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Distributed Adaptive Control of Pinning Synchronization in Complex Dynamical Networks with Non-delayed and Delayed Coupling

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Abstract: This paper discusses the distributed adaptive control schemes for pinning synchronization in complex dynamical network with non-delayed and delayed coupling. An effective distributed adaptive strategy to adjust simultaneously coupling strength and feedback gains is designed based on information of the non-delayed network's configuration. For a special case where the information of delay is available, a distributed adaptive scheme to tune the coupling weights of non-delayed coupling network is proposed by using the delayed feedback controllers. Based on the small-world network and scale-free network, simulation examples are given to demonstrate the effectiveness of theoretical analysis.

Keywords: Complex dynamical network, delayed feedback controller, distributed adaptive update law, Non-delayed and delayed coupling, pinning synchronization.

1. INTRODUCTION

Due to the remarkable contribution of Watts, Barabási et al. on random networks, the small-world [1] and scale-free [2] network models have been applied extensively to study more realistic complex networks. Actually, complex networks are ubiquitous in our daily life, such as the Internet, WWW, electric power grids, neural networks and scientific citation networks [3], etc. Some significant concepts describing the dynamical behaviors of complex network are proposed endlessly and investigated deeply on variety of science and engineering fields in the past few decades. Synchronization, which is one of numerous dynamical behaviors for complex networks, has attracted more and more attentions from researchers in different discipline. Since the ground-breaking work [4] of Pecora and Carroll was published, a great deal of research results on chaos synchronization and control have been reported [5-12]. Many kinds of synchronization protocols have been proposed, such as complete synchronization [13], projective synchronization [14], generalized synchronization [15], lag synchronization [16,17] and so on. In order to synchronize a dynamical complex network, introducing some control schemes have become common views. However, it is literally impossible to add controllers to every node in very large-scale network because of the computational complexity and cost effectiveness. To reduce the number of controllers, an idea using some local feedback injections was applied successfully to a part of network's nodes, which is well known as pinning control [10,18-23].

Time-delayed coupling is omnipresent in nature and cannot be particularly ignored in long distance communication and traffic congestions. Hence, to simulate more realistic network, time-delay must be taken into account. Most of reports on synchronization of dynamical networks are assumed that there exists the information communication of node only at time t [10,11] or at time $t-\tau$ [24,25], respectively. In [26,27], authors investigated the delay-dependent synchronization criteria for the fuzzy complex dynamical networks and coupled discrete-time neural networks with interval time-varying delays, respectively. Even so, how to realize synchronization between different networks is still challenging and open work. However, the simplification does not satisfy the characteristics of real world, i.e. there is the information exchange of nodes not only at time t but also at time $t-\tau$. Recently, a general complex dynamical network with non-delayed and delayed coupling, which describes dynamical behaviors between two complex networks, was proposed and some results had been reported [28-32].

The above-mentioned studies on complex dynamical networks with non-delayed and delayed coupling focused mainly on how to realize synchronization. In this paper, our motivations are to propose some control strategies to tune parameters of network by using pinning synchronization of network. Here, we should be careful of the influence of delayed coupling which is absolutely ignored. In the following, the affect of delayed coupling is illustrated. Furthermore, how to use synchronization to

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tune parameters of networks, to the best of our knowledge, there are few results concerning it for networks with non-delayed and delayed coupling. Recently, in [33,34], the distributed adaptive control gains and coupling weights were proposed and their adaptive update laws were worth to use for reference. Differing from [33], we draw the delayed coupling term into the network in this paper. On the other hand, it is significant to consider the cost of coupling strength and feedback gains in a real network. Generally, they are hoped to be smallest possible. To achieve synchronization, the bigger feedback gains are required if the coupling strength is too small. How to achieve their balance is challenging question. In this paper, the adaptive technique is used to solve this question. The coupling strength and feedback gains are tuned simultaneously to achieve their balance by designed adaptive laws. Besides, we also design adaptive tune of coupling weights by delayed feedback controllers. The main contributions of this paper are twofold: (i) a distributed adaptive non-delayed coupling strength and its update law is designed under adaptive pinning scheme; (ii) using fully the information on size of delay, the distributed adaptive non-delayed coupling weights and their update laws are proposed under delayed pinning scheme. The simulations indicate the effectiveness on our distributed adaptive control designs.

The rest of this paper is organized as follows: In Section 2, some preliminaries, lemmas and assumption are briefly outlined. The main theorems for pinning synchronization on complex dynamical networks with non-delayed and delayed coupling are given in Section 3, 4. In Section 5, two numerical examples on small-world and scale-free networks are simulated to demonstrate the theoretical analysis. Conclusions are drawn in Section 6.

Notation: Throughout this paper, the following standard notations are used. $\mathbb{R}^N$ denotes the N-dimensional Euclidean space. $\mathbb{R}^{N\times N}$ be $N\times N$ real matrices, $I_N \in \mathbb{R}^{N\times N}$ be an N-dimensional identity matrix. A^{T} and A^{-1} be the transpose and inverse of matrix A, respectively. For real symmetric matrices X and Y, the notation $X<Y$ $(X\le Y)$ means $X-Y$ is negative definite (respectively, negative semi-definite). $\lambda_{\max}(\cdot)$ denotes the maximum eigenvalue of a real symmetric matrix. $\mathrm{diag}(\cdots)$ stands for a block-diagonal matrix, $\otimes$ denotes the Kronecker product. The matrices are assumed to have compatible dimensions if they are not explicitly stated.

2. MODEL DESCRIPTION AND PRELIMINARIES

In this paper, we consider a complex dynamical network consisting of N identical codes with linearly diffusive coupling. The model with non-delayed and delayed coupling is described by

$$\dot{x}_i(t) = f(x_i(t),t) + c_1\sum_{j=1}^{N} a_{ij}\Gamma x_j(t) + c_2\sum_{j=1}^{N} b_{ij}\Gamma x_j(t-\tau),\quad i=1,2,\cdots,N, \tag{1}$$

where $x_i(t)=(x_{i1}(t),x_{i2}(t),\cdots,x_{in}(t))^{\mathrm{T}}\in\mathbb{R}^n$, is the state vector of the ith node, $f:\mathbb{R}^n\times\mathbb{R}^+\to\mathbb{R}^n$ is a continuously differentiable vector function standing for the activity of an individual subsystem; $\tau>0$ is the coupling delay; $c_1>0,\ c_2>0$ are the coupling strengths; $\Gamma = \mathrm{diag}(\gamma_1,\gamma_2,\cdots,\gamma_n)\in\mathbb{R}^{n\times n}$ is a positive definite diagonal inner coupling matrix in which $\gamma_j>0$ if and only if two nodes can communicate through the jth state, otherwise $\gamma_j=0$; $A=(a_{ij})_{N\times N}$ and $B=(b_{ij})_{N\times N}$ are the coupling configuration matrices representing the coupling weights and topological structure for non-delayed configuration and delayed one, where a_{ij} and b_{ij} are defined as follow: if there exits a connection between node i and node j $(i\ne j)$, then $a_{ij}=a_{ji}>0,\ b_{ij}=b_{ji}>0$; otherwise $a_{ij}=a_{ji}=0,\ b_{ij}=b_{ji}=0$; and the diagonal elements are defined by

$$a_{ii} = -\sum_{j=1,j\ne i}^{N} a_{ij},\quad b_{ii} = -\sum_{j=1,j\ne i}^{N} b_{ij},$$

where $i,j=1,2,\cdots,N.$

Suppose that $C([-\tau,0],\mathbb{R}^n)$ be the Banach space of continuous functions with the norm

$$\|\phi\| = \sup_{-\tau\le t\le 0}\|\phi(t)\|.$$

The initial conditions of functional differential equations (1) are given by $x_i(t)=\phi_i(t)\in C([-\tau,0],\mathbb{R}^n)$. Under the initial conditions, we always assume that (1) has an unique solution.

Note that a solution $s(t)$ of an isolated node satisfies

$$\dot{s}(t) = f(s(t),t), \tag{2}$$

in which $s(t)$ may be an equilibrium point, a periodic orbit, or even a chaotic orbit. Throughout this paper, the undirected networks are considered.

The aim of this paper is to find some appropriate schemes such that the solutions of network (1) synchronize with the solution of (2), in the sense that

$$\lim_{t\to\infty}\|x_i(t)-s(t)\| = 0,\quad i=1,2,\cdots,N. \tag{3}$$

In the rest of this paper, we need the following assumption and some lemmas:

Assumption 1 [33]**:** There exist a constant diagonal matrix $\Delta=\mathrm{diag}(\delta_1,\delta_2,\cdots,\delta_n)$ and $\eta>0$ such that

$$(x-y)^{\mathrm{T}}(f(x,t)-f(y,t)-\Delta(x-y)) \le -\eta(x-y)^{\mathrm{T}}(x-y),$$

where all $x,y\in\mathbb{R}^n$ and $t>0$.

In what follows, we assume that $\Delta = H\Gamma$ in which $H=\mathrm{diag}(h_1,h_2,\cdots,h_n)$.

Remark 1: Assumption 1 is so-called QUAD condition on vector field and very mild. For example, the condition is satisfied if $\partial f_i/\partial x_j\,(i,j=1,2,\cdots,n)$ are uniformly bounded and Γ is positive definite. It includes many well-known systems, such as Chua's oscillators, Rössler system, Lorenz system, Chen system, and Lü system.

Without loss of generality, we only consider the connected network; otherwise, we may consider the synchronization on each connected component of network separately, i.e., cluster synchronization. It is well known that the coupling configuration matrix of undirected network has a simple eigenvalue 0 and all the other eigenvalues are negative if and only if the network is connected [33,38].

Lemma 1 [35]**:** If $A=(a_{ij})_{N\times N}$ is irreducible and satisfies $a_{ij}=a_{ji}\ge 0,\ i\ne j;\ a_{ii}=-\sum_{j=1,j\ne i}^{N}a_{ij},\ i,j=1,2,\cdots,N$. Then, for any constant $\xi>0$, all eigenvalues of the matrix $A-\Xi$ are negative definite, where $\Xi=\mathrm{diag}(\xi,0,\cdots,0)$.

Lemma 2 [36]**:** Let $Q(x)$ and $R(x)$ be two symmetric matrices, matrix $S(x)$ has appropriate dimension. Then the linear matrix inequality (LMI)

$$\begin{pmatrix} Q(x) & S(x) \\ S(x)^T & R(x) \end{pmatrix} < 0$$

is equivalent to one of the following conditions:

(i) $Q(x)<0,\ R(x)-S(x)^T Q(x)^{-1}S(x)<0;$

(ii) $R(x)<0,\ Q(x)-S(x)R(x)^{-1}S(x)^T<0.$

Lemma 3 [37]**:** For any matrices A, B, C, D with appropriate dimensions and scalar k, the Kronecker product $\otimes$ satisfies:

(i) $(kA)\otimes B=A\otimes(kB)=k(A\otimes B);$

(ii) $(A+B)\otimes C=(A\otimes C)+(B\otimes C);$

(iii) $(A\otimes B)(C\otimes D)=(AC)\otimes(BD);$

(iv) $(A\otimes B)^T=A^T\otimes B^T.$

For the given network structures, it is difficult or even impossible to ensure the network (1) to realize self synchronization since the existence of delayed coupling terms. However, the time-delay cannot be ignored absolutely. So, there is an acceptable strategy that controllers may be applied to force the network to synchronize. On the other hand, due to the very large-scale nodes of complex network, it is impossible to add controllers to every node. To reduce the number of controlled nodes, the pinning control scheme is considered. Under the strategy of pinning control, there are two questions to be solved: (i) what kind of intrinsic parameters of network may be used to achieve synchronization? (ii) How large should be the suitable values of parameters and pinning controller's feedback gains to realize synchronization? To answer above two questions, we observe that the non-delayed coupling strength and weights may be utilized to synchronize network. Usually, it gives much larger coupling strength, weights and feedback gains than those needed in practice. So, it is not quite practical. A better way is to use adaptive approach to tune them. In the following, we will discuss them in detail.

3. DISTRIBUTED ADAPTIVE COUPLING STRENGTH VIA ADAPTIVE PINNING SCHEME

In this section, to achieve the synchronization of network (1), the pinning strategy is applied if the network is not self-synchronized. Suppose that there are l controlled nodes, where l is a positive integer satisfies $1\le l\le N$. Without loss of generality, rearrange the order of the nodes in the network, and let the first l nodes be controlled. Consequently, based on network (1), the pinning controlled network with the time-varying non-delayed coupling strength is described by:

$$\begin{aligned} \dot{x}_i(t) &= f(x_i(t),t)+c_1(t)\sum_{j=1}^{N}a_{ij}\Gamma x_j(t) \\ &\quad +c_2\sum_{j=1}^{N}b_{ij}\Gamma x_j(t-\tau)+u_i(t), \quad i=1,2,\cdots,l, \\ \dot{x}_i(t) &= f(x_i(t),t)+c_1(t)\sum_{j=1}^{N}a_{ij}\Gamma x_j(t) \\ &\quad +c_2\sum_{j=1}^{N}b_{ij}\Gamma x_j(t-\tau), \quad i=l+1,\cdots,N, \end{aligned} \tag{4}$$

where the time-varying coupling strength $c_1(t)>0$ is the differentiable function, and $u_i(t)$ is the feedback controller. Substituting (2) into (4), one yields the error network as follows:

$$\begin{aligned} \dot{e}_i(t) &= f(x_i(t),t)-f(s(t),t)+c_1(t)\sum_{j=1}^{N}a_{ij}\Gamma e_j(t) \\ &\quad +c_2\sum_{j=1}^{N}b_{ij}\Gamma e_j(t-\tau)+u_i(t), \quad i=1,2,\cdots,l, \\ \dot{e}_i(t) &= f(x_i(t),t)-f(s(t),t)+c_1(t)\sum_{j=1}^{N}a_{ij}\Gamma e_j(t) \\ &\quad +c_2\sum_{j=1}^{N}b_{ij}\Gamma e_j(t-\tau), \quad i=l+1,\cdots,N, \end{aligned} \tag{5}$$

where $e_i(t)=x_i(t)-s(t)$. The adaptive feedback controllers and their update laws are designed as:

$$\begin{aligned} &u_i(t)=-d_i(t)\Gamma e_i(t), \quad \dot{d}_i(t)=\alpha_i e_i^{\mathrm{T}}(t)\Gamma e_i(t), \\ &d_i(0)\ge 0, \quad \alpha_i>0, \quad i=1,2,\cdots,l. \end{aligned} \tag{6}$$

For the time-varying non-delayed coupling strength $c_1(t)$, the distributed adaptive law is given by:

$$\begin{aligned} &\dot{c}_1(t)=\sigma\sum_{i=1}^{N}\sum_{j=1,j\ne i}^{N}a_{ij}(e_i(t)-e_j(t))^{\mathrm{T}}\Gamma(e_i(t)-e_j(t)), \\ &c_1(0)\ge 0, \quad \sigma>0. \end{aligned} \tag{7}$$

Theorem 1: Suppose that Assumption 1 holds, Γ is a positive definite diagonal matrix. The undirected controlled network (5) is globally synchronized to (2) under the adaptive feedback controllers (6) and distributed adaptive laws (7).

Proof: Consider the Lyapunov functional candidate:

$$
\begin{aligned}
V(t)=&\frac{1}{2}\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)e_i(t)+\frac{(c_1(t)-\bar{c}_1)^2}{4\sigma}\\
&+\sum_{i=1}^{l}\frac{(d_i(t)-\bar{d}_i)^2}{2\alpha_i}+r\sum_{i=1}^{N}\int_{t-\tau}^{t}e_i^{\mathrm{T}}(s)\Gamma e_i(s)ds,
\end{aligned}\tag{8}
$$

where $\bar{c}_1$, $\bar{d}_i$, and r are positive constant to be determined below. Let $\bar{D}=\mathrm{diag}(\bar{d}_1,\bar{d}_2,\cdots,\bar{d}_l,0,\cdots,0)\in\mathbb{R}^{N\times N}$, $e(t)=(e_1^{\mathrm{T}}(t),e_2^{\mathrm{T}}(t),\cdots,e_N^{\mathrm{T}}(t))^{\mathrm{T}}$.

Differentiating $V(t)$ along the trajectories of (5), we have

$$
\begin{aligned}
\dot{V}(t)=&\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)\left[f(x_i(t),t)-f(s(t),t)\right]\\
&+c_1(t)\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}e_i^{\mathrm{T}}(t)\Gamma e_j(t)\\
&+c_2\sum_{i=1}^{N}\sum_{j=1}^{N}b_{ij}e_i^{\mathrm{T}}(t)\Gamma e_j(t-\tau)\\
&-\sum_{i=1}^{l}d_i(t)e_i^{\mathrm{T}}(t)\Gamma e_i(t)+\sum_{i=1}^{l}(d_i(t)-\bar{d}_i)e_i^{\mathrm{T}}(t)\Gamma e_i(t)\\
&+\frac{c_1(t)-\bar{c}_1}{2}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}a_{ij}(e_i(t)\\
&-e_j(t))^{\mathrm{T}}\Gamma(e_i(t)-e_j(t))\\
&+r\sum_{i=1}^{N}\left[e_i^{\mathrm{T}}(t)\Gamma e_i(t)-e_i^{\mathrm{T}}(t-\tau)\Gamma e_i(t-\tau)\right].
\end{aligned}
$$

In view of $\sum_{j=1}^{N}a_{ij}=0$ and $a_{ij}=a_{ji}$, one obtains

$$
\begin{aligned}
&\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}a_{ij}(e_i(t)-e_j(t))^{\mathrm{T}}\Gamma(e_i(t)-e_j(t))\\
&=-2\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}e_i^{\mathrm{T}}(t)\Gamma e_j(t).
\end{aligned}
$$

By Assumption 1, it follows that

$$
\begin{aligned}
\dot{V}(t)\le&-\eta\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)e_i(t)+\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)H\Gamma e_i(t)\\
&+\bar{c}_1\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}e_i^{\mathrm{T}}(t)\Gamma e_j(t)-\sum_{i=1}^{l}\bar{d}_ie_i^{\mathrm{T}}(t)\Gamma e_i(t)\\
&+c_2\sum_{i=1}^{N}\sum_{j=1}^{N}b_{ij}e_i^{\mathrm{T}}(t)\Gamma e_j(t-\tau)\\
&+r\sum_{i=1}^{N}\left[e_i^{\mathrm{T}}(t)\Gamma e_i(t)-e_i^{\mathrm{T}}(t-\tau)\Gamma e_i(t-\tau)\right]\\
=&-\eta e^{\mathrm{T}}(t)(I_N\otimes I_n)e(t)\\
&+e^{\mathrm{T}}(t[I_N\otimes(H\Gamma)+\bar{c}_1(A\otimes\Gamma)\\
&-\bar{D}\otimes\Gamma+r(I_N\otimes\Gamma)]e(t)-c_2e^{\mathrm{T}}(t)(B\otimes\Gamma)e(t-\tau)\\
&-re^{\mathrm{T}}(t-\tau)(I_N\otimes\Gamma)e(t-\tau)\\
=&-\eta e^{\mathrm{T}}(t)(I_N\otimes I_n)e(t)+(e^{\mathrm{T}}(t),e^{\mathrm{T}}(t-\tau))Z\\
&\times(e^{\mathrm{T}}(t),e^{\mathrm{T}}(t-\tau))^{\mathrm{T}},
\end{aligned}
$$

where

$$
Z=\begin{pmatrix}\Pi & -\frac{c_2}{2}(B\otimes\Gamma)\\ -\frac{c_2}{2}(B\otimes\Gamma)^{\mathrm{T}} & -r(I_N\otimes\Gamma)\end{pmatrix},
$$

where $\Pi=I_N\otimes(H\Gamma)+\bar{c}_1(A\otimes\Gamma)-\bar{D}\otimes\Gamma+r(I_N\otimes\Gamma)$.

Note that $-r(I_N\otimes\Gamma)<0$, by Lemma 2, $Z<0$ is equivalent to

$$
\begin{aligned}
&I_N\otimes(H\Gamma)+(\bar{c}_1A-\bar{D})\otimes\Gamma+r(I_N\otimes\Gamma)\\
&+\frac{c_2^2}{4r}(BB^{\mathrm{T}}\otimes\Gamma)<0.
\end{aligned}\tag{9}
$$

By Lemma 1, it follows that all eigenvalues of $\bar{c}_1A-\bar{D}$ are negative. Since $B=B^{\mathrm{T}}$, Γ is a positive definite diagonal matrix, one obtains

$$
\begin{aligned}
&I_N\otimes(H\Gamma)+(\bar{c}_1A-\bar{D})\otimes\Gamma+r(I_N\otimes\Gamma)+\frac{c_2^2}{4r}(BB^{\mathrm{T}}\otimes\Gamma)\\
&\le\left[h+\lambda+r+\frac{c_2^2\rho^2(B)}{4r}\right](I_N\otimes\Gamma),
\end{aligned}\tag{10}
$$

where $h=\max_{1\le j\le n}\{h_j\}$, $\lambda=\lambda_{\max}(\bar{c}_1A-\bar{D})<0$, $\rho(B)>0$ is the spectral radius of B.

Taking $r=c_2\rho(B)/2$, we have

$$
\begin{aligned}
&\left[h+\lambda+r+\frac{c_2^2\rho^2(B)}{4r}\right](I_N\otimes\Gamma)\\
&=[h+\lambda+r+c_2\rho(B)](I_N\otimes\Gamma).
\end{aligned}
$$

Therefore, choosing suitable $\bar{c}_1$, $\bar{d}_i$ such that $h+\lambda+r+c_2\rho(B)<0$. Then,

$$
[h+\lambda+r+c_2\rho(B)](I_N\otimes\Gamma)<0.\tag{11}
$$

From (9)-(11), we can conclude that

$$
\dot{V}(t)\le-\eta e^{\mathrm{T}}(t)(I_N\otimes I_n)e(t).\tag{12}
$$

By (12), we have $\dot{V}(t)\le0$ and $\dot{V}(t)=0$ if and only if $e(t)=0$. Hence, the set $\mathscr{M}=\{e(t)=0,\ d_i(t)=\bar{d}_i,\ c_1(t)=\bar{c}_1\}$ is the largest invariant set contained in $\mathscr{D}=\{e(t):\dot{V}(t)=0\}$ for system (5). According to LaSalle's invariance principle [39], for any initial condition, every solution of system (5) asymptotically converges to $\mathscr{M}$ as $t\to\infty$, i.e., $\|e_i(t)\|\to0$, $d_i(t)\to\bar{d}_i$, $c_1(t)\to\bar{c}_1$, where $\bar{d}_i$, $(i=1,2,\cdots,l)$, $\bar{c}_1$ are positive constants. It can conclude that globally asymptotical synchronization of network (4) is achieved. The proof is completed.

Remark 2: In model (4), we only introduce the time-varying non-delayed coupling strength $c_1(t)$ and design its adaptive update law (7). From another perspective, it is feasible that we modify the fixed delayed coupling strength c_2 to time-varying one (e.g., $c_2(t)$) and design corresponding adaptive update law similar to (7). However, we find a remarkable phenomenon that delayed time-varying coupling strength converges gradually to zero with time evolution by the numerical simulations. Hence, we think the delayed coupling as desynchronizing factor. On the other hand, we emphasize specially the time-delay cannot be ignored in a real network. It is main reason why we do not introduce time-varying delayed coupling strength.

4. DISTRIBUTED ADAPTIVE COUPLING WEIGHTS VIA DELAYED PINNING SCHEME

Recall the definitions of the topological configuration matrices A and B in network (1). In this section, we consider the adjustment of non-delayed coupling configuration matrix $A=(a_{ij})_{N\times N}$.

Construct the time-varying matrix $A(t)=(a_{ij}(t))_{N\times N}$ where $a_{ij}(t)$ is the differentiable function with respect to t and satisfies the following properties:

(i) $a_{ij}(t)=a_{ji}(t)$ for all t, $i,j=1,2,\cdots,N$;

(ii) When $i\neq j$, $a_{ij}(t)>0$ for all t if and only if $a_{ij}>0$. Otherwise, $a_{ij}(t)=0$ for all t if and only if $a_{ij}=0$;

(iii) The diagonal element $a_{ii}(t)=-\sum_{j=1,j\neq i}^{N}a_{ij}(t)$ which ensure that $\sum_{j=1}^{N}a_{ij}(t)=0$ for all t and $i=1,2,\cdots,N$.

The corresponding time-varying non-delayed and fixed delayed coupling network model can be written as

$$\begin{aligned}\dot{x}_i(t)&=f(x_i(t),t)+c_1\sum_{j=1}^{N}a_{ij}(t)\Gamma x_j(t)\\&\quad+c_2\sum_{j=1}^{N}b_{ij}\Gamma x_j(t-\tau),\quad i=1,2,\cdots,N.\end{aligned}\tag{13}$$

According to the definition of $a_{ij}(t)$, the topological structures of model (13) are same as model (1) and only non-delayed coupling weights are modified. Hence, we can design the adaptive update law to tune connective weights of non-delayed coupling network's nodes.

Remark 3: Similar to formation of non-delayed time-varying coupling matrix $A(t)$, we can also construct a delayed time-varying coupling matrix $B(t)$. However, just like the delayed coupling is thought as desynchronizing factor in Remark 2, we also find that non-zero elements of time-varying delayed coupling will converge to zero with time evolution by numerical simulations. Due to the existence of time-delay, we have to consider only the adjustment of non-delayed coupling matrix.

In what follows, the pinning strategy is introduced to realize the synchronization of network (13). Thus, the pinning controlled network is described by

$$\begin{aligned}\dot{x}_i(t)&=f(x_i(t),t)+c_1\sum_{j=1}^{N}a_{ij}(t)\Gamma x_j(t)\\&\quad+c_2\sum_{j=1}^{N}b_{ij}\Gamma x_j(t-\tau)+u_i(t),\quad i=1,2,\cdots,l,\\\dot{x}_i(t)&=f(x_i(t),t)+c_1\sum_{j=1}^{N}a_{ij}(t)\Gamma x_j(t)\\&\quad+c_2\sum_{j=1}^{N}b_{ij}\Gamma x_j(t-\tau),\quad i=l+1,\cdots,N,\end{aligned}\tag{14}$$

where the time-varying coupling weights $a_{ij}(t)>0$ $(i\neq j)$ is differentiable function, and $u_i(t)$ is the feedback controller. Substituting (2) into (13), we obtain the error network as follows:

$$\begin{aligned}\dot{e}_i(t)&=f(x_i(t),t)-f(s(t),t)+c_1\sum_{j=1}^{N}a_{ij}(t)\Gamma e_j(t)\\&\quad+c_2\sum_{j=1}^{N}b_{ij}\Gamma e_j(t-\tau)+u_i(t),\quad i=1,2,\cdots,l,\\\dot{e}_i(t)&=f(x_i(t),t)-f(s(t),t)+c_1\sum_{j=1}^{N}a_{ij}(t)\Gamma e_j(t)\\&\quad+c_2\sum_{j=1}^{N}b_{ij}\Gamma e_j(t-\tau),\quad i=l+1,\cdots,N,\end{aligned}\tag{15}$$

where $e_i(t)=x_i(t)-s(t)$. For a special case where the information of delay τ is available, so we consider the delayed feedback controllers as follows:

$$u_i(t)=-c_1d_i\Gamma e_i(t)-c_2k_i\Gamma e_i(t-\tau),\tag{16}$$

where d_i, k_i $(i=1,2,\cdots,l)$ are positive constants. Similar the delayed feedback technique had been used in [9].

For the time-varying coupling weight $a_{ij}(t)$, we design the following distributed adaptive law:

$$\begin{aligned}&\dot{a}_{ij}(t)=\sigma_{ij}(e_i(t)-e_j(t))^{\mathrm{T}}\Gamma(e_i(t)-e_j(t)),\\&a_{ij}(0)=a_{ji}(0)\geq 0,\quad i\neq j,i,\quad j=1,2,\cdots,N,\end{aligned}\tag{17}$$

where $a_{ij}=a_{ji}$ are positive constants.

Theorem 2: Suppose that Assumption 1 holds and Γ is a positive definite diagonal matrix, then the undirected complex network (14) is globally synchronized under the delayed feedback controllers (16) and distributed adaptive laws (17).

Proof: Consider the Lyapunov functional candidate:

$$\begin{aligned}V(t)&=\frac{1}{2}\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)e_i(t)+\frac{c_1}{4}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{(a_{ij}(t)-\bar{a}_{ij})^2}{\sigma_{ij}}\\&\quad+r\sum_{i=1}^{N}\int_{t-\tau}^{t}e_i^{\mathrm{T}}(s)\Gamma e_i(s)ds,\end{aligned}\tag{18}$$

where $\bar{a}_{ij}=\bar{a}_{ji}(i\neq j)$ are nonnegative constants, and $\bar{a}_{ij}=0$ if and only if $a_{ij}(t)=0$, r is positive constant to be determined.

The derivative of $V(t)$ along the trajectories of (15) obtains that

$$
\begin{aligned}
\dot{V}(t) = \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)[f(x_i(t),t) - f(s(t),t) \\
&+c_1\sum_{j=1}^{N} a_{ij}(t)\Gamma e_j(t) + c_2\sum_{j=1}^{N} b_{ij}\Gamma e_j(t-\tau)] \\
&-c_1\sum_{i=1}^{l} d_i e_i^{\mathrm{T}}(t)\Gamma e_i(t) - c_2\sum_{i=1}^{l} k_i e_i^{\mathrm{T}}(t)\Gamma e_i(t-\tau) \\
&+\frac{c_1}{2}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}(a_{ij}(t)-\overline{a}_{ij})(e_i(t) \\
&-e_j(t))^{\mathrm{T}}\Gamma(e_i(t)-e_j(t)) \\
&+r\sum_{i=1}^{N}\left[e_i^{\mathrm{T}}(t)\Gamma e_i(t) - e_i^{\mathrm{T}}(t-\tau)\Gamma e_i(t-\tau)\right].
\end{aligned} \tag{19}
$$

From Assumption 1, one has

$$
\begin{aligned}
&\sum_{i=1}^{N} e_i^{\mathrm{T}}(t)[f(x_i(t),t) - f(s(t),t)] \\
&\leq -\eta\sum_{i=1}^{N} e_i^{\mathrm{T}}(t)e_i(t) + \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)H\Gamma e_i(t).
\end{aligned} \tag{20}
$$

Constructing a Laplace matrix $\overline{A} = (\overline{a}_{ij})_{N\times N}$ with $\overline{a}_{ii} = -\sum_{j=1,j\neq i}^{N}\overline{a}_{ij}$, one obtains

$$
\begin{aligned}
&\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}(a_{ij}(t)-\overline{a}_{ij})(e_i(t)-e_j(t))^{\mathrm{T}}\Gamma(e_i(t)-e_j(t)) \\
&= -2\sum_{i=1}^{N}\sum_{j=1}^{N}(a_{ij}(t)-\overline{a}_{ij})e_i^{\mathrm{T}}(t)\Gamma e_j(t).
\end{aligned} \tag{21}
$$

Combining (19)-(21), it follows that

$$
\begin{aligned}
\dot{V}(t) &\leq -\eta\sum_{i=1}^{N} e_i^{\mathrm{T}}(t)e_i(t) + \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)H\Gamma e_i(t) \\
&+c_1\sum_{i=1}^{N}\sum_{j=1}^{N}\overline{a}_{ij}e_i^{\mathrm{T}}(t)\Gamma e_j(t) \\
&+c_2\sum_{i=1}^{N}\sum_{j=1}^{N} b_{ij}e_i^{\mathrm{T}}(t)\Gamma e_j(t-\tau) \\
&-c_1\sum_{i=1}^{l} d_i e_i^{\mathrm{T}}(t)\Gamma e_i(t) - c_2\sum_{i=1}^{l} k_i e_i^{\mathrm{T}}(t)\Gamma e_i(t-\tau) \\
&+r\sum_{i=1}^{N}\left[e_i^{\mathrm{T}}(t)\Gamma e_i(t) - e_i^{\mathrm{T}}(t-\tau)\Gamma e_i(t-\tau)\right] \\
&= -\eta e^{\mathrm{T}}(t)(I_N\otimes I_n)e(t) + e^{\mathrm{T}}(t)[I_N\otimes(H\Gamma) \\
&+c_1(\overline{A}-D)\otimes\Gamma + r(I_N\otimes\Gamma)]e(t) \\
&+c_2 e^{\mathrm{T}}(t)[(B-K)\otimes\Gamma]e(t-\tau) \\
&-re^{\mathrm{T}}(t-\tau)(I_N\otimes\Gamma)e(t-\tau),
\end{aligned}
$$

where $e(t) = (e_1^{\mathrm{T}}(t), e_2^{\mathrm{T}}(t), \cdots, e_N^{\mathrm{T}}(t))^{\mathrm{T}}$, and

$$
\begin{aligned}
D &= \mathrm{diag}(d_1, d_2, \cdots, d_l, 0, \cdots, 0), \\
K &= \mathrm{diag}(k_1, k_2, \cdots, k_l, 0, \cdots, 0).
\end{aligned}
$$

In the following, we can employ the similar steps as in Theorem 1 to complete the proof, so we omit it.

Remark 4: In the fixed coupling strengths and topological structures of network (14), the non-delayed coupling weights can be tuned slightly to achieve synchronization. It is noted that the delayed information should be fully available. Hence, by introducing the delayed feedback controllers, the pinning feedback gains can be lower than ones only using memoryless state-feedback controllers.

5. NUMERICAL SIMULATION

In this section, we provide two simulation examples to verify the theoretical analysis. In our simulation examples, small-world and scale-free complex networks are applied to guarantee connectivity in the designed network. Here, we consider the complex dynamical network (1) that consists of N identical Chen system. Denote the state of the i-th node by $x_i = (x_{i1}, x_{i2}, x_{i3})$, then the individual node dynamics are represented as:

$$
\begin{cases}
\dot{x}_{i1} = 35(x_{i2} - x_{i1}), \\
\dot{x}_{i2} = -7x_{i1} - x_{i1}x_{i3} + 28x_{i2}, \\
\dot{x}_{i3} = x_{i1}x_{i2} - 3x_{i3},
\end{cases}
$$

which has a chaotic attractor (See Fig. 1). In [10], Chen system has been verified to satisfy Assumption 1.

5.1. Distributed adaptive coupling strength

In this subsection, we investigate the global synchronization of the pinning controlled network (4) with 100 Chen oscillators, where 5 adaptive pinning controllers are used and inner coupling matrix $\Gamma = \mathrm{diag}(1,2,1)$. The non-delayed coupling network is presented by a BA network where $N = 100$, $m_0 = m = 2$, which contains about 200 connections (See Fig. 2). The delayed coupling network is described by a WS network where $N = 100$, $p = 0.05$ (See Fig. 3). The parameters are set as $\tau = 0.5$, $c_2 = 2$, $\sigma = 0.2$, $\alpha_1 = \cdots = \alpha_5 = 0.16$, and initial conditions are totally randomly chosen between 0 and 3. According to Fig. 4 and Fig. 5, synchronization is

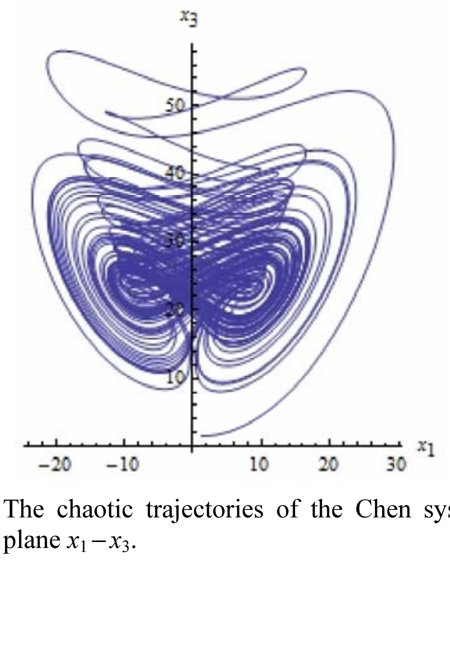


Fig. 1. The chaotic trajectories of the Chen system on plane $x_1 - x_3$.

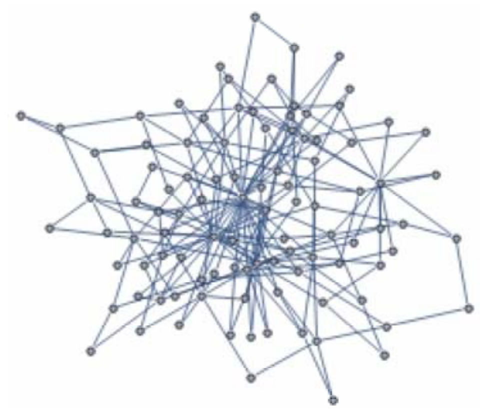


Fig. 2. A scale-free network contains 100 nodes.

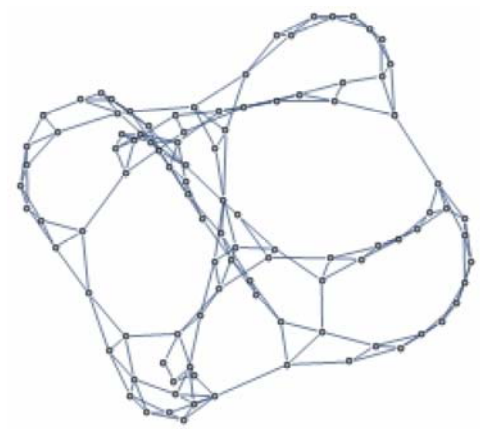
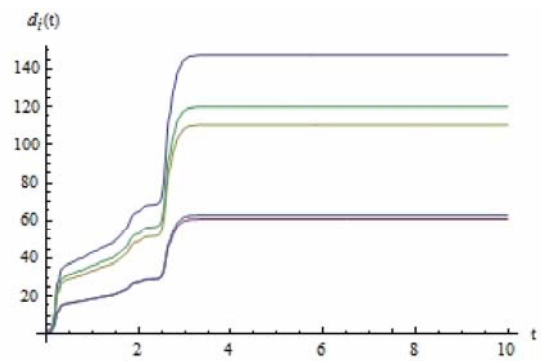
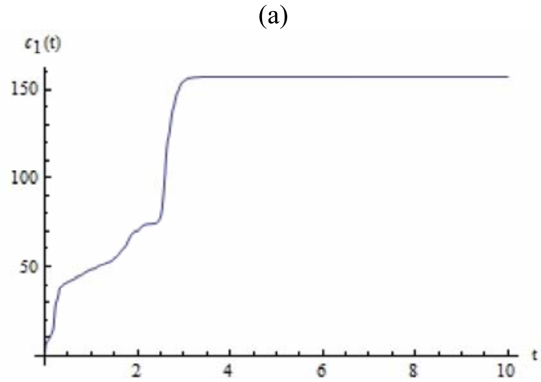


Fig. 3. A small-world network contains 100 nodes.

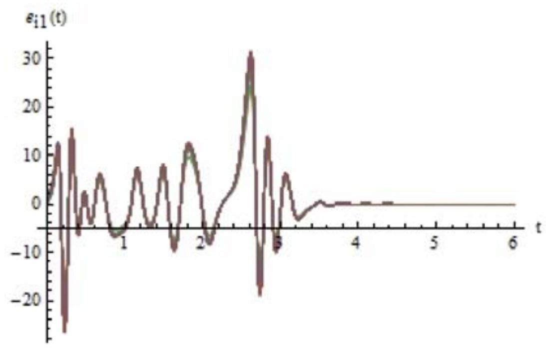


(a)

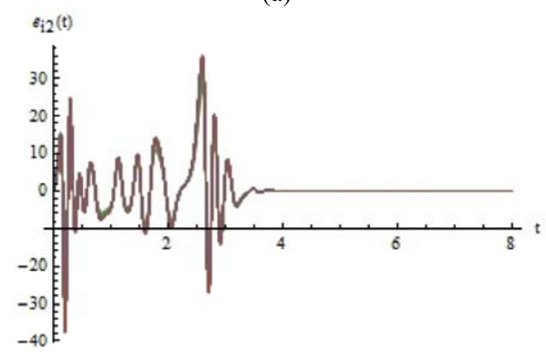


(b)

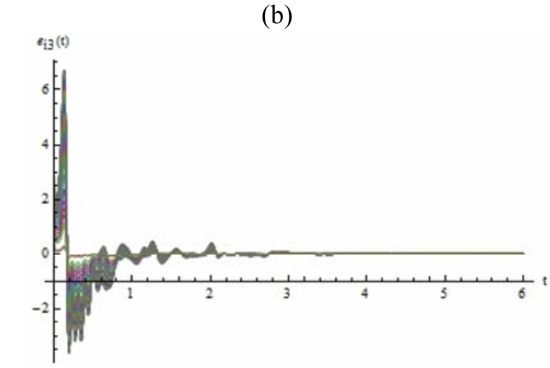
Fig. 4. (a) States of adaptive feedback gains $d_i(t)$ of network (5), i = 1, 2, …5; (b) States of adaptive coupling strength $c_1(t)$ of network (5).



(a)



(b)



(c)

Fig. 5. Synchronization errors $e_{i1}(t)$, $e_{i2}(t)$, $e_{i3}(t)$ of network (5) with $\tau = 0.5$, i = 1, 2, …,100.

asymptotically achieved, the adaptive coupling strength and feedback gains of pinning controller asymptotically converge to constant values.

5.2. Distributed adaptive coupling weights

In this case, the pinning synchronization of network (14) with 50 Chen oscillators is considered, where 3 delayed feedback pinning controllers are utilized and $\Gamma = \mathrm{diag}(1,2,1)$. On the basis of a WS random network ($N = 50$, $p = 0.03$) which contains about 100 connections, we generate the time-varying non-delayed coupling configuration matrix $A(t)$ according to the explanation of $A(t)$ in Section 4. The delay coupling network is described by a BA network ($N = 50$, $m_0 = m = 2$) which also contains about 100 connections. The parameters of network (14) is given by $c_1 = 125$, $c_2 = 2$, $\tau = 0.5$. For connected node i and j, $\sigma_{ij} = \sigma_{ji} = 0.05$, $a_{ij}(0) = a_{ji}(0) = 1$. The feedback gains d_i and k_i of pinning controllers can be chosen as arbitrary positive real numbers. In this

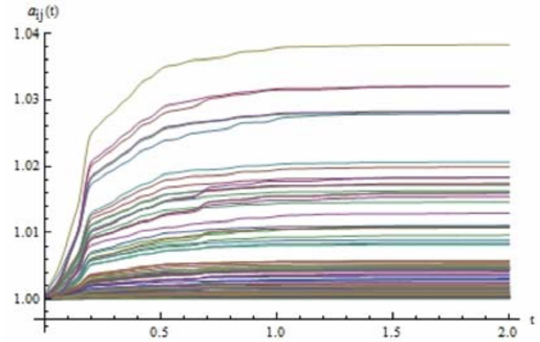
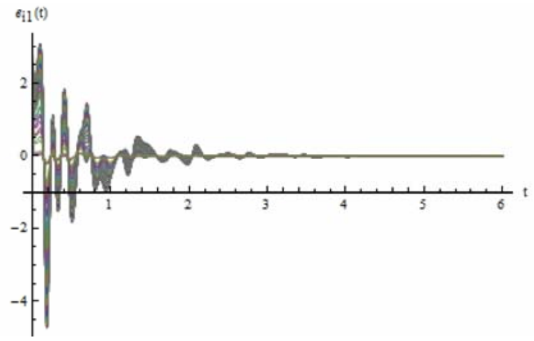
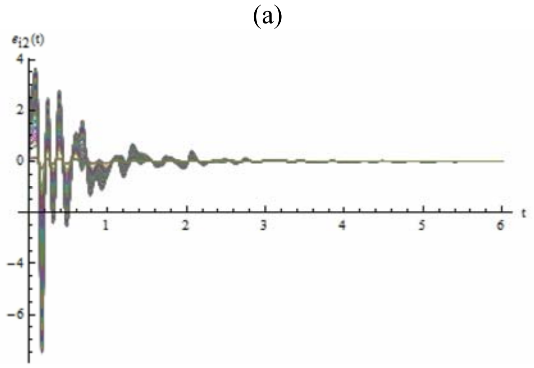


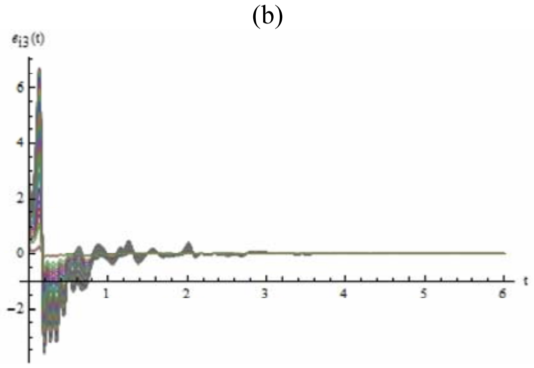
Fig. 6. Evolution of adaptive weights $a_{ij}(t)$.



(a)



(b)



(c)

Fig. 7. Synchronization errors $e_{i1}(t)$, $e_{i2}(t)$, $e_{i3}(t)$ of network (14) with $\tau = 0.5$, $i = 1, 2, \ldots, 50$.

simulation, we set them as $d_1 = 8.016$, $d_2 = 9.145$, $d_3 = 6.845$, $k_1 = 6.877$, $k_2 = 7.919$, $k_3 = 8.413$. The rests of initial conditions are taken randomly between 0 and 3. As displayed in Figs. 6 and 7, global synchronization is realized and coupling weights of non-delayed coupling network converge to steady-state value that are slightly higher than their initial value 1.

6. CONCLUSIONS

In this paper, two kinds of the distributed adaptive control schemes for pinning synchronization in the undirected complex networks with delayed and non-delayed coupling are proposed. For the first control scheme, the coupling strength and feedback gains can be adjusted simultaneously to achieve synchronization. It is noted that the delay information is available, the second pinning scheme that the coupling weights are adaptively tuned via delayed feedback controllers is represented. Finally, two numerical examples are designed to illustrate the validity and effectiveness of established theory. The simulations indicate that above theoretic results can be applied to realize synchronization in large-scale networks.

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