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Consensus of Multiple Euler-Lagrange Systems Using One Euler-Lagrange System's Velocity Measurements

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Abstract: This brief paper studies the stationary consensus of multiple Euler-Lagrange systems with nonlinear protocols. Two consensus protocols are given to guarantee that positions and velocities of multiple Euler-Lagrange systems reach consensus. Proposed protocols need only the relative position measurements and the velocity measurements of one Euler-Lagrange system. Finally, numerical simulations are given to illustrate the theoretical results.

Keywords: Consensus, euler-Lagrange system, multi-agent system, nonlinear protocol.

1. INTRODUCTION

Consensus problems of multiple Euler-Lagrange (EL) systems have been intensively investigated in recent years. This research interest is mainly because that EL systems can describe a large class of mechanical systems, including autonomous vehicles, power systems, walking robots, robotic manipulators and rigid bodies [1–4].

Various distributed controllers were proposed to tackle the consensus control problems of multiple EL systems. By using Lyapunov theory and Matrosov theory, three distributed controllers were proposed to demonstrate the stability of networked EL systems [5]. The authors in [6] proposed a model-independent cross-coupled controller for position synchronization of multi-axis motions. Considering the effects of communication delays, the protocols with time-delays were studied in [7–10]. The consensus algorithms for multiple EL systems without using neighbors' velocity information were presented in [11–13]. Some consensus algorithms for heterogeneous EL systems were developed in [14, 15]. The authors in [16–18] studied the consensus problem of networked EL systems, in which the parametric uncertainties were considered. The leader-following consensus of multiple EL systems was also studied recently [19].

Related to the consensus of networked EL systems are the consensus problems in multi-agent systems [20–27]. Under the assumption that the velocity of the active leader can not be measured in real time, distributed observers were designed for leader-following control in [20]. The consensus algorithms for double-integrator dynamics in four cases were proposed in [21]. Using local information, the authors in [22] presented the leader-following consensus protocols for both fixed and switching interaction topologies. The nonlinear protocols for leaderless and leader-following were developed in [23]. The authors in [24] developed a thermodynamic framework for addressing consensus problems for nonlinear multi-agent systems with switching topologies. In [25], some sufficient conditions guaranteeing exponential consensus for directed networks were presented. The reference [26] studied the second-order leader-following consensus problem of nonlinear multi-agent systems without assuming that the interaction diagraph is strongly connected or contains a directed spanning. Two protocols, which make all agents asymptotically reach consensus while accomplishing some tasks, were introduced in [27].

It is well known that retrieval of the velocity information of all EL systems is generally difficult in practical situations due to technology limitations and external environment conditions. For example, in order to save cost, space and weight, some EL systems are not equipped with velocity sensors [11]. Hence, these EL systems can not obtain any velocity information, and the protocols proposed in [6] and [7] can not be implemented. On the other hand, some EL systems may miss velocity information un-

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der special environments, or the obtained velocity measurements are inaccurate [28, 29]. By introducing an auxiliary system, an observer or a distributed filter, consensus algorithms without velocity measurements were derived in [11–13]. However, these control strategies make consensus protocols complex and improve the control costs. In order to deal with above problems, this paper presents two simple consensus protocols, in which only one EL systems's velocity measurements and relative position measurements between EL systems are needed, to solve the consensus problems of multiple EL systems.

In addition, the nonlinear factor inevitably exists as a nonlinear function of a variable between EL systems in the measurements. For instance, due to the limitation of observational technique and the inaccuracy of model parameters, the velocity $\dot{q}_i$ of the EL system may be unobservable in some cases, and a nonlinear function $f(\dot{q}_i)$ of the velocity $\dot{q}_i$ can be observed [24, 25]. On the other hand, since EL systems are always subjected to some uncertain nonlinear factors, the relative position $h(q_j - q_i)$ can be measured instead of $q_j - q_i$ during information exchange. Accordingly, in order to achieve high control performance, nonlinear control protocols should be considered in the consensus problem of multiple EL systems.

The main contribution of this paper is to provide two simple consensus protocols, in which only relative position information between EL systems and one EL system's velocity information are needed, to guarantee that all interconnected EL systems reach stationary consensus. The proposed protocol can also solve the tracking problem for multiple EL systems where only one EL system knows the desired position. The rest of the paper is organized as follows. Section 2 states the problem formulations. Section 3 gives the main consensus results of multiple EL systems. Two simulation examples are given to show the effectiveness of the proposed control algorithms in Section 4. Finally, Section 5 presents a brief conclusion to this paper.

Some mathematical notations to be used throughout this paper are given below. Let R define a set of real numbers; $R^{m\times 1}$ be the m-dimensional real column vector; $R^{m\times m}$ be the set of $m\times m$ real matrices; and $\bar{n} = \{1, 2, \cdots, n\}$ be an index set. $\lambda_m(A)$ and $\lambda_M(A)$ represent the minimum and maximum eigenvalues of matrix A, respectively. $\|x\|$ denotes the Euclidean norm of vector x. For any function $f : R_{\geq 0} \to R^m$, the L_∞-norm is defined as $\|f\|_\infty = \sup_{t\geq 0}\|f(t)\|$, and the L_2-norm as $\|f\|_2^2 = \int_0^\infty \|f(t)\|^2\,dt$. The L_∞ spaces are defined as the sets $f : R_{\geq 0} \to R^m : \|f\|_\infty < \infty$ and $R_{\geq 0} \to R^m : \|f\|_2 < \infty$. $A \succeq 0$ denotes that matrix A is positive definite.

2. PROBLEM FORMULATIONS

Consider n EL systems described by the following equations

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) = \tau_i + \iota_i, \quad i \in \bar{n}, \quad (1)$$

where $q_i = (q_{i1}, \cdots, q_{im})^\top \in R^{m\times 1}$ is the vector of generalized coordinates, $M_i(q_i) \in R^{m\times m}$ is the symmetric positive-finite inertial matrix, $C_i(q_i, \dot{q}_i)\dot{q}_i \in R^{m\times 1}$ is the vector of Coriolis and centrifugal torques, $G_i(q_i)$ is the gravitational torques and it is compensated [5], [8]. $\tau_i + \iota_i \in R^m$ is the vector for torques produced by the actuators associated with the ith EL system.

The control objective is to design protocol $\tau_i + \iota_i$ which can guarantee that positions and velocities of all EL systems in (1) achieve stationary consensus, namely, $\lim_{t\to\infty}\|q_i - q_j\| = 0$, $\lim_{t\to\infty}\|\dot{q}_i\| = 0$, $i, j \in \bar{n}$. The protocol ι_i is used to compensate the effects of $G_i(q_i)$ on the dynamics, so choose $\iota_i = G_i(q_i)$. Control input τ_i is introduced to satisfy the objective of the consensus control of system (1) as defined above. The design of τ_i is as follows:

(i) Suppose that τ_1 for the EL system, labeled as 1, is $-Bf(\dot{q}_1) + c_{12}h(q_2 - q_1)$. Matrix $B = diag(b_1(t), b_2(t), \cdots, b_m(t))$ is symmetric positive definite and satisfies that $0 < b \leq b_i(t) < +\infty$, $\dot{b}_i(t)$ is bounded. Communication link $c_{12} = c_{21} > 0$ denotes the connection between the first and the second EL system. Nonlinear function f and h satisfy the following Hypothesis 1 and Hypothesis 2, respectively.

(ii) For $i = 2, \cdots, n-1$, suppose that τ_i for the EL system, labeled as i, is $c_{i,i-1}h(q_{i-1} - q_i) + c_{i,i+1}h(q_{i+1} - q_i)$, in which $c_{i,i-1} = c_{i-1,i} > 0$ denotes there is a connection between the ith and the $(i-1)th$ EL system.

(iii) Similarly, suppose that τ_n for the EL system, labeled as n, is $c_{n,n-1}h(q_{n-1} - q_n)$ where $c_{n,n-1} = c_{n-1,n} > 0$. Based on above discussions, the protocol $\tau_i + \iota_i$ is given as

$$\tau_i + \iota_i = \begin{cases} -Bf(\dot{q}_1) + c_{12}h(q_2 - q_1) \\ \qquad + G_1(q_1),\ i = 1, \\ c_{i,i-1}h(q_{i-1} - q_i) + c_{i,i+1}h(q_{i+1} - q_i), \\ \qquad + G_i(q_i),\ i = 2, \cdots, n-1, \\ c_{n,n-1}h(q_{n-1} - q_n) + G_n(q_n),\ i = n. \end{cases} \quad (2)$$

Remark 1: Control protocol (2) has the following features. Firstly, only relative position measurements between two neighbored EL systems and the velocity measurements of the first EL system are required. This is different from existing protocols in [3] and [5]. Secondly, protocol (2) does't need auxiliary systems [11], or a distributed filter and an observer [12]. Furthermore, in order to better reflect some real situations, the nonlinear information change is considered in protocol (2).

Protocol (2) has some limitations or drawbacks. Firstly, system (1) will take a longer time to achieve consensus since only the first EL system can obtain the velocity measurements. Secondly, system (1) has the non-uniform rate of convergence among EL systems. In other word, since the other $n-1$ EL systems don't obtain the velocity measurements, the first EL system has higher convergence rate than the others.

Now some fundamental properties for system (1) will be presented.

Property 1: Assume that $0 \prec \lambda_m(M_i(q_i))I \preceq M_i(q_i) \prec \lambda_M(M_i(q_i))I \prec \infty$, $\|C_i(q_i,\dot{q}_i)\| \le k_c\|\dot{q}_i\|$, $k_c > 0$.

Property 2: Under an appropriate definition of the matrix $C_i(q_i,\dot{q}_i)$, $\dot{M}_i(q_i) - 2C_i(q_i,\dot{q}_i)$ is skew-symmetric. Therefore, for a given vector $r \in R^m$, it is easy to verify that $r^\top(\dot{M}_i(q_i) - 2C_i(q_i,\dot{q}_i))r = 0$.

Property 3 [7]**:** Consider a mechanical system of the form (1). Assume that $\dot{q}_i, \ddot{q}_i \in L_\infty$, then the time derivative of its Coriolis matrix $C_i(q_i,\dot{q}_i)$ is bounded.

Two hypotheses and two lemmas are needed in the subsequent analysis.

Hypothesis 1: (1) $f_l(z) = 0 \Leftrightarrow z = 0$.
(2) $\frac{f_l(z_1)-f_l(z_2)}{z_1-z_2} \ge k > 0, \forall z_1 \ne z_2 \in R$.
(3) $f_l(z)$ is continuously differentiable function, and $f_l'(z)$ denotes the differentiation with respect to z.

Hypothesis 2: (1) $h_l(-z) = -h_l(z), \forall z \in R$.
(2) $h_l(z) = 0 \Leftrightarrow z = 0$.
(3) $(z_j - z_i)h_l(z_j - z_i) > 0, \forall z_j \ne z_i \in R$.
(4) $h_l(z)$ is continuously differentiable function.

Lemma 1 [30]**:** If $w : R \to R$ is a uniformly continuous function for $t \ge 0$ and if $\lim_{t\to\infty}\int_0^t |w(\lambda)|d\lambda$ exists and is finite, then $\lim_{t\to\infty} w(t) = 0$.

Lemma 2 [30]**:** Let $t \to g(t)$ be a differentiable function with a finite limit as $t \to \infty$. If $\dot{g}(t)$ is uniformly continuous, then $\lim_{t\to\infty}\dot{g}(t) = 0$.

3. POSITIONS OF EL SYSTEMS REACH CONSENSUS

In this section, the control protocols to guarantee that all EL systems asymptotically reach stationary consensus is derived based on the Lyapunov stability theory and the Barbălat lemma.

Theorem 1: Consider system (1) with protocol (2) and assume that Hypotheses 1-2 hold. Then, $\lim_{t\to\infty}\|q_j - q_i\| = 0$, $\lim_{t\to\infty}\|\dot{q}_i\| = 0$, $i, j \in \bar{n}$.

Proof: Select the Lyapunov function candidate

$$
\begin{aligned}
V(q_i,\dot{q}_i) &= \frac{1}{2}\sum_{i=1}^{n}\dot{q}_i^\top M_i(q_i)\dot{q}_i \\
&\quad + \sum_{i=1}^{n-1}\sum_{l=1}^{m} c_{i,i+1}\int_0^{q_{i+1,l}-q_{il}} h_l(s)ds.
\end{aligned}
$$

In the following proof, we have used the fact that $\dot{M}_i(q_i) - 2C_i(q_i,\dot{q}_i)$ is skew-symmetric, and the equality $\sum_{i=1}^{n-1} c_{i,i+1}(\dot{q}_{i+1}-\dot{q}_i)^\top h(q_{i+1}-q_i) = \sum_{i=2}^{n} c_{i-1,i}\dot{q}_i^\top h(q_i - q_{i-1}) - \sum_{i=2}^{n-1} c_{i,i+1}\dot{q}_i^\top h(q_{i+1}-q_i) - c_{12}\dot{q}_1^\top h(q_2-q_1) = \sum_{i=2}^{n-1} c_{i-1,i}\dot{q}_i^\top h(q_i - q_{i-1}) - \sum_{i=2}^{n-1} c_{i,i+1}\dot{q}_i^\top h(q_{i+1}-q_i) + c_{n-1,n}\dot{q}_n^\top h(q_n - q_{n-1}) - c_{12}\dot{q}_1^\top h(q_2-q_1)$.

Differentiating $V(q_i,\dot{q}_i)$ with respect to time along the solution of (1) yields

$$
\begin{aligned}
\dot{V} &= \sum_{i=1}^{n}\dot{q}_i^\top M_i(q_i)\ddot{q}_i + \frac{1}{2}\sum_{i=1}^{n}\dot{q}_i^\top \dot{M}_i(q_i)\dot{q}_i \\
&\quad + \sum_{i=1}^{n-1}\sum_{l=1}^{m} c_{i,i+1}h_l(q_{i+1,l}-q_{il})(\dot{q}_{i+1,l}-\dot{q}_{il}) \\
&= \dot{q}_1^\top\big(-C_1(q_1,\dot{q}_1)\dot{q}_1 - Bf(\dot{q}_1) + c_{12}h(q_2-q_1)\big) \\
&\quad + \dot{q}_n^\top\big(-C_n(q_n,\dot{q}_n)\dot{q}_n + c_{n,n-1}h(q_{n-1}-q_n)\big) \\
&\quad + \sum_{i=2}^{n-1}\dot{q}_i^\top M_i(q_i)\ddot{q}_i + \frac{1}{2}\dot{q}_1^\top \dot{M}_1(q_1)\dot{q}_1 \\
&\quad + \frac{1}{2}\dot{q}_n^\top \dot{M}_n(q_n)\dot{q}_n + \frac{1}{2}\sum_{i=2}^{n-1}\dot{q}_i^\top \dot{M}_i(q_i)\dot{q}_i \\
&\quad + \sum_{i=1}^{n-1} c_{i,i+1}(\dot{q}_{i+1}-\dot{q}_i)^\top h(q_{i+1}-q_i) \\
&= \frac{1}{2}\dot{q}_1^\top\big(\dot{M}_1(q_1) - 2C_1(q_1,\dot{q}_1)\big)\dot{q}_1 + \frac{1}{2}\dot{q}_n^\top\big(\dot{M}_n(q_n) \\
&\quad - 2C_n(q_n,\dot{q}_n)\big)\dot{q}_n + \dot{q}_1^\top\big(-Bf(\dot{q}_1) + c_{12}h(q_2 \\
&\quad - q_1)\big) + c_{n,n-1}\dot{q}_n^\top h(q_{n-1}-q_n) + \sum_{i=2}^{n-1}\dot{q}_i^\top \\
&\quad \big(-C_i(q_i,\dot{q}_i)\dot{q}_i + c_{i,i-1}h(q_{i-1}-q_i) + c_{i,i+1} \\
&\quad \times h(q_{i+1}-q_i)\big) + \frac{1}{2}\sum_{i=2}^{n-1}\dot{q}_i^\top \dot{M}_i(q_i)\dot{q}_i + \sum_{i=2}^{n-1} c_{i-1,i} \\
&\quad \times \dot{q}_i^\top h(q_i - q_{i-1}) - \sum_{i=2}^{n-1} c_{i,i+1}\dot{q}_i^\top h(q_{i+1}-q_i) \\
&\quad + c_{n-1,n}\dot{q}_n^\top h(q_n - q_{n-1}) - c_{12}\dot{q}_1^\top h(q_2-q_1) \\
&= -\dot{q}_1^\top Bf(\dot{q}_1) \le -bk\dot{q}_1^\top\dot{q}_1. \qquad (3)
\end{aligned}
$$

The inequality (3) is obtained based on the property of $f_l(z)$ and the property of matrix B.

We can conclude from inequality (3) that $\dot{V}(q_i,\dot{q}_i) \le 0$ and $\int_0^t bk\dot{q}_1^\top\dot{q}_1\,ds \le V(q_i(0),\dot{q}_i(0)) - V(q_i,\dot{q}_i)$. Hence, $\lim_{t\to\infty}\int_0^t bk\dot{q}_1^\top\dot{q}_1\,ds$ exists and is finite. Note that $M_1(q_1)\ddot{q}_1 = -C_1(q_1,\dot{q}_1)\dot{q}_1 - Bf(\dot{q}_1) + c_{12}h(q_2-q_1)$ and Property 1, we get that $2\dot{q}_1^\top\ddot{q}_1$ is bounded. By applying Lemma 1, we have $\lim_{t\to\infty}\|\dot{q}_1\| = 0$ since $\dot{q}_1^\top\dot{q}_1$ is uniformly continuous in time.

Furthermore, taking the derivative on both sides of equality $M_1(q_1)\ddot{q}_1 = -C_1(q_1,\dot{q}_1)\dot{q}_1 - Bf(\dot{q}_1) + c_{12}h(q_2-q_1)$, we get that $\dot{M}_1(q_1)\ddot{q}_1 + M_1(q_1)\frac{d}{dt}\ddot{q}_1 = -\frac{d}{dt}C_1(q_1,\dot{q}_1)\dot{q}_1 - C_1(q_1,\dot{q}_1)\ddot{q}_1 - \frac{d}{dt}(Bf(\dot{q}_1)) + c_{12}\frac{d}{dt}h(q_2 - q_1)$. Using the fact that $\dot{M}_1(q_1) = C_1(q_1,\dot{q}_1) + C_1^\top(q_1,\dot{q}_1)$, $\dot{q}_1, \ddot{q}_1 \in L_\infty$ together with Property 1, Property 3 and Hypotheses 1-2 allows us to conclude that $\frac{d}{dt}\ddot{q}_1$ is bounded.

By applying Lemma 2, we have $\lim_{t\to\infty}\|\ddot{q}_1\| = 0$ since $\ddot{q}_1$ is uniformly continuous in time. Taking the limit as $t \to \infty$ on both sides of equality $\|M_1(q_1)\ddot{q}_1 + C_1(q_1,\dot{q}_1)\dot{q}_1\| = \| -Bf(\dot{q}_1) + c_{12}h(q_2 - q_1)\|$ gives rise to the expression $\lim_{t\to\infty}\|M_1(q_1)\ddot{q}_1 + C_1(q_1,\dot{q}_1)\dot{q}_1\| = \lim_{t\to\infty}\| -Bf(\dot{q}_1) + c_{12}$ $h(q_2 - q_1)\|$. We can conclude that $\lim_{t\to\infty}\|q_2 - q_1\| = 0$ from $\lim_{t\to\infty}\|\ddot{q}_1\| = 0$, $\lim_{t\to\infty}\|\dot{q}_1\| = 0$ and the properties of f, h, $M_1(q_1)$ and $C_1(q_1,\dot{q}_1)$. It is easy to see that $\dot{q}_2 - \dot{q}_1$ is uniformly continuous in time, so $\lim_{t\to\infty}\|\dot{q}_2 - \dot{q}_1\| = 0$. As a result, it follows that $\lim_{t\to\infty}\|\dot{q}_2\| = 0$.

By following the similar analysis, we can get the result $\lim_{t\to\infty}\|\ddot{q}_2\| = 0$ and $\lim_{t\to\infty}\|q_2 - q_3\| = 0$. Then, the same method of analysis can be used to get the final results $\lim_{t\to\infty}\|\dot{q}_i\| = 0$, $\lim_{t\to\infty}\|q_i - q_{i-1}\| = 0$, namely, $\lim_{t\to\infty}\|q_j - q_i\| = 0$, $i, j \in \bar{n}$.

Remark 2: Protocol τ_i is only one of protocols to solve the consensus problem of system (1), and we are looking for some other protocols. In protocol τ_i, when the first EL system's velocity information is measurable, system (1) can reach consensus. In other situations, we can not determine the convergence of system (1) due to the complexity of the analysis.

4. POSITIONS OF EL SYSTEMS CONVERGE TO THE DESIRED POSITION

In Section 3, system (1) can achieve consensus under protocol (2). However, in that case we only know that $q_1 = q_2 =, \cdots, = q_n$ as $t \to \infty$, and don't know where the position q_i converge. In order to guarantee that the state q_i in system (1) converges to a desired position q_0, namely, $q_1 = q_2 =, \cdots, = q_n = q_0$ as $t \to \infty$, we need to design another protocol, which can be denoted by $\tau_i^* + \iota_i^*$.

Similarly, the input ι_i^* is used to compensate the effects of $G_i(q_i)$ on the dynamics, so choose $\iota_i^* = G_i(q_i)$. The protocol $\tau_i^* + \iota_i^*$ is chosen as

$$\tau_i^* + \iota_i^* = \begin{cases} -Bf(\dot{q}_1) + c_{12}h(q_2 - q_1) \\ \qquad +G_1(q_1),\ i = 1, \\ c_{i,i-1}h(q_{i-1} - q_i) + c_{i,i+1}h(q_{i+1} - q_i), \\ \qquad +G_i(q_i),\ \ i = 2, \cdots, n-1, \\ c_{n,n-1}h(q_{n-1} - q_n) \\ \qquad +c_{n0}h(q_0 - q_n) + G_n(q_n),\ i = n. \end{cases} \tag{4}$$

In protocol (4), the state $q_0 \in R^m$ denotes the desired position state. Here, $c_{n0} > 0$ denotes that only the nth EL system knows the stationary position q_0 directly.

Theorem 2: Consider system (1) with protocol (4) and assume that Hypotheses 1-2 hold. Then, $\lim_{t\to\infty}\|q_i - q_0\| = 0$, $\lim_{t\to\infty}\|\dot{q}_i\| = 0$, $i \in \bar{n}$.

Proof: Let $\hat{q}_i = q_i - q_0$, then system (1) with protocol $\tau_i^* + \iota_i^*$ can be written as

$$\begin{cases} M_1(q_1)\ddot{\hat{q}}_1 + C_1(q_1,\dot{q}_1)\dot{\hat{q}}_1 = -Bf(\dot{\hat{q}}_1) \\ \qquad +c_{12}h(\hat{q}_2 - \hat{q}_1), \\ M_i(q_i)\ddot{\hat{q}}_i + C_i(q_i,\dot{q}_i)\dot{\hat{q}}_i = c_{i,i-1}h(\hat{q}_{i-1} - \hat{q}_i) \\ \qquad +c_{i,i+1}h(\hat{q}_{i+1} - \hat{q}_i), i = 2,3,\cdots,n-1, \\ M_n(q_n)\ddot{\hat{q}}_n + C_n(q_n,\dot{q}_n)\dot{\hat{q}}_n = c_{n,n-1}h(\hat{q}_{n-1} - \hat{q}_n) \\ \qquad -c_{n0}h(\hat{q}_n). \end{cases} \tag{5}$$

Select the Lyapunov function as

$$\begin{aligned} V(\hat{q}_i,\dot{\hat{q}}_i) &= \frac{1}{2}\sum_{i=1}^{n}\dot{\hat{q}}_i^\top M_i(q_i)\dot{\hat{q}}_i + c_{n0}\sum_{l=1}^{m}\int_0^{\hat{q}_{nl}} h_l(s)ds \\ &\quad + \sum_{i=1}^{n-1}\sum_{l=1}^{m} c_{i,i+1}\int_0^{\hat{q}_{i+1,l}-\hat{q}_{il}} h_l(s)ds. \end{aligned}$$

Differentiating $V(\hat{q}_i,\dot{\hat{q}}_i)$ with respect to time along the solution of (5) yields

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{n}\dot{\hat{q}}_i^\top M_i(q_i)\ddot{\hat{q}}_i + \frac{1}{2}\sum_{i=1}^{n}\dot{\hat{q}}_i^\top \dot{M}_i(q_i)\dot{\hat{q}}_i \\ &\quad + \sum_{i=1}^{n-1}\sum_{l=1}^{m} c_{i,i+1}h_l(\hat{q}_{i+1,l} - \hat{q}_{il})(\dot{\hat{q}}_{i+1,l} - \dot{\hat{q}}_{il}) \\ &\quad + c_{n0}\sum_{l=1}^{m} h_l(\hat{q}_{nl})\dot{\hat{q}}_{nl} \\ &= \dot{\hat{q}}_1^\top\big(-C_1(q_1,\dot{q}_1)\dot{\hat{q}}_1 - Bf(\dot{\hat{q}}_1) + c_{12}h(\hat{q}_2 - \hat{q}_1)\big) \\ &\quad + \dot{\hat{q}}_n^\top\big(-C_n(q_n,\dot{q}_n)\dot{\hat{q}}_n + c_{n,n-1}h(\hat{q}_{n-1} - \hat{q}_n) \\ &\quad - c_{n0}h(\hat{q}_n)\big) + \sum_{i=2}^{n-1}\dot{\hat{q}}_i^\top M_i(q_i)\ddot{\hat{q}}_i \\ &\quad + \frac{1}{2}\dot{\hat{q}}_1^\top \dot{M}_1(q_1)\dot{\hat{q}}_1 + \frac{1}{2}\dot{\hat{q}}_n^\top \dot{M}_n(q_n)\dot{\hat{q}}_n \\ &\quad + \frac{1}{2}\sum_{i=2}^{n-1}\dot{\hat{q}}_i^\top \dot{M}_i(q_i)\dot{\hat{q}}_i + \sum_{i=1}^{n-1} c_{i,i+1}(\dot{\hat{q}}_{i+1} - \dot{\hat{q}}_i)^\top \\ &\quad \times h(\hat{q}_{i+1} - \hat{q}_i) + c_{n0}\dot{\hat{q}}_n^\top h(\hat{q}_n) \\ &= \frac{1}{2}\dot{\hat{q}}_1^\top\big(\dot{M}_1(q_1) - 2C_1(q_1,\dot{q}_1)\big)\dot{\hat{q}}_1 + \frac{1}{2}\dot{\hat{q}}_n^\top\big(\dot{M}_n(q_n) \\ &\quad - 2C_n(q_n,\dot{q}_n)\big)\dot{\hat{q}}_n + \dot{\hat{q}}_1^\top\big(-Bf(\dot{\hat{q}}_1)c_{12} \\ &\quad \times h(\hat{q}_2 - \hat{q}_1)\big) + c_{n,n-1}\dot{\hat{q}}_n^\top h(\hat{q}_{n-1} - \hat{q}_n) \\ &\quad + \sum_{i=2}^{n-1}\dot{\hat{q}}_i^\top\big(-C_i(q_i,\dot{q}_i)\dot{\hat{q}}_i + c_{i,i-1}h(\hat{q}_{i-1} - \hat{q}_i) \\ &\quad + c_{i,i+1}h(\hat{q}_{i+1} - \hat{q}_i)\big) + \frac{1}{2}\sum_{i=2}^{n-1}\dot{\hat{q}}_i^\top \dot{M}_i(q_i)\dot{\hat{q}}_i \\ &\quad + \sum_{i=2}^{n-1} c_{i-1,i}\dot{\hat{q}}_i^\top h(\hat{q}_i - \hat{q}_{i-1}) - \sum_{i=2}^{n-1} c_{i,i+1}\dot{\hat{q}}_i^\top h(\hat{q}_{i+1} \\ &\quad - \hat{q}_i) + c_{n-1,n}\dot{\hat{q}}_n^\top h(\hat{q}_n - \hat{q}_{n-1}) - c_{12}\dot{\hat{q}}_1^\top h(\hat{q}_2 - \hat{q}_1) \\ &= -\dot{\hat{q}}_1^\top Bf(\dot{\hat{q}}_1) \le -bk\dot{\hat{q}}_1^\top \dot{\hat{q}}_1. \end{aligned}$$

By following a similar analysis presented in Theorem 1, one has $\lim_{t\to\infty}\|\dot{\hat{q}}_i\| = 0$, $\lim_{t\to\infty}\|\hat{q}_i - \hat{q}_{i-1}\| = 0$. Note that $\ddot{\hat{q}}_n$ is uniformly continuous in time, then $\lim_{t\to\infty}\|\ddot{\hat{q}}_n\| = 0$. Taking the limit as $t \to \infty$ on both sides of $\|M_n(q_n)\ddot{\hat{q}}_n + C_n(q_n, \dot{q}_n) \times \dot{\hat{q}}_n\| = \|c_{n,n-1}h(\hat{q}_{n-1} - \hat{q}_n) - c_{n0}h(\hat{q}_n)\|$, and noting the Property 1, Hypothesis 2 and $\lim_{t\to\infty}\|\hat{q}_{n-1} - \hat{q}_n\| = 0$, it gives that $\lim_{t\to\infty}\|\hat{q}_n\| = 0$, namely, $\lim_{t\to\infty}\|q_i - q_0\| = 0$, $i \in \bar{n}$.

5. NUMERICAL SIMULATIONS

In this section, numerical simulations consisting of three identical two-link manipulators are performed to show the effectiveness of the proposed protocols. The dynamics of the two-link manipulators are given as follows [1]:

$$M(q_1)\ddot{q}_1 + C(q_1, \dot{q}_1)\dot{q}_1 = \tau_1,$$

where

$$M(q_1) = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}, q_1 = \begin{bmatrix} q_{11} \\ q_{12} \end{bmatrix}, \tau_1 = \begin{bmatrix} \tau_{11} \\ \tau_{12} \end{bmatrix},$$

$$C(q_1, \dot{q}_1) = \begin{bmatrix} -d\dot{q}_{12} & -d(\dot{q}_{11} + \dot{q}_{12}) \\ d\dot{q}_{11} & 0 \end{bmatrix},$$

with $H_{11} = a_1 + 2a_3\cos q_{12} + 2a_4\sin q_{12}$, $H_{12} = H_{21} = a_2 + a_3\cos q_{12} + a_4\sin q_{12}$, $H_{22} = a_2$, $d = a_3\sin q_{12} - a_4\cos q_{12}$, $a_1 = I_1 + m_l l_{c1}^2 + I_e + m_e l_{ce}^2 + m_e l_1^2$, $a_2 = I_e + m_e l_{ce}^2$, $a_3 = m_e l_1 l_{ce}\cos\delta_e$, $a_4 = m_e l_1 l_{ce}\sin\delta_e$.

In the following simulations, the parameters remain unchanged and can be chosen as $m_l = 1$, $l_1 = 1$, $m_e = 2$, $\delta_e = \frac{\pi}{6}$, $I_1 = 0.12$, $l_{c1} = 0.5$, $I_e = 0.25$, $l_{ce} = 0.6$, $f(\dot{q}_1) = \dot{q}_1$, $h(z) = z + 0.5sin(z)$, matrix $B = diag(3,4)$, $c_{12} = c_{21} = c_{23} = c_{32} = c$, $c_{30} = 0.5$. The initial conditions for three manipulators are chosen as $(q_{11}, q_{12}, \dot{q}_{11}, \dot{q}_{12})^\top = (-0.21, -0.22, 0.28, 0.29)^\top$, $(q_{21}, q_{22}, \dot{q}_{21}, \dot{q}_{22})^\top = (-0.02, 0.31, -0.33, 0.2)^\top$ and $(q_{31}, q_{32}, \dot{q}_{31}, \dot{q}_{32})^\top = (-0.4, 0.26, 0.27, -0.3)^\top$.

Example 1: In this case, we consider three manipulators with protocol (2) and $c = 1$:

$$\begin{cases} M(q_1)\ddot{q}_1 = -C(q_1, \dot{q}_1)\dot{q}_1 - B\dot{q}_1 + ch(q_2 - q_1), \\ M(q_2)\ddot{q}_2 = -C(q_2, \dot{q}_2)\dot{q}_2 + ch(q_1 - q_2) \\ \qquad\qquad +ch(q_3 - q_2), \\ M(q_3)\ddot{q}_3 = -C(q_3, \dot{q}_3)\dot{q}_3 + ch(q_2 - q_3). \end{cases} \tag{6}$$

Fig. 1 shows the consensus process of q_{i1} and q_{i2}, $i = 1,2,3$, in system (6). Fig. 2 illustrates that the velocities $\dot{q}_{i1}$ and $\dot{q}_{i2}$, $i = 1,2,3$, in system (6) converge to zero. We can see from Figs. 1-2 that three two-link manipulators can reach stationary consensus.

Example 2: Under protocol (4), we consider three manipulators with the desired state $(q_{01}, q_{02}, \dot{q}_{01}, \dot{q}_{02})^\top = (1, 0.8, 0, 0)^\top$ and $c = 3$:

$$\begin{cases} M(q_1)\ddot{q}_1 = -C(q_1, \dot{q}_1)\dot{q}_1 - B\dot{q}_1 + ch(q_2 - q_1), \\ M(q_2)\ddot{q}_2 = -C(q_2, \dot{q}_2)\dot{q}_2 + ch(q_1 - q_2) \\ \qquad\qquad +ch(q_3 - q_2), \\ M(q_3)\ddot{q}_3 = -C(q_3, \dot{q}_3)\dot{q}_3 + ch(q_2 - q_3) \\ \qquad\qquad +c_{30}h(q_0 - q_3). \end{cases} \tag{7}$$

Fig. 3 shows that q_{i1} and q_{i2}, $i = 1,2,3$ in system (7) converge to 1 and 0.8, respectively. Fig. 4 shows that $\dot{q}_{i1}$ and $\dot{q}_{i2}$ in system (7) converge to 0. Figs. 3-4 illustrate that the states of three manipulators in system (7) approach $(1, 0.8, 0, 0)^\top$.

Furthermore, let two manipulators get velocity information in system (7) and the parameters remain unchanged. So, we get the following controlled system:

$$\begin{cases} M(q_1)\ddot{q}_1 = -C(q_1, \dot{q}_1)\dot{q}_1 - B\dot{q}_1 + ch(q_2 - q_1), \\ M(q_2)\ddot{q}_2 = -C(q_2, \dot{q}_2)\dot{q}_2 - B\dot{q}_2 + ch(q_1 - q_2) \\ \qquad\qquad +ch(q_3 - q_2), \\ M(q_3)\ddot{q}_3 = -C(q_3, \dot{q}_3)\dot{q}_3 + ch(q_2 - q_3) \\ \qquad\qquad +c_{30}h(q_0 - q_3). \end{cases} \tag{8}$$

Figs. 5-6 show that the states of three manipulators in system (8) approach $(1, 0.8, 0, 0)^\top$ within a short period of time. Clearly, all states in system (8) have faster convergent speed than states in (7) because there are two manipulators can get velocity information in (8).

It can be seen from Figs. 1-4 that there are oscillatory transient behaviors. This happens because only the first

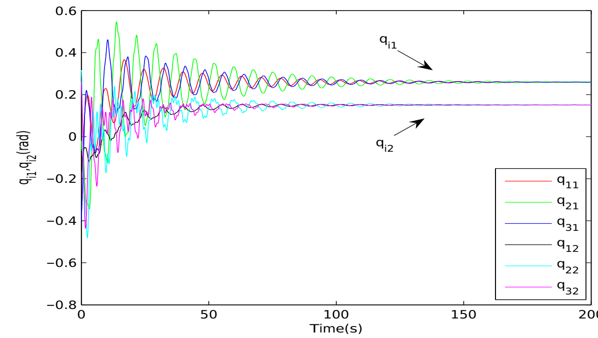

Fig. 1. Consensus of q_{i1} and q_{i2} in (6).

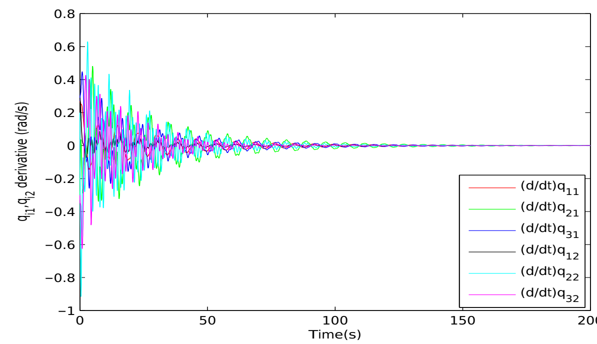

Fig. 2. Derivatives of q_{i1} and q_{i2} in (6) converge to 0.

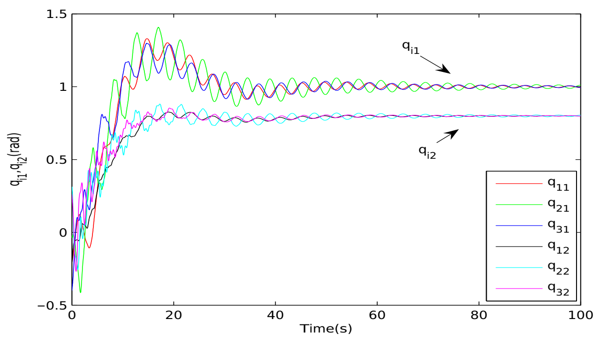

Fig. 3. Joint angles q_{i1} and q_{i2} in (7) converge to 1 and 0.8, respectively.

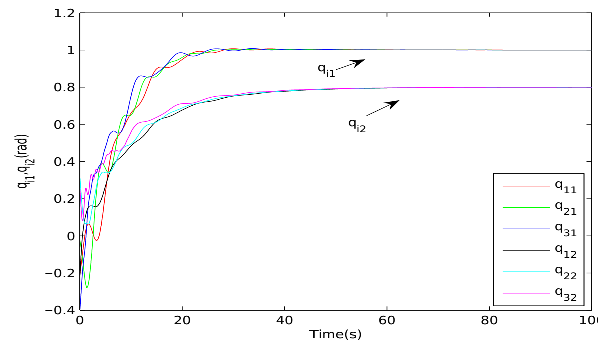

Fig. 5. Joint angles q_{i1} and q_{i2} in (8) converge to 1 and 0.8, respectively.

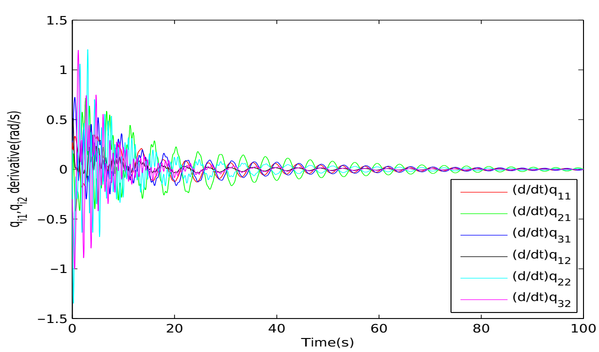

Fig. 4. Derivatives of q_{i1} and q_{i2} in (7) converge to 0.

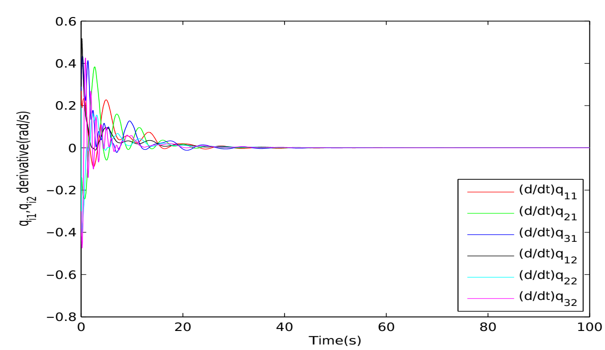

Fig. 6. Derivatives of q_{i1} and q_{i2} in (8) converge to 0.

EL system can obtain velocity measurements and dissipates the energy. This oscillatory transient behaviors can be reduced by two ways. Firstly, let more *EL* systems have velocity gains in practical applications. For example, let the first and the last *EL* systems obtain velocity measurements in protocol (2) or (4). System (8) and Figs.5-6 show this case. Secondly, choose suitable velocity gain matrix $B = diag(b_1(t), b_2(t), \cdots, b_m(t))$ by numerical simulations.

6. CONCLUSIONS

The consensus of multiple *EL* systems with nonlinear control protocols, in which only relative position information between *EL* systems and one *EL* system's velocity information are needed, has been studied. Two protocols were given to solve the consensus problems of networked *EL* systems. Numerical simulations were used to illustrate the theoretical results. In future work, we focus on some other topologies and derive convergence conditions to guarantee that the networked *EL* systems reach consensus with given topologies.

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