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# Synchronization for Fractional Order Chaotic Systems with Uncertain Parameters

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**Abstract:** This paper deals with the synchronization of fractional order chaotic systems with uncertain parameters. An adaptive control law consisting of fractional order feedback and sliding mode control is proposed. The two fractional order chaotic systems can be synchronized and the uncertain parameters can be identified under the controller and the update rule proposed. Illustrative example and numerical simulation results are provided to demonstrate the effectiveness of the control method proposed.

**Keywords:** Chaos, fractional order systems, parameter identification, synchronization.

## 1. INTRODUCTION

Fractional order systems have become a hot research field in recent years. Due to the fact that the fractional order modeling can describe the real-world physical phenomena more reasonably and accurately than the classical integer order calculus [1, 2], the fractional order systems play an important role in the research of nonlinear systems, where many physical and biological phenomena can be given natural interpretations by the fractional order models. Up to now, the control of fractional order linear and nonlinear systems has been investigated in a lot of researches with the focus on fractional differential equations, fractional order systems stability analysis [3–8], controller design [9–11], as well as numerical solutions and simulations [12, 13]. Furthermore, many traditional chaotic systems have been generalized into their corresponding fractional order circumstances, where chaotic behaviorsare found under certain constrains on the parameters of the fractional order systems [14], such as fractional order Chua's system [15], fractional order Lorenz's system [16] and fractional order Volta's system [17].

Chaotic phenomenon existes in real-world generally and has been applied into many applications. The most representative applications are the synchronization of chaotic systems and secret communication. Furthermore, chaos has also been found in biology, economy and sociology, with more and more fractional chaotic systems explored.

Synchronization of chaotic systems is an important field in nonlinear system research. For integer order chaotic systems, the synchronization control has been fully investigated and there are various control methods proposed to deal with the synchronization of chaotic systems, which include linear control [18], nonlinear state feedback control [19], robust control [20], sliding mode control [21] and so on. On the other hand, researches on the synchronization of fractional order chaotic systems has also been developed. However, the major of the existed works for the synchronization of fractional order systems are mainly restricted in the area where classical control methods for integer order systems are applied directly, based on the intrinsic stability relations between fractional and integer order nonlinear systems. Moreover, the synchronization of fractional order chaotic systems with incommensurate orders [22] and between different chaotic systems [23] hasalso been studied.

Nevertheless, the aforementioned works on the synchronization of chaotic systems mainly consider the chaotic system without uncertain parameters. In practical applications, systems are often with uncertain parameters that provide new challenge to the synchronization. Therefore, synchronization of chaotic systems with uncertain parameters is a significant research issue. To deal with this problem, some control methods have been proposed, such as active control [24], sliding mode control [25], backstepping control design [16], and so on [27–30].

In this paper, we concern with the synchronization of fractional order chaotic systems with uncertain parameters. The master system and the slave system are with the same fractional derivative order, and the slave system is with some uncertain parameters. Based on the steepest descent method [31], a parameter update rule is adopted

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to identify the uncertain parameters. An adaptive control law with the combination of nonlinear state feedback control and sliding mode control is proposed, and under this controller, the two fractional order chaotic systems are synchronized and the uncertain parameters are identified eventually.

The organization of this paper is as follows. In section 2, fractional calculus properties and adaptive parameter identification update rule is introduced. In section 3, the adaptive sliding mode control design is presented. In section 4, illustrative example and numerical simulation results are provided.

## 2. PROBLEM FORMULATION AND PRELIMINARIES

### 2.1. Fractional calculus and its properties

**Definition 1:** Let $f:[a,b]\to R$ and $f\in L^1[a,b]$ . The Riemann-Liouville fractional derivative of order is defined as:

$$D^\alpha f(t)=\frac{1}{\Gamma(n-\alpha)}\frac{d^n}{dt^n}\int_0^t\frac{f(\tau)}{(t-\tau)^{\alpha-n+1}}d\tau, \tag{1}$$

where $n-1<\alpha<n$ and $\Gamma(\cdot)$ is the Gamma function. In this paper we employ $D$ for representing the classical integer differential $D^1 f(t)=df(t)/dt$ .

Furthermore, there are some properties for the fractional calculus [1]:

For $\alpha=n$ , where $n$ is an integer, the fractional order derivative coincides with the integer order derivative. Particularly, when $\alpha=0$ , it appears as the identity operator, i.e.,

$$D^\alpha D^{-\alpha}f(t)=D^0 f(t)=f(t). \tag{2}$$

Fractional operator is a linear operator

$$D^\alpha[af(t)+bg(t)]=aD^\alpha f(t)+bD^\alpha g(t), \tag{3}$$

where $a$ and $b$ are real constants.

For fractional orders $\alpha,\beta\in(0,1]$ and $\alpha+\beta\in(0,1]$

$$D^\alpha D^\beta f(t)=D^{\alpha+\beta}f(t), \tag{4}$$

### 2.2. Adaptive parameter identification

There are various methods proposed to identify uncertain parameters in nonlinear systems, some of which have been applied into system identification. In this paper, we employ a simple classical method, which has been used to synchronize chaotic systems successfully, and this method is based on the steepest descent method.

Consider a pair of systems consisting of drive system and response system respectively. The drive system is described as

$$\dot{x}=f(x,\langle c_j\rangle)\quad i=1,2,...,m \tag{5}$$

and the response system is

$$\dot{y}=g(y,\langle c_j'\rangle)+Bu\quad i=1,2,...,m, \tag{6}$$

where the system state $x\in\mathbb{R}^n$ , $y\in\mathbb{R}^n$ and $l\le m$ ; the functions $f=[f_1,f_2,\ldots,f_n]^\top$ and $g=[g_1,g_2,\ldots,g_n]^\top$ are continuous nonlinear system functions; $c_j$ are the known arameters in the drive system, and $c_j'$ are unknown parameters in the response system, which need to be identified; $B$ is the control matrix and $u$ is the control input.

Define the system error $e=y-x$ , then the two systems above is said to be synchronized if

$$\lim_{t\to\infty}\|e(t)\|=\|y(t)-x(t)\|=0 \tag{7}$$

under arbitrary initial conditions $x(0)$ and $y(0)$ .

Choose the dynamical system synchronization error $E(c_j',t)=(y(t)-x(t))^2$ , by the steepest descent method, the unknown parameters $c_j'$ evolve in the way such that they make $E$ minimum. The evolution law of the unknown parameters is described in the following theorem:

**Theorem 1** (Maybhate [31])**:** For the two systems above, the uncertain parameters in the response system can be identified by the following update rule, while the two systems can be synchronized under the controller $u$

$$\dot{c}_j'=\delta(y_i(t)-x_i(t))\frac{\partial g_i}{\partial c_j'}, \tag{8}$$

where $\delta$ is a real constant which will have an impact on the convergence speed of the identification.

Theorem 1 claims that the uncertain parameters can be identified while the error system can be synchronized. For further details, please refer to the reference [31].

## 3. SYNCHRONIZATION OF FRACTIONAL ORDER CHAOTIC SYSTEMS

Consider a pair of fractional order chaotic systems. The drive system is

$$D^\alpha x=f(x,\langle c_j\rangle),\quad j=1,2,\ldots,m \tag{9}$$

and the response system with unknown parameters is

$$D^\alpha y=g(y,\langle c_j'\rangle)+u(x,y)\quad j=1,2,\ldots,m, \tag{10}$$

where the system state $x\in\mathbb{R}^n$ , $y\in\mathbb{R}^n$ and $\alpha\in(0,1]$ is the order of the fractional derivative of the chaotic systems; $f=[f_1,f_2,\ldots,f_n]^\top$ and $g=[g_1,g_2,\ldots,g_n]^\top$ are smooth continuous nonlinear system functions; $c_j$ are the known parameters in the drive system, and $c_j'$ are the unknown parameters in the response system, which need to be identified, and $u(x,y)$ is the controller.

The system error is defined as $e=y-x$ , and the systems (9) and (10) are said to be synchronized under arbitrary initial conditions $x(0)$ and $y(0)$ , if

$$\lim_{t\to\infty}\|e(t)\|=\|y(t)-x(t)\|=0. \tag{11}$$

To achieve a synchronization of the two chaotic systems above, we rewrite the controller in the form $u = u_1 + u_2$ , where

$$u_1 = (D^{\alpha-1} - I)g(y) + f(x), \tag{12}$$

where $I = D^0$ is the identity operator. Then take fractional derivative of order $1-\alpha$ on both sides of the equation (10), we obtain the integer order differential equation of the system (10)

$$\begin{aligned} D^{1-\alpha}D^{\alpha}y = D^1 y = \dot{y} &= D^{1-\alpha}g(y) + D^{1-\alpha}u \\ &= D^{1-\alpha}g(y) + D^{1-\alpha}(D^{\alpha-1} - I)g(y) \\ &\quad + D^{1-\alpha}f(x) + D^{1-\alpha}u_2 \\ &= g(y) + D^{1-\alpha}f(x) + D^{1-\alpha}u_2 \end{aligned} \tag{13}$$

With this integer order form of the response system (10), the uncertain parameters in the response system can be identified according to the update rule given in equation (8).

For the error system

$$D^{\alpha}e = D^{\alpha}(y-x) = D^{\alpha}y - D^{\alpha}x = g(y) + u - f(x) \tag{14}$$

take fractional derivative of order $1-\alpha$ on both sides of the above equation, we obtain

$$\begin{aligned} D^{1-\alpha}D^{\alpha}e = \dot{e} &= D^{1-\alpha}g(y) + D^{1-\alpha}u - D^{1-\alpha}f(x) \\ &= D^{1-\alpha}g(y) + D^{1-\alpha}(D^{\alpha-1} - I)g(y) \\ &\quad + D^{1-\alpha}f(x) + D^{1-\alpha}u_2 - D^{1-\alpha}f(x) \\ &= g(y) + D^{1-\alpha}u_2. \end{aligned} \tag{15}$$

Let $D^{1-\alpha}u_2 = Bv$ , where $B = [0,1,0,\ldots,0]^{\top}$ is the control matrix, and $v$ is the external control. Then we have $u_2 = D^{\alpha-1}Bv$ , therefore the equation (15) becomes

$$\dot{e} = g(y) + D^{1-\alpha}u_2 = g(y) + Bv. \tag{16}$$

**Remark 1:** To the best of our knowledge, most of the fractional order chaotic systems that are verified to have chaotic behaviors in theory and application are constrained to the case of $\alpha \in (0,1)$ , and most of the researches on the control of the chaotic systems also mainly consider this case. Although the restriction on the system order, our work can deal with most of the fractional chaotic systems with theoretical rigorousness, and in the view of application-oriented, we think that the consideration of $\alpha \in (0,1)$ can provide a satisfactory result.

For the case of $\alpha > 1$ , which is planned to be considered in sequel works, here we give a sketch of our idea. For $1-\alpha < 0$ , use the property

$$\begin{aligned} {}_0D_t^{-q}\,{}_0D_t^{p}f(t) &= {}_0D_t^{p-q}f(t) \\ &\quad - \sum_{j=1}^{m}[{}_0D_t^{p-j}f(t)]_{|t=0}\frac{t^{q-j}}{\Gamma(1+q-j)}, \end{aligned}$$

where $n-1 < q < n$ and $m-1 < p < m$ .

Let the term $\sum_{j=1}^{m}[{}_0D_t^{p-j}f(t)]_{|t=0}\frac{t^{q-j}}{\Gamma(1+q-j)}$ be denoted as $m_f(t)$ , and let the controller $u = u_1 + u_2 + u_3$ , where $u_1$ and $u_2$ remain same as in the case of $\alpha \in (0,1)$ and $D^{1-\alpha}u_3 = m_g(t) - m_y(t) + m_x(t)$ . Take $D^{1-\alpha}$ on both sides of the error system (14), consequently we have the same equation (16) of integer order, and then the method and procedure remain same as dealt in the case of $\alpha \in (0,1)$ .

For system (16), we design a sliding mode controller $v$ to synchronize the fractional order systems. Design a classical sliding surface $s(e) = 0$ , such that

$$s(e) = e_n + \sum_{i=1}^{n-1}\rho_i e_i, \tag{17}$$

where the sliding surface parameters $\rho_i$ are selected to be positive such that $\lambda^{n-1} + \rho_{n-1}\lambda^{n-2} + \ldots + \rho_1$ is Hurwitz, namely, all its roots lie in the open left half-plane. Choose the reaching law $\dot{s}(e) = -q \cdot \mathrm{sign}(s)$ such that the surface reaching condition $s\dot{s} < 0$ is satisfied, where

$$\mathrm{sign}(s) = \begin{cases} 1 & s > 0 \\ 0 & s = 0 \\ -1 & s < 0, \end{cases} \tag{18}$$

then we obtain

$$\begin{aligned} \dot{s}(e) &= \frac{\partial s}{\partial e}(g(y) + Bv) \\ &= \frac{\partial s}{\partial e}g(y) + \frac{\partial s}{\partial e}(Bv) = -q \cdot \mathrm{sign}(s), \end{aligned} \tag{19}$$

thus we have the following theorem:

**Theorem 2:** For error system (16), if we choose the sliding mode controller

$$v = \left(\frac{\partial s}{\partial e}B\right)^{-1}\left(-q \cdot \mathrm{sign}(s) - \frac{\partial s}{\partial e}g(y)\right), \tag{20}$$

then the error system is stabilized at its equilibrium.

**Proof:** Choose Lyapunov function

$$V(e) = \frac{1}{2}e^2 = \frac{1}{2}\left(e_n + \sum_{i=1}^{n-1}\rho_i e_i\right)^2 \tag{21}$$

by considering $v$ in equation (20), then its derivative is

$$\begin{aligned} \dot{v}(e) &= \left(e_n + \sum_{i=1}^{n-1}\rho_i e_i\right)\left(\dot{e}_n + \sum_{i=1}^{n-1}\rho_i \dot{e}_i\right) \\ &= s\left(\frac{\partial s}{\partial e}g(y) + \frac{\partial s}{\partial e}(Bv)\right) \\ &= -s \cdot q \cdot \mathrm{sign}(s) \le 0 \end{aligned} \tag{22}$$

when $\dot{V}(e) = 0$ , it implies that $s(e) = 0$ , that is ,the system trajectory is enforced on the sliding surface, thus the system error is stabilized asymptotically on the sliding surface, since that the roots of characteristic polynomial are all of negative real parts, according to the definition of the sliding surface in (17). This completes the proof.

Therefore, the fractional order chaotic systems (9) and (10) can be synchronized under the controllers $u_1$ in equation (12) and $u_2 = D^{\alpha-1}Bv$ , where $v$ is given in equation (20). Since the synchronization problem has been converted into an integer form in equation (16) under the controller $u_1$ and $u_2$ , we can apply the method declared in Theorem 1, and then uncertain parameters in the response system (10) can be identified by the update rule given in equation (8).

## 4. ILLUSTRATIVE EXAMPLE AND SIMULATIONS

To verify the control law proposed in the last section, we give an example of fractional order chaotic system to illustrate its effectiveness. Consider the well-known Lorenz system, and it has been confirmed theoretically and practically that the fractional order Lorenz system is still a chaotic system.

For fractional order Lorenz system, the drive system

$$\begin{aligned} D^\alpha x_1 &= f_1(x) = -a(x_1 - x_2), \\ D^\alpha x_2 &= f_2(x) = bx_1 - x_2 - x_1x_3, \\ D^\alpha x_3 &= f_3(x) = x_1x_2 - cx_3, \end{aligned} \tag{23}$$

where $\alpha \in (0,1)$ is the order of fractional derivative, the parameters are known as $a = 10$ , $b = 28$ and $c = 8/3$. Under the controller $u$ proposed in the last section, we obtain the response system

$$\begin{aligned} D^\alpha y_1 &= D^{\alpha-1}(-\beta(y_1 - y_2)) + f_1(x), \\ D^\alpha y_2 &= D^{\alpha-1}(by_1 - y_2 - y_1y_3) + f_2(x) + D^{\alpha-1}v, \\ D^\alpha y_3 &= D^{\alpha-1}(y_1y_2 - \gamma y_3) + f_3(x), \end{aligned} \tag{24}$$

where the sliding mode controller $v$ is

$$\begin{aligned} v =& -q \cdot \operatorname{sign}(s) - \rho(-\beta(y_1 - y_2)) \\ & -(y_1y_2 - \gamma y_3) - (by_1 - y_2 - y_1y_3), \end{aligned} \tag{25}$$

where the sliding surface is $s(e) = \rho e_1 + e_2 + e_3$ . The two unknown parameters $\beta$ and $\gamma$ can be identified under the following update rule

$$\begin{aligned} \dot{\beta} &= -\delta_1(y_1 - x_1)(y_2 - y_1), \\ \dot{\gamma} &= -\delta_2(y_3 - x_3)(-y_3), \end{aligned} \tag{26}$$

where the constants $\delta_i$ are control parameters, and different choices of the parameter $\delta_i$ can have an impact on the convergence speed of the parameter identification.

Fig. 1 and Fig. 2 show the convergence of the error system and the parameters identification, when initial conditions $x(0) = [1,2,3]^\top$, $y(0) = [4,5,6]^\top$ , $\beta(0) = 20$, $\gamma(0) = 20$, $q = 3$, $\rho = 2$, $\delta_1 = 9$, $\delta_2 = 7$, and $\alpha = 0.98$. In figure 2, the parameter estimates converge at $\beta = 9.9989$ and $\gamma = 2.6661$.

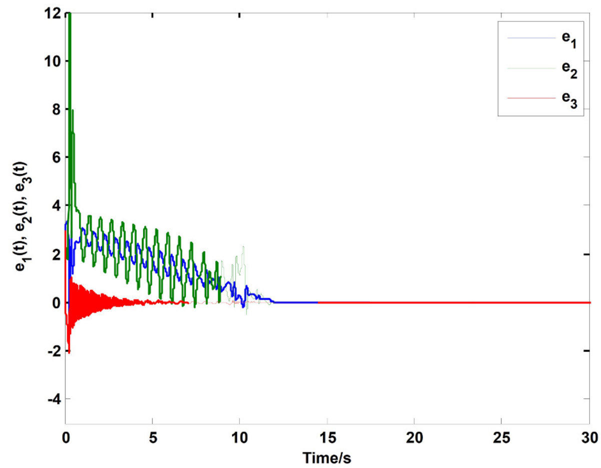


Fig. 1. Convergence of error system: case 1.

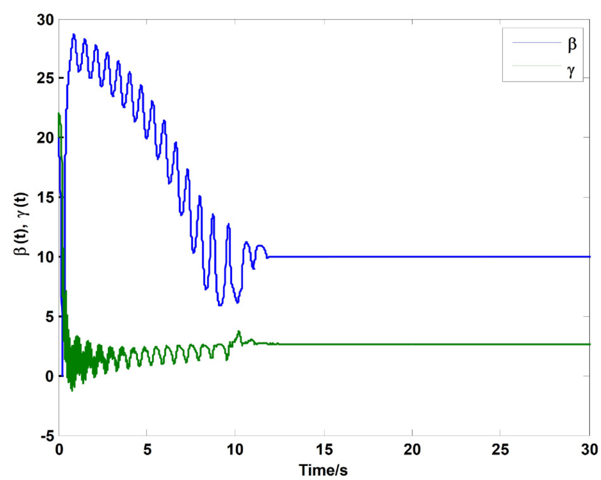


Fig. 2. Convergence of parameter identification: case 1.

Fig. 3 and Fig. 4 show the convergence of the error system and the parameters identification, when initial conditions $x(0) = [-3,2,-1]^\top$ , $y(0) = [7,8,9]^\top$ , $\beta(0) = 10$ , $\gamma(0) = 15$ , $q = 3$ , $\rho = 2$ , $\delta_1 = 3$ , $\delta_2 = 2$ , and $\alpha = 0.97$ . In figure 2, the parameter estimates converge at $\beta = 9.9954$ and $\gamma = 2.6675$ .

From Fig. 1 to Fig. 4, it is obvious that the two fractional order Lorenz systems are synchronized by the controller $u$ proposed in this paper. Enlarging the parameter $\delta_i$ can enhance the convergence speed. The uncertain parameters $\beta$ and $\gamma$ can be well identified under arbitrary initial conditions, which confirms the effectiveness of our control method.

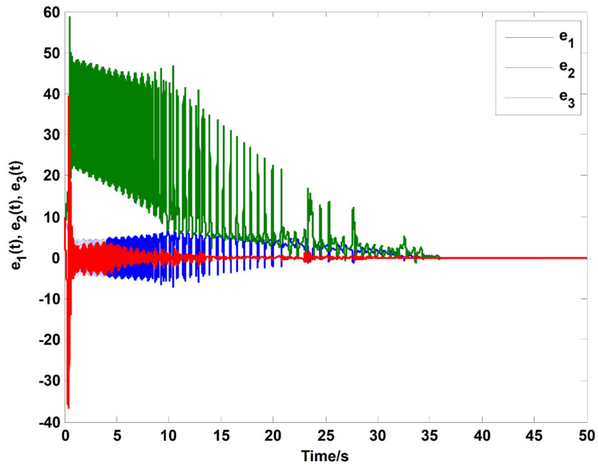


Fig. 3. Convergence of error system: case 2.

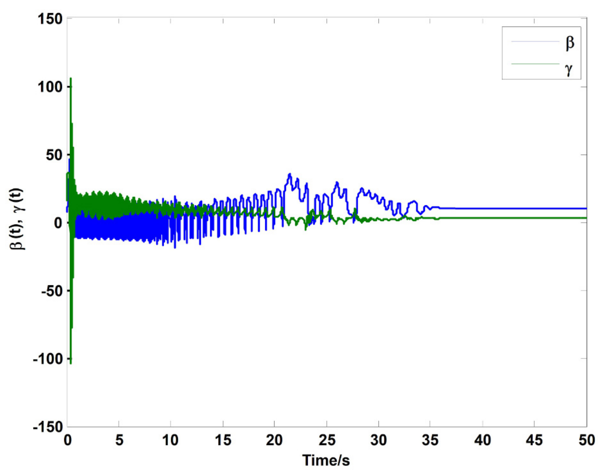


Fig. 4. Convergence of parameter identification: case 2.

## 5. CONCLUSIONS

This paper considers the synchronization of two fractional order chaotic systems with uncertain parameters. Based on the fractional calculus properties, a fractional order nonlinear state feedback controller is proposed to transform the fractional order error system into its integer form, then a sliding mode controller is designed to stabilize the error system. The uncertain parameters can be well identified by the update rule under the proposed controller. Illustrative example and simulation results are provided to demonstrate our control method.

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