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Stability Criterion of Linear Delayed Impulsive Differential Systems with Impulse Time Windows

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Abstract: In this paper, we study the uniform stability of linear delayed differential equationswith impulse time windows. By means of Lyapunov functions and Razumikhin technique combined with classification discussion technique, the criterion of uniform stability is obtained, which may be used to discuss others stability of delayed differential equations with impulse time win-dows. Two examples are given to illustrate the effectiveness of the theoretic result.

Keywords: Impulsive differential systems, impulse time windows, uniform stability.

1. INTRODUCTION

Due to the wide applications of impulsive differential systems in many fields such as control technology, electrical engineering, medicine, biology and so forth, they have attracted many researchers' attention (see [1–9]). Since time delay exists in many fields of our society, sys-tems with time delay has received amounts of attention in the past decades years (see [7–35]).

There have been many results on stability analysis of delayed differential equations by means of the method of Lyapunov functions and Razumikhin technique. For example, Zhang and Sun [10] get some results for the stability of linear delayed differential equations by using Lyapunov functions and analysis technique. Liu *et al.*. [19] studied stability problems of linear and nonlinear impulsive systems with time delay. The criteria of uniform stability and uniform asymptotic stability for Takagi-Sugeno (T-S) model of fuzzy delay systems with impulse were obtained in [12]. Chen *et al.*. [23] worked out the problem of exponential stability of T-S fuzzy delay systems with delayed impulses. The global exponential stability of impulsive delayed linear differential equations was investigated by Zhou and Wu [13]. It should be noticed that the impulses of these studied systems all occurred at the fixed-time points. However, on some conditions, the impulses may occur in a little range of time, i.e. impulse time windows. To the authors' best knowledge, there have been few published results at present. As a result, it is necessary to study the delayed differential equations with impulse time windows.

In this paper, our purpose is to study the uniform stability of linear delayed differential equations with impulse time windows. By means of Lyapunov functions and Razumikhin technique combined with other analysis techniques, the criterion of uniform stability is obtained. The methods presented in this paper may be used to discuss others stability of delayed differential equations with impulse time windows, for example, asymptotical stability and exponential stability, which will promote the development of impulsive control technology. Finally, two numerical examples are given to illustrate the effectiveness of the proposed stability criterion.

The rest of this paper is organized as follows. In Section 2, we introduce some basic notations and definitions. We establish uniform stability criterion for linear delayed impulsive differential systems with impulse time windows in Section 3. Two numerical examples are discussed to illustrate the theoretic result in Section 4. Finally, conclusions are given in Section 5.

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2. PRELIMINARIES

The following notations and definitions will be used in this paper. Let R be the set of real numbers, $R+$ be the set of nonnegative real numbers and R^n be the space of n-dimensional column vectors $x = col(x_1, x_2, \cdots x_n)$ with the Euclidean norm. For $a, b \in R$ with a $<$ b and $S \subseteq R^n$, we define the following classes of functions. $PC([a,b],S) = \{\phi : [a,b] \to S | \phi(t) = \phi(t^+), \forall t \in [a,b]; \phi(t^-)$ exists in S, $\forall t \in [a,b]$ and $\phi(t^-) = \phi(t)$ for all but at most a finite number of points $t \in [a,b]\}$. Let $J_\tau = PC([-\tau,0],R^n)$, for $\psi \in J_\tau$, the norm of ψ is defined by $|\psi| = sup_{-\tau \le s \le 0} \|\psi(s)\|$ and $x_t = J_\tau$ is defined by $x_t(s) = x(t+s)$ for $s \in [-\tau, 0]$, where $\tau > 0$ is a constant. For any $\alpha > 0$, let $PC(\alpha) = \{\phi \in J_\tau : |\phi| < \alpha\}$.

For any given $\theta \ge t_0$ and $\in J_\tau$, an impulsive linear delayed differential system with fixed-time impulse is given by:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bx(t-\tau), & t \ge \theta,\ t \ne \tau_k, \\ \triangle x(t) = x(t) - x(t^-) = Cx(t^-), & t = \tau_k, k \in N_+, \\ x(\theta + s) = \varphi(s), & s \in [-\tau, 0], \end{cases} \tag{1}$$

where $x \in R^n$, $A, B, C \in R^{n \times n}$, $\tau > 0$, $\theta \ge t_0$ and $0 \le t_0 = \tau_0 < \tau_1 < \tau_2 < \cdots < \tau_k < \cdots$, $\tau_k \to \infty$ for $k \to \infty$, N_+ is the set of positive integer. In this system, the impulses occur at the fixed-time points, which have been studied in many papers.

Next, the following impulsive delayed system with impulse time windows will be investigated, in which the impulses will occur in some time intervals.

$$\begin{cases} \dot{x}(t) = Ax(t) + Bx(t-\tau), & \\ \qquad t \ge \theta, t \ne \tau_k, \tau_k \in \left[\tau_k^l, \tau_k^r\right), & \\ \triangle x(t) = x(t) - x(t^-) = Cx(t^-), & t = \tau_k, k \in N_+, \\ x(\theta + s) = \varphi(s), & s \in [-\tau, 0], \end{cases} \tag{2}$$

where $0 \le t_0 = \tau_0^l = \tau_0 = \tau_0^r \le \tau_1^l < \tau_1^r \le \tau_2^l < \tau_2^r \le \cdots \le \tau_k^l < \tau_k^r \le \cdots$, $\tau_k^r \to \infty$ for $k \to \infty$ and the impulse time τ_k is any value of fixed-time windows $\left[\tau_k^l, \tau_k^r\right)$, i.e. $\tau_k \in \left[\tau_k^l, \tau_k^r\right)$.

Definition 1 [3]**:** The function $V : [t_0, \infty) \times R^n \to R_+$ belongs to class v_0 if the following conditions are true.

(i) $V(t,x)$ is locally Lipschitzian in $x \in R_n$ and is continuous on each of the sets $[\tau_{k-1}, \tau_k)$, $V(t,0) \equiv 0$ for all $t \ge t_0$.

(ii) For each $k = 1, 2, \ldots$, there exist finite limits

$$\lim_{(t,y) \to (\tau_k^-, x)} V(t,y) = V(\tau_k^-, x),$$

$$\lim_{(t,y) \to (\tau_k^+, x)} V(t,y) = V(\tau_k^+, x)$$

with $V\left(\tau_k^+, x\right) = V(\tau_k, x)$ being satisfied.

Definition 2 [3]**:** Let $V \in v_0$, for $t \in (\tau_{k-1}, \tau_k)$, the upper right-hand derivative of V is defined by

$$D^+V(t, x(t)) = \lim_{h \to 0^+} sup \frac{1}{h} \{V(t+h, x(t+h)) \\ - V(t, x(t))\}.$$

Definition 3 [12]**:** Suppose that any $\theta \ge t_0$ and $\varepsilon > 0$, there exists a $\delta = \delta(\theta, \varepsilon) > 0$ such that $\|\phi\| < \delta\,(t \ge \theta)$ implies that $\|x(t, \theta, \phi)\| < \varepsilon$. Then, the trivial solution of (2) is said to be stable.

The trivial solution of (2) is said to be uniformly stable if δ is independent of θ.

3. MAIN RESULTS

Theorem 1: Let $\lambda_1 > 0$ and $\lambda_2 > 0$ be the smallest and the largest eigenvalues of symmetric and positive matrix P, λ_3 and λ_4 be respectively the largest eigenvalueslmensiona of $P^{-1}(A^TP + PA + PP)$ and $P^{-1}B^TB$, $0 < \lambda_5 < 1$ be the largest eigenvalues of $P^{-1}[(I+C)^T P((I+C)]$, where I is the identity matrix. Then the origin solution of (2) is uniformly stable if

$$(\lambda_3 + \frac{\lambda_4}{\lambda_5})(\tau_k^r - \tau_{k-1}^r) < -\ln\lambda_5, \tag{3}$$

for $k = 1, 2, 3, \ldots$.

Proof: For any $\varepsilon > 0$, there is a $\delta = \delta(\varepsilon)$ such that $\delta < \sqrt{\frac{\lambda_1 \lambda_5}{\lambda_2}}\varepsilon$. Let $x(t) = x(t, \theta, \phi)$ be any solution of the linear impulsive systems (2) through (θ, ϕ).

Construct a Lyapunov function

$$V(t, x(t)) = x^T(t)Px(t) \in v_0,$$

then, $\lambda_1 \|x\|^2 \le V(t,x(t)) \le \lambda_2 \|x\|^2$. When $t \ne \tau_k$, $k \in N_+$, we have

$$\begin{aligned} & D^+V(t, x(t)) \\ & = x^T(t)\left[A^TP + PA\right]x(t) + 2x^T(t-\tau)PBx(t) \\ & \le x^T(t)\left[A^TP + PA + PP\right]x(t) \\ & \qquad + x^T(t-\tau)B_TBx(t-\tau) \\ & \le \lambda_3 V(t, x(t)) + \lambda_4 V(t-\tau, x(t-\tau)). \end{aligned} \tag{4}$$

Obviously, for any $\theta \ge t_0$, $t \in [\theta - \tau, \theta]$ and $\phi \in PC(\delta)$, there exists a $\alpha \in [-\tau, 0]$ such that $t = \theta + \alpha$ implies that

$$\begin{aligned} & V(t, x(t)) = V(\theta + \alpha, x(\theta + \alpha)) \\ & \le \lambda_2 \|\phi(\alpha)\|^2 \le \lambda_2 \delta^2 \le \frac{\lambda_2}{\lambda_5}\delta^2. \end{aligned}$$

Let $\theta \in [\tau_{m-1}^r, \tau_m^r)$ for some $m \in N$. The following inequality should be firstly proved.

$$V(t, x(t)) \le \frac{\lambda_2}{\lambda_5}\delta^2, \quad \theta \le t < \tau_m^r. \tag{5}$$

Two cases are possible:

Case i: $\theta \in [\tau_{m-1}^r, \tau_m)$

In this case, we will consider two parts according to the value of t.

(a) $t \in [\theta, \tau_m)$

We prove that

$$V(t, x(t)) \leq \frac{\lambda_2}{\lambda_5}\delta^2, \quad \theta \leq t < \tau_m. \tag{6}$$

If inequality (6) is not true, then there is a $\xi \in (\theta, \tau_m)$, such that

$$V(\theta\ , x(\theta)) \leq \lambda_2\delta^2 < \frac{\lambda_2}{\lambda_5}\delta^2 < V(\xi, x(\xi)), \tag{7}$$

where we define

$$\xi = inf\{t \in (\theta, \tau_m) | V(t, x(t)) > \frac{\lambda_2}{\lambda_5}\delta^2\}.$$

Then by employing the continuance of $V(t, x(t))$ in $[\tau_{m-1}^r, \tau_m)$, there exists a $t_1 \in (\theta, \xi]$ such that

$$\begin{aligned} &V(t_1\ , x(t_1\ )) = \frac{\lambda_2}{\lambda_5}\delta^2 \\ &V(t, x(t)) \leq \frac{\lambda_2}{\lambda_5}\delta^2, \qquad \theta - \tau \leq t \leq t_1. \end{aligned} \tag{8}$$

From the inequality (7), we know that there exists a $s_1 \in [\theta, t_1)$ such that

$$\begin{aligned} &V(s_1, x(s_1)) = \lambda_2\delta^2, \\ &V(t, x(t)) \geq \lambda_2\delta^2, \qquad s_1 \leq t \leq t_1. \end{aligned} \tag{9}$$

Therefore, from (8) and (9), we have

$$V(t+s, x(t+s)) \leq \frac{\lambda_2}{\lambda_5}\delta^2 \leq \frac{1}{\lambda_5}V(t, x(t)),\ s \in [-\tau\ , 0],$$

for $t \in [s_1, t_1]$. As a result, $V(t-\tau, x(t-\tau)) \leq (1/\lambda_5)V(t, x(t))$, then for $t \in [s_1, t_1]$,

$$\mathrm{D^+V(t, x(t))} \leq (\lambda_3 + \frac{\lambda_4}{\lambda_5})V(t, x(t)). \tag{10}$$

Now, integrating (10) in $t \in [s_1, t_1]$, we get

$$\int_{s_1}^{t_1} \frac{\mathrm{D^+V(t, x(t))}}{V(t, x(t))}dt = \int_{V(s_1, x(s_1))}^{V(t_1, x(t_1))} \frac{\mathrm{du}}{u} = -\ln\lambda_5.$$

On the other hand, we can conclude that

$$\begin{aligned} \int_{s_1}^{t_1} \frac{\mathrm{D^+V(t, x(t))}}{V(t, x(t))}dt &\leq \int_{s_1}^{t_1} \left(\lambda_3 + \frac{\lambda_4}{\lambda_5}\right)dt \\ &\leq \int_{\tau_{m-1}^r}^{\tau_m^r} \left(\lambda_3 + \frac{\lambda_4}{\lambda_5}\right)dt < -\ln\lambda_5. \end{aligned}$$

A contradiction is obviously obtained. So (6) is proved.

(b) $t \in [\tau_m, \tau_m^r)$

From (6) and the given conditions, we obtain

$$\begin{aligned} V(\tau_m, x(\tau_m)) &= x^T(\tau_m^-)(I+C)^T P(I+C)x(\tau_m^-) \\ &\leq \lambda_5 V\left(\tau_m^-, x\left(\tau_m^-\right)\right) \leq \lambda_2\delta^2. \end{aligned}$$

Then, by employing the same way of (a), we can prove

$$V(t, x(t)) \leq \frac{\lambda_2}{\lambda_5}\delta^2, \qquad \tau_m \leq t < \tau_m^r. \tag{11}$$

Finally, the inequality (5) can be concluded from inequalities (6) and (11).

Case ii: $\theta \in [\tau_m, \tau_m^r)$

We prove that

$$V(t, x(t)) \leq \frac{\lambda_2}{\lambda_5}\delta^2, \qquad \theta \leq t < \tau_m^r. \tag{12}$$

Making use of the same technique of (a) in case i, it is easy to prove (5).

From the discussion of the above two cases, inequality (5) is obtained.

Next, we prove that

$$V(t, x(t)) \leq \frac{\lambda_2}{\lambda_5}\delta^2, \qquad \tau_m^r \leq t < \tau_{m+1}^r. \tag{13}$$

Two cases will be considered.

Case 1: $t \in [\tau_m^r, \tau_{m+1})$

In this case, we first prove that

$$V(t, x(t)) \leq \frac{\lambda_2}{\lambda_5}\delta^2, \qquad \tau_m \leq t < \tau_{m+1}. \tag{14}$$

From the inequalities of (11) and (12), we can easily obtain the inequality (14) by utilizing the similar way of the proof of (6).

Since $[\tau_m^r, \tau_{m+1}) \subset [\tau_m, tau_{m+1})$, we can conclude that

$$V(t, x(t)) \leq \frac{\lambda_2}{\lambda_5}\delta^2, \qquad \tau_m^r \leq t < \tau_{m+1}.$$

Case 2: $t \in [\tau_{m+1}, \tau_{m+1}^r)$

From (14) we can obtain

$$\begin{aligned} &V(\tau_{m+1}, x(\tau_{m+1})) \\ &= x^T(\tau_{m+1}^-)(I+C)^T P(I+C)x(\tau_{m+1}^-) \\ &\leq \lambda_5 V\left(\tau_{m+1}^-, x\left(\tau_{m+1}^-\right)\right) \leq \lambda_2\delta^2. \end{aligned}$$

Then, using the similar way of the proof of (6), it is easy to get

$$V(t, x(t)) \leq \frac{\lambda_2}{\lambda_5}\delta^2, \qquad hskip1pc\tau_{m+1} \leq t < \tau_{m+1}^r.$$

Therefore, the inequality (13) is proved by the discussion of the above two cases.

By simple induction, we can obtain

$$V(t,x(t)) \le \frac{\lambda_2}{\lambda_5}\delta^2, \quad \tau^r_{m+k} \le t < \tau^r_{m+k+1}$$

for $k = 0,1,2,\cdots$.

As a result, we get the following inequality

$$V(t,x(t)) \le \frac{\lambda_2}{\lambda_5}\delta^2,$$

for any $t \ge \theta$. It then follows that

$$\lambda_1\|x\|^2 \le V(t,x(t)) = x^T(t)Px(t) \le \frac{\lambda_2}{\eta}\delta^2,\ t \ge \theta,$$

which implies

$$\|x\| \le \sqrt{\frac{\lambda_2}{\lambda_1\eta}}\delta < \varepsilon,\ t \ge \theta.$$

So the origin solution of (2) is uniformly stable.

Remark 1: In Theorem 1, we have supposed that $P^{-1}(A^TP+PA+PP)$ exists the largest real eigenvalue λ_3, $P^{-1}B^TB$ exists the largest real eigenvalue λ_4. Since P is positive definite, there exists a positive definite symmetric matrix M such that $M^2 = MM = P$. Thus, $P^{-1}Q = M^{-1}M^{-1}Q = M^{-1}[M^{-1}QM^{-1}]M$ and $P^{-1}Q$ and $M^{-1}QM^{-1}$ have the same set of eigenvalues. If Q is symmetric as in the paper's case, $M^{-1}Q\,M^{-1}$ is also symmetric and hence has only real eigenvalues. As a result, $P^{-1}Q$ has only real eigenvalues. In this theorem, the matrices $A^TP+PA+PP$ and B^TB are symmetric and P is positive definite, so the matrices $P^{-1}\left(A^TP+PA+PP\right)$ and $P^{-1}B^TB$ should have only real eigenvalues and hence have the largest eigenvalues.

Remark 2: If let $\tau_k = \tau^l_k = \tau^r_k$ and $\tau^r_k < \tau^l_{k+1}$ for any $k \in N_+$, i.e. the time of impulses is fixed, similar result of (1) has been derived in [10]. Furthermore, the condition $(\lambda_3+\frac{\lambda_4}{\lambda_5})(\tau_k-\tau_{k-1}) < -\ln\lambda_5$ of Theorem 3.1 in [10] restrained the distance of two impulses being less than $\frac{-\ln\lambda_5}{\lambda_3+\frac{\lambda_4}{\lambda_5}}$, while in this theorem, the distance of two impulses can be equal and greater than $\frac{-\ln\lambda_5}{\lambda_3+\frac{\lambda_4}{\lambda_5}}$, Obviously, this result is more general and effective.

Remark 3: In Theorem 1, we only try to investigate the uniform stability of simple linear delayed differential equations with impulse time windows. It is worth noting that the uniform asymptotical stability and exponential stability on this equation may be obtained by utilizing the similar ways in this theorem. Moreover, the stability of the nonlinear and general delayed differential systems with impulse time windows may also be worked out in the future.

4. NUMERICAL EXAMPLES

In this section, we carry out two numerical example to illustrate the above theoretical result.

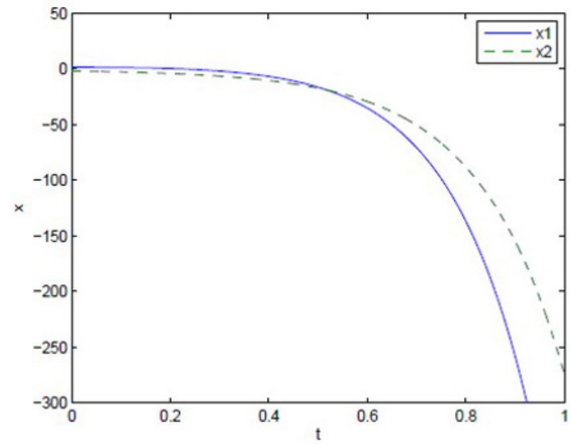

Fig. 1. System without impulses of Example 1.

Example 1: Consider the following linear delayed system with impulse time windows:

$$\begin{cases} \dot{x}(t) = \begin{pmatrix} 2 & \frac{5}{3} \\ \frac{1}{2} & 4 \end{pmatrix} x(t) + \begin{pmatrix} 3 & 5 \\ \frac{1}{3} & 1 \end{pmatrix} x(t-\tau), \\ \qquad t \ne \tau_k, \tau_k \in \left[\tau^l_k, \tau^r_k\right), \\ x(t) = \begin{pmatrix} \frac{3}{5} & 0 \\ 0 & \frac{3}{5} \end{pmatrix} x(t^-), \qquad \mathrm{t} = \tau_\mathrm{k}, \mathrm{k} \in \mathrm{N}_+ \\ \mathrm{x}(t_0+\mathrm{s}) = \varphi(\mathrm{s}) \qquad \mathrm{s} \in [-\tau, 0], \end{cases} \tag{15}$$

Let $\mathrm{P} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\tau^r_{k+1} - \tau^l_{k+1} = 0.007$, $\tau^l_{k+1} - \tau^r_k = 0.002$ and

$$\phi_1(t) = \begin{cases} 0, & t \in [-0.1, 0], \\ 1.5, & t = 0, \end{cases}$$

$$\phi_2(t) = \begin{cases} 0, & t \in [-0.1, 0], \\ -1.8, & t = 0, \end{cases}$$

using the notations of Theorem 1, we obtain that $\lambda_3 = 9.9486$, $\lambda_4 = 35.0604$, $\lambda_5 = 0.36$ and $\tau^r_{k+1} - \tau^r_k = 0.009$, thus

$$(\lambda_3 + \frac{\lambda_4}{\lambda_5})(\tau^r_{k+1} - \tau^r_k) < -\ln\lambda_5.$$

As a result, the system (15) is uniformly stable. It can be illustrated by Figs. 1 and 2. From $\tau^r_{k+1} - \tau^l_{k+1} = 0.007$, $\tau^l_{k+1} - \tau^r_k = 0.002$, we can conclude $\tau^r_{k+1} - \tau^l_k = 0.016$,which implies that the length of impulsive interval $\tau_{k+1} - \tau_k$ is any value of interval $[0.002, 0.016]$.

Example 2: Consider the following mass-spring damper mechanical system in [11, 12]:

$$\mathscr{M}\ddot{p}(t) + \mathscr{D}(\dot{p}(t))\dot{p}(t) + \mathscr{S}p(t) = \mathscr{I}u(t-\tau(t)), \tag{16}$$

where $p(t)$ is the relative position of the mass, $\mathscr{M}$ is themass of the system, $\mathscr{D}(\dot{p}(t))$ is the damping coefficient the damper, $\mathscr{S}$ is the stiffness of the spring, $\mathscr{I}$ is the input coefficient, $u(t-\tau(t))$ is the delayed external force.

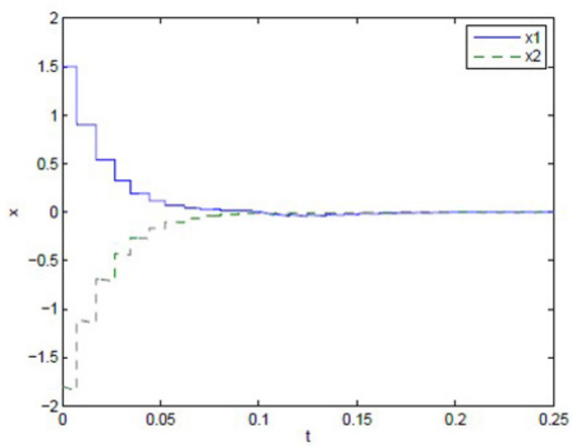

Fig. 2. Impulsive system with impulse time windows of Example 1.

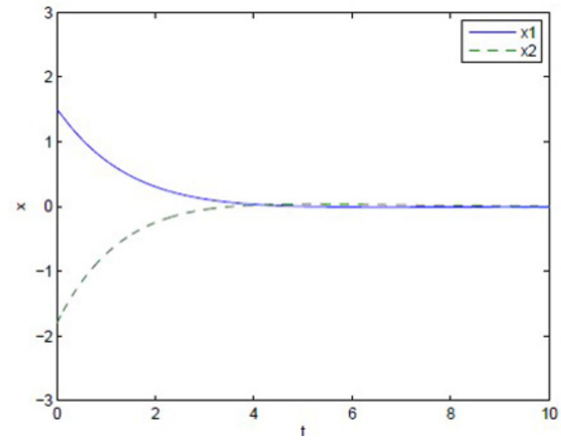

Fig. 3. System without impulses of Example 2.

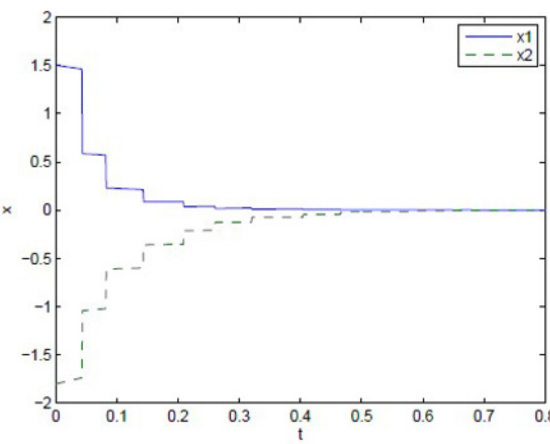

Fig. 4. Impulsive system with impulse time windows of Example 1.

Choosing $x(t) = [\dot{p}(t),\ p(t)]^T$ with $x_1(t) = \dot{p}(t)$ and $x_2(t) = p(t)$, and let $\mathscr{M} = 1$, $\mathscr{D}(\dot{p}(t)) = 0.5$, $\mathscr{S} = 0.1$, $\mathscr{I} = 1$, $u(t-\tau(t)) = -0.7x_1(t-\tau) - 0.5x_2(t-\tau)$, then, we have the equations of mass-spring-damper mechanical system with additional delayed impulses shown in the following equation.

$$\begin{cases} \dot{x}(t) = \begin{pmatrix} -0.5 & 0.1 \\ 1 & 0 \end{pmatrix} x(t) \\ \qquad + \begin{pmatrix} -0.7 & -0.5 \\ 0 & 0 \end{pmatrix} x(t-\tau) \\ \qquad t \neq \tau_k, \tau_k \in \left[\tau_k^l, \tau_k^r\right), \\ x(t) = \begin{pmatrix} 0.4 & 0 \\ 0 & 0.4 \end{pmatrix} x(t^-), \qquad t = \tau_k, k \in \mathrm{N}_+ \\ \mathrm{x}(t_0 + \mathrm{s}) = \varphi(\mathrm{s}) \qquad \mathrm{s} \in [-\tau, 0], \end{cases}$$

Let $\mathrm{P} = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.5 \end{pmatrix}$, $\tau_{k+1}^r - \tau_{k+1}^l = 0.05$, $\tau_{k+1}^l - \tau_k^r = 0.009$ and

$$\phi_1(t) = \begin{cases} 0, & t \in [-0.1, 0], \\ 1.5, & t = 0, \end{cases}$$

$$\phi_2(t) = \begin{cases} 0, & t \in [-0.1, 0], \\ -1.8, & t = 0, \end{cases}$$

using the notations of Theorem 1, we obtain that $\lambda_3 = 2.1858$, $\lambda_4 = 5.4$, $\lambda_5 = 0.36$ and $\tau_{k+1}^r - \tau_k^r = 0.009$, thus

$$(\lambda_3 + \frac{\lambda_4}{\lambda_5})(\tau_{k+1}^r - \tau_k^r) < -\ln\lambda_5.$$

As a result, the system (16) is uniformly stable. It can be illustrated by Figs. 3 and 4. From $\tau_{k+1}^r - \tau_{k+1}^l = 0.05$, $\tau_{k+1}^l - \tau_k^r = 0.009$, we can conclude $\tau_{k+1}^r - \tau_k^l = 0.118$,which implies that the length of impulsive interval $\tau_{k+1} - \tau_k$ is any value of interval $[0.009, 0.118]$.

Remark 4: The current research results mainly focus on fixed impulsive instants, and there exist few results on variable impulsive instants. In this paper, we consider that an impulsive effect can occur at any time of an interval, which implies that impulsive instants are not fixed. At the same time, it is the initial research for the stability analysis of delayed systems with state-dependent impulses.

5. CONCLUSION

The uniform stability of delayed impulsive differential equations with impulse time windows has been investigated in this paper. From the obtained result, it could be seen that the uniform stability on this system could not be changed under some conditions, even when the dist ance of two impulses were enlarged under special conditions. This implied that using more less impulses could control system to be stable. Through the analyses of two numerical examples, it could be seen that the presented result was effective.

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