)\delta) Q_j x(t-(j-1)\delta) \\ &- x^{\mathrm{T}}(t-j\delta) Q_j x(t-j\delta] \\ &+ x^{\mathrm{T}}(t-(k-1)\delta) Q_k x(t-(k-1)\delta) \\ &- (1-\mu) x^{\mathrm{T}}(t-\tau(t)) Q_k x(t-\tau(t)) \\ =& \zeta^{\mathrm{T}}(t) \Xi_4(k) \zeta(t),\end{aligned} \tag{17}$$

$$\begin{aligned}\dot{V}_5(x_t) =& \dot{x}^{\mathrm{T}}(t) \left[\delta^2 \sum_{j=0}^{m} Z_j \right] \dot{x}(t) - \delta \int_{t-\delta}^{t} \dot{x}^{\mathrm{T}}(s) Z_0 \dot{x}(s) ds \\ &- \delta \sum_{j=1}^{m} \int_{t-j\delta}^{t-(j-1)\delta} \dot{x}^{\mathrm{T}}(s) Z_j \dot{x}(s) ds.\end{aligned} \tag{18}$$

For the case of $\tau(t) \notin [(k-1)\delta, k\delta]$ and $\tau(t) \in [(k-1)\delta, k\delta]$, $1 \le k \le m$, applying Jensen inequality and Lemma 2 (Peng-Park's integral inequality) to deal with the last item in (18), respectively, it can be deduced for $\begin{bmatrix} Z_k & \hat{S}_k \\ * & Z_k \end{bmatrix} \ge 0$, where $\hat{S}_k = \sum_{i=1} h_i S_{ik}$, that

$$\begin{aligned}&-\delta \sum_{j=1}^{m} \int_{t-j\delta}^{t-(j-1)\delta} \dot{x}^{\mathrm{T}}(s) Z_j \dot{x}(s) ds \\ &\le \sum_{j=1, j\neq k}^{m} \nu_1^{\mathrm{T}}(t) \begin{bmatrix} -Z_j & Z_j \\ * & -Z_j \end{bmatrix} \nu_1(t) \\ &+ \varpi_1^{\mathrm{T}}(t) \begin{bmatrix} -Z_k & Z_k - \hat{S}_k & \hat{S}_k \\ * & -2Z_k + \mathrm{He}(\hat{S}_k) & Z_k - \hat{S}_k \\ * & * & -Z_k \end{bmatrix} \varpi_1(t) \\ &= \sum_{i=1}^{r} h_i \zeta^{\mathrm{T}}(t) \Xi_5(i,k) \zeta(t),\end{aligned} \tag{19}$$

where $\nu_1(t) = [x^{\mathrm{T}}(t-(j-1)\delta), x^{\mathrm{T}}(t-j\delta)]^{\mathrm{T}}$, $\varpi_1(t) = [x^{\mathrm{T}}(t-(k-1)\delta), x^{\mathrm{T}}(t-\tau(t)), x^{\mathrm{T}}(t-k\delta)]^{\mathrm{T}}$.

On the other hand, it follows from Lemma 3 that

$$\begin{aligned}&-\delta \int_{t-\delta}^{t} \dot{x}^{\mathrm{T}}(s) Z_j \dot{x}(s) ds \\ &\le \nu_2^{\mathrm{T}}(t) \begin{bmatrix} -4Z_0 & -2Z_0 & 6Z_0 \\ * & -4Z_0 & 6Z_0 \\ * & * & -12Z_0 \end{bmatrix} \nu_2(t) \\ &= \zeta^{\mathrm{T}}(t) \Xi_0 \zeta(t),\end{aligned} \tag{20}$$

where $\nu_2(t) = \left[x^{\mathrm{T}}(t), x^{\mathrm{T}}(t-\delta), \frac{1}{\delta} \int_{t-\delta}^{t} x^{\mathrm{T}}(s) ds \right]^{\mathrm{T}}$.

By (13)-(20), the following inequality holds

$$\dot{V}(t,x_t)|_{\{\tau(t)\in[(k-1)\delta,k\delta]\}} \le \sum_{i=1}^{r} h_i \zeta^{\mathrm{T}}(t) \Xi(i,k) \zeta(t), \tag{21}$$

where $\Xi(i,k)$ is defined in Theorem 1.

In what follows, the nominal system (6) with the augmented vector $\zeta(t)$ can be rewritten as:

$$0 = \sum_{i=1}^{r} h_i \Gamma_i \zeta(t),$$

where Γ_i $(i = 1,2,\cdots,r)$ are defined in Theorem 1.

Therefore, the asymptotic stability conditions for the nominal system (6) can be represented by

$$\begin{aligned}&\sum_{i=1}^{r} h_i \zeta^{\mathrm{T}}(t) \Xi(i,k) \zeta(t) < 0, \\ &\textit{subject to}: \ 0 = \sum_{i=1}^{r} h_i \Gamma_i \zeta(t).\end{aligned} \tag{22}$$

By Finsler's lemma, for any matrix Y with appropriate dimension, the conditions in (22) are equivalent to

$$\sum_{i=1}^{r} h_i \zeta^{\mathrm{T}}(t) [\Xi(i,k) + \mathrm{He}(Y\Gamma_i)] \zeta(t) < 0. \tag{23}$$

Then, it follows from (21), (22), (23) and LMIs (10) that $\dot{V}(t,x_t)|_{\{\tau(t)\in[(k-1)\delta,\ k\delta]\}} < 0$. This means

$$\dot{V}(t,x_t)|_{\{\tau(t)\in[(k-1)\delta,k\delta]\}} < -\gamma \|x(t)\|^2$$

for a sufficiently small $\gamma > 0$. Therefore, by Lyapunov-Krasovskii stability theorem [26], the nominal system (6) with any delay $\tau(t)$ satisfying (2) and (3) is globally asymptotically stable. This completes the proof.

For the uncertain T-S fuzzy system (6), replacing A_i and A_{di} with $A_i + HF(t)E_i$ and $A_{di} + HF(t)E_{di}$ in (10), the following result can be easily derived by applying Lemma 4 and Schur complement [27]. Thus, it is omitted here.

Theorem 2: Given a positive integer m, scalars $\tau \ge 0$, μ, and $\delta = \frac{\tau}{m}$, then the uncertain T-S system

(6) with the time-delay $\tau(t)$ satisfying (2) and (3) is asymptotically stable if there exist scalars $\varepsilon_{ik} > 0$ $(i=1,\cdots,r;\ k=1,\cdots,m)$, symmetric positive matrices $P=\begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix}$, $X=[X_{ij}]_{m\times m}$, Q_j, Z_0, Z_j, $R_l=\begin{bmatrix} R_{1l} & R_{2l} \\ * & R_{3l} \end{bmatrix}$ and any matrices Y and $S_{ij}\,(i=1,2,\cdots,r; j=1,2,\cdots,m;\ l=1,2,\cdots,m-1)$ with appropriate dimensions, such that the following LMIs hold for $i=1,2,\cdots,r$ and $k=1,2,\cdots,m$:

$$\begin{bmatrix} \Xi(i,k)+\mathrm{He}(Y\Gamma_i) & YH & \varepsilon_{ik}(e_2E_i^{\mathrm{T}}+e_1E_{di}^{\mathrm{T}}) \\ * & -\varepsilon_{ik}I & 0 \\ * & * & -\varepsilon_{ik}I \end{bmatrix} < 0, \quad (24)$$

$$\Upsilon(i,k) \ge 0, \quad (25)$$

where $\Xi(i,k)$, Γ_i and $\Upsilon(i,k)$ are defined in Theorem 1.

Remark 1: Based on delay-partitioning approach, the new LKF (12) is different from those in [5,11,14,18,20-22,28] on account of the $[X_{ij}]_{m\times m}$-dependent integral item is considered. In addition, the relationships between the augmented state vectors $[x^{\mathrm{T}}(t),\ x^{\mathrm{T}}(t-\delta),\ x^{\mathrm{T}}(t-2\delta),\ \cdots, x^{\mathrm{T}}(t-m\delta)]^{\mathrm{T}}$ have been fully taken into account by employing such a $[X_{ij}]_{m\times m}$-dependent LKF, which combing with the Finsler's lemma will be helpful to reduce the conservativeness of the derived conditions. This will be demonstrated later by numerical examples.

Remark 2: For $\tau(t)\in[(k-1)\delta,\ k\delta]$ $(1\le k\le m)$, a tighter bounding inequality, i.e., Peng-Park's integral inequality (Lemma 2), is employed to effectively estimate the time-varying delay-dependent integral term $-\delta\int_{t-k\delta}^{t-(k-1)\delta} \dot{x}^{\mathrm{T}}(s)Z_k\dot{x}(s)ds$. Since (i) no free weighting matrices are employed and (ii) none of time-varying delay-dependent useful items are ignored, some improvements in both computational efficiency and performance behavior may be expected while inheriting the advantages of delay-partitioning method [14]. On the other hand, Seuret-Wirtinger's integral inequality (Lemma 3), that is shown less conservative than previous inequalities often based on Jensen's theorem, is adopted to effectively estimate the integral term $-\delta\int_{t-\delta}^{t} \dot{x}^{\mathrm{T}}(s)Z_0\dot{x}(s)ds$.

Remark 3: In the proof of Theorem 1, some fuzzy-weighting matrices $\hat{S}_k=\sum_{i=1}^{r} h_iS_{ik}$ are introduced to consider the relationships of the T-S fuzzy models, which will lead to less conservative results [11].

Remark 4: The vector $e_s s=\cdots m+$ defined in (9) plays a crucial role in representing the derivative of the augmented LKF in a unified framework of state vector augmentation, without listing out each elements of the large-scale symmetric block-matrix (see appendix) one by one. It's worth mentioning that, the LMIs-based stability criteria in e_s-form can be directly implemented by Matlab LMI Toolbox, for example, the term $e_2P_1e_{m+4}^{\mathrm{T}}$ in (10) indicates that one of $(2,m+4)$'s elements in LMI (10) is the matrix P_1.

Finally, in the case of the time-varying delay $\tau(t)$ being non-differentiable or unknown $\dot{\tau}(t)$, setting $Q_k=0$ $(Q_j\ne 0, j=1,\cdots,k-1)$ in Theorem 2, we have the following corollary.

Corollary 1: Given a positive integer m, scalars $\tau\ge 0$ and $\delta=\frac{\tau}{m}$, then the uncertain T-S system (6) with the time-delay $\tau(t)$ satisfying (2) is asymptotically stable if there exist scalars $\varepsilon_{ik}>0$ $(i=1,\cdots,r; k=1,\cdots,m)$, symmetric positive matrices $P=\begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix}$, $X=[X_{ij}]_{m\times m}$, Z_0, Z_j, $R_l=\begin{bmatrix} R_{1l} & R_{2l} \\ * & R_{3l} \end{bmatrix}$ and any matrices Y and S_{ij} $(i=1,2,\cdots,r;\ j=1,2,\cdots,m;\ l=1,2,\cdots,m-1)$ with appropriate dimensions, such that the following LMIs hold for $i=1,2,\cdots,r$ and $k=1,2,\cdots,m$:

$$\begin{bmatrix} \tilde{\Xi}(i,k)+\mathrm{He}(Y\Gamma_i) & YH & \varepsilon_{ik}(e_2E_i^{\mathrm{T}}+e_1E_{di}^{\mathrm{T}}) \\ * & -\varepsilon_{ik}I & 0 \\ * & * & -\varepsilon_{ik}I \end{bmatrix} < 0, \quad (26)$$

$$\Upsilon(i,k)\ge 0, \quad (27)$$

where

$$\tilde{\Xi}(i,k)=\sum_{j=0}^{3}\Xi_j+\tilde{\Xi}_4(k)+\Xi_5(i,k) + e_{m+4}\left(\delta^2\sum_{j=0}^{m}Z_j\right)e_{m+4}^{\mathrm{T}}$$

with Γ_i, $\Upsilon(i,k)$, $\Xi_0,\cdots,\Xi_3$ and $\Xi_5(i,k)$ are defined in Theorem 1 and

$$\tilde{\Xi}_4(k)=\sum_{j=1}^{k-1}\left[e_{j+1}Q_je_{j+1}^{\mathrm{T}}-e_{j+2}Q_je_{j+2}^{\mathrm{T}}\right].$$

4. NUMERICAL EXAMPLE

This section gives two examples to demonstrate the effectiveness of the proposed approach. For comparisons, we study the T-S fuzzy system (6) with fuzzy rules investigated in recent publications [5,11,14,18-22,28].

Example 1: Consider the following time-delayed nonlinear system [5,11,14,18,20-22,28]:

$$\begin{cases} \dot{x}_1(t)=0.5[1-\sin^2(\theta(t))]x_2(t)-x_1(t-\tau(t)) \\ \qquad -[1+\sin^2(\theta(t))]x_1(t), \\ \dot{x}_2(t)=\mathrm{sgn}(|\theta(t)|-\pi/2)[0.9\cos^2(\theta(t))-1]x_1(t-\tau(t)) \\ \qquad -x_2(t-\tau(t))-[0.9+0.1\cos^2(\theta(t))]x_2(t), \end{cases}$$

which can be exactly expressed as a T-S fuzzy system (6) with the following rules [14,22,28]:

$$R^1: \text{ If } \theta(t) \text{ is } \pm\pi/2, \text{ then } x(t)=A_1x(t)+A_{d1}x(t-\tau(t));$$
$$R^2: \text{ If } \theta(t) \text{ is } 0, \text{ then } x(t)=A_2x(t)+A_{d2}x(t-\tau(t)), \quad (28)$$

where

$$A_1=\begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix},\quad A_{d1}=\begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},$$

Table 1. Maximum allowable delay bounds of τ for different μ: Example 1.

μ	0	0.1	≥ 1
Tian *et al.* [28]	1.597	-	0.721
Lien *et al.* [5]	1.5973	1.484	0.831
Li *et al.*[20]	1.5974	1.4847	0.982
Liu *et al.* [21]	1.5974	1.4957	1.2642
Peng *et al.*[22]	1.8034	-	0.9899
Kwon *et al.* [18]	1.6609	1.5332	1.2696
Zeng *et al.* [11] (*m*=3)	2.0002	1.8090	1.3631
Theorem 1 (*m*=3)	2.3359	2.1698	1.6381
Theorem 1 (*m*=4)	2.4870	2.3279	1.8274
[11] improved by (*m*=3)	> 16.8 %	> 19.9 %	> 20.2 %

$$A_2 = \begin{bmatrix} -1 & 0.5 \\ 0 & -1 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} -1 & 0 \\ 0.1 & -1 \end{bmatrix}.$$

The membership functions for above rules 1 and 2 are

$$h_1(\theta(t)) = \sin^2(\theta(t)), \qquad h_2(\theta(t)) = \cos^2(\theta(t)), \tag{29}$$

where $\theta(t) = x_1(t)$.

For different μ, the Maximum allowable delay bounds of the time-varying delay computed by Theorem 1 with m=3,4 are listed in Table 1. For comparison, the upper bounds obtained by the conditions in [5,11,18,20-22,28] are also tabulated in Table 1, where "–" denotes that the results are not provided in these papers. It is clear that the method proposed in this paper is less conservative than those in [5,11,18,20-22,28]. It is also concluded that the conservatism is gradually reduced with the increase of m. With initial state conditions $[1,-1]^{\mathrm{T}}$, Fig. 1 shows the simulation results of the state responses of the T-S fuzzy system (28) with $0 \le \tau(t) \le 2.4870$ listed in Table 1; and the phase portrait of system (28) is given in Fig. 2. It shows from the simulation results (Figs. 1 and 2) that the maximum allowable delay bounds of τ listed in Table 1 are capable of guaranteeing asymptotical stability of the considered system (28).

Example 2: Consider the following uncertain T-S fuzzy system [5,11,19,21]

$$\dot{x}(t) = \sum_{i=1}^{2} h_i\{(\theta(t))[A_i(t)x(t) + A_{di}(t)x(t-\tau(t))\}, \tag{30}$$

where

$$A_1 = \begin{bmatrix} -2 & 1 \\ 0.5 & -1 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} -1.6 & 0 \\ 0 & -1 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 1.6 & 0 \\ 0 & 0.05 \end{bmatrix},$$

$$E_{d1} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1.6 & 0 \\ 0 & -0.05 \end{bmatrix},$$

$$E_{d2} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad H = \begin{bmatrix} 0.03 & 0 \\ 0 & -0.03 \end{bmatrix},$$

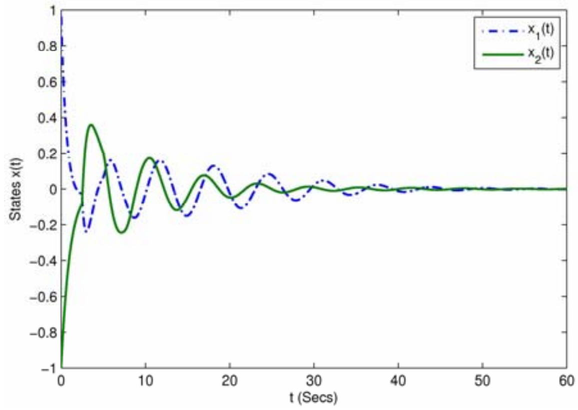

Fig. 1. The state responses of system (28).

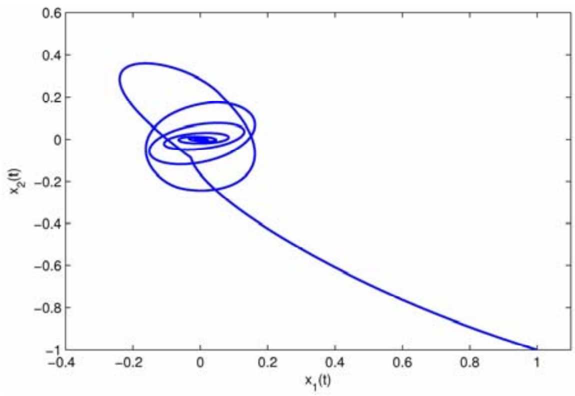

Fig. 2. The phase portrait of system (28).

and the membership functions are the same in (29).

For different μ, by utilizing Theorem 2, Corollary 1 and the conditions in [5,11,19,21], the upper bounds that guarantee the robust stability of the considered system are summarized in Table 2. It can be concluded that the result proposed in this paper is less conservative than those in [5,11,19,21]. Meanwhile, it is shown in Table 2 that the conservatism is gradually reduced with the increase of m. With initial state conditions $[1,-1]^{\mathrm{T}}$ and the unknown matrix function $F(t) = \mathrm{diag}\{\sin t, \cos t\}$, Fig. 3 shows the simulation results of the state responses of the system (30) with $0 \le \tau(t) \le 1.6425$ listed in Table 2; and the phase portrait of (30) is given in Fig. 4. It also shows from the simulation results (Figs. 3 and 4) that the maximum allowable delay bounds of τ listed in Table 2 are capable of guaranteeing asymptotically robust stability of the considered system (30).

Table 2. Maximum allowable delay bounds of τ for different μ: Example 2.

μ	0	0.1	0.5	Unknown
Li *et al.* [19]	0.950	0.892	0.637	–
Lien *et al.* [5]	1.168	1.122	0.934	0.499
Liu *et al.* [21]	1.192	1.155	1.100	1.050
Zeng *et al.* [11] (*m*=2)	1.390	1.318	1.132	1.127
Theorem 2 (*m*=2)	1.4737	1.4182	1.2916	1.2299
Theorem 2 (*m*=3)	1.6425	1.5990	1.4923	1.4182
[11] improved by (*m*=2)	>6.0 %	>8.9 %	>14.1 %	>9.1 %

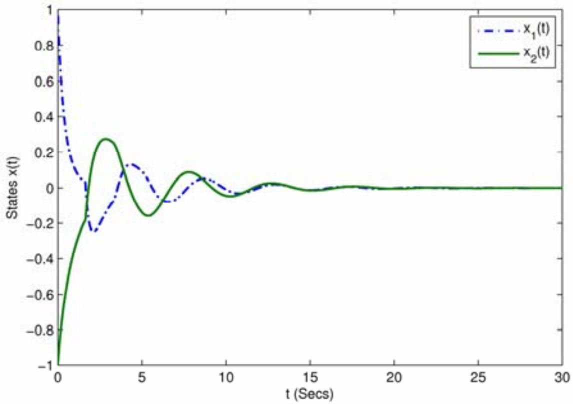

Fig. 3. The state responses of system (30).

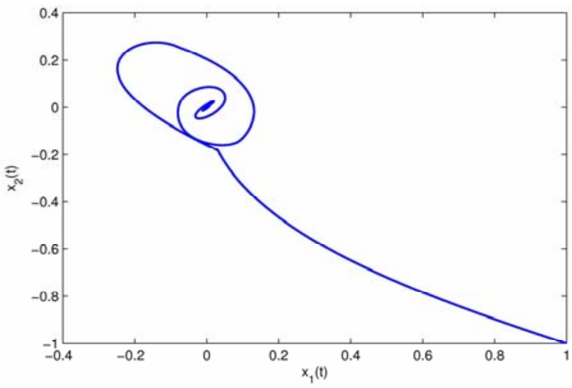

Fig. 4. The phase portrait of system (30).

5. CONCLUSION

The robust stability criteria for uncertain T-S fuzzy systems with time-varying delay have been investigated in this paper by delay-partitioning approach, Finsler's lemma and LMIs approach. An appropriate LKF is established in the framework of state vector augmentation. Then, by virtue of employing Seuret-Wirtinger's integral inequality and Peng-Park's integral inequality to effectively deal with (time-varying) delay-dependent integral items, none of any useful time-varying items are arbitrarily ignored, therefore, less conservative results can be expected. Two numerical examples have been given to demonstrate that the proposed result is an improvement over existing ones.

APPENDIX A

Theorem 1 is in the form of e_s defined in (9), another conventional form (i.e., the large-scale symmetric block-matrix form) of Theorem 1 is also given as follows:

Theorem 1': Given a positive integer m, scalars $\tau \geq 0$, μ, and $\delta = \frac{\tau}{m}$, then the nominal system (6) with the time-delay $\tau(t)$ satisfying (2) and (3) is asymptotically stable if there exist symmetric positive matrices $P = \begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix}$, $R_l = \begin{bmatrix} R_{1l} & R_{2l} \\ * & R_{3l} \end{bmatrix}$, $X = [X_{ij}]_{m\times m}$, Q_j, Z_0, Z_j, and any matrices Y_1, Y_2, Y_3 and $S_{ij}(i = 1, 2, \cdots, r;\ j = 1, 2, \cdots, m;\ l = 1, 2, \cdots, m-1)$ with appropriate dimensions, such that the following LMIs hold for $i = 1, 2, \cdots, r$ and $k = 1, 2, \cdots, m$:

$$\Pi + \Psi_{ik} + \Lambda_k + \Phi_i < 0, \tag{A.1}$$

$$\Upsilon(i,k) = \begin{bmatrix} Z_k & S_{ik} \\ * & Z_k \end{bmatrix} \geq 0, \tag{A.2}$$

where

$$\Pi = (\pi_{lj})_{(m+4)\times(m+4)} + (\pi_{lj})^{\mathrm{T}}_{(m+4)\times(m+4)},$$
$$\Psi_{ik} = (\psi_{lj})_{(m+4)\times(m+4)} + (\psi_{lj})^{\mathrm{T}}_{(m+4)\times(m+4)},$$
$$\Lambda_k = \mathrm{diag}\{\Lambda_{k1}, \Lambda_{k2}, \cdots, \Lambda_{k,m+4}\},$$

$$\Phi_i = \begin{bmatrix} \mathrm{He}(Y_1 A_{di}) & Y_1 A_i + A_{di}^{\mathrm{T}} Y_2 & 0 & \cdots & 0 & 0 & 0 & -Y_1 + A_{di}^{\mathrm{T}} Y_3^{\mathrm{T}} \\ * & \varphi_1 & \chi_1 & \cdots & 0 & 0 & 6Z_0 + \delta P_3 & P_1 - Y_2 + A_i^{\mathrm{T}} Y_3^{\mathrm{T}} \\ * & * & \varphi_2 & \cdots & 0 & 0 & 6Z_0 - \delta P_3 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ * & * & * & \cdots & \varphi_m & \chi_m & 0 & 0 \\ * & * & * & \cdots & * & \varphi_{m+1} & 0 & 0 \\ * & * & * & \cdots & * & * & -12Z_0 & \delta P_2^{\mathrm{T}} \\ * & * & * & \cdots & * & * & * & \left(\delta^2 \sum_{j=0}^{m} Z_j\right) - \mathrm{He}(Y_3) \end{bmatrix}$$

with

$$\pi_{lj} = \begin{cases} X_{1,j-1}, & l = 2, 2 \leq j \leq m+1, \\ X_{l-1,j-1} - X_{l-2,j-2}, & 3 \leq l \leq m+1, l \leq j \leq m+1, \\ X_{l-2,m}, & 3 \leq l \leq m+2, 2 \leq j = m+2, \\ 0, & otherwise, \end{cases}$$

$$\psi_{lj} = \begin{cases} -Z_k + S_{ik}, & l = j = 1, \\ Z_k - S_{ik}^T, & l = 1, j = k+1, \\ Z_k - S_{ik}, & l = 1, j = k+2, \\ -Z_k + S_{ik}, & l = k+1, j = k+2, \\ 0, & otherwise, \end{cases}$$

$$\varphi_j = \begin{cases} R_{11} - Z_1 - 4Z_0 + \mathrm{He}(P_2 + Y_2 A_i), & j = 1, \\ R_{31} + R_{12} - R_{11} - Z_1 - Z_2 - 4Z_0, & j = 2, \\ R_{3,j-2} + R_{1,j-1} - R_{3,j-3} - R_{1,j-2} - Z_j - Z_{j-1}, & 3 \leq j \leq m-1, \\ R_{3,m-1} - R_{3,m-2} - R_{1,m-1} - Z_m - Z_{m-1}, & j = m, \\ -R_{3,m-1} - Z_m, & j = m+1, \end{cases}$$

$$\chi_j = \begin{cases} R_{21} + Z_1 - 2Z_0 - P_2, & j = 1, \\ R_{2,j} - R_{2,j-1} + Z_j, & 2 \le j \le m-1, \\ -R_{2,m-1} + Z_m, & j = m, \end{cases}$$

$$\Lambda_{kj} = \begin{cases} -(1-\mu)Q_k, & j = 1, \\ Q_1, & j = 2, \\ Q_{j-1} - Q_{j-2}, & 3 \le j \le k+1, \\ 0, & otherwise. \end{cases}$$

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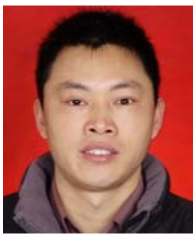

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