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Observer-based Adaptive Arbitrary Switching Fuzzy Tracking Control for a Class of Switched Nonlinear Systems

Chun-Yan Wang and Xiao-Hong Jiao*

Abstract: This paper focuses on the output tracking problem for a class of uncertain switched nonlinear systems with arbitrary switching signal and unavailable states. By designing a proper reduced-order observer and introducing fuzzy approximation, a systematic adaptive domination recursive method is presented to explicitly construct a common smooth dynamic output feedback controller, such that under arbitrary switching, all error variables of the corresponding closed-loop switched systems are semi-globally, uniformly, and ultimately bounded (SGUUB), as well as the tracking error may be adjusted to a small neighborhood of the origin. Compared with the existing results, the proposed control scheme overcomes the restrictions on nonlinear functions of systems only relating to the output variable and switching signal subjecting to certain dwell time. Its effectiveness and feasibility are demonstrated by both a numerical example and a chemical system.

Keywords: Adaptive control, arbitrary switching, fuzzy control, observer, switched system.

1. INTRODUCTION

Switched systems consist of a family of switched subsystems and a rule signal which specifies switching among them [1]. As an important class of hybrid systems in control field, stability analysis and control synthesis of switched systems have caught increasing attention for last decade and plenty of interesting results have been put forward, see e.g., [1-5] and the references therein. According to specific operation of switched systems, two different control modes have been developed: restricted switching and arbitrary switching. When the switching signal is observable and can be designed as a part of control input, strong stability results can be derived by restricted switching, such as [6-9]. On the contrary, when the switching signal is unknown or uncontrollable, arbitrary switching control mode becomes the key to ensure the stabilizability of switched systems.

Due to complex structures of nonlinear systems, it is still challenging to derive conditions to stabilize the switched nonlinear systems under arbitrary switching. Around the key issue of how to design a controller independent of the switching signal to achieve the arbitrarily switching control, great efforts have been made by researchers for a class of switched lower triangular systems, see [10-15]. Among them, Wang *et al*. [11] developed a method based on existing results to construct a common robust stabilizing controller for a class of uncertain switched lower-triangular systems under arbitrary switching. It overcomes the structural restrictions on functions of [12] in obtaining the common virtual control. Also under arbitrary switching, a systemic adaptive recursive control method was further proposed in [13] to design a common adaptive state feedback controller for nonlinearly parameterized switched systems in lower-triangular forms, which is concerned essentially with drawbacks of lacking the detailed forms of common virtual control laws and relating to the switching signal for the given adaptive laws in [14]. The discontinuity of the designed adaptive controller in [15] was also defeated at the same time.

However, all these progresses made for switched nonlinear systems under arbitrary switching are based on the assumption that the system states are measurable. In fact, it's not easy to get the informations of them directly in most practical systems due to difficulties in measurement or some economic considerations. This adds a further complication to the design of the controller for switched systems, especially under arbitrary switching. According to specific structure and assumption, there has been some detailed researches on the design of dynamic output feedback controllers for each switched subsystem, such as [7,16,17]. Their implementations, however, depend more on the design of switching rules, which is difficult to be achieved for switched systems with uncontrollable switching. Designing a common output feedback controller is still an open problem for switched systems. Some researchers are working on it. For a class of switched linear systems with unknown inputs, the exponentially-converging observation of the state vector was given for any arbitrary switching sequence in [18]. The observable switching signals and no consideration of the activated delays between the plant and the corresponding observer

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Chun-Yan Wang is with the College of Electrical Engineering and College of Sciences, Yanshan University, Qinhuangdao 066004, China (e-mail: wcy@ysu.edu.cn).

Xiao-Hong Jiao is with the College of Electrical Engineering, Yanshan University, Qinhuangdao 066004, China (e-mail: jiaoxh @ysu.edu.cn).

* Corresponding author.

are the central premise. With unknown switching signal, Chiang *et al*. [19] designed a static robust output feedback controller to stabilize a class of nonlinear systems in switching parameter forms, where the nonlinearities are assumed to be only related to the output variable. Based on the same assumption, with unobservable switching signal and unmeasurable states, Wu *et al*. [20] addressed a design method of a common dynamic output feedback controller under arbitrary switching for switched stochastic nonlinear systems. Except for a standard Wiener process, there are no other unknown factors to be considered.

In view of the above analysis, there still has been no a report to adaptive output feedback control designs under arbitrary switching for switched nonlinear systems without any prior information about switching signals, state variables and uncertainties. The main challenge is how to design a common feedback function to update the adaptive law and then how to construct an observer and a smooth adaptive controller independent of any switching signals to achieve adaptive output feedback stabilization under arbitrary switching. To this end, based on our early efforts in stabilization problem of switched nonlinear systems with arbitrary switching [11,13], this paper focuses on the observer-based output feedback tracking problem for a class of switched nonlinear systems. The main contributions are that: 1) A common adaptive fuzzy dynamic output feedback controller independent of any switching information is designed explicitly for a class of uncertain switched nonlinear systems through a systemic adaptive fuzzy recursive process. 2) The instability of the observer error systems, which is caused by asynchronous switching between on-line switched subsystem and the related switched observer, is avoided by designing a common observer for the whole switched systems.

This paper is organized as follows: In Section 2, the observer-based adaptive fuzzy output feedback control problem is formulated for a class of switched systems. In Section 3, a systematic procedure is developed for the design of a common smooth adaptive dynamic output feedback controller. It is proved that under arbitrary switching, all signals of the corresponding closed-loop switched systems are SGUUB. The output tracking error can be adjusted to converge to a neighborhood of origin as small as possible. The effectiveness and feasibility are validated by simulation of two examples in Section 4. Section 5 shows the conclusion.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider the trajectory tracking control problem for a class of switched nonlinear systems in forms:

$$\begin{cases} \dot{x}_i = x_{i+1} + f_{i\sigma(t)}(\bar{x}_n), \quad i = 1, 2, \cdots, n-1, \\ \dot{x}_n = u + f_{n\sigma(t)}(\bar{x}_n), \\ y = x_1, \end{cases} \tag{1}$$

where $x = \bar{x}_n = [x_1, \cdots, x_n]^T \in R^n$ and $u, y \in R$ are system state, input and output variables, respectively. The right-continuous function $\sigma(t)$: $[0,+\infty] \to M = \{1,\cdots,m\}$ is a piecewise constant switching signal. Moreover, $\sigma(t) = k$ implies just the subsystem k is active. But the time instant of being activated is unknown. For all $i \in N = \{1,\cdots,n\}$, $k \in M$, $f_{ik}(\cdot)$ are unknown nonlinear smooth functions. y_r is the reference signal whose time derivatives are continuous and bounded. The states x_i are assumed not to be measurable except for x_1.

Remark 1: It should be noted that system (1) is not a lower triangular system indeed. In fact, it is a general single input single output nonlinear switched system with definite control gain. Only to derive the expected controller by backstepping recursive design conveniently, we express it in the form of (1).

Due to unknown switching signals and unmeasurable states being further considered in (1), this paper tries to design an observer-based adaptive fuzzy output feedback controller, such that under arbitrary switching, all signals of the closed-loop switched system are guaranteed to be SGUUB and the output can converge to a neighborhood of the reference signal y_r as small as possible.

Remark 2: The observer-based dynamic output feedback control is an important issue in control field to solve the problem of system states unavailable for measurement, and for various non-switched nonlinear systems, many related significant results have been reported, such as [21-24]. However, for switched nonlinear systems with uncontrollable switching laws, the observer-based dynamic output feedback control problem is far from studied due to difficulty in seeking a common dynamic controller. This paper just proposes a systemic approach of constructing such a controller to stabilize a class of nonlinear switched systems under arbitrary switching.

In view of the unknown nonlinear functions in system (1), a fuzzy logic system(FLS) is introduced to approximate them with the following lemma:

Lemma 1: For any given continuous function $f(x)$ on a compact set $\Omega \in R^n$, there exists a FLS $\hat{f}(x) = \Theta^T \Psi(x)$ such that $\forall \varepsilon > 0$, $\sup_{x\in\Omega} |f(x) - \Theta^T \Psi(x)| \le \varepsilon$, here $\Theta^T = [\theta_1,\cdots,\theta_N]$ is the weight vector, $\Psi(x) = [\psi_1(x), \cdots, \psi_N(x)]$ is fuzzy basic function vector, whose element is defined

$$\psi_j(x) = \prod_{i=1}^{n} \mu_{A_i^j}(x_i) / \sum_{j=1}^{N} \prod_{i=1}^{n} \mu_{A_i^j}(x_i), \tag{2}$$

where N is the number of fuzzy rules, and $\mu_{A_i^j}(x_i)$ is the membership function of fuzzy variable, which is usually chosen as a Gaussian function. More details can be referred in [25].

From Lemma 1, the FLS can be used to approximate any continuous functions on a compact space based on proper fuzzy rules. Due to this approximation capability and structure, the unknown functions $f_{ik}(\bar{x}_i)$ in system (1) can be approximated by a FLS with the same structures in fuzzy basic function and membership function for different switched subsystems, that is, for $\forall k \in M$, there are given constants $\varepsilon_{ik} > 0$, such that

$$f_{ik}(x) = \Theta_{ik}^T \Psi_i(x) + \varepsilon_{ik}, \quad i \in N. \tag{3}$$

What's more, in order to achieve the tracking control under arbitrary switching and lighten the calculation burden, the only parameter to be estimated is defined as:

$$\phi = \max\{\|\Theta_{ik}\|^2, i \in N, k \in M\}. \tag{4}$$

3. FUZZY ADAPTIVE ARBITRARY SWITCHING TRACKING CONTROL

In this section, by fuzzy adaptive domination recursive design, the constructing condition of a common smooth adaptive controller for switched systems (1) will be developed based on a reduced-order observer, such that, under arbitrary switchings, all signals in the closed-loop switched system are guaranteed to be bounded and the tracking error is as small as possible.

3.1. Observer-based fuzzy errors switched systems

First, as in [26], the following reduced-order observer is introduced to estimate the unmeasured states of switched systems (1)

$$\begin{cases} \dot{\hat{\xi}}_i = \hat{\xi}_{i+1} + l_{i+1}y - l_i(\hat{\xi}_1 + l_1 y), \quad i = 1, 2, \cdots, n-2, \\ \dot{\hat{\xi}}_{n-1} = u - l_{n-1}(\hat{\xi}_1 + l_1 y). \end{cases} \tag{5}$$

Obviously, this observer is unrelated to any switching signals. Let $e = [e_1, \cdots e_{n-1}]^T$ be the observer error by

$$e_i = x_{i+1} - \hat{\xi}_i - l_i y, \quad i = 1, \cdots, n-1, \tag{6}$$

which means each unmeasured state x_{i+1} is estimated by $\hat{\xi}_i + l_i y$ if e_i converges to zero. From (1), (5) and (6), we have the errors equation

$$\dot{e} = (A + LC)e + F_k(x), \tag{7}$$

where

$$A = \begin{pmatrix} 0 & I_{(n-2)\times(n-2)} \\ 0 & 0 \end{pmatrix}_{(n-1)\times(n-1)}, \quad C = [-1, 0, \cdots, 0]_{1\times(n-1)}$$

$$\begin{aligned} F_k(x) &= [f_{2k} - l_1 f_{1k}, \cdots, f_{nk} - l_{n-1} f_{1k}]^T \\ &= \bar{\Theta}_{n-1,k}^T \bar{\Psi}_{n-1} - L\Theta_{1k}^T \Psi_1 + \bar{\varepsilon}_k \end{aligned}$$

I is a unit matrix, choose the observer gain matrix $L = [l_1, \cdots, l_{n-1}]^T$ to make $A + LC$ be strict Hurwitz, and

$$\bar{\Theta}_{n-1,k} = \operatorname{diag}\{\Theta_{2k}, \cdots, \Theta_{nk}\}, \quad \bar{\Psi}_{n-1}^T = [\Psi_2^T, \cdots, \Psi_n^T],$$
$$\bar{\varepsilon}_k = [\varepsilon_{2k} - l_1 \varepsilon_{1k}, \cdots, \varepsilon_{nk} - l_{n-1}\varepsilon_{1k}]^T.$$

Further, for any given proper positive scalars γ, γ_0, there exist positive-definite matrices P, Q to satisfy the following LMI based on Schur compliment lemma

$$\begin{bmatrix} PA + A^T P + MC + C^T M^T + nI/\gamma + Q & P \\ P & -\gamma_0 I/3 \end{bmatrix} < 0, \tag{8}$$

where M is a matrix with proper dimension. Obviously, the gain matrix is obtained by $L = P^{-1}M$.

For the positive-definite function $V_e = e^T P e/2$, the time derivative along the solutions of (7) is

$$\dot{V}_e = \frac{1}{2} e^T (PA + A^T P + MC + C^T M^T) e + e^T P F_k(x). \tag{9}$$

By using the Young's inequality and fuzzy approximation (3), one has

$$e^T P F_k(x) \le \frac{3}{2\gamma_0} \|e^T P\|^2 + \frac{\gamma_0}{2}\left(\|\bar{\Theta}_{n-1,k}^T \bar{\Psi}_{n-1}\|^2 + \|L\Theta_{1k}^T \Psi_1\|^2 + \|\bar{\varepsilon}_k\|^2 \right). \tag{10}$$

By virtue of the definition of fuzzy basic function vector in (2), the fuzzy approximation (3) and the property of Gaussian function, we know that $\Psi_i^T \Psi_i \le 1$, $\forall i \in N$. Hence, it further gives from Cauchy inequality that

$$\|\bar{\Theta}_{n-1,k}^T \bar{\Psi}_{n-1}\|^2 \le \sum_{i=2}^{n} \|\Theta_{ik}\|^2 \Psi_i^T \Psi_i \le (n-1)\phi, \tag{11}$$

$$\|L\Theta_{1k}^T \Psi_1\|^2 \le \|L\|^2 \|\Theta_{1k}\| \Psi_1^T \Psi_1 \le \|L\|^2 \phi. \tag{12}$$

Associating with (8)-(12) yields

$$\dot{V}_e \le -\frac{1}{2} e^T \left(Q + \frac{n}{\gamma} I \right) e + c_0, \tag{13}$$

where $c_0 = \max_{k\in M}\{\gamma_0[(n-1)\phi + \|L\|^2 \phi + \|\bar{\varepsilon}_k\|^2]/2\}$.

Remark 3: In this paper, a state observer such as (5) is proposed for the whole switched systems rather than for each switched subsystem. By doing so, the delays arising from asynchronous switching between switched subsystems and the associated observers are avoided. In addition, the restrictions on the unknown functions (see (11) in [19] and (27) in [20]) are removed by the application of FLS and the property of Gaussian function.

3.2. Adaptive arbitrary switching feedback control design

Under arbitrary switching, the detailed design procedure of adaptive output feedback controller based on observer (5) is showed in the following steps:

Step 1: Define the tracking error as:

$$z_1 = y - y_r. \tag{14}$$

From (1), (3), (4), (6), and (14), the time derivative of positive-definite function $V_{z_1} = z_1^2/2$ satisfies

$$\dot{V}_{z_1} \le z_1\left(\hat{\xi}_1 + l_1 y - \dot{y}_r\right) + \frac{z_1^2}{2}(\gamma + \gamma_1\phi + \gamma_1) + \frac{e_1^2}{2\gamma} + \frac{1+\varepsilon_{1k}^2}{2\gamma_1}.$$

View $\hat{\xi}_1$ as the virtual control, and define

$$z_2 = \hat{\xi}_1 - \alpha_1(y, y_r, \hat{\phi}) - \dot{y}_r. \tag{15}$$

The common virtual control law α_1 being chosen as

$$\alpha_1 = -\rho_1 z_1 - l_1 y - \frac{\gamma}{2} z_1 - \frac{\gamma_1}{2} z_1 \hat{\phi} - \frac{\gamma_1}{2} z_1 \tag{16}$$

gives that

$$\dot{V}_{z_1} \le z_1 z_2 - \rho z_1^2 + \frac{\gamma_1}{2} z_1^2 \tilde{\phi} + \frac{1}{2\gamma} e^T e + c_1, \tag{17}$$

where ρ_1, γ, γ_1 are positive design constants, $\varepsilon_1 = \max_{k \in M}\{\varepsilon_{1k}\}$, $c_1 = (1+\varepsilon_1^2)/2\gamma_1$. The estimate error $\tilde{\phi} = \phi - \hat{\phi}$, $\hat{\phi}$ is estimation of unknown constant ϕ.

Remark 4: The given virtual control law α_1 in (16) is independent of any switching signals. This independence is the key to eventually construct the common smooth controller for the switched systems under arbitrary switching signals. In addition, α_1 is free of any information on the unknown nonlinear functions $f_{1k}(x)$ except the estimation about the minimum norm ϕ, see (4). Comparing with ref. [11], such design idea simplifies the acquisition of common virtual control laws and reduces the computational burden in following procedure.

Step 2: From (3)-(6) and (15)-(16), the time derivative of positive-definite function $V_{z_2} = z_2^2/2$ is

$$\dot{V}_{z_2} = z_2\left[\hat{\xi}_2 - \ddot{y}_r + \Pi_2 - \frac{\partial \alpha_1}{\partial \hat{\phi}}\dot{\hat{\phi}} - \frac{\partial \alpha_1}{\partial y}(e_1 + \Theta_{1k}^T \Psi_1 + \varepsilon_{1k})\right], \tag{18}$$

where

$$\Pi_2 = l_2 y - l_1(\hat{\xi}_1 + l_1 y) - \frac{\partial \alpha_1}{\partial y_r}\dot{y}_r - \frac{\partial \alpha_1}{\partial y}(\hat{\xi}_1 + l_1 y).$$

Using Young's inequality, there are constants γ, $\gamma_2 > 0$ such that

$$\begin{aligned} &-z_2\frac{\partial \alpha_1}{\partial y}(e_1 + \Theta_{1k}^T \Psi_1 + \varepsilon_{1k}) \\ &\le \frac{1}{2\gamma}e^T e + \frac{z_2^2}{2}\left(\frac{\partial \alpha_1}{\partial y}\right)^2[\gamma + \gamma_2(1+\phi)] + c_2, \end{aligned}$$

where $c_2 = (1+\varepsilon_1^2)/2\gamma_2$. View $\hat{\xi}_2$ as the virtual control in (18), and define

$$z_3 = \hat{\xi}_2 - \alpha_2(y, y_r, \dot{y}_r, \hat{\xi}_1, \hat{\phi}) - \ddot{y}_r. \tag{19}$$

Then the common virtual control law α_2 chosen as

$$\alpha_2 = -\rho_2 z_2 - z_1 - \Pi_2 - \frac{z_2^2}{2}\left(\frac{\partial \alpha_1}{\partial y}\right)^2[\gamma + \gamma_2(1+\phi)] + t_1 \tag{20}$$

makes the time derivative of V_{z2} with a positive design parameter ρ_2 further meet

$$\begin{aligned} \dot{V}_{z_2} &\le z_2\left[z_3 - z_1 - \rho_2 z_2 + \frac{\gamma_2 z_2}{2}\left(\frac{\partial \alpha_1}{\partial y}\right)^2\tilde{\phi} + z_2\left(t_1 - \frac{\partial \alpha_1}{\partial \hat{\phi}}\dot{\hat{\phi}}\right)\right] \\ &\quad + \frac{e^T e}{2\gamma} + c_2. \end{aligned} \tag{21}$$

Remark 5: Considering $\dot{\hat{\phi}}$ is eventually a function of state x and its estimation, the term $\frac{\partial \alpha_1}{\partial \hat{\phi}}\dot{\hat{\phi}}$ can not be dealt with like [21,27], which is incorrect for $n \ge 3$. A tuning function t_1 should be introduced to compensate this term from this step.

Step j $(3 \le j \le n-1)$**:** Similarly, the common virtual control law α_{j-1} can be obtained based on the control design in the first $j-1$ recursive steps. Via defining

$$z_j = \hat{\xi}_{j-1} - \alpha_{j-1}(y, y_r, \cdots, y_r^{(j-2)}, \hat{\xi}_1, \cdots, \hat{\xi}_{j-2}, \hat{\phi}) - y_r^{(j-1)} \tag{22}$$

the time derivative of positive-definite function $V_{z_j} = z_j^2/2$ with (3)-(6) satisfies

$$\begin{aligned} \dot{V}_{z_j} &\le z_j\left[\hat{\xi}_j - y_r^{(j)} + \Pi_j\right] + \frac{z_j^2}{2}\left(\frac{\partial \alpha_{j-1}}{\partial y}\right)^2\left[\gamma + \gamma_j(1+\phi)\right] \\ &\quad - z_j\frac{\partial \alpha_{j-1}}{\partial \hat{\phi}}\dot{\hat{\phi}} + \frac{e^T e}{2\gamma} + c_j, \end{aligned}$$

where $c_j = (1+\varepsilon_1^2)/2\gamma_j$, $\gamma_j > 0$ is a design constant, and

$$\begin{aligned} \Pi_j &= l_j y - \left(l_{j-1} + \frac{\partial \alpha_{j-1}}{\partial y}\right)(\hat{\xi}_1 + l_1 y) \\ &\quad - \sum_{l=1}^{j-1}\frac{\partial \alpha_{j-1}}{\partial y_r^{(l-1)}}y_r^{(l)} - \sum_{l=1}^{j-2}\frac{\partial \alpha_{j-1}}{\partial \hat{\xi}_l}\dot{\hat{\xi}}_l. \end{aligned}$$

Introducing the coordinate transformation

$$z_{j+1} = \hat{\xi}_j - \alpha_j(y, y_r, \cdots, y_r^{(j-1)}, \hat{\xi}_1, \cdots, \hat{\xi}_{j-1}, \hat{\phi}) - y_r^{(j)} \tag{23}$$

with the common virtual control law chosen as

$$\begin{aligned} \alpha_j &= -\rho_j z_j - z_{j-1} - \Pi_j \\ &\quad - \frac{z_j}{2}\left(\frac{\partial \alpha_{j-1}}{\partial y}\right)^2[\gamma + \gamma_j(1+\phi)] + t_{j-1} \end{aligned} \tag{24}$$

results in

$$\begin{aligned} \dot{V}_{z_j} &\le z_j z_{j+1} - z_j z_{j-1} - \rho_j z_j^2 + \frac{\gamma_j}{2}z_j^2\left(\frac{\partial \alpha_{j-1}}{\partial y}\right)^2\tilde{\phi} \\ &\quad + z_j\left(t_{j-1} - \frac{\partial \alpha_{j-1}}{\partial \hat{\phi}}\dot{\hat{\phi}}\right) + \frac{1}{2\gamma}e^T e + c_j, \end{aligned} \tag{25}$$

where ρ_j is a positive design parameter and t_{j-1} is an introduced tuning function to be given later.

Step n: In this final step, the common actual control input u and the adaptive law $\dot{\hat{\phi}}$ will be constructed explicitly for switched systems (1). First, from (3)-(6) and (23), the common control input u chosen as

$$\begin{aligned} u &= -\rho_n z_n - z_{n-1} - \Pi_n \\ &\quad - \frac{z_n}{2}\left(\frac{\partial \alpha_{n-1}}{\partial y}\right)^2[\gamma + \gamma_n(1+\phi)] + t_{n-1} \end{aligned} \tag{26}$$

makes the time derivative of positive-definite function $V_{z_n} = z_n^2/2$ satisfy

$$\dot{V}_{z_n} \le -z_n z_{n-1} - \rho_n z_n^2 + \frac{\gamma_n}{2}z_n^2\left(\frac{\partial \alpha_{n-1}}{\partial y}\right)^2\tilde{\phi} \tag{27}$$

$$+z_n\left(t_{n-1}-\frac{\partial\alpha_{n-1}}{\partial\hat{\phi}}\dot{\hat{\phi}}\right)+\frac{e^Te}{2\gamma}+c_n,$$

where $c_n=(1+\varepsilon_1^2)/2\gamma_n$, $\gamma_n>0$ is a constant, ρ_n is a positive design parameter and the tuning function t_{n-1} will be given later, and

$$\Pi_n=-\left(l_{n-1}+\frac{\partial\alpha_{n-1}}{\partial y}\right)(\hat{\xi}_1+l_1y)$$
$$-\sum_{l=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_r^{(l-1)}}y_r^{(l)}-\sum_{l=1}^{n-2}\frac{\partial\alpha_{n-1}}{\partial\hat{\xi}_l}\dot{\hat{\xi}}_l-y_r^{(n)}.$$

Then define the adaptive law for $\hat{\phi}$ and the tuning functions for $t_j(j=1,\cdots,n-1)$ with $\mu>0$ by

$$\dot{\hat{\phi}}=\frac{r}{2}\left[\gamma_1z_1^2+\sum_{j=2}^{n}\gamma_jz_j^2\left(\frac{\partial\alpha_{j-1}}{\partial y}\right)^2\right]-\mu r\hat{\phi}, \tag{28}$$

$$t_j=\frac{\partial\alpha_j}{\partial\hat{\phi}}\left[\frac{r}{2}z_1^2+\sum_{l=2}^{j+1}\gamma_lz_l^2\left(\frac{\partial\alpha_{l-1}}{\partial y}\right)^2-\mu r\hat{\phi}\right]$$
$$+\frac{\gamma_{j+1}r}{2}z_{j+1}\left(\frac{\partial\alpha_j}{\partial y}\right)^2\sum_{l=1}^{j-1}\frac{\partial\alpha_l}{\partial\hat{\phi}}z_{l+1}. \tag{29}$$

Applying inequalities (13), (17), (21), (25) and (27), the time derivative of a common Lyapunov function candidate $V=V_e+V_{z_1}+\cdots+V_{z_n}+\tilde{\phi}^2/2r$ chosen for the augmented switched systems (1), (5) and (7) satisfies

$$\begin{aligned}\dot{V}&\le-\frac{1}{2}e^TQe-\sum_{j=1}^{n}\rho_jz_j^2+\frac{\mu}{2}(\tilde{\phi}^2+\phi^2)+\sum_{j=0}^{n}c_j\\&\le-\rho V+c,\end{aligned} \tag{30}$$

where

$$c=\mu\phi^2/2+\textstyle\sum_{j=0}^{n}c_j,$$
$$\rho=\min\{\lambda_{\min}(Q)/\lambda_{\max}(P),2\rho_j,\mu r,j\in N\},\ r$$

is a positive design constant.

It follows from (30) that $\dot{V}<0$ on $V=\delta\ge c/\rho$, which means $V\le\delta$ is an invariant set, i.e., if $V(0)\ge c/\rho$ then $V(t)$ decreases until $V(t)\le c/\rho$, if $V(0)<c/\rho$, from $0\le V(t)\le[V(0)-c/\rho]e^{-\rho t}+c/\rho$ gained by (30), $V(t)\le c/\rho$. for all $t\ge0$. Hence, for any bounded initial conditions, variables $\chi=[e^T,z^T,\tilde{\phi}]^T$ will eventual converge to the compact set $\Xi=\{e^TQe+\sum_{i=1}^{n}z_i^2+\tilde{\phi}^2/r\le2c/\rho\}$ Moreover, all signals x, $\hat{x}$, $\hat{\phi}$, u are SGUUB by virtue of the relationships (6), (14), (15), (19), (22), (23) and the choices of the virtual control law $\alpha_i(i=1,\cdots,n-1)$.

Remark 6: From above analysis, the choice of initial value $\chi(0)$ doesn't affect the convergence size of variables χ what's more, even if the exact scope of tracking error is unknown due to unknown parameter ϕ, it is possible to make the compact set χ as small as possible by appropriately increasing ρ(increasing the design parameters ρ_j, r, and adjusting γ_0, γ) and decreasing c (decreasing the design parameters γ_0, ε_1^2, and increasing γ_j). Thus, the tracking error in the closed-loop switched systems can converge to an arbitrary small neighborhood of origin. However, being sensitive to the parameters, the undesirable high gain controller, and even the unsolvable problem about LMI (8) may be caused by adjusting these design parameters simply to go in for smaller tracking error. In practical engineers, the tracking error tends to be tuned as small as possible by limiting the gain of controller to an acceptable value.

So far, the above analysis and design procedure can be summarized in the following theorem:

Theorem 1: For a class of switched nonlinear systems described by (1), based on a reduced-order observer (5), a common adaptive dynamic output feedback controller is constructed explicitly in the form (26) and (28) by a fuzzy recursive domination design method, such that under arbitrary switching, all signals in the corresponding closed-loop switched system remain SGUUB for any finite initial conditions. Moreover, the output tracking error converges to an expected small neighborhood of the origin by adjusting design parameters appropriately.

Remark 7: In the development of adaptive output feedback design method under arbitrary switching based upon a common reduced-order observer, only one newly introduced unknown parameter ϕ needs to be estimated. Such a method not only reduces the computational burden, but also is a need to achieve adaptive arbitrary switching control for the switched system in this paper. And the inappropriate expression of adaptive laws being related to the switching signal proposed in [14] under arbitrary switching is righted simultaneously.

4. SIMULATION ANALYSIS

In this section, two examples are given to show the effectiveness and the applicability of the proposed control method for a class of general switched nonlinear systems with arbitrary switching.

Example 1: A numerical example with three switched subsystems is considered for the design of the adaptive dynamic output feedback controller for the switched system (1) with $n=3$. The functions $f_{ik}(\cdot)$ are unknown and break the limit of only depending on output y in [19,20]. In simulation, they are chosen as

$$f_{11}=-x_1^2e^{-x_3},\quad f_{21}=x_1^2x_2/(1+x_3^2),\quad f_{31}=x_2\sin x_1,$$
$$f_{12}=x_1^2x_2-x_2+x_3,\quad f_{22}=x_1^2\sin x_2,\quad f_{32}=x_2^2x_3,$$
$$f_{13}=x_2e^{-0.2x_1},\quad f_{23}=x_1x_3\arctan x_2,\quad f_{33}=x_2x_3.$$

In order to design the controller u by the developed control method, first of all, five membership functions are chosen to approximate $f_{ik}(\cdot)$, $i=1,2,3$:

$$\mu_{A_i^j}(x_i)=\exp\left[-\frac{(x_i+1.5-0.5j)^2}{2}\right],\quad j=1,\cdots,5.$$

Then from (5), the following reduced-observer is needed:

$$\begin{cases}\dot{\hat{\xi}}_1 = \hat{\xi}_2 + l_2 y - l_1(\hat{\xi}_1 + l_1 y) \\ \dot{\hat{\xi}}_2 = u - l_2(\hat{\xi}_1 + l_1 y)\end{cases}$$

By solving LMI (8) with $\gamma = 3,\ \gamma_0 = 9,$ we get

$$P = \begin{bmatrix} 3.0838 & -1.8535 \\ -1.8535 & 2.0347 \end{bmatrix}, \quad M = \begin{bmatrix} 6.2444 \\ 1.4269 \end{bmatrix},$$

$$Q = \begin{bmatrix} 4.6071 & 0 \\ 0 & 0.8034 \end{bmatrix}, \quad L = P^{-1}M = \begin{bmatrix} 5.4062 \\ 5.6259 \end{bmatrix}.$$

Finally, the common fuzzy adaptive controller is constructed by (26), (28) and (29) with parameters, $\rho_1 = 0.3,\ \rho_2 = 0.2,\ \rho_3 = 0.3,\ \gamma_1 = \gamma_2 = \gamma_3 = r = 0.1,\ \mu =$ 1.2, and the given tracking reference signal $y_r =$ $0.5[\cos t + \sin(0.1t)]$. The simulation is run with initial conditions $x(0) = [0.1, 0.1, -0.1]^T$, $\hat{\xi}(0) = [0.2, 0.3]^T$, $\hat{\phi}(0)$ $= 0.1$ under a random switching signal, see Fig. 1(a). The corresponding simulation results are shown in Fig. 1(b)-(f), respectively. From these figures, it is seen that even though there are unknown switching signal and unavailable states in system, the proposed adaptive fuzzy control approach can effectively realize the output feedback tracking control with the designed common controller for the corresponding closed-loop switched systems under arbitrary switching. The boundedness of signals x, $\hat{\xi}$, $\hat{\phi}$ and u is guaranteed, and specially, the tracking error $z_1 = x_1 - y_r$ can converge to a neighborhood of the origin as small as possible. Certainly, from simulation, it is worth mentioning that the main purpose of the example is the output feedback tracking control, which is well achieved by the developed adaptive control

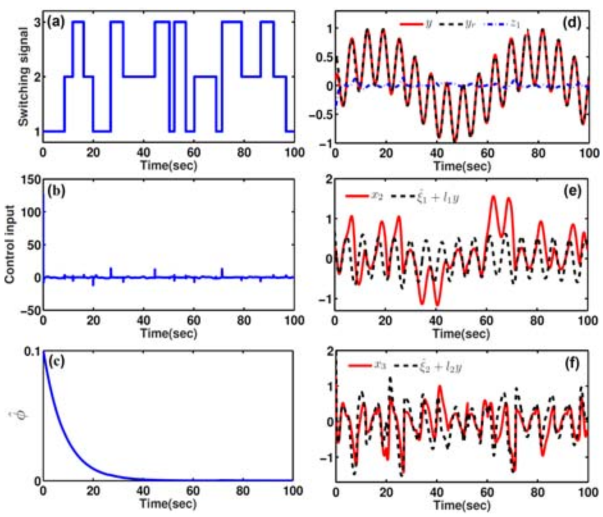

Fig. 1. (a) A random switching signal; (b) Response curve of the common control input u; (c) The estimation of parameter ϕ; (d) The trajectories of the system output y, reference signal y_r and tracking error z_1; (e) The trajectories of the state x_2 and its reconstructed value $\hat{\xi}_1 + l_1 y$; (f) The trajectories of the state x_3 and its reconstructed value $\hat{\xi}_2 + l_2 y$.

technique even though there are relatively large errors between real values of the unavailable states and their estimations. Such high observer errors are related to the design constant c_0, which is affected by the values of design parameters γ_0 and γ in LIM (8), the use of upper bound about $\|\Theta_{ik}\|^2$ and $\Psi_i(\bar{x}_i)^T \Psi_i(\bar{x}_i)$ in design process (11) and (12), and the choices of the approximation errors ε_{ik} in (3).

Example 2: A chemical system about the continuous stirred tank reactor(CSTR) is taken into account to further illustrate the feasibility of our proposed control method, whose dynamic process can be modelled as a switched nonlinear system [28]:

$$\begin{cases}\dot{C}_A = \dfrac{q_k}{V}(C_{Afk} - C_A) - a_0 \exp\left(-\dfrac{E}{RT}\right) C_A, \\ \dot{T} = \dfrac{q_k}{V}(T_{fk} - T) - a_1 \exp\left(-\dfrac{E}{RT}\right) C_A + a_2(T_C - T),\end{cases} \tag{31}$$

where the switching signal $k = 1, 2$. C_A, T, T_C denote the concentration of reactant, reactor temperature and coolant temperature, respectively. For both modes, the expected nominal operating conditions are $T_C^* = 300\ K$, $C_A^* = 0.5\ mol/L$ and $T^* = 350\ K$. In addition, the detailed physical meanings and the nominal values of the other parameters can be available in [28].

In fact, the concrete or even accurate forms of the system dynamic description cannot be obtained easily in engineering practice, which results in the model state feedback linearization proposed in [28] being achieved difficultly. Hence, in the case of removing the temperature sensor and the supervisory mechanism from detecting the reactor temperature T and the position of the selector valve, the control objective of system (32) is based on the developed adaptive fuzzy dynamic output feedback control method, design an observer-based common controller to realize the reactant concentration C_A and the reactor temperature T converging to a neighborhood of their expected values as small as possible under arbitrarily switching the position of the selector valve. To this end, define the states $x_1 = C_A - C_A^*,\ x_2 = T - T^*,$ the control input $u = a_2(T_C - T_C^*)$ and measurable output y $= x_1$, the CSTR system (31) can be transformed into the switched system (1) with $n = 2$ and

$$\begin{aligned}
&f_{11} = 0.5(1 - x_1) - a_0 g(\bar{x}_2) - x_2, \\
&f_{21} = -2.592x_2 - 104.6 - a_1 g(\bar{x}_2), \\
&f_{12} = 2(0.25 - x_1) - a_0 g(\bar{x}_2) - x_2, \\
&f_{22} = -4.092x_2 - 104.6 - a_1 g(\bar{x}_2), \\
&g(\bar{x}_2) = (x_1 + 0.5)\exp[-8750/(x_2 + 350)].
\end{aligned}$$

Choose the same membership functions as Example 1 to approximate unknown functions $f_{ik}(\cdot)$. According to Theorem 1, the designed adaptive dynamic controller is

$$\dot{\hat{\xi}} = u - l_1(\hat{\xi} + l_1 y),$$

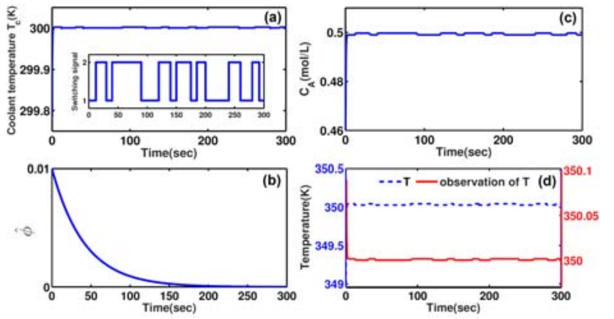

Fig. 2. (a) Response curve of the coolant temperature T_C under a random switching signal between two modes; (b) The estimation of parameter ϕ; (c) The response curve of reactant concentration C_A; (d) The trajectories of reactor temperature T and its observation value $\hat{\xi}_1 + l_1 y + T^*$.

$$\dot{\hat{\phi}} = \frac{r}{2}\left[\gamma_1 z_1^2 + \gamma_2 z_2^2\left(\frac{\partial \alpha_1}{\partial y}\right)^2\right] - \mu r \hat{\phi},$$

$$u = -\rho_2 z_2 - z_1 - \Pi_2 - \frac{z_2}{2}\left(\frac{\partial \alpha_1}{\partial y}\right)^2 (\gamma + \gamma_2 \gamma_2 \hat{\phi}) + \frac{\partial \alpha_1}{\partial \hat{\phi}} \dot{\hat{\phi}}.$$

By solving LMI (8) with $\gamma = 3$, $\gamma_0 = 1$, the observer gain is l_1=2.6208. The design parameters of controller are chosen as $\rho_1 = \rho_2 = 0.1$, $\gamma_1 = \gamma_2 = 1$, $r = 0.02$, $\mu = 1.2$, the tracking reference signal is $y_r = 0$.

The simulation results are obtained with the initial states $x(0) = [-0.04, -1]^T$, $\hat{\xi}(0) = 0.2$, $\hat{\phi}(0) = 0.01$ shown in Fig. 2. It can be seen that although the position of the selector valve is switched randomly, by appropriately manipulating the coolant temperature T_C, variables C_A and T can converge to their expected values in an acceptable smaller error, which satisfies the control objective of the CSTR system.

5. CONCLUSION

For a class of uncertain switched nonlinear systems, this paper provided a novel constructing method of a common adaptive fuzzy smooth dynamic controller and solved the output feedback tracking control problem under arbitrary switching by a recursive scheme. Both theoretical analysis and simulation validation of two examples showed that under arbitrary switching, all signals were ensured to be SGUUB and with proper selections of the controller parameters, the tracking error might be adjusted to be a neighborhood of the origin as small as possible. The developed method improved and enriched the existing results. Certainly, it should be mentioned that relative larger observation error will likely appear in the control process due to the application of the bound about the fuzzy approximated information to unknown functions. More perfect adaptive output feedback design methods, as mentioned in [29], being adopted to stabilize the switched non-linear systems with control delays, unknown disturbance and arbitrary switching is an objective in our next work.

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Chun-Yan Wang received her M.S. degree in Operational Research and Cybernetics from Yanshan University in 2004. She is a teacher in College of Sciences and pursuing a Ph.D. degree in Control Science and Control Engineering of Yanshan University. Her research interests include robust and adaptive control of switched systems.

Xiao-Hong Jiao received her Ph.D. degree in Mechanical Engineering from Sophia University, Tokyo, Japan, in 2004. She is a Professor with the Institute of Electrical Engineering, Yanshan University, Qinhuangdao, China. Her current research interests include robust adaptive control of nonlinear systems and time-delay systems and applications to hybrid distributed generation systems and automotive powertrain.