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An Output Feedback Coordinated Tracking Controller for High-order Linear Systems

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Abstract: This paper proposes a distributed coordinated tracking controller for a group of high-order linear systems. The proposed controller is based on the disturbance observer which is a robust output feedback controller. It is noted that agents are allowed to use only outputs of their neighbors and the proposed controller uses only relative measurements. Since only outputs are exchanged among agents, the amount of information exchange does not depend on the dimension of agents' dynamics. Numerical simulations are included to validate the theory.

Keywords: Distributed coordinated tracking, disturbance observer, multi-agent systems, output feedback.

1. INTRODUCTION

During the last two decades, control problems on multi-agent systems which consist of a group of systems have attracted a number of researchers. Major research efforts have been put into the problems of consensus and synchronization, formation control, distributed optimization, and distributed estimation, etc., and a vast amount of results can be found in the literature. See, e.g., [1-4], the survey paper [5], and references therein for more details.

Among the problems mentioned above, the consensus problem is considered in this paper. In particular, we consider the consensus problem with a leader, called the coordinated tracking problem or leader-following problem, of which the objective is to design a controller so that each agent in the group called follower tracks the trajectory of an agent called leader. Previous results on this topic are summarized as follows. In [6,7], tracking controllers based on distributed observers which estimate the velocity of the leader are proposed. Note that these results cover second order systems and more importantly they require the assumption that the leader's acceleration input is known to all followers which is not realistic. In [8], a bounded control is proposed for the case where the state of the leader is available to only a limited number of followers. Although time varying interaction networks can be covered, the controller requires estimates of the time derivative of other agents' states which is not easy to implement. Recently, variable structure based coordinated tracking controllers are presented in [9] where the controller does not use the velocity (for the first order agents) or acceleration (for the second order agents) information of the other agents. Note that the work [9] requires that agents should exchange not only positions but also velocities with their neighbors.

In this paper, we present an output feedback coordinated tracking controller for high order linear systems. Compared to the results mentioned above, the present work considers a more realistic case where the followers do not know the input to the leader, agents are allowed to exchange only outputs, and no absolute measurements are used. It is noted that a preliminary version of the current paper has been published in [10], where a group of double integrators are considered and the design of controller requires the knowledge of network topology.

The proposed controller is based on the disturbance observer [11] which has been known as a robust output feedback controller and widely used in industry; see [12-16] and references therein. Precisely, the disturbance observer developed in [16] is modified to solve the problem. Moreover, we present a constructive design procedure to determine the controller parameters.

The rest of this paper is organized as follows. In Section 2, we formulate the problem and state the key assumptions. In Section 3, an output feedback distributed coordinated tracking controller is presented and a rigorous stability proof is given. Numerical simulation results are given in Section 4 and Section 5 concludes the paper.

Notation: For a matrix M, m_{ij} represents the (i, j) component of M. I_k denotes the identity matrix of dimension k. $0_k \in \mathbb{R}^k$ and $1_k \in \mathbb{R}^k$ represent column vectors with all their elements being 0 and 1, respectively. For two matrices A and B, $A \otimes B$ denotes

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the Kronecker product. Given a matrix A, $\lambda_{\max}(A)$ ($\lambda_{\min}(A)$, resp.) denotes the maximum (minimum, resp.) eigenvalue of A. The i th-order time derivative of x is denoted by $x^{(i)}$. A polynomial $c(s)=s^n+c_{n-1}s^{n-1}+\cdots+c_0$ is said to be a Hurwitz polynomial if all roots of $c(s)=0$ have negative real parts.

Notions from graph theory: A weighted graph $\mathscr{G}$ consists of vertex set $V(\mathscr{G})=\{v_1,\cdots,v_N\}$, edge set $E(\mathscr{G})\subseteq V(\mathscr{G})\times V(\mathscr{G})$, and weight W where v_i denotes the i th agent, N represents the number of agents, i.e., $|V(\mathscr{G})|$, and $E(\mathscr{G})$ is a set of ordered pairs of vertices called edge satisfying $(v_i,v_j)\in E(\mathscr{G})$ if and only if information flows from the i th agent to the j th agent. Weight $W: V(\mathscr{G})\times V(\mathscr{G})\to\mathbb{R}_+$ is a mapping that assigns weights to edges of $\mathscr{G}$ and satisfies $W(v_i,v_j)\neq 0$ if and only if $(v_i,v_j)\in E(\mathscr{G})$. A subgraph $\hat{\mathscr{G}}$ of $\mathscr{G}$ is a graph such that $V(\hat{\mathscr{G}})\subset V(\mathscr{G})$, $E(\hat{\mathscr{G}})\subset E(\mathscr{G})\bigcap(V(\hat{\mathscr{G}})\times V(\hat{\mathscr{G}}))$. The weights of the subgraph $\hat{\mathscr{G}}$ are defined to have the same values as those of $\mathscr{G}$. Let the adjacency matrix $A=[a_{ij}]$ be an $N\times N$ nonnegative matrix and its element is defined as $a_{ii}=0$, $a_{ij}=W(v_j,v_i)$ if $i\neq j$. The graph $\mathscr{G}$ is called undirected if $A=A^T$, and directed otherwise. The Laplacian matrix L of a graph is defined as $L=D-A$, where $D=\mathrm{diag}(A1_N)$.

2. PROBLEM FORMULATION

We consider a multi-agent system composed of N agents. The dynamics of the ith agent is described by

$$\begin{aligned}\dot{x}^i &= A_g x^i + B_g u^i,\\ y^i &= C_g x^i,\end{aligned}\tag{1}$$

where $x^i\in\mathbb{R}^n$ is the state of the i th agent, $u^i\in\mathbb{R}$ the input, $y^i\in\mathbb{R}$ the output,

$$A_g=\left[\begin{array}{c|c}0_{n-1} & I_{n-1}\\ \hline \multicolumn{2}{c}{a^\top}\end{array}\right]\in\mathbb{R}^{n\times n},\quad a=[a_0,\cdots,a_{n-1}]^\top,$$

$$B_g=\begin{bmatrix}0_{n-1}\\ 1\end{bmatrix}\in\mathbb{R}^n,\quad C_g=\begin{bmatrix}1, & 0_{n-1}^\top\end{bmatrix}\in\mathbb{R}^{1\times n}.$$

Each agent is supposed to follow the trajectory of the leader whose dynamics is given by

$$\begin{aligned}\dot{\bar{x}} &= A_g\bar{x}+B_g\bar{u},\\ \bar{y} &= C_g\bar{x},\end{aligned}\tag{2}$$

where $\bar{u}\in\mathbb{R}$ is the control input to the leader, $\bar{x}\in\mathbb{R}^n$ the state, $\bar{y}\in\mathbb{R}$ the output. It is assumed that the leader does not receive any information from its neighbors. For simplicity, the leader is labeled as $N+1$.

Assumption 1: The control input $\bar{u}$ to the leader is continuously differentiable for all t, and $\bar{u}$ and $\dot{\bar{u}}$ are uniformly bounded, i.e., $|\bar{u}|<\bar{u}^+$, and $|\dot{\bar{u}}|<\dot{\bar{u}}^+$.

The graph which models the interaction among $N+1$ agents is denoted by $\mathscr{G}$ and its subgraph associated with agents $1,\ldots,N$ is denoted by $\mathscr{G}^F$. Here the superscript F is used to indicate that the agents $1,\ldots,N$ are followers. In what follows, the agents $1,\ldots,N$ are called followers.

The weighted adjacency matrix and its Laplacian associated with $\mathscr{G}$ are denoted by $A\in\mathbb{R}^{(N+1)\times(N+1)}$ and $L\in\mathbb{R}^{(N+1)\times(N+1)}$, respectively. We define $L^\star\in\mathbb{R}^{N\times N}$ by a submatrix of L with the last column and last row removed. It should be noted that $L^\star$ is different from the Laplacian associated to the graph $\mathscr{G}^F$.

Assumption 2: The graph $\mathscr{G}$ is fixed and contains a directed spanning tree, and the graph $\mathscr{G}^F$ is undirected and connected. The eigenvalues of the $L^\star$, denoted by $\lambda_i(L^\star)$, $i=1,\cdots,N$, are bounded by known values, i.e., $\lambda^-\le\lambda_i(L^\star)\le\lambda^+$, $\forall i=1,\cdots,N$.

Assumption 2 means that at least one follower receives information from the leader and thus at least one $a_{i(N+1)}$ of A is nonzero. It is noted that L has one simple zero eigenvalue with associated eigenvector 1_{N+1} and that $L^\star$ is symmetric and positive definite.

The problem under consideration is to design an output feedback controller for distributed coordinated tracking. Precisely, we would like to design an (dynamic) output feedback controller of the form

$$\begin{aligned}\dot{\chi}^i &= F(\chi^i,z^i),\\ u^i &= G(\chi^i,z^i),\quad i=1,\ldots,N,\\ z^i &= \sum_{j=1}^{N+1}a_{ij}(y^j-y^i),\end{aligned}\tag{3}$$

which solves the practical distributed coordinated tracking problem in the sense that for a given $\varepsilon>0$ it holds that

$$\limsup_{t\to\infty}\|x^i(t)-\bar{x}(t)\|\le\varepsilon,\quad i=1,\ldots,N,\tag{4}$$

while all signals in the closed-loop systems are bounded.

Note that each agent has its own controller but the structure is the same for all agents, and that the controller uses only its neighbors' information, namely, we consider distributed controllers. It is emphasized that the leader is allowed to be driven by a nonzero input and no follower in the group can access the leader's input $\bar{u}$, which are distinct features of the current work compared to a number of existing results; see [6,7] and the papers mentioned in [5].

The control objective, expressed by the condition (4), is to make the followers' states converge to the vicinity of the leader's trajectory. Although the convergence condition is weakened compared to the case where asymptotic convergence to $\bar{x}(t)$ is concerned, the information available to the controller is only the output x_1^j of its neighbors which makes the problem challenging.

We close this section by emphasizing that by the internal model principle the best we can expect is the coordinated tracking in the practical sense because each agent can not use the control input $\bar{u}$ applied to the leader so that the resulting pattern of state trajectory is not predictable.

3. MAIN RESULT

The proposed distributed coordinated tracking controller is based on the disturbance observer developed recently [15,16] and is given by

$$\begin{aligned} \dot{p}^i &= A_\tau p^i + B_p u^i, \\ \dot{q}^i &= A_\tau q^i - B_q z^i, \\ u^i &= p_1^i - \dot{q}_n^i - \beta^\top q^i = p_1^i + \frac{\alpha_0}{\tau^n}\left(q_1^i + z^i\right) - \beta^\top q^i, \end{aligned} \tag{5}$$

where

$$A_\tau = \left[-\alpha_\tau \;\middle|\; \begin{matrix} I_{n-1} \\ 0_{n-1}^\top \end{matrix}\right] \in \mathbb{R}^{n\times n}, \quad B_p = \begin{bmatrix} 0_{n-1} \\ \dfrac{\alpha_0}{\tau^n} \end{bmatrix} \in \mathbb{R}^n,$$

$$B_q = \alpha_\tau = \left[\frac{\alpha_{n-1}}{\tau}, \cdots, \frac{\alpha_0}{\tau^n}\right]^\top \in \mathbb{R}^n,$$

$$\beta = [\beta_0, \cdots, \beta_{n-1}]^\top \in \mathbb{R}^n.$$

Note that the input to the controller is z^i which is the weighted sum of the relative measurements and this means that the proposed controller is an output feedback controller. We emphasize that the control input to the leader $\bar{u}$ is not used. The constants $\alpha_0, \ldots, \alpha_{n-1}$, β_0, $\ldots, \beta_{n-1}$, and τ are controller parameters and all parameters except τ are selected following the procedure described below.

Controller Design Procedure:

1) Choose $\beta_0, \ldots, \beta_{n-1}$ such that the polynomial $s^n + \beta_{n-1}s^{n-1} + \cdots + \beta_0$ is Hurwitz.
2) Take $\alpha_1, \ldots, \alpha_{n-1}$ such that the polynomial $s^{n-1} + \alpha_{n-1}s^{n-2} + \cdots + \alpha_1$ is Hurwitz.
3) Choose $\alpha_0^\dagger > 0$ such that the polynomial $s^n + \alpha_{n-1}s^{n-1} + \cdots + \alpha_0$ is Hurwitz for all $0 < \alpha_0 \le \alpha_0^\dagger$.
4) Choose $\alpha_0^\ddagger > 0$ such that the Nyquist plot of $G_c(s) = \alpha_0 / [s(s^{n-1} + \alpha_{n-1}s^{n-2} + \cdots + \alpha_1)]$ is located in the right side of the disk whose diameter has endpoints $-1/\lambda^-$ and $-1/\lambda^+$ in the complex plane.
5) Choose $\alpha_0 = \min\{\alpha_0^\dagger, \alpha_0^\ddagger\}$.

Remark 1: Note that we can always find the constants $\alpha_0^\dagger$ and $\alpha_0^\ddagger$ in the controller design procedure. Indeed, $\alpha_0^\dagger$ can be chosen from the root locus $1 + kG_c(s) = 0$ with respect to $k > 0$, and the existence of $\alpha_0^\ddagger$ is guaranteed by the boundedness of the Nyquist plot to the negative real axis. The existence of $\alpha_0^\ddagger$ is exploited to prove the absolute stability, using circle criterion [17], of a dynamics which explains the convergence of the controller states to their steady state values.

Remark 2: The reasoning behind the proposed controller (5) is the following. As proved rigorously in [15,16], the disturbance observer based controller can make an uncertain closed-loop system behave as if it is a disturbance free nominal closed-loop system provided that design parameters are chosen appropriately. In our case, we apply this ability to the dynamics of $\tilde{x}^i = x^i - \bar{x}$ so that it behaves approximately the same as $\dot{\tilde{x}}^i = A_{11}\tilde{x}^i$ where A_{11} is a Hurwitz matrix whose characteristic equation is given by $s^n + \beta_{n-1}s^{n-1} + \ldots + \beta_0$. See (8) for the structure of A_{11}.

With the parameters chosen above, we can investigate the stability of the closed-loop system and choose the last controller parameter τ which will be chosen sufficiently small. First of all, we rewrite the closed-loop system as follows:

Lemma 1: In the coordinates given by

$$\begin{aligned} \tilde{x}^i &= x^i - \bar{x}, \\ \xi_j^i &= \left(q_j^i + (z^i)^{(j-1)}\right)/\tau^{n-j}, \\ \eta_j^i &= \tau^{j-1}\left((p_1^i)^{(j-1)} - (q_n^i)^{(j)}\right), \end{aligned}$$

the dynamics of the ith agent and its controller becomes

$$\begin{aligned} \dot{\tilde{x}}_j^i &= \tilde{x}_{j+1}^i, \quad j = 1, \ldots, n-1, \\ \dot{\tilde{x}}_n^i &= \sum_{j=1}^n a_{j-1}\tilde{x}_j^i + u^i - \bar{u}, \\ \dot{\xi}_j^i &= -\frac{\alpha_{n-j}}{\tau}\xi_1^i + \frac{1}{\tau}\xi_{j+1}^i, \quad j = 1, \cdots, n-1, \\ \dot{\xi}_n^i &= -\frac{\alpha_0}{\tau}\xi_1^i + (z^i)^{(n)}, \\ \dot{\eta}_j^i &= \frac{1}{\tau}\eta_{j+1}^i, \quad j = 1, \cdots, n-1, \\ \dot{\eta}_n^i &= -\frac{1}{\tau}\sum_{j=1}^n \alpha_{j-1}\eta_j^i + \frac{\alpha_0}{\tau}\left(u^i + (z^i)^{(n)}\right). \end{aligned} \tag{6}$$

Proof: One can easily verify the relations for $\tilde{x}_j^i$, ξ_j^i, $j = 1, \ldots, n$, and η_j^i, $j = 1, \ldots, n-1$, by straightforward computations. In order to derive the dynamics of η_n^i, we first compute

$$\begin{aligned} \dot{p}_1^i &= -\frac{\alpha_{n-1}}{\tau}p_1^i + p_2^i \\ \ddot{p}_1^i &= -\frac{\alpha_{n-1}}{\tau}\dot{p}_1^i - \frac{\alpha_{n-2}}{\tau^2}p_1^i + p_3^i \\ &\vdots \\ \left(p_1^i\right)^{(n-1)} &= -\sum_{j=1}^{n-1}\frac{\alpha_{n-j}}{\tau^j}\left(p_1^i\right)^{n-j-1} + p_n^i \end{aligned}$$

and

$$\dot{q}_n^i = -\frac{\alpha_0}{\tau^n}q_1^i - \frac{\alpha_0}{\tau^n}z^i$$

$$\ddot{q}_n^i = -\frac{\alpha_{n-1}}{\tau}\dot{q}_n^i - \frac{\alpha_0}{\tau^n}\left(q_2^i + \dot{z}^i\right)$$
$$\vdots$$
$$\left(q_n^i\right)^{(n-1)} = -\sum_{j=1}^{n-2}\frac{\alpha_{n-j}}{\tau^j}\left(q_n^i\right)^{(n-j-1)} - \frac{\alpha_0}{\tau^n}\left(q_{n-1}^i + (z^i)^{(n-2)}\right).$$

In addition, we have

$$(p_1^i)^{(n)} = -\sum_{j=1}^{n}\frac{\alpha_{n-j}}{\tau^j}(p_1^i)^{(n-j)} + \frac{\alpha_0}{\tau^n}u^i,$$
$$(q_n^i)^{(n)} = -\sum_{j=1}^{n}\frac{\alpha_{n-j}}{\tau^j}(q_n^i)^{(n-j)} - \frac{\alpha_0}{\tau^n}(z^i)^{(n-1)},$$
$$(q_n^i)^{(n+1)} = -\sum_{j=1}^{n}\frac{\alpha_{n-j}}{\tau^j}(q_n^i)^{(n-j+1)} - \frac{\alpha_0}{\tau^n}(z^i)^{(n)}.$$

From these computations, we compute $\dot{\eta}_n^i$ as

$$\begin{aligned}\dot{\eta}_n^i &= \tau^{n-1}\left(\left(p_1^i\right)^{(n)} - \left(q_n^i\right)^{(n+1)}\right)\\ &= -\tau^{n-1}\sum_{j=1}^{n}\frac{\alpha_{n-j}}{\tau^j}\left(\left(p_1^i\right)^{(n-j)} - \left(q_n^i\right)^{(n-j+1)}\right)\\ &\quad + \frac{\alpha_0}{\tau}\left(u^i + \left(z^i\right)^{(n)}\right)\\ &= -\frac{1}{\tau}\sum_{j=1}^{n}\alpha_{j-1}\eta_j^i + \frac{\alpha_0}{\tau}\left(u^i + \left(z^i\right)^{(n)}\right),\end{aligned}$$

which completes the proof.

We rewrite the control input of the i th agent as

$$\begin{aligned}u^i &= \eta_1^i - \sum_{j=1}^{n}\beta_{j-1}\left(\tau^{n-j}\xi_j^i - (z^i)^{(j-1)}\right)\\ &= \eta_1^i - \sum_{j=1}^{n}\beta_{j-1}\left(\tau^{n-j}\xi_j^i + \sum_{k=1}^{N}l_{ik}^\star\tilde{x}_j^k\right),\end{aligned} \tag{7}$$

where we applied the relation $z^i = \sum_{k=1}^{N+1}a_{ik}(y^k - y^i) = -\sum_{k=1}^{N}l_{ik}^\star\tilde{x}_1^k$.

To proceed, we compute the quasi-steady state of ξ_j^i and η_j^i (the imaginary steady state values of ξ_j^i and η_j^i when τ is zero and the plant states and inputs are frozen), denoted by $\bar{\xi}^i{}_j, \bar{\eta}^i{}_j$, respectively, as follows:

$$\bar{\xi}_1^i = \cdots = \bar{\xi}_n^i = 0, \quad \bar{\eta}_2^i = \cdots = \bar{\eta}_n^i = 0, \tag{9}$$
$$\bar{\eta}_1^i = -\sum_{j=1}^{n}(\beta_{j-1} + a_{j-1})\tilde{x}_j^i + \sum_{j=1}^{n}\beta_{j-1}\sum_{k=1}^{N}l_{ik}^\star\tilde{x}_j^k + \bar{u}. \tag{10}$$

Indeed, we first rewrite the dynamics of ξ_j^i and η_j^i as

$$\tau\dot{\xi}_j^i = -\alpha_{n-j}\xi_1^i + \xi_{j+1}^i, \quad j = 1,\cdots,n-1,$$
$$\tau\dot{\xi}_n^i = -\alpha_0\xi_1^i + \tau(z^i)^{(n)},$$
$$\tau\dot{\eta}_j^i = \eta_{j+1}^i, \quad j = 1,\cdots,n-1,$$
$$\tau\dot{\eta}_n^i = -\sum_{j=1}^{n}\alpha_{j-1}\eta_j^i + \alpha_0\left(u^i + \left(z^i\right)^{(n)}\right).$$

Taking $\tau = 0$ in the dynamics of ξ_j^i, $j = 1,\ldots,n$, and η_j^i, $j = 1,\ldots,n-1$, results in (9). To derive (10), we consider the dynamics of η_n^i. Taking $\tau = 0$ and applying (9) yield

$$\bar{\eta}_1^i = u^i + (z^i)^{(n)}.$$

Substituting the relations

$$(z^i)^{(n)}\Big|_{\tau=0} = -\sum_{k=1}^{N}l_{ik}^\star\left(\sum_{j=1}^{n}a_{j-1}\tilde{x}_j^k + u^k\Big|_{\tau=0} - \bar{u}\right)$$
$$u^k\Big|_{\tau=0} = \eta_1^k - \sum_{j=1}^{n}\beta_{j-1}\sum_{m=1}^{N}l_{km}^\star\tilde{x}_j^m,$$

one has

$$\begin{aligned}\bar{\eta}_1^i &= \bar{\eta}_1^i - \sum_{j=1}^{n}\beta_{j-1}\sum_{m=1}^{N}l_{im}^\star\tilde{x}_j^m\\ &\quad - \sum_{k=1}^{N}l_{ik}^\star\left(\sum_{j=1}^{n}a_{j-1}\tilde{x}_j^k + \bar{\eta}_1^k - \sum_{j=1}^{n}\beta_{j-1}\sum_{m=1}^{N}l_{km}^\star\tilde{x}_j^m - \bar{u}\right)\end{aligned}$$

from which we have

$$\alpha = [\alpha_{n-1},\cdots,\alpha_0]^\top,\quad \alpha_< = [\alpha_0,\cdots,\alpha_{n-1}]^\top,\quad \beta_\flat = [0,\beta_0,\cdots,\beta_{n-2}]^\top,\quad a_\flat = [0,a_0,\cdots,a_{n-2}]^\top,\quad \Delta_\tau^k = \mathrm{diag}\{\tau^k,\ldots,1\},$$

$$A_{11} = \left[\begin{array}{c|c}0_{n-1} & I_{n-1}\\ \hline \multicolumn{2}{c}{-\beta^\top}\end{array}\right],\quad \tilde{A}_{12} = \begin{bmatrix}0_{(n-1)\times n}\\ -\beta^\top\Delta_\tau^{n-1}\end{bmatrix},\quad \tilde{A}_{13} = \begin{bmatrix}0_{(n-1)\times n}\\ C_g\end{bmatrix},\quad A_{22} = \left[\begin{array}{c|c}-\alpha & \begin{matrix}I_{n-1}\\ 0_{n-1}^\top\end{matrix}\end{array}\right]\quad \tilde{A}_{31} = \begin{bmatrix}\beta_\flat^\top + a_\flat^\top - (\beta_{n-1} + a_{n-1})\beta^\top\\ 0_{(n-1)\times n}\end{bmatrix},$$

$$A_{32} = \left[\begin{array}{c|c}\multicolumn{2}{c}{0_{(n-1)\times n}}\\ \hline 0_{n-1}^\top & -\alpha_0\beta_{n-1}\end{array}\right],\quad \tilde{A}_{32} = \begin{bmatrix}-(\beta_{n-1} + a_{n-1})\beta^\top\Delta_\tau^{n-1}\\ 0_{(n-2)\times n}\\ -\alpha_0[\beta_0\ \cdots\ \beta_{n-2}\ \ 0]\Delta_\tau^{n-1}\end{bmatrix}\quad A_{33} = \left[\begin{array}{c|c}0_{n-1} & I_{n-1}\\ \hline \multicolumn{2}{c}{-\alpha_<^\top + \alpha_0 C_g}\end{array}\right],\quad \tilde{A}_{33} = \begin{bmatrix}(\beta_{n-1} + a_{n-1})C_g\\ 0_{(n-1)\times n}\end{bmatrix}, \tag{8}$$

$$\tilde{F}_{21} = \begin{bmatrix}0_{(n-1)\times n}\\ -\beta^\top\end{bmatrix},\quad \tilde{F}_{22} = \begin{bmatrix}0_{(n-1)\times n}\\ -\beta^\top\Delta_\tau^{n-1}\end{bmatrix},\quad \tilde{F}_{23} = \begin{bmatrix}0_{(n-1)\times n}\\ C_g\end{bmatrix},\quad \tilde{F}_{31} = \begin{bmatrix}\beta_\flat^\top - \beta_{n-1}\beta^\top\\ 0_{(n-1)\times n}\end{bmatrix},\quad F_{32} = \left[\begin{array}{c|c}\multicolumn{2}{c}{0_{(n-1)\times n}}\\ \hline 0_{n-1}^\top & -\alpha_0\beta_{n-1}\end{array}\right]$$

$$\tilde{F}_{32} = \begin{bmatrix}-\beta_{n-1}\beta^\top\Delta_\tau^{n-1}\\ 0_{(n-2)\times n}\\ -\alpha_0[\beta_0\ \cdots\ \beta_{n-2}\ \ 0]\Delta_\tau^{n-1}\end{bmatrix},\quad F_{33} = \begin{bmatrix}0_{(n-1)\times n}\\ \alpha_0 C_g\end{bmatrix},\quad \tilde{F}_{33} = \begin{bmatrix}\beta_{n-1}C_g\\ 0_{(n-1)\times n}\end{bmatrix}.$$

$$0=\sum_{k=1}^{N} l_{ik}^{\star}\left(\sum_{j=1}^{n}\left(\beta_{j-1}+a_{j-1}\right)\tilde{x}_j^k +\bar{\eta}_1^k-\sum_{j=1}^{n}\beta_{j-1}\sum_{m=1}^{N} l_{km}^{\star}\tilde{x}_j^m-\bar{u}\right). \tag{11}$$

We stack (11) for all $i=1,\cdots,N$ to have

$$0_N= L^{\star}\begin{bmatrix}\sum_{j=1}^{n}(\beta_{j-1}+a_{j-1})\tilde{x}_j^1+\bar{\eta}_1^1-\sum_{j=1}^{n}\beta_{j-1}\sum_{k=1}^{N} l_{1k}^{\star}\tilde{x}_j^k-\bar{u}\\ \vdots \\ \sum_{j=1}^{n}(\beta_{j-1}+a_{j-1})\tilde{x}_j^N+\bar{\eta}_1^N-\sum_{j=1}^{n}\beta_{j-1}\sum_{k=1}^{N} l_{Nk}^{\star}\tilde{x}_j^k-\bar{u}\end{bmatrix}$$

from which the relation (10) follows since $L^{\star}$ is a positive definite matrix by Assumption 2.

Now, we are ready to write the whole closed-loop system. Let U be an orthogonal matrix (i.e., $UU^{\top}=I$) such that $UL^{\star}U^{\top}=\mathrm{diag}\{\lambda_1(L^{\star}),\cdots,\lambda_N(L^{\star})\}=:\Lambda$, $\tilde{\eta}_j^i=\eta_j^i-\bar{\eta}_j^i$, and $\chi^i=[\tilde{x}_1^i,\cdots,\tilde{x}_n^i,\xi_1^i,\cdots,\xi_n^i,\tilde{\eta}_1^i,\cdots,\tilde{\eta}_n^i]^{\top}$. Define $\chi=[(\chi^1)^{\top},\cdots,(\chi^N)^{\top}]^{\top}\in\mathbb{R}^{3nN}$ and

$$\zeta^i=(U_i\otimes I_{3n})\chi, \tag{12}$$

where U_i is the i th row of U. Note that we do not define $\tilde{\xi}_j^i$ since $\bar{\xi}_j^i=0$.

Lemma 2: With ζ^i defined in (12), the whole closed-loop system is rewritten as

$$\dot{\zeta}=\left[I_N\otimes\bar{A}-\Lambda\otimes\bar{F}\right]\zeta+B_{\zeta}\dot{\bar{u}}, \tag{13}$$

where $\zeta=[(\zeta^1)^{\top},\cdots,(\zeta^N)^{\top}]^{\top}\in R^{3nN}$,

$$\bar{A}=\begin{bmatrix}A_{11} & \tilde{A}_{12} & \tilde{A}_{13}\\ 0_{n\times n} & \frac{1}{\tau}A_{22} & 0_{n\times n}\\ \tilde{A}_{31} & \frac{1}{\tau}A_{32}+\tilde{A}_{32} & \frac{1}{\tau}A_{33}+\tilde{A}_{33}\end{bmatrix}\in\mathbb{R}^{3n\times 3n},$$

$$\bar{F}=\begin{bmatrix}0_{n\times n} & 0_{n\times n} & 0_{n\times n}\\ \tilde{F}_{21} & \tilde{F}_{22} & \tilde{F}_{23}\\ \tilde{F}_{31} & \frac{1}{\tau}F_{32}+\tilde{F}_{32} & \frac{1}{\tau}F_{33}+\tilde{F}_{33}\end{bmatrix}\in\mathbb{R}^{3n\times 3n},$$

$$B_{\zeta}=-(U\otimes I_{3n})\left(1_N\otimes[0_{2n}^{\top},1,0_{n-1}^{\top}]^{\top}\right)\in R^{3nN}$$

and other matrices and vectors are defined in (8).

Proof: At first, we compute the dynamics of χ^i as

$$\dot{\tilde{x}}_j^i=\tilde{x}_{j+1}^i,\quad j=1,\cdots,n-1,$$

$$\dot{\tilde{x}}_n^i=\tilde{\eta}_1^i-\sum_{j=1}^{n}\beta_{j-1}\left(\tau^{n-j}\xi_j^i+\tilde{x}_j^i\right),$$

$$\dot{\xi}_j^i=-\frac{\alpha_{n-j}}{\tau}\xi_1^i+\frac{1}{\tau}\xi_{j+1}^i,\quad j=1,\cdots,n-1,$$

$$\dot{\xi}_n^i=-\frac{\alpha_0}{\tau}\xi_1^i-\sum_{k=1}^{N} l_{ik}^{\star}\left(\tilde{\eta}_1^k-\sum_{j=1}^{n}\beta_{j-1}\left(\tau^{n-j}\xi_j^k+\tilde{x}_j^k\right)\right),$$

$$\begin{aligned}\dot{\tilde{\eta}}_1^i&=-(\beta_{n-1}+a_{n-1})\beta_0\tilde{x}_1^i\\&\quad+\sum_{j=1}^{n-1}\left(\beta_{j-1}+a_{j-1}-(\beta_{n-1}+a_{n-1})\beta_j\right)\tilde{x}_{j+1}^i\\&\quad-(\beta_{n-1}+a_{n-1})\sum_{j=1}^{n}\beta_{j-1}\tau^{n-j}\xi_j^i\\&\quad+(\beta_{n-1}+a_{n-1})\tilde{\eta}_1^i+\frac{1}{\tau}\tilde{\eta}_2^i-\dot{\bar{u}}\\&\quad-\sum_{k=1}^{N} l_{ik}^{\star}\left(-\beta_{n-1}\beta_0\tilde{x}_1^k+\sum_{j=1}^{n-1}(\beta_{j-1}-\beta_{n-1}\beta_j)\tilde{x}_{j+1}^k\right.\\&\quad\left.-\beta_{n-1}\sum_{j=1}^{n}\beta_{j-1}\tau^{n-j}\xi_j^k+\beta_{n-1}\tilde{\eta}_1^k\right),\end{aligned}$$

$$\dot{\tilde{\eta}}_j^i=\frac{1}{\tau}\tilde{\eta}_{j+1}^i,\quad j=2,\cdots,n-1,$$

$$\begin{aligned}\dot{\tilde{\eta}}_n^i&=-\frac{1}{\tau}\sum_{j=2}^{n}\alpha_{j-1}\tilde{\eta}_j^i-\frac{\alpha_0}{\tau}\left(\sum_{j=1}^{n}\beta_{j-1}\tau^{n-j}\xi_j^i\right)\\&\quad-\frac{\alpha_0}{\tau}\sum_{k=1}^{N} l_{ik}^{\star}\left(\tilde{\eta}_1^k-\sum_{j=1}^{n}\beta_{j-1}\tau^{n-j}\xi_j^k\right),\end{aligned}$$

$$u^i=\tilde{\eta}_1^i-\sum_{j=1}^{n}(\beta_{j-1}+a_{j-1})\tilde{x}_j^i-\sum_{j=1}^{n}\beta_{j-1}\tau^{n-j}\xi_j^i+\bar{u},$$

which can be written compactly as

$$\dot{\chi}=\left[I_N\otimes\bar{A}-L^{\star}\otimes\bar{F}\right]\chi+B_{\chi}\dot{\bar{u}}, \tag{14}$$

where $B_{\chi}=-1_N\otimes[0_{2n}^{\top},1,0_{n-1}^{\top}]^{\top}$. Applying (12), we have

$$\dot{\zeta}=\left[I_N\otimes\bar{A}-\Lambda\otimes\bar{F}\right]\zeta+B_{\zeta}\dot{\bar{u}},$$

which completes the proof.

To proceed, we present the properties guaranteed by the controller parameters obtained from the controller design procedure.

Lemma 3: Let $\alpha_0,\ldots,\alpha_{n-1}$ be the constants chosen from **Controller Design Procedure**, $G_c(s)=C_g(sI-A_{33})^{-1}B_g=1/[s(s^{n-1}+\alpha_{n-1}s^{n-2}+\cdots+\alpha_1)]$, and $A_{\tilde{\eta}}=A_{33}-\lambda^{-}F_{33}$ (A_{33} and F_{33} defined in (8)). Then, the following properties hold true.

1) For each $\lambda\in[\lambda^{-},\lambda^{+}]$, the polynomial $s^n+\alpha_{n-1}s^{n-1}+\cdots+\alpha_1 s+\lambda\alpha_0$ is Hurwitz.
2) The transfer function $Z_T(s)=1+(\lambda^{+}-\lambda^{-})G_c(s)(1+\lambda^{-}G_c(s))^{-1}$ is strictly positive real.
3) There exists a symmetric positive definite matrix $P_{\tilde{\eta}}\in\mathbb{R}^{n\times n}$, a row vector $M\in\mathbb{R}^{1\times n}$, a number $W\in\mathbb{R}$, and a positive scalar δ such that

$$P_{\tilde\eta}A_{\tilde\eta}+A_{\tilde\eta}^\top P_{\tilde\eta}=-M^\top M-\delta P_{\tilde\eta}$$
$$P_{\tilde\eta}B_g\alpha_0=(\lambda^+-\lambda^-)C_g^\top-M^\top W$$
$$W^\top W=2.$$

Proof: We firstly note that $s^{n-1}+\alpha_{n-1}s^{n-2}+\cdots+\alpha_1$ is Hurwitz (coefficients $\alpha_{n-1},\cdots,\alpha_1$ are obtained from the second step in **Controller Design Procedure**). With these coefficients and α_0 which is also chosen from the procedure, the assertions are proved as follows. From the circle criterion (see, e.g., [17, Theorem 7.2]), the fourth step in the controller design procedure guarantees that the transfer function $G_c(s)(1+\lambda G_c(s))^{-1}$, whose denominator is $s^n+\alpha_{n-1}s^{n-1}+\cdots+\alpha_1 s+\lambda\alpha_0$, is asymptotically stable for any $\lambda\in[\lambda^-,\lambda^+]$, and that transfer function

$$\begin{aligned}Z_T(s)&=(1+\lambda^+G_c(s))(1+\lambda^-G_c(s))^{-1}\\&=1+(\lambda^+-\lambda^-)G_c(s)(1+\lambda^-G_c(s))^{-1}\end{aligned}$$

is strictly positive real. This means that properties 1) and 2) hold true.

The property 3) follows from Kalman-Yakubovich-Popov Lemma (see, e.g., [17, Lemma 6.3]) since the transfer function $Z_T(s)$ admits a minimal realization $(A_{\tilde\eta},B_g\alpha_0,(\lambda^+-\lambda^-)C_g,1)$, i.e., $Z_T(s)=1+(\lambda^+-\lambda^-)C_g(sI-A_{\tilde\eta})^{-1}B_g\alpha_0$. This completes the proof.

Now, we state the main result of this paper.

Theorem 1: For the multi-agent system with agents described by (1) and (2), suppose that Assumptions 1 and 2 hold true. For any given $\varepsilon>0$, there exists $0<\tau^*<1$ such that for any $0<\tau<\tau^*$, the distributed controller (5) solves the practical coordinated tracking problem, i.e., $\limsup_{t\to\infty}\|x^i(t)-\bar x(t)\|\le\varepsilon$, $i=1,\ldots,N$, and all signals in the closed-loop systems are bounded.

Proof: We start the proof by investigating the stability of (13). Since the dynamics (13) is decoupled, it is sufficient to consider the dynamics for ζ^i. Towards this, let $\zeta_1^i,\zeta_2^i,\zeta_3^i\in\mathbb{R}^n$ be such that $\zeta^i=[(\zeta_1^i)^\top,(\zeta_2^i)^\top,(\zeta^i)_3^\top]^\top$ and define $\theta_1=\zeta_1^i$, $\theta_2=\zeta_2^i$, $\theta_3=\zeta_3^i$ to simplify notation. Consider the Lyapunov function candidate defined by

$$V(t)=\theta^\top P\theta:=\theta_1^\top P_1\theta_1+\gamma\theta_2^\top P_2\theta_2+\theta_3^\top P_3\theta_3, \tag{15}$$

where $\theta=[\theta_1^\top\ \theta_2^\top\ \theta_3^\top]^\top$, $P=\mathrm{diag}\{P_1,\gamma P_2,P_3\}$, and P_1, P_2, and P_3 are symmetric positive definite matrices such that

$$\begin{aligned}&P_1A_{11}+A_{11}^\top P_1=-I_n,\\&P_2A_{22}+A_{22}^\top P_2=-I_n,\\&P_3A_{\tilde\eta}+A_{\tilde\eta}^\top P_3=-M^\top M-\delta P_3,\\&P_3B_g\alpha_0=(\lambda^+-\lambda^-)C_g^\top-M^\top W,\\&W^\top W=2.\end{aligned}\tag{16}$$

Note that the existence of P_1 and P_2 follows from the stability of A_{11} and A_{22}, and Lemma 3 guarantees the existence of P_3, M, W, and δ. The constant γ is defined by $\gamma=1+\frac{2}{\delta}\bar\lambda$ where $\bar\lambda=\max_{\lambda_i\in[\lambda^-,\lambda^+]}\lambda_{\max}((A_{32}-\lambda_iF_{32})^\top P_3(A_{32}-\lambda_iF_{32}))$.

We compute the time derivative of $V(t)$ as

$$\begin{aligned}\dot V(t)=&-\theta_1^\top\theta_1-\frac{\gamma}{\tau}\theta_2^\top\theta_2-\frac{1}{\tau}\theta_3^\top(M^\top M+\delta P_3)\theta_3\\&+2\theta_1^\top P_1\tilde A_{12}\theta_2+2\theta_1^\top P_1\tilde A_{13}\theta_3-2\gamma\lambda_i\theta_2^\top P_2\tilde F_{21}\theta_1\\&-2\gamma\lambda_i\theta_2^\top P_2\tilde F_{22}\theta_2-2\gamma\lambda_i\theta_2^\top P_2\tilde F_{23}\theta_3\\&+2\theta_3^\top P_3(\tilde A_{31}-\lambda_i\tilde F_{31})\theta_1+\frac{2}{\tau}\theta_3^\top P_3(A_{32}-\lambda_iF_{32})\theta_2\\&+2\theta_3^\top P_3(\tilde A_{32}-\lambda_i\tilde F_{32})\theta_2-\frac{2}{\tau}\theta_3^\top P_3(\lambda_i-\lambda^-)F_{33}\theta_3\\&+2\theta_3^\top P_3(\tilde A_{33}-\lambda_i\tilde F_{33})\theta_3-2\theta_3^\top P_3U_i1_NC_g^\top\dot{\bar u}.\end{aligned}$$

Applying Young's inequality and arranging terms, we obtain

$$\begin{aligned}\dot V(t)\le&-\frac{1}{2}\theta_1^\top\theta_1-\frac{\gamma}{\tau}\theta_2^\top\theta_2-\frac{\delta}{\tau}\theta_3^\top P_3\theta_3\\&\underline{-\frac{1}{\tau}\theta_3^\top M^\top M\theta_3-\frac{2}{\tau}\theta_3^\top P_3(\lambda_i-\lambda^-)F_{33}\theta_3}\\&+\theta_2^\top Q_2(\lambda_i,\tau)\theta_2+\theta_3^\top Q_3(\lambda_i,\tau)\theta_3\\&+\frac{\delta}{2\tau}\theta_3^\top P_3\theta_3+\frac{2}{\tau\delta}\theta_2^\top(A_{32}-\lambda_iF_{32})^\top P_3(A_{32}\\&-\lambda_iF_{32})\theta_2+\mu^*\left(\dot{\bar u}^+\right)^2,\end{aligned}$$

where μ^* is a positive constant defined shortly. Also, $Q_2(\lambda_i,\tau)\in\mathbb{R}^{n\times n}$ and $Q_3(\lambda_i,\tau)\in\mathbb{R}^{n\times n}$ are given by

$$\begin{aligned}Q_2(\lambda_i,\tau)=&8\tilde A_{12}^\top P_1^2\tilde A_{12}+8\gamma^2\lambda_i^2P_2\tilde F_{21}\tilde F_{21}^\top P_2\\&+\gamma^2\lambda_i^2P_2^2+\tilde F_{22}^\top\tilde F_{22}+\gamma^2\lambda_i^2P_2\tilde F_{23}\tilde F_{23}^\top P_2\\&+(\tilde A_{32}-\lambda_i\tilde F_{32})^\top(\tilde A_{32}-\lambda_i\tilde F_{32}),\\Q_3(\lambda_i,\tau)=&8\tilde A_{13}^\top P_1^2\tilde A_{13}+I_n\\&+8P_3(\tilde A_{31}-\lambda_i\tilde F_{31})(\tilde A_{31}-\lambda_i\tilde F_{31})^\top P_3\\&+2P_3^2+(\tilde A_{33}-\lambda_i\tilde F_{33})^\top(\tilde A_{33}-\lambda_i\tilde F_{33})\\&+\frac{(N+1)^2}{4\mu^*}P_3C_g^\top C_gP_3.\end{aligned}$$

Note that the forth line of the bound for $\dot V(t)$ comes from

$$\begin{aligned}&\frac{2}{\tau}\theta_3^\top P_3^{\frac12}P_3^{\frac12}(A_{32}-\lambda_iF_{32})\theta_2\\&\le\frac{\delta}{2\tau}\theta_3^\top P_3\theta_3+\frac{2}{\tau\delta}\theta_2^\top(A_{32}-\lambda_iF_{32})^\top P_3(A_{32}-\lambda_iF_{32})\theta_2.\end{aligned}$$

We define

$$\bar Q_2=\max_{0<\tau\le1,\lambda^-\le\lambda_i\le\lambda^+}\|Q_2(\lambda_i,\tau)\|,$$
$$\bar Q_3=\max_{0<\tau\le1,\lambda^-\le\lambda_i\le\lambda^+}\|Q_3(\lambda_i,\tau)\|.$$

Moreover, by applying (16) and taking $W=\sqrt{2}$, we obtain a bound for the underlined part in $\dot{V}(t)$ as

$$\begin{aligned}
&-\frac{1}{\tau}\theta_3^\top M^\top M\theta_3-\frac{2}{\tau}\theta_3^\top P_3(\lambda_i-\lambda^-)F_{33}\theta_3\\
&\le -\frac{1}{\tau}\theta_3^\top M^\top M\theta_3-\frac{2}{\tau}\theta_3^\top P_3(\lambda_i-\lambda^-)B_g\alpha_0 C_g\theta_3\\
&\quad -\frac{2}{\tau}\theta_3^\top C_g^\top(\lambda_i-\lambda^-)(\lambda_i-\lambda^+)C_g\theta_3\\
&= -\frac{1}{\tau}\theta_3^\top M^\top M\theta_3\\
&\quad -\frac{2}{\tau}\theta_3^\top(\lambda_i-\lambda^-)((\lambda^+-\lambda^-)C_g^\top-M^\top W)C_g\theta_3\\
&\quad -\frac{2}{\tau}\theta_3^\top C_g^\top(\lambda_i-\lambda^-)(\lambda_i-\lambda^+)C_g\theta_3\\
&= -\frac{1}{\tau}\theta_3^\top\left(M-(\lambda_i-\lambda^-)WC_g\right)^\top\left(M-(\lambda_i-\lambda^-)WC_g\right)\theta_3\\
&\le 0.
\end{aligned}$$

Now, from the definition of γ and the bounds obtained above, it follows that

$$\begin{aligned}
\dot{V}(t) &\le -\frac{1}{2}\theta_1^\top\theta_1-\frac{1}{\tau}\theta_2^\top\theta_2-\frac{\delta}{2\tau}\theta_3^\top P_3\theta_3\\
&\quad +\bar{Q}_2\theta_2^\top\theta_2+\bar{Q}_3\theta_3^\top\theta_3+\mu^*(\dot{\bar{u}}^+)^2\\
&\le -\frac{1}{2}\theta_1^\top\theta_1-\left(\frac{1}{\tau}-\bar{Q}_2\right)\theta_2^\top\theta_2\\
&\quad -\left(\frac{\delta\lambda_{\min}(P_3)}{2\tau}-\bar{Q}_3\right)\theta_3^\top\theta_3+\mu^*(\dot{\bar{u}}^+)^2.
\end{aligned}$$

If we take $\mu^*=\frac{\lambda_{\min}(P)\varepsilon^2}{2\lambda_{\max}(P)(\dot{\bar{u}}^+)^2N^2}$, then there exist $\bar{\tau}_1>0$ and $\bar{\tau}_2>0$ such that

$$\frac{1}{\bar{\tau}_1}-\bar{Q}_2\ge\frac{1}{2},\quad \frac{\delta\lambda_{\min}(P_3)}{2\bar{\tau}_2}-\bar{Q}_3\ge\frac{1}{2}.$$

Therefore, it follows that for any given $\varepsilon>0$, if $\tau<\tau^*:=\min\{\bar{\tau}_1,\bar{\tau}_2\}$, then

$$\dot{V}(t)\le-\rho V+\mu^*(\dot{\bar{u}}^+)^2,\quad \rho:=\frac{1}{2\lambda_{\max}(P)}. \tag{17}$$

From the comparison lemma [17], we have

$$\begin{aligned}
V(t) &\le e^{-\rho t}V(0)+\int_0^t e^{-\rho(t-\sigma)}\mu^*(\dot{\bar{u}}^+)^2 d\sigma\\
&= e^{-\rho t}V(0)+\frac{1}{\rho}\mu^*(\dot{\bar{u}}^+)^2(1-e^{-\rho t}),
\end{aligned}$$

from which we have $\limsup_{t\to\infty}V(t)\le\frac{1}{\rho}\mu^*(\dot{\bar{u}}^+)^2$, and $\limsup_{t\to\infty}\theta^\top P\theta\le\frac{1}{\rho}\mu^*(\dot{\bar{u}}^+)^2$. Hence, from the definition of μ^*, it follows that $\limsup_{t\to\infty}\|\zeta^i\|\le\frac{\varepsilon}{N}$ and that $\limsup_{t\to\infty}\|\zeta\|\le\varepsilon$. Since

$$\begin{aligned}
\limsup_{t\to\infty}\|\chi\| &= \limsup_{t\to\infty}\|(U\otimes I_{3n})^{-1}(U\otimes I_{3n})\chi\|\\
&\le\|(U\otimes I_{3n})^{-1}\|\cdot\limsup_{t\to\infty}\|\zeta\|,
\end{aligned}$$

we conclude that $\limsup_{t\to\infty}\|\chi^i\|\le\varepsilon$ and that $\limsup_{t\to\infty}\|x^i-\bar{x}\|=\limsup_{t\to\infty}\|\tilde{x}^i\|\le\varepsilon$, which completes the proof.

4. SIMULATION

We consider five followers and one leader which are third order systems given by (1) with $a=[1,2,-1]^\top$. The network topology among agents is shown in Fig. 1 whose Laplacian matrix is given by

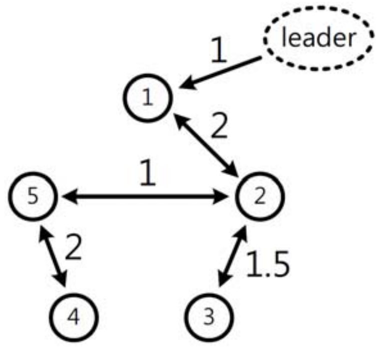

Fig. 1. Network topology used in simulation.

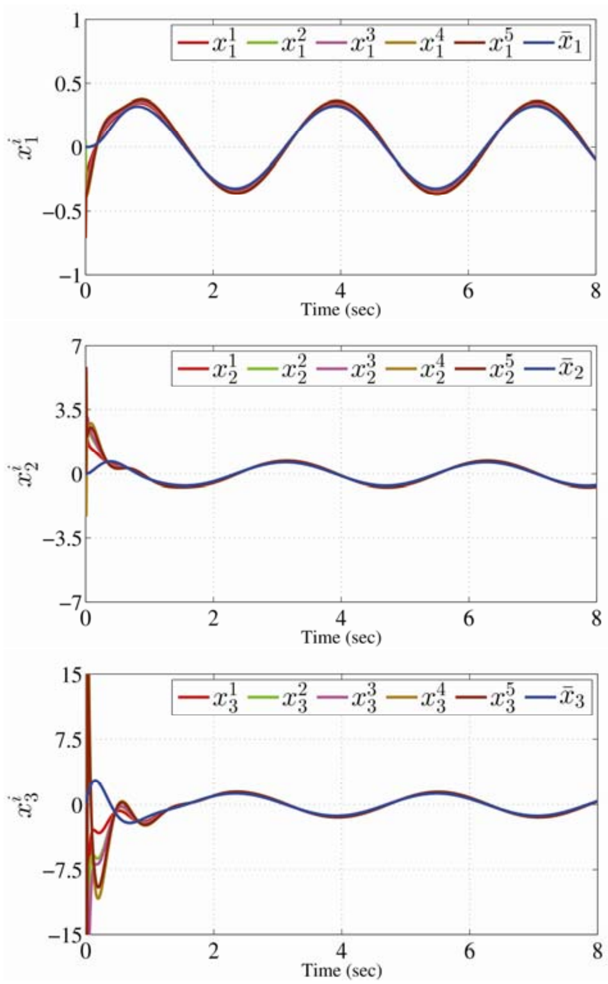

Fig. 2. Practical coordinated tracking of five followers under the proposed control.

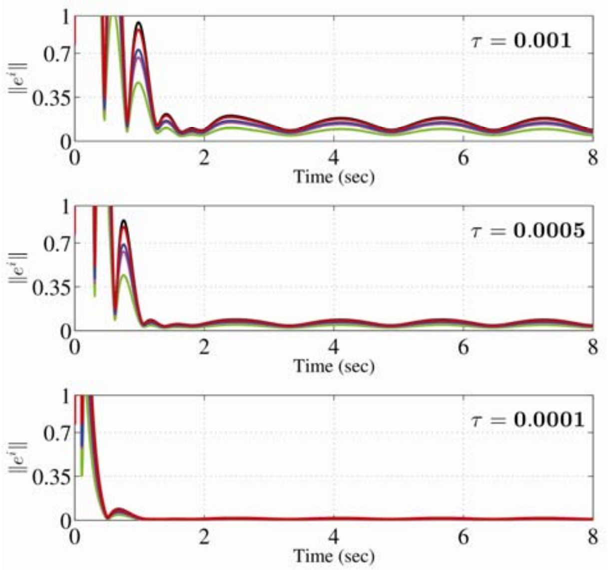

Fig. 3. Tracking error $\| e^i \|$ for three cases. Steady state tracking error becomes smaller as τ decreases.

$$L = \begin{bmatrix} 3 & -2 & 0 & 0 & 0 & -1 \\ -2 & 4.5 & -1.5 & 0 & -1 & 0 \\ 0 & -1.5 & 1.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & -2 & 0 \\ 0 & -1 & 0 & -2 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The leader's dynamics is given by

$$\begin{aligned} \dot{\bar{x}} &= A_g \bar{x} + B_g \bar{u}, \\ \bar{u} &= K[r(t) - \bar{x}_1, \dot{r}(t) - \bar{x}_2, \ddot{r}(t) - \bar{x}_3]^\top \end{aligned} \tag{18}$$

in which $r(t) = 0.3 \sin 2t$ and the gain matrix K is selected such that eigenvalues of $A_g - B_g K$ are located at $-10/3 \pm j10\sqrt{3}/3$ and $-10/3$. Controller parameters are chosen as $\alpha = [2, 2, 0.1]^\top$, $\beta = [1, 5, 10]^\top$, and $\tau = 0.001$ along **Controller Design Procedure** given in Section 3. The initial state of all followers are randomly selected but that of leader is set to be zero. Fig. 2 illustrates that all states of agents converge to the vicinity of leader's trajectory. To illustrate that the practical coordinated tracking is achieved by the proposed controller, we present Fig. 3 for which the simulation is conducted for different values of τ, while all parameters and initial states are the same. From the result, one can deduce that ultimate bound of the tracking error $(\| e^i \| = \| x^i - \bar{x} \|)$ becomes smaller as τ decreases, which indicates that the desired steady state error bound ε can be obtained by adjusting $\tau < \tau^*$.

5. CONCLUSIONS

This paper investigated the distributed coordinated tracking problem for a group of high-order linear systems. As a solution, we have proposed a distributed coordinated tracking controller which is based on disturbance observer. The controller requires only the weighted sum of its neighbor's output, thus the amount of information exchange does not depend on the system order. Future research topics include extensions to the case with switching network and nonlinear agents.

REFERENCES

[1] R. Olfati-Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems," *IEEE Trans. on Automatic Control*, vol. 54, no. 7, pp. 1601-1607, 2009.

[2] W. Ren, R. W. Beard, and E. M. Atkins, "Information consensus in multivehicle cooperative control," *IEEE Control Systems Magazine*, vol. 27, no. 2, pp. 71-82, 2007.

[3] W. Ren and R. W. Beard, *Distributed Consensus in Multi-vehicle Cooperative Control: Theory and Applications*, Springer, 2010.

[4] N. E. Leonard, D. A. Paley, R. Sepulchre, M. Fratantoni, and R. E. Davis, "Collective motion, sensor networks, and ocean sampling," *Proc. of the IEEE*, vol. 95, no. 1, pp. 48-74, 2007.

[5] Y. Cao, W. Yu, W. Ren, and G. Chen, "An overview of recent progress in the study of distributed multi-agent coordination," *IEEE Trans. on Industrial Informatics*, vol. 9, no. 1, pp. 427-438, 2013.

[6] Y. Hong, J. Hu, and L. Gao, "Tracking control for multi-agent consensus with an active leader and variable topology," *Automatica*, vol. 42, no. 7, pp. 1177-1182, 2006.

[7] Y. Hong, G. Chen, and L. Bushnell, "Distributed observers design for leader-following control of multi-agent networks," *Automatica*, vol. 44, no. 3, pp. 846-850, 2008.

[8] W. Ren, "Consensus tracking under directed interaction topologies: algorithms and experiments," *IEEE Trans. Control Systems Technology*, vol. 18, no. 1, pp. 230-237, 2010.

[9] Y. Cao and W. Ren, "Distributed coordinated tracking with reduced interaction via a variable structure approach," *IEEE Trans. on Automatic Control*, vol. 57, no. 1, pp. 33-48, 2012.

[10] J. Back, K. Ryu, and S. Kim, "An output feedback controller for practical distributed coordinated tracking of double integrators," *Proc. Int. Conf. on Control, Automation and Systems*, pp. 1348-1351, 2012.

[11] K. Ohnishi, "A new servo method in mechatronics," *Trans. of Japanese Society of Electrical Engineers* (in Japanese), vol. 107-D, pp. 83-86, 1987.

[12] T. Umeno and Y. Hori, "Robust servosystem design with two degrees of freedom and its application to novel motion control of robot manipulators," *IEEE Trans. Industrial Electronics*, vol. 40, no. 5, pp. 473-485, 1993.

[13] Y. Choi, K. Yang, W. K. Chung, H. R. Kim, and I. H. Suh, "On the robustness and performance of disturbance observers for second-order systems," *IEEE Trans. on Automatic Control*, vol. 48, no. 2, pp. 315-320, 2003.

[14] H. Shim and Y. Joo, "State space analysis of disturbance observer and a robust stability condition," *Proc. of the IEEE Conference on Decision and Control*, pp. 2193-2198, 2007.
[15] J. Back and H. Shim, "Adding robustness to nominal output feedback controllers for uncertain nonlinear systems: a nonlinear version of disturbance observer," *Automatica*, vol. 44, no. 10, pp. 2528-2537, 2008.
[16] J. Back and H. Shim, "An inner-loop robust transient performance for uncertain MIMO nonlinear systems," *IEEE Trans. on Automatic Control*, vol. 54, no. 7, pp. 1601-1607, 2009.
[17] H. Khalil, *Nonlinear Systems*, 3rd ed., Prentice Hall, New Jersey, 2002.

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