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Stability of Impulsive Systems with Time Window via Comparison Method

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Abstract: The stability of impulsive systems with time window is studied via comparison method. Two theorems are obtained to determine the different impulsive time windows for stable and unstable continuous dynamical systems, respectively. The effectiveness of the theoretical results are illustrated by two numerical examples.

Keywords: Comparison method, impulsive system, stability, time window.

1. INTRODUCTION

The study of impulsive systems has been assuming a greater importance since many evolution processes in nature are characterized by the fact that at certain moments of time they experience an abrupt change of states. These systems have important applications in various fields, such as mechanical systems, network systems, sampled-data systems, and control systems with communication constraints (see [1-3]). On the other hand, impulsive control based on impulsive systems can provide an efficient way to deal with plants that cannot endure continuous control inputs. The stability analysis of ordinary differential equations with impulsive effects at fixed time has been the subject of many investigations in recent years (see [4-18]). For example, [9] and [10] considered the asymptotic stability and stability of impulsive control systems, respectively. In [13], Xie investigated the stabilization and synchronization of Lorenz systems by impulsive control.

It should be noticed that the impulses of these studied systems all occurred at the fixed-time points. But in some real impulsive control system, it is not easy to ensure that the impulsive input is exactly at a fixed time, that is to say, the impulses may occur in a little range of time therefore we just need to consider impulsive effect in an interval, which we call it impulsive time window.

It is well known that comparison methods play an important role in the theory of impulsive differential equations, hence in this paper, we mainly investigate stability of impulsive system with time window via comparison method. The rest of the paper is organized as follows. In Section 2, we introduce the impulsive systems with time windows and some preliminaries. In Section 3, we give two main theorems via comparison methods; one is for stable continuous dynamical system, another is for unstable ones. And then we give two numerical examples to show the effectiveness of the obtained results in Section 4. Finally, we conclude our results.

2. PROBLEM STATEMENT AND PRELIMINARIES

Let $R=(-\infty,+\infty)$ be the set of real numbers, R^n be the space of n-dimensional real column vectors and $Z_+=\{1,2,\cdots\}$ be the set of positive integer numbers. $K=\{\varphi:R_+\to R_+$ is continuous, monotone strictly increasing and $\varphi(0)=0\}$. $PC[D,F]$ is the set of functions $\psi:D\to F$ which are continuous for $t\in D$, $t\neq t_k$, have discontinuous of first kind at the points t_k and are left continuous. In this paper, we consider the following impulsive control system:

$$\begin{cases}\dot{x}=f(t,x), & t\neq t_k, t_k\in[t_k^l,t_k^r)\\ x(t_k^+)=U(k,x(t_k)) & \\ x(t_0^+)=x_0, & k=1,2,\cdots,\end{cases}\tag{1}$$

where $x\in R^n$ is state variable. $f\in PC[R_+\times R^n,R^n)$, $[t_k^l,t_k^r)(k=1,2,\cdots)$ are fixed-time windows and $t_k\in[t_k^l,t_k^r)$, $k\in Z_+$. We assume that $t_0<t_1^l\le t_1<t_1^r<t_2^l\le t_2<t_2^r<\cdots<t_k^l\le t_k<t_k^r<\cdots$.

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Definition 1: If for each $\varepsilon > 0$, $t_0 \in R_+$, there is a $\delta = \delta(t_0, \varepsilon) > 0$ such that $\|x_0\| < \delta$ implies $\|x(t)\| < \varepsilon$. Then the solution of (1) is said to be stable.

If the solution of (1) is stable and for each $\varepsilon > 0$, $t_0 \in R_+$, there is a $\delta = \delta(t_0) > 0$ and $T = T(t_0, \varepsilon)$ such that $\|x_0\| < \delta$ implies $\|x(t)\| < \varepsilon$ for $t \geq t_0 + T$. Then the solution of (1) is said to be asymptotically stable.

Definition 2: Let $V \in V_0$, and assume that

$$\begin{cases} D^+V(t,x) \leq g(t, V(t,x)), & t \neq t_k, t_k \in [t_k^l, t_k^r) \\ V(t, x + U(k,x)) \leq \psi_k(V(t,x)), \end{cases} \quad (2)$$

where $g : R_+ \times R \to R$ is continuous in $(t_{k-1}, t_k) \times R$ and for each $x \in R$, $k \in Z_+$,

$$\lim_{(t,y) \to (t_k^+, x)} g(t,y) = g(t_k^+, x)$$

exists. $\psi_k : R_+ \to R_+$ is non-decreasing. Then the following system

$$\begin{cases} \dot{w} = g(t,x), & t \neq t_k, t_k \in [t_k^l, t_k^r) \\ w(t_k^+) = \psi_k(w(t_k)), & \\ w(t_0^+) = w_0, & k = 1, 2, \cdots \end{cases} \quad (3)$$

is the comparison system of (1).

Lemma 1: Assume that

1) $f(t,0) = 0$, $g(t,0) = 0$ and $U(k,0) = 0$ for all k;
2) $V : R_+ \times S_\rho \to R_+$, $\rho > 0$, $V \in V_0$, $D^+V(t,x) \leq g(t, V(t,x))$, $t \neq t_k, t_k \in [t_k^l, t_k^r)$;
3) There exists a $\rho_0 > 0$ such that $x \in S_{\rho_0}$ implies that $x + U(k,x) \in S_\rho$ for all k and $V(t, x + U(k,x)) \leq \psi_k(V(t,x))$, $t = t_k$, $t_k \in [t_k^l, t_k^r)$; $x \in S_{\rho_0}$
4) $\beta(\|x\|) \leq V(t,x) \leq \alpha(\|x\|)$ on $R_+ \times S_\rho$, where $\alpha(\cdot)$, $\beta(\cdot) \in K$.

If the trivial solution of comparison system (3) is stable (resp. asymptotically stable), then the trivial solution of (1) is also stable (resp. asymptotically stable).

Proof: From condition 1 we know that the trivial solutions of both (1) and (3) exist.

Firstly, we prove the trivial solution of (1) is stable.

Let us suppose that the trivial solution of (3) is stable and let $0 < \eta < \min(\rho, \rho_0)$, then there exist an $\varepsilon_1(t_0, \eta) > 0$ such that $0 \leq w_0 < \varepsilon_1$ implies $w(t, t_0, w_0) < \beta(\eta)$ where $t \geq t_0$ and $w(t, t_0, w_0)$ is an arbitrary solution of (3). Let us choose $w_0 = \alpha(\|x\|)$ and let $\varepsilon_2 = \varepsilon_2(\eta)$ such that $\alpha(\varepsilon_2) < \beta(\eta)$. Let $\varepsilon = \min(\varepsilon_1, \varepsilon_2)$, then we have the following claim:

Claim 1: For any solution $x(t, t_0, x_0)$ of (1), if $\|x_0\| \leq \varepsilon$, then $\|x(t)\| \leq \eta$ for $t \geq t_0$.

If claim is not true then there are k and a solution $x_1(t) = x_1(t, t_0, x_0)$ of (1) satisfying

$$\eta \leq \|x_1(\tau_1)\| \text{ and } \|x_1(t)\| < \eta \text{ for } t \in [t_0, t_k),\ t_k \in [t_k^l, t_k^r), \quad (4)$$

where $\|x_0\| < \varepsilon$, $\tau_1 > t_0$ and $\tau_1 \in (t_k, t_{k+1}]$.

From $0 < \eta < \min(\rho, \rho_0)$ we know $0 < \eta < \rho_0$, then from condition 3 we have $\|x_1(t_k) + U(k, x_1(t_k))\| < \rho$. Therefore there exists a $\tau_2 \in (t_k, \tau_1]$, $t_k \in [t_k^l, t_k^r)$, such that $\eta < \|x_1(\tau_2)\| < \rho$. Followed conditions 2 and 3, we have

$$V(t, x_1) \leq w_{\max}(t, t_0, w_0),\ w_0 = \alpha(\|x_0\|),\ t \in [t_0, \tau_2], \quad (5)$$

where $w_{\max}(t, t_0, x_0)$ is the maximal solution of (3). Followed condition 4 and recall that any solution of (3), $w(t, t_0, x_0) < \beta(\eta)$, we have

$$\begin{aligned} \beta(\eta) &\leq \beta(\|x_1(\tau_2)\|) \leq V(\tau_2, x_1(\tau_2)) \\ &\leq w_{\max}(t, t_0, x_0) < \beta(\eta), \end{aligned} \quad (6)$$

which leads to a contradiction. Hence, Claim 1 is true and the trivial solution of system (1) is stable.

Secondly, we prove the trivial solution of (1) is asymptotically. We just to prove the trivial solution of (1) is attractive.

Let us suppose that the trivial solution of (3) is asymptotically. Following the proof of stable case, we know that the trivial solution of (1) is stable. Let $\varepsilon_3 = \varepsilon(t_0, \min(\rho, \rho_0))$ then the stable property leads to

$$\|x_0\| < \varepsilon_3 \text{ implies } \|x(t)\| < \rho,\ t \geq t_0. \quad (7)$$

Let $0 < \eta < \min(\rho, \rho_0)$, $t_0 \in R_+$ and because the trivial solution of (3) is attractive, there exist $\varepsilon_4 = \varepsilon_4(t_0) > 0$ and $T = T(t_0, \eta) > 0$ such that

$$0 \leq w_0 < \varepsilon_4 \text{ implies } w(t, t_0, w_0) < \beta(\eta),\ t \geq t_0 + T. \quad (8)$$

Let us choose $\varepsilon_0 = \min(\varepsilon_3, \varepsilon_4)$ and let $\|x_0\| < \varepsilon_0$, then we have

$$\begin{aligned} \beta(\|x(t)\|) &\leq V(t, x(t)) \leq w_{\max}(t, t_0, \alpha(\|x_0\|)) \\ &< \beta(\eta), t \geq t_0 + T, \end{aligned} \quad (9)$$

which proves the trivial solution of (1) is asymptotically stable.

3. MAIN RESULTS

Theorem 1: If the comparison system is as follows:

$$\begin{cases} \dot{w} = -\sigma(t) h(w), & t \neq t_k, t_k \in [t_k^l, t_k^r), \\ w(t_k^+) = \psi_k(w(t_k)), & \\ w(t_0^+) = w_0, & k = 1, 2, \cdots, \end{cases} \quad (10)$$

where $\sigma \in PC[R_+, R_+]$, ψ_k, $h \in K$. Then the trivial solution of the system (1) is

(i) stable if

$$\int_\theta^{\psi_{k+1}(\theta)} \frac{1}{h(s)} ds - \int_{t_k^r}^{t_{k+1}^l} \sigma(s) ds \leq 0, \quad (11)$$

for some $\rho > 0$ and all $\theta \in (0, \rho]$;

(ii) asymptotically stable if

$$\int_\theta^{\psi_{k+1}(\theta)} \frac{1}{h(s)} ds - \int_{t_k^r}^{t_{k+1}^l} \sigma(s) ds \leq -\varpi, \text{ for } \varpi > 0. \quad (12)$$

Proof: (i) Let $w(t,t_0,w_0)$ be any solution of system (10), then $w(t)$ is non-increasing in $(t_k,t_{k+1}]$, $k\in N$, $w(t_{k+1})<w(t_k^+)$. It follows from (10) that for $t\in(t_k, t_{k+1}]$, $k\in N$, we have

$$\int_{w(t_k^+)}^{w(t)}\frac{1}{h(s)}ds=-\int_{t_k}^{t}\sigma(s)ds \tag{13}$$

and follows from (11) that

$$\begin{aligned}\int_{w(t_{k+1})}^{w(t_{k+1}^+)}\frac{1}{h(s)}ds&=\int_{w(t_{k+1}^+)}^{\psi_{k+1}(w(t_{k+1}))}\frac{1}{h(s)}ds\\&\le\int_{t_k^r}^{t_{k+1}^l}\sigma(s)ds\le\int_{t_k}^{t_{k+1}}\sigma(s)ds,\end{aligned} \tag{14}$$

then we have

$$\int_{w(t_k^+)}^{w(t_{k+1}^+)}\frac{1}{h(s)}ds=\int_{w(t_k^+)}^{w(t)}\frac{1}{h(s)}ds+\int_{w(t)}^{w(t_{k+1}^+)}\frac{1}{h(s)}ds\le 0. \tag{15}$$

From which and since $h\in K$, we know that $w(t_{k+1}^+)\le w(t_k^+)$. Let us assume that $0\le t_0<t_1$ and choose $\delta=\delta(t_0)>0$ such that if $w_0\in(0,\delta)$, then $\psi_1(w_0)<\rho$. Since $\psi_1\in K$, we have

$$w(t_1^+)=\psi_1(w_0)<\rho. \tag{16}$$

Since $w(t)$ is non-increasing in $(t_1,t_2]$, it follows from (16) that

$$w(t_2)\le w(t_1^+)<\rho. \tag{17}$$

Using mathematical induction we immediately know that if $w_0<d$, then $w(t)<\rho$, for all $t>t_0$. This proves the stability of the trivial solution.

(ii) From the proof of (i), we know $w(t_{k+1}^+)\le w(t_k^+)$ and $w(t)$ is decreasing in $(t_k,t_{k+1}]$, hence we only need to prove $\lim_{k\to\infty} w(t_k^+)=0$.
If it is false, then there is such a $\eta>0$ that $w(t_k^+)>\eta$. Since the sequence $w(t_k^+)$ is non-increasing and $h\in K$, we have

$$h(\eta)\le h(w(t_{k+1}^+))\le h(w(t_k^+)), \tag{18}$$

and it follows from (14) and (17) that

$$\int_{w(t_k^+)}^{w(t_{k+1}^+)}\frac{1}{h(s)}ds\le\frac{w(t_{k+1}^+)-w(t_k^+)}{h(\eta)}, \tag{19}$$

then we have

$$w(t_{k+1}^+)\le w(t_k^+)-h(\eta)\varpi, \tag{20}$$

Using mathematical induction we can get

$$w(t_{k+1}^+)\le w(t_0^+)-(k+1)h(\eta)\varpi. \tag{21}$$

It follows $w(t)>0$ that we have the contradiction: $\lim_{k\to\infty} w(t_{k+1}^+)<0$. Therefore the trivial solution of the comparison system (1) is asymptotically stable.

Corollary 1: If $g(t,w)=-qw$, $q>0$, $\psi_k(w)=dw$, $d>0$, then the trivial solution of (1) is

(i) stable if

$\ln d\le q(t_{k+1}^l-t_k^r)$;

(ii) asymptotically stable if

$\ln\gamma d\le q(t_{k+1}^l-t_k^r)$, $\gamma>1$.

Theorem 2: If the comparison system is as follows:

$$\begin{cases}\dot w=\sigma(t)h(w), & t\ne t_k, t_k\in[t_k^l,t_k^r),\\ w(t_k^+)=\psi_k(w(t_k)),\\ w(t_0^+)=w_0, & k=1,2,\cdots,\end{cases} \tag{22}$$

where $\sigma\in PC[R_+,R_+]$, ψ_k, $h\in K$. Then the trivial solution of the system (1) is

(i) stable if

$$\int_{\theta}^{\psi_{k+1}(\theta)}\frac{1}{h(s)}ds+\int_{t_k^l}^{t_{k+1}^r}\sigma(s)ds\le 0, \tag{23}$$

for some $\rho>0$ and all $\theta\in(0,\rho]$;

(ii) asymptotically stable if

$$\int_{\theta}^{\psi_{k+1}(\theta)}\frac{1}{h(s)}ds+\int_{t_k^l}^{t_{k+1}^r}\sigma(s)ds\le-\varpi,\ \text{for}\ \varpi>0. \tag{24}$$

Proof: (i) Let $w(t,t_0,w_0)$ be any solution of system, it follows from (22) that for $t\in(t_k,t_{k+1}]$, $k\in N$, we have

$$\int_{w(t_k^+)}^{w(t)}\frac{1}{h(s)}ds=\int_{t_k}^{t}\sigma(s)ds, \tag{25}$$

and follows from (25) that

$$\begin{aligned}\int_{w(t_{k+1})}^{w(t_{k+1}^+)}\frac{1}{h(s)}ds&=\int_{w(t_{k+1}^+)}^{\psi_{k+1}(w(t_{k+1}))}\frac{1}{h(s)}ds\\&\le-\int_{t_k^l}^{t_{k+1}^r}\sigma(s)ds\le-\int_{t_k}^{t_{k+1}}\sigma(s)ds,\end{aligned} \tag{26}$$

then we have

$$\int_{w(t_k^+)}^{w(t_{k+1}^+)}\frac{1}{h(s)}ds=\int_{w(t_k^+)}^{w(t)}\frac{1}{h(s)}ds+\int_{w(t)}^{w(t_{k+1}^+)}\frac{1}{h(s)}ds\le 0. \tag{27}$$

From which and since $h\in K$, we know that $w(t_{k+1}^+)\le w(t_k^+)$.
The rest of proof is similarly of the proof of Theorem 1.

Corollary 2: If $g(t,w)=qw$, $q>0$, $\psi_k(w)=dw$, $d>0$, then the trivial solution of (1) is

(i) stable if

$\ln d\le -q(t_{k+1}^r-t_k^l)$;

(ii) asymptotically stable if

$\ln\gamma d\le -q(t_{k+1}^r-t_k^l)$, $\gamma>1$.

Remark 1: Theorem 1 needs the continuous dynamics of system to be stable, but Theorem 2 allows the continuous dynamics to be unstable. Theorem 2 shows

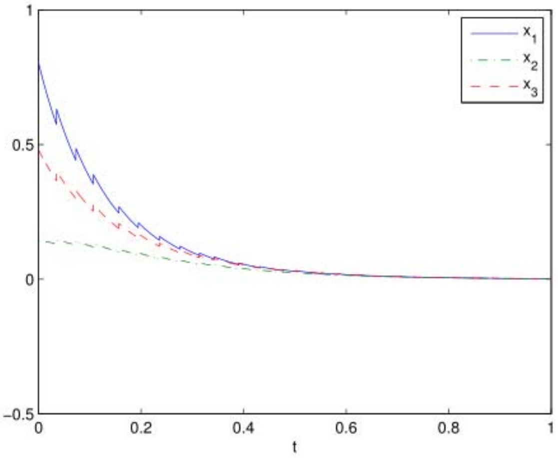

Fig. 1. Stable results of Example 1 with $\kappa = 0.1$.

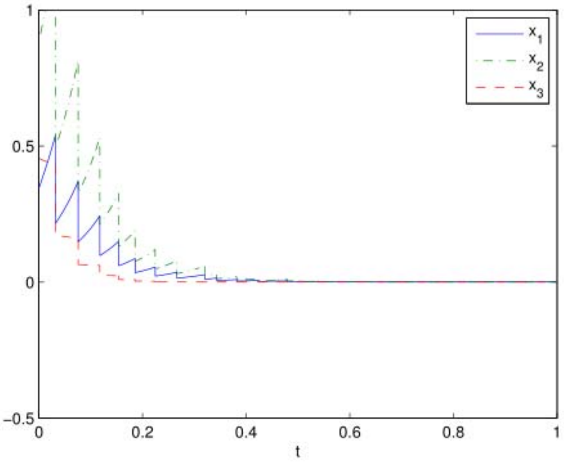

Fig. 2. Stable results of Example 1 with $\kappa = 0.1$.

that an unstable system with time window can be successfully stabilized by impulse.

Remark 2: The current research results mainly focus on impulses all occurred at the fixedtime points, and there exist few results on impulsive effect occurred at any time of an interval. We get Theorem 1 and Theorem 2 to determine the different impulsive time windows for stable and unstable continuous dynamical systems, respectively.

4. NUMERICAL EXAMPLE

In this section, we carry out two numerical example to illustrate the theoretical results above.

Example 1: Consider the impulsive system given by

$$\begin{cases} \dot{x} = Ax + \phi(x), & t \neq t_k, t_k \in [t_k^l, t_k^r), \\ x(t_k^+) = (I+B)x(t_k), \end{cases}$$

where $x^T = (x_1, x_2, x_3)$, and

$$A = \begin{pmatrix} -10 & 2 & 0 \\ 2 & -10 & 0 \\ 0 & 0 & -8 \end{pmatrix}, \quad \phi(x) = \begin{pmatrix} 0 \\ -x_1x_3 \\ x_1x_2 \end{pmatrix},$$

$$B = \begin{pmatrix} \kappa & 0 & 0 \\ 0 & \kappa & 0 \\ 0 & 0 & \kappa \end{pmatrix}.$$

Let $V(t,x) = x^T x$, then

$$-q = \lambda_{\max}(A + A^T) = -16,$$
$$d = \lambda_{\max}((I+B)(I+B^T)) = (1+\kappa)^2.$$

Noting that

$$\begin{cases} D^+V(t,x) \le -qV(t,x), \\ V(t_i^+, x+Bx) \le dV(t_i, x) \le (1+\kappa)^2 V(t_i, x), \end{cases}$$

we choose $t_k^r - t_k^l = 0.02$, $t_{k+1}^l - t_k^r = 0.02$, where $t_k \in [t_k^l, t_k^r)$ is random, then from Corollary 3.2, we get

$$(1+\kappa)^2 < \exp(16 \times 0.02) < 1.1735.$$

Obviously when $(1+\kappa) < 0$, the system is asymptotically stable, therefore if $\kappa < 0.1735$, the impulsive system is asymptotically stable.

Example 2: The impulsively controlled Lorenz system is given by

$$\begin{cases} \dot{x} = Ax + \phi(x), & t \neq t_k, t_k \in [t_k^l, t_k^r), \\ x(t_k^+) = (I+B)x(t_k), \end{cases}$$

where $x^T = (x_1, x_2, x_3)$, and

$$A = \begin{pmatrix} -10 & 10 & 0 \\ 28 & -1 & 0 \\ 0 & 0 & -8/3 \end{pmatrix}, \quad \phi(x) = \begin{pmatrix} 0 \\ -x_1x_3 \\ x_1x_2 \end{pmatrix},$$

$$B = \begin{pmatrix} \kappa & 0 & 0 \\ 0 & \kappa & 0 \\ 0 & 0 & \kappa \end{pmatrix},$$

then

$$q = \lambda_{\max}(A + A^T) = 28.05,$$
$$d = \lambda_{\max}((I+B)(I+B^T)) = (1+\kappa)^2,$$

we choose $t_k^r - t_k^l = 0.02$, $t_{k+1}^l - t_k^r = 0.02$, where $t_k \in [t_k^l, t_k^r)$ is random, then from Corollary 3.4, we get

$$(1+\kappa)^2 < \exp(-28.05 \times 0.06) < 0.1858.$$

Therefore if $-1.4311 < \kappa < -0.5689$, the impulsively controlled Lorenz system is asymptotically stable.

5. CONCLUSION

In this paper, we have studied the stability of impulsive systems with time window via comparison method. Two theorems are obtained to determine the different impulsive time windows for stable and unstable continuous dynamical systems, respectively. Finally, two numerical examples and their numerical simulations are given to illustrate the effectiveness of the theoretical results.

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