International Journal of Control, Automation and Systems 15(3) (2017) 1107-1116
http://dx.doi.org/10.1007/s12555-014-0171-8

ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Quantized Control for Uncertain Singular Markovian Jump Linear Systems with General Incomplete Transition Rates

Jing Xie, Yong-Gui Kao*, Cai-Hong Zhang, and Hamid Reza Karimi

Abstract: Quantization is indeed a natural way to take into consideration in the control design of the complexity constraints for the controller as well as the communication constraints in the information exchange between the controller and the plant. This paper is devoted to investigating quantized state-feedback control problems for a class of continuous-time uncertain singular Markovian jump linear systems (CUSMJLSs) with generally uncertain transition rates (GUTRs) and input quantization. In this case, each transition rate can be completely unknown or only its estimate value is known. First, input quantization is introduced, then by introducing new matrix inequality conditions, sufficient conditions are formulated for quantized state-feedback control of CSMJLUSs with GUTRs and input quantization. Finally, a numerical example is presented to illustrate the effectiveness and efficiency of the proposed results.

Keywords: Generally uncertain transition rate, input quantization, quantized control, singular Markovian jump system.

1. INTRODUCTION

Singular systems, also referred to as descriptor systems, generalized state-space systems, differential-algebraic systems or semi-state systems, are more appropriate to describe the behaviors of some practical systems, such as economic systems, chemical process, circuit systems, electric systems, robotic systems, space navigation systems, biological systems and networked control systems (see [1–3] and the references therein). Markovian jump systems(MJSs) are popular in modelling many practical systems subject to abrupt failures or changes in structures and parameters [4–9]. Recently, more and more researchers have focused on the problem of stochastic stability and stochastic admissibility for singular Markovian jump systems(SMJSs) [10–17]. [13] discussed the robust control with bounded transition rates(TRs) via proportional-derivative state feedback controllers; [11] derived exponential mean-square stability of time-delay singular systems with Markovian switching and nonlinear perturbations. Every TR in above mentioned literatures is required to be completely known as a prior.

As is well known, in many practical engineering applications, not all the TRs could be precisely determined or estimated because of expensive cost or other factors. Therefore, analysis and synthesis problem for normal MJSs with uncertain TRs have attracted increasing interests [18–20]. [19] have investigated the stochastic stability robustness for continuous-time and discrete-time MJSs with upper bounded TRs. [14] dealt with the problem of delay-dependent H_∞ control also with bounded TRs. In these cases, the precise value of each TR does not required to be known, but its bounds (upper bound and lower bound) are known. However, in some practical cases, it is very hard or even impossible to derive the bound of each TR. Hence, the idea for stochastic stability of MJS with partly known transition TRs is developed [21]. Then, partly unknown TRs for MJSs were involved in [12, 18, 21–24]. What's more, filtering problems and neural networks for MJSs are also investigated in [25, 26] and [27]. [28] considered delay-dependent H_∞ filtering for discrete-time SMJSs with time-varying delay and partially unknown transition probabilities. Unfortunately, the complete knowledge of every TR in the above mentioned models is either exactly known or completely unknown, which may be too restrictive in many practical situations. [29] proposed another description for the uncertain TRs, which is called generally uncertain TRs (GUTRs). In this kind of

Manuscript received April 30, 2014; revised December 24, 2014, June 5, 2015, September 14, 2015, and February 26, 2016; accepted July 5, 2016. Recommended by Associate Editor Yingmin Jia under the direction of Editor Yoshito Ohta. This work was supported by the National Natural Science Foundations of China (61473097) and the Qingdao Postdoctoral Application Research Project (2015117).

Jing Xie is with the College of Automation Engineering, Qingdao University of Technology, 2 Changjiang Road, Huangdao District, Qingdao 266520, China (e-mail: tiantian1210x@163.com). Yong-Gui Kao is with the Department of Mathematics, Harbin Institute of Technology(Weihai), 2 Wenhua West Road, Weihai 264209, China (e-mail: ygkao2008@gmail.com). Cai-Hong Zhang is with the College of Automation and Electrical Engineering of Qingdao University, Qingdao University, 308 Ningxia Road, Qingdao 266071, China (e-mail: rainbow823@163.com). Hamid Reza Karimi is with the Department of Mechanical Engineering, Politecnico di Milano, 20156 Milan, Italy (e-mail: hamidreza.karimi@polimi.it).
* Corresponding author.

GUTR model, each transition rate can be completely unknown or only its estimate is known, which makes this model be applicable to more practical cases. In fact, both bounded uncertain TR models and partly uncertain TR models are the special cases of GUTR models.

On the other hand, information processing devices, such as analog-to-digital and digital-to-analog converters, have been widely employed in many modern engineering practice and brought some benefits, such as lower cost, reduced weight and power, simple installation and maintenance. At the same time, some new problems may also occur such as server deterioration of system performance or even system instability. Signal quantization always exists in computer-based control systems and should be fully considered in such cases. Therefore, the feedback stabilization problem is probed by utilizing dynamic quantizers [30, 31], and static quantizers [32–34]. In addition, it is also applied to filter design [35] and H_∞ control design [36]. In practice, quantization errors have adverse effects on the network control systems modeled as MJSs. Xiao *et al.* [37] addressed the stabilization problem for single-input discrete MJSs via mode dependent quantized state feedback, but the TRs are assumed to be completely known. Li *et al.* [38] concerned with the robust quantized state-feedback controller design for delayed Markoivan jump linear systems with generally incomplete TRs. To the best of our knowledge, the problem of SMJSs with GUTRs has not been investigated. Motivated by the stabilization study of SMJSs with GUTRs in [39], it is of great importance and challenging to extend the results of [38] to SMJSs with GUTRs, which is the main purpose of our research.

In this paper, we will investigate the regularity and stochastic stability problems for continuous-time uncertain singular Markovian jump linear systems with GUTRs and input quantization. In Section 2, the SMJ model with GUTRs is formulated, while some definitions and lemmas are stated. In Section 3, by introducing several new inequality conditions, sufficient conditions are derived for the stochastic stability of CSMJLUSs with GUTRs and input quantization. In Section 4, a numerical example is provided to illustrate the feasibility and applicability of the developed results. Section 5 concludes the paper.

Notation: In this paper, $\Re^n$ and $\Re^{n\times m}$ denote n- dimensional Euclidean space and the set of all $n\times m$ real matrices, respectively. $\mathbb{Z}$ denotes the set of integers. The notation $P>0$ ($P\geq 0$) means that the matrix P is a real symmetric and positive definite (semi-positive-definite) matrix. $(\Omega,\mathbb{F},\mathbb{P})$ is a complete probability space, where Ω represents the space, $\mathbb{F}$ is the σ-algebra of the sample space and $\mathbb{P}$ is the probability measure on $\mathbb{F}$. $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. A^{T} denotes the transpose of A. I and 0 represent the identity matrix and a zero matrix in appropriate dimension, respectively. The ∞-norm and 1-norm for a vector $x\in\Re^n$ are defined as $|x|_\infty=\max_{1\leq i\leq n}\{|x_i|\}$ and $|x|_1=\sum_{j=1}^n|x_j|$, respectively. While the ∞-norm for a matrix $A=(a_{ij})_{n\times n}$ is given by $\|A\|_\infty=\max\{\sum_{j=1}^n|a_{1j}|,\cdots,\sum_{j=1}^n|a_{nj}|\}$. The sign function $\mathrm{sgn}(x)$ ($x\in\Re^n$) means that $[\mathrm{sgn}(x_1),\cdots,\mathrm{sgn}(x_n)]^{\mathrm{T}}$, where $\mathrm{sgn}(x_i)=1$ ($x_i>0$), $\mathrm{sgn}(x_i)=0$ ($x_i=0$), $\mathrm{sgn}(x_i)=-1$ ($x_i<0$). In symmetric block matrices, $*$ represents a term that is induced by symmetry.

2. PRELIMINARIESE

Consider the following continuous-time uncertain singular Markovian jump linear systems(CUSMJLSs) with GUTRs and input quantization:

$$\begin{cases} E(r_t)\dot{x}(t)=(A(r_t)+\Delta A(r_t))x(t)+(B(r_t)\\ \qquad\qquad +\Delta B(r_t))q(u(t)),\\ x_0=x(0), \end{cases} \tag{1}$$

where $x(t)\in\Re^n$ is the system state, $u(t)\in\Re^m$ is the control input. The matrix $E(r_t)$ is singular, with $\mathtt{rank}(E(r_t))=r<n$. $A(r_t)$ and $B(r_t)$ are real constant matrices of appropriate dimensions, $\Delta A(r_t)$ and $\Delta B(r_t)$ are parameter uncertainties. The mode jumping process $\{r_t,t\geq 0\}$ is a continuous-time Markovian process with right continuous trajectories and taking values in a finite set $\mathbb{S}=\{1,2,\cdots,s\}$. It governs the switching among the different system modes with the following mode transition probabilities:

$$\mathbb{P}r\{r_{t+\Delta}=j|r_t=i\}=\begin{cases}\pi_{ij}\Delta+o(\Delta), & j\neq i;\\ 1+\pi_{ij}\Delta+o(\Delta), & j=i;\end{cases} \tag{2}$$

where $\Delta>0$, $\lim_{\Delta\to 0}\frac{o(\Delta)}{\Delta}=0$, and $\pi_{ij}\geq 0$ ($i\neq j$) is the TR from mode i at time t to mode j at time $t+\Delta$, and there is $\pi_{ii}=-\sum_{j=1,j\neq i}^s\pi_{ij}\leq 0$.

The mode TR matrix $\Pi\triangleq(\pi_{ij})_{s\times s}$ is considered to be generally uncertain. For instance, the GUTR matrix of system (1) with s operation modes may be like that

$$\begin{bmatrix}\hat{\pi}_{11}+\Delta_{11} & ? & ? & \cdots & ?\\ ? & ? & \hat{\pi}_{23}+\Delta_{23} & \cdots & \hat{\pi}_{2s}+\Delta_{2s}\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ ? & \hat{\pi}_{s2}+\Delta_{s2} & ? & \cdots & ?\end{bmatrix}, \tag{3}$$

where $\hat{\pi}_{ij}$ and $\Delta_{ij}\in[-\delta_{ij},\delta_{ij}]$ ($\delta_{ij}\geq 0$), represent the estimate value and estimate error of the uncertain TR π_{ij} respectively, while $\hat{\pi}_{ij}$ and δ_{ij} are known. "?" represents the complete unknown transition rate π_{ij}, which means its estimate value $\hat{\pi}_{ij}$ and estimate error bound δ_{ij} are unknown.

Now, there are the following definitions. For any $i\in\mathbb{S}$, the set U^i denotes as $U^i=U^i_{\mathrm{k}}\cup U^i_{\mathrm{uk}}$, with $U^i_{\mathrm{k}}\triangleq\{j:\pi_{ij}\ \mathtt{is\ known\ for}\ j\in\mathbb{S}\}$ and $U^i_{\mathrm{uk}}\triangleq\{j:\pi_{ij}\ \mathtt{is\ unknown\ for}\ j\in\mathbb{S}\}$. Moreover, if $U^i_{\mathrm{k}}\neq\emptyset$, it is further described as $U^i_{\rm k} = \{k^i_1, \cdots, k^i_{m_i}\}$, where m_i represents the number of elements in $U^i_{\rm k}$.

According to the properties of TRs and the definitions of $U^i_{\rm k}$, $U^i_{\rm uk}$, we could take the following assumes.

Assumption 1: If $U^i_{\rm k} \neq \mathbb{S}$ and $i \in U^i_{\rm k}$, then $\hat{\pi}_{ij} - \delta_{ij} \geq 0$ $(\forall j \in U^i_{\rm k}, j \neq i)$, $\hat{\pi}_{ii} + \delta_{ii} \leq 0$, and $\sum_{j \in U^i_{\rm k}} \hat{\pi}_{ij} \leq 0$.

Assumption 2: If $U^i_{\rm k} = \mathbb{S}$, then $\hat{\pi}_{ij} - \delta_{ij} \geq 0$ $(\forall j \in \mathbb{S}, j \neq i)$, $\hat{\pi}_{ii} = -\sum^{s}_{j=1, j\neq i} \hat{\pi}_{ij} \leq 0$, and $\delta_{ii} = \sum^{s}_{j=1, j\neq i} \delta_{ij} > 0$.

Assumption 3: If $U^i_{\rm k} \neq \mathbb{S}$ and $i \notin U^i_{\rm k}$, then $\hat{\pi}_{ij} - \delta_{ij} \geq 0$ $(\forall j \in U^i_{\rm k})$.

Remark 1: The above assumptions are reasonable, since they are the direct results from the properties of the TRs (e.g $\pi_{ij} \geq 0$ $(\forall i, j \in \mathbb{S}, i \neq j)$, and $\pi_{ii} = -\sum^{s}_{j=1, j\neq i} \pi_{ij} \leq 0$).

For the sake of simplicity, we write $E_i \triangleq E(r_t = i), A_i \triangleq A(r_t = i), \Delta A_i \triangleq \Delta A(r_t = i), B_i \triangleq B(r_t = i), \Delta B_i \triangleq \Delta B(r_t = i)$ for $\forall i \in \mathbb{S}$. Then system (1) can be described by

$$\begin{cases} E_i \dot{x}(t) = (A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)q(u(t)), \\ x_0 = x(0), \end{cases} \tag{4}$$

The following assumptions are assumed to be valid:

Assumption 4: The pair (A_i, B_i) is controllable, and the matrix B_i is full row rank for any $i \in \mathbb{S}$.

Remark 2: According to the property of full row rank matrices, there exists the following conclusion. For any matrix $B_i \in \mathbb{R}^{n\times m}$, B_i is full row rank, if and only if there exist full column rank matrix $\mathbb{X}_i \in \mathbb{R}^{m\times n}$ such that $B_i\mathbb{X}_i = I$. This conclusion will be utilized to design the state-feedback gain matrix in the following.

Assumption 5: $\Delta A_i = D_i\Lambda_i(t)F_i, \Delta B_i = B_iM_i\Xi_i(t)N_i$ and $\|M_i\Xi_i(t)N_i\|_\infty \leq \psi$, where $0 \leq \psi < 1$, parameter matrices D_i, F_i, M_i and N_i are known with appropriate dimensions, $\Lambda_i(t)$ and $\Xi_i(t)$ are time-varying uncertain matrices satisfying $\Lambda^{\rm T}_i(t)\Lambda_i(t) \leq I$ and $\Xi^{\rm T}_i(t)\Xi_i(t) \leq I$.

In addition, the quantizer $q(\cdot)$ is defined by an operator function $round(\cdot)$ that rounds towards the nearest integer, i.e. $q(u(t)) = \mu \cdot round(\frac{u(t)}{\mu})$. Where $\mu > 0$ is called a quantizing level of the quantizer. In computer-based control systems, the value of μ depends on the sampling accuracy and is known a priori. $q(\cdot)$ is the uniform quantizer with the fixed level μ. Define $e_\mu = q(u(t)) - u(t)$, since each component of e_μ is bounded by the half of the quantizing level μ thus we have $|e_\mu|_\infty \leq \frac{\mu}{2}$.

Remark 3: In the above description of the quantizer, we use a quantizing level μ to bound the error, which is relative simple, but can represent many kinds of quantizers. However, there are many different forms that are possible for a quantizer, and many-well known results have considered the quantized control problems, such as [30] adopted a different point of the feedback quantizer by introducing a positive integer and a "strictly causal" function which is continuous from the left everywhere and maintains a constant value on each interval. A symmetric quantizer is assumed, which is static and time-invariant, to stabilize the given systems or to achieve certain performance with the coarsest quantization density in [32]. In our future work, we will utilizing other kinds of quantizers [30, 32] to deal with this state-feedback control problem and its relative items, such as the filter design [35], H_∞ control design [36] and so on.

Our objective in this paper is to design a state feedback control law as

$$u(t) = K_ix(t) + u_{ic}, \tag{5}$$

such that the closed-loop system is stochastically stable, where $K_i > 0$ and u_{ic} will be designed in the following results. Then the quantizer $q(u(t))$ is rewritten as

$$q(u(t)) = u(t) + e_\mu = K_ix(t) + u_{ic} + e_\mu. \tag{6}$$

Remark 4: The nonlinear part of the controller u_{ic} in (5) is designed against the effect of signal quantization, and the linear part $K_ix(t)$ is proposed to deal with model uncertainties and unknown TRs for guaranteeing the stochastic stability.

Now, substituting (8) into the CUSMJLSs (6), the closed-loop dynamic system can be obtained as follows

$$\begin{aligned} E_ix(t) =& (A_i + \Delta A_i)x(t) \\ &+ (B_i + \Delta B_i)(K_ix(t) + u_{ic} + e_\mu). \end{aligned} \tag{7}$$

The following lemmas and definitions are useful in deriving the main results.

Definition 1: i) System (1) is called regular, if for any $i \in \mathbb{S}$, there exists a constant $z \in \mathbb{Z}$ such that $\det(zE_i - A_i) \neq 0$. ii) System (1) is called impulse-free, if $\deg\det(zE_i - A_i) = \text{rank}(E_i)$ for any $i \in \mathbb{S}$.

Definition 2: i) System (1) is said to be stochastically stable if there exists a constant $\varepsilon(x_0, r_0) > 0$, such that the following inequality holds

$$\mathbb{E}\{\int^{+\infty}_0 x^{\rm T}(t)x(t){\rm d}t \mid x_0, r_0\} < \varepsilon(x_0, r_0),$$

for any initial condition $x_0 \in \Re^n$ and $r_0 \in \mathbb{S}$. ii) System (1) is said to be stochastically admissible if it is regular, impulse-free and stochastically stable.

Lemma 1 [41]**:** Given a symmetric matrix Π and matrices M, N with appropriate dimensions, then $\Pi + MF(t)N + N^{\rm T}F^{\rm T}(t)M^{\rm T} < 0$ for all $F(t)$ satisfying $F^{\rm T}(t)F(t) \leq I$, if and only if there exists a scalar $\varepsilon > 0$ such that the following inequality holds: $\Pi + \varepsilon MM^{\rm T} + \varepsilon^{-1}N^{\rm T}N < 0$.

Lemma 2 [10]**:** System (1) is regular and stochastically stable if there exist nonsingular matrices $P(i)$, $\forall i \in \mathbb{S}$

such that

$$\begin{cases} E_i^{\mathrm{T}} P_i = P_i E_i \geq 0; \\ A_i^{\mathrm{T}} P_i + P_i A_i + \sum_{j=1}^{s} \pi_{ij} E_j^{\mathrm{T}} P_j < 0. \end{cases} \tag{8}$$

Lemma 3 [40]**:** Given any real number ε and any matrix Q, the matrix inequality $\varepsilon(Q+Q^{\mathrm{T}}) \leq \varepsilon^2 T + QT^{-1}Q^{\mathrm{T}}$ holds for any matrix $T > 0$.

Lemma 4 (Shur Complement [2])**:** Let $Q = Q^{\mathrm{T}}$, S and $R = R^{\mathrm{T}}$ be matrices of appropriate dimensions, then $R < 0$ and $Q - SR^{-1}S^{\mathrm{T}} < 0$ is equivalent to $\begin{bmatrix} Q & S \\ * & R \end{bmatrix} < 0$.

3. MAIN RESULTS

The purpose of this section is to study the stochastic stability problem for system (7) with GUTRs and input quantization. The controller is constructed as (5) by two parts. The nonlinear part u_{ic} is proposed to eliminate the effect of input quantization. The linear part $K_i x(t)$ is designed by solving several matrix inequalities for achieving the stochastic stability against model uncertainties and unknown transition rates. We also derive sufficient conditions of the regularity and stochastic stability for the CUSMJLSs (4) with (6).

Theorem 1: Consider CUSMJLSs (1) with GUTR (3) satisfying Assumption 1 to Assumption 5. For any $i, j, l \in \mathbb{S}$, if there exist constants $\varepsilon_i > 0$, $\sigma_i > 0$ and parameter matrices X_i, V_i, $T_{ijl} > 0$, $W_{ij} > 0$, $Q_{ij} > 0$; such that $E_i X_i = X_i E_i^{\mathrm{T}} \geq 0$ $(\forall i \in \mathbb{S})$ and one of the following three cases holds.

Case I: $U_{\mathrm{k}}^i \neq \mathbb{S}$, $i \in U_{\mathrm{k}}^i$ and $U_{\mathrm{k}}^i = \{k_1^i, \cdots, k_{m_i}^i\}$, the following inequalities (9) hold

$$\begin{cases} E_l X_l E_l^{\mathrm{T}} - E_j X_j E_j^{\mathrm{T}} \geq 0, \qquad \exists\, l \in U_{\mathrm{uk}}^i,\ \forall\, j \in U_{\mathrm{uk}}^i; \\ \begin{bmatrix} \Psi_{11}(i) & D_i & (E_iX_i+V_i\phi_i^{\mathrm{T}})F_i^{\mathrm{T}} & H_i^{\mathrm{T}}N_i^{\mathrm{T}} & B_iM_i & L_1(i) \\ * & -\frac{1}{\varepsilon_i}I & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon_i I & 0 & 0 & 0 \\ * & * & * & -\sigma_i I & 0 & 0 \\ * & * & * & * & -\frac{1}{\sigma_i}I & 0 \\ * & * & * & * & * & -L_2(i) \end{bmatrix} < 0, \end{cases} \tag{9}$$

where

$$\Psi_{11}(i) = A_i(E_iX_i + V_i\phi_i^{\mathrm{T}}) + (E_iX_i + V_i\phi_i^{\mathrm{T}})A_i^{\mathrm{T}} - H_i^{\mathrm{T}}B_i^{\mathrm{T}} - B_iH_i + \sum_{j \in U_{\mathrm{k}}^i} [\hat{\pi}_{ij}(E_jX_jE_j^{\mathrm{T}} - E_lX_lE_l^{\mathrm{T}}) + \frac{1}{4}\delta_{ij}^2 T_{ijl}],$$

$$L_1(i) = \left[E_{k_1^i}X_{k_1^i}E_{k_1^i}^{\mathrm{T}} - E_lX_lE_l^{\mathrm{T}}, \cdots, E_{k_{m_i}^i}X_{k_{m_i}^i}E_{k_{m_i}^i}^{\mathrm{T}} - E_lP_lE_l^{\mathrm{T}} \right],$$

$$L_2(i) = \mathrm{diag}\{T_{ik_1^i l}, \cdots, T_{ik_{m_i}^i l}\}.$$

Case II: $U_{\mathrm{k}}^i = \mathbb{S}$, the following inequalities (10) hold

$$\begin{bmatrix} \bar{\Psi}_{11}(i) & D_i & (E_iX_i+V_i\phi_i^{\mathrm{T}})F_i^{\mathrm{T}} & H_i^{\mathrm{T}}N_i^{\mathrm{T}} & B_iM_i & M_1(i) \\ * & -\frac{1}{\varepsilon_i}I & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon_i I & 0 & 0 & 0 \\ * & * & * & -\sigma_i I & 0 & 0 \\ * & * & * & * & -\frac{1}{\sigma_i}I & 0 \\ * & * & * & * & * & -M_2(i) \end{bmatrix} < 0, \tag{10}$$

where

$$\bar{\Psi}_{11}(i) = A_i(E_iX_i + V_i\phi_i^{\mathrm{T}}) + (E_iX_i + V_i\phi_i^{\mathrm{T}})A_i^{\mathrm{T}} - H_i^{\mathrm{T}}B_i^{\mathrm{T}} - B_iH_i + \sum_{j=1, j\neq i}^{s} [\hat{\pi}_{ij}(E_jX_jE_j^{\mathrm{T}} - E_iX_iE_i^{\mathrm{T}}) + \frac{1}{4}\delta_{ij}^2 W_{ij}],$$

$$M_1(i) = \big[E_1X_1E_1^{\mathrm{T}} - E_iX_iE_i^{\mathrm{T}}, \cdots, E_{i-1}X_{i-1}E_{i-1}^{\mathrm{T}} - E_iX_iE_i^{\mathrm{T}}, E_{i+1}X_{i+1}E_{i+1}^{\mathrm{T}} - E_iX_iE_i^{\mathrm{T}}, \cdots, E_sX_sE_s^{\mathrm{T}} - E_iP_iE_i^{\mathrm{T}}\big],$$

$$M_2(i) = \mathrm{diag}\{W_{i1}, \cdots, W_{i(i-1)}, W_{i(i+1)}, \cdots, W_{is}\}.$$

Case III: $U_{\mathrm{k}}^i \neq \mathbb{S}$, $i \notin U_{\mathrm{k}}^i$ and $U_{\mathrm{k}}^i = \{k_1^i, \cdots, k_{m_i}^i\}$, the following inequalities (11) hold

$$\begin{cases} E_i X_i E_i^{\mathrm{T}} - E_j X_j E_j^{\mathrm{T}} \geq 0, \qquad \forall\, j \in U_{\mathrm{uk}}^i; \\ \begin{bmatrix} \tilde{\Psi}_{11}(i) & D_i & (E_iX_i+V_i\phi_i^{\mathrm{T}})F_i^{\mathrm{T}} & H_i^{\mathrm{T}}N_i^{\mathrm{T}} & B_iM_i & N_1(i) \\ * & -\frac{1}{\varepsilon_i}I & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon_i I & 0 & 0 & 0 \\ * & * & * & -\sigma_i I & 0 & 0 \\ * & * & * & * & -\frac{1}{\sigma_i}I & 0 \\ * & * & * & * & * & -N_2(i) \end{bmatrix} < 0, \end{cases} \tag{11}$$

where

$$\tilde{\Psi}_{11}(i) = A_i(E_iX_i + V_i\phi_i^{\mathrm{T}}) + (E_iX_i + V_i\phi_i^{\mathrm{T}})A_i^{\mathrm{T}} - H_i^{\mathrm{T}}B_i^{\mathrm{T}} - B_iH_i + \sum_{j \in U_{\mathrm{k}}^i} [\hat{\pi}_{ij}(E_jX_jE_j^{\mathrm{T}} - E_iX_iE_i^{\mathrm{T}}) + \frac{1}{4}\delta_{ij}^2 Q_{ij}],$$

$$N_1(i) = \left[E_{k_1^i}X_{k_1^i}E_{k_1^i}^{\mathrm{T}} - E_iX_iE_i^{\mathrm{T}}, \cdots, E_{k_{m_i}^i}X_{k_{m_i}^i}E_{k_{m_i}^i}^{\mathrm{T}} - E_iP_iE_i^{\mathrm{T}} \right],$$

$$N_2(i) = \mathrm{diag}\{Q_{ik_1^i}, \cdots, Q_{ik_{m_i}^i}\}.$$

Then the CUSMJLS (1) is regular and stochastically stable with GUTR (3) and the quantizer (8) designed as

$$\begin{cases} u(t) = K_i x(t) + u_{ic}, \quad q(u(t)) = u(t) + e_\mu, \\ K_i = -H_i(E_iX_i + V_i\phi_i^{\mathrm{T}})^{-1}, \end{cases} \tag{12}$$

where $\phi_i \in \mathbb{R}^{n\times(n-r)}$ is full column rank satisfying $E_i\phi_i = 0$, $|e_\mu|_\infty \leq \frac{\mu}{2}$, $0 \leq \psi < 1$ and $u_{ic} = -\frac{\mu(1+\psi)}{2(1-\psi)}\mathrm{sgn}(B_i^{\mathrm{T}}(E_iX_i + V_i\phi_i^{\mathrm{T}})^{-1}x(t))$.

Proof: Firstly, we choose parameter matrices $P_i^{-1} > 0$ for any $i \in \mathbb{S}$ such that $E_i^{\mathrm{T}}P_i^{-1} = P_i^{-1}E_i \geq 0$. Then taking the Lyapunov function candidate $V(t) = x^{\mathrm{T}}(t)E_i^{\mathrm{T}}P_i^{-1}x(t)$ for (9), the weak infinitesimal operator $\mathfrak{J}_a^x[\cdot]$ of the process $\{x(t), r_t, t \geq 0\}$ for plant (7) at the point $\{t, x, i\}$ is given by

$$\begin{aligned} \mathfrak{J}_a^x[V] &= \dot{x}^{\mathrm{T}}(t)E_i^{\mathrm{T}}P_i^{-1}x(t) + x^{\mathrm{T}}(t)P_i^{-1}E_i\dot{x}(t) \\ &\quad + x^{\mathrm{T}}(t)(\sum_{j=1}^{s}\pi_{ij}E_j^{\mathrm{T}}P_j^{-1})x(t) \\ &= x^{\mathrm{T}}(t)\Big\{[A_i + (B_i + \Delta B_i)K_i]^{\mathrm{T}}P_i^{-1} + P_i^{-1}[A_i \\ &\quad + (B_i + \Delta B_i)K_i]\Big\}x(t) + x^{\mathrm{T}}(t)\Delta A_i^{\mathrm{T}}(t)P_i^{-1}x(t) \\ &\quad + x^{\mathrm{T}}(t)P_i^{-1}\Delta A_i x(t) + (u_{ic} + e_\mu)^{\mathrm{T}}(B_i \end{aligned}$$

$$+\Delta B_i)^{\mathrm{T}}P_i^{-1}x(t)+x^{\mathrm{T}}(t)P_i^{-1}(B_i+\Delta B_i)(u_{ic}+e_\mu)+x^{\mathrm{T}}(t)(\sum_{j=1}^{s}\pi_{ij}E_j^{\mathrm{T}}P_j^{-1})x(t). \tag{13}$$

According to the design of the quantizer as (12) and Assumption 5, one can obtain that

$$\begin{aligned}&(u_{ic}+e_\mu)^{\mathrm{T}}(B_i+\Delta B_i)^{\mathrm{T}}P_i^{-1}x(t)+x^{\mathrm{T}}(t)P_i^{-1}(B_i\\&\quad+\Delta B_i)(u_{ic}+e_\mu)\\&=-2\cdot\frac{\mu(1+\psi)}{2(1-\psi)}|B_i^{\mathrm{T}}P_i^{-1}x(t)|_1+e_\mu^{\mathrm{T}}B_i^{\mathrm{T}}P_i^{-1}x(t)\\&\quad+x^{\mathrm{T}}(t)P_i^{-1}B_ie_\mu+(u_{ic}+e_\mu)^{\mathrm{T}}N_i^{\mathrm{T}}\Xi_i^{\mathrm{T}}(t)M_i^{\mathrm{T}}B_i^{\mathrm{T}}\\&\quad\times P_i^{-1}x(t)+x^{\mathrm{T}}(t)P_i^{-1}B_iM_i\Xi_i(t)N_i(u_{ic}+e_\mu)\\&\le[-\frac{\mu(1+\psi)}{1-\psi}+2|e_\mu|_\infty+2\|N_i^{\mathrm{T}}\Xi_i^{\mathrm{T}}(t)M_i^{\mathrm{T}}\|_\infty\\&\quad\times(|u_{ic}|_\infty+|e_\mu|_\infty)]|B_i^{\mathrm{T}}P_i^{-1}x(t)|_1\\&\le\left[-\frac{\mu(1+\psi)}{1-\psi}+2\cdot\frac{\mu}{2}+2\psi\cdot(\frac{\mu(1+\psi)}{2(1-\psi)}\right.\\&\quad\left.+\frac{\mu}{2})\right]\cdot|B_i^{\mathrm{T}}P_i^{-1}x(t)|_1=0.\end{aligned} \tag{14}$$

Substituting (14) into (13), by Assumption 5 we have

$$\begin{aligned}\mathfrak{J}_a^x[V]&\le x^{\mathrm{T}}(t)\Big\{[A_i+(B_i+\Delta B_i)K_i]^{\mathrm{T}}P_i^{-1}+P_i^{-1}[A_i\\&+(B_i+\Delta B_i)K_i]\Big\}x(t)+x^{\mathrm{T}}(t)F_i^{\mathrm{T}}\Lambda_i^{\mathrm{T}}(t)D_i^{\mathrm{T}}P_i^{-1}x(t)\\&+x^{\mathrm{T}}(t)P_i^{-1}D_i\Lambda_i(t)F_ix(t)+x^{\mathrm{T}}(t)(\sum_{j=1}^{s}\pi_{ij}E_j^{\mathrm{T}}P_j^{-1})x(t).\end{aligned}$$

Secondly, according to Lemma 2, we know that the system (9) will be stochastically stable when it satisfies that $\mathfrak{J}_a^x[V]<0$, that is to say, the following inequality holds

$$\begin{aligned}&[A_i+(B_i+\Delta B_i)K_i]^{\mathrm{T}}P_i^{-1}+P_i^{-1}[A_i+(B_i\\&+\Delta B_i)K_i]+F_i^{\mathrm{T}}\Lambda_i^{\mathrm{T}}(t)D_i^{\mathrm{T}}P_i^{-1}+P_i^{-1}D_i\Lambda_i(t)F_i\\&+\sum_{j=1}^{s}\pi_{ij}E_j^{\mathrm{T}}P_j^{-1}<0.\end{aligned} \tag{15}$$

According to Lemma 2 in [42] and Definition 6 in [43], we can deduce that the above inequality (17) is equivalent to

$$\begin{aligned}&P_i[A_i+(B_i+\Delta B_i)K_i]^{\mathrm{T}}+[A_i+(B_i+\Delta B_i)K_i]P_i\\&+P_iF_i^{\mathrm{T}}\Lambda_i^{\mathrm{T}}(t)D_i^{\mathrm{T}}+D_i\Lambda_i(t)F_iP_i\\&+\sum_{j=1}^{s}\pi_{ij}P_jE_j^{\mathrm{T}}<0.\end{aligned} \tag{16}$$

By setting $P_i=E_iX_i+V_i\phi_i^{\mathrm{T}}$ ($\forall i\in\mathbb{S}$) in (18) where $E_i\phi_i=0$, it is easy to obtain that

$$P_iE_i^{\mathrm{T}}=E_iP_i^{\mathrm{T}}=E_iX_iE_i^{\mathrm{T}}\ge 0, \tag{17}$$

$$\begin{aligned}&(E_iX_i+V_i\phi_i^{\mathrm{T}})A_i^{\mathrm{T}}+A_i(E_iX_i+V_i\phi_i^{\mathrm{T}})+(E_iX_i\\&+V_i\phi_i^{\mathrm{T}})K_i^{\mathrm{T}}(B_i+\Delta B_i)^{\mathrm{T}}+(B_i+\Delta B_i)K_i(E_iX_i\\&+V_i\phi_i^{\mathrm{T}})+(E_iX_i+V_i\phi_i^{\mathrm{T}})F_i^{\mathrm{T}}\Lambda_i^{\mathrm{T}}(t)D_i^{\mathrm{T}}+D_i\Lambda_i(t)\\&\cdot F_i(E_iX_i+V_i\phi_i^{\mathrm{T}})+\sum_{j=1}^{s}\pi_{ij}E_jX_jE_j^{\mathrm{T}}<0.\end{aligned} \tag{18}$$

Define that $K_i=-H_i(E_iX_i+V_i\phi_i^{\mathrm{T}})^{-1}$. Substituting K_i into (20), it finally holds that

$$\begin{aligned}&(E_iX_i+V_i\phi_i^{\mathrm{T}})A_i^{\mathrm{T}}+A_i(E_iX_i+V_i\phi_i^{\mathrm{T}})-H_i^{\mathrm{T}}(B_i\\&+B_iM_i\Xi_i(t)N_i)^{\mathrm{T}}-(B_i+B_iM_i\Xi_i(t)N_i)H_i\\&+(E_iX_i+V_i\phi_i^{\mathrm{T}})F_i^{\mathrm{T}}\Lambda_i^{\mathrm{T}}(t)D_i^{\mathrm{T}}+D_i\Lambda_i(t)F_i(E_iX_i\\&+V_i\phi_i^{\mathrm{T}})+\sum_{j=1}^{s}\pi_{ij}E_jX_jE_j^{\mathrm{T}}<0.\end{aligned} \tag{19}$$

Now using Lemma 1, there exist constants $\varepsilon_i>0$ and $\sigma_i>0$ such that (21) could be amplified that

$$\begin{aligned}&(E_iX_i+V_i\phi_i^{\mathrm{T}})A_i^{\mathrm{T}}+A_i(E_iX_i+V_i\phi_i^{\mathrm{T}})-H_i^{\mathrm{T}}B_i^{\mathrm{T}}\\&-B_iH_i+\frac{1}{\sigma_i}H_i^{\mathrm{T}}N_i^{\mathrm{T}}N_iH_i+\sigma_iB_iM_iM_i^{\mathrm{T}}B_i^{\mathrm{T}}\\&+\varepsilon_iD_iD_i^{\mathrm{T}}+\frac{1}{\varepsilon_i}(E_iX_i+V_i\phi_i^{\mathrm{T}})F_i^{\mathrm{T}}F_i(E_iX_i+V_i\phi_i^{\mathrm{T}})\\&+\sum_{j=1}^{s}\pi_{ij}E_jX_jE_j^{\mathrm{T}}<0.\end{aligned} \tag{20}$$

Lastly, three cases should be considered to discuss the last part $\sum_{j=1}^{s}\pi_{ij}E_jX_jE_j^{\mathrm{T}}$ in (20). And we will prove that (20) could be guaranteed by inequalities (9)-(11) in Theorem 1, respectively.

Case I: $U_{\mathrm{k}}^i=\mathbb{S}$ and $i\in U_{\mathrm{k}}^i$. If the second condition holds of (9), that is to say, there exist $l\in U_{\mathrm{uk}}^i$ such that $E_jX_jE_j^{\mathrm{T}}\le E_lX_lE_l^{\mathrm{T}}$ ($\forall j\in U_{\mathrm{uk}}^i$), then we have

$$\begin{aligned}\sum_{j=1}^{s}\pi_{ij}E_jX_jE_j^{\mathrm{T}}&=\sum_{j\in U_{\mathrm{k}}^i}\pi_{ij}E_jX_jE_j^{\mathrm{T}}+\sum_{j\in U_{\mathrm{uk}}^i}\pi_{ij}E_jX_jE_j^{\mathrm{T}}\\&\le\sum_{j\in U_{\mathrm{k}}^i}\pi_{ij}E_jX_jE_j^{\mathrm{T}}-\sum_{j\in U_{\mathrm{k}}^i}\pi_{ij}E_lX_lE_l^{\mathrm{T}}=\sum_{j\in U_{\mathrm{k}}^i}\hat{\pi}_{ij}(E_jX_jE_j^{\mathrm{T}}\\&-E_lX_lE_l^{\mathrm{T}})+\sum_{j\in U_{\mathrm{k}}^i}\Delta_{ij}(E_jX_jE_j^{\mathrm{T}}-E_lX_lE_l^{\mathrm{T}}).\end{aligned}$$

And using Lemma 3, there exists parameter matrices $T_{ijl}>0$ ($j\in U_{\mathrm{k}}^i$ and $j\neq i$) such that

$$\begin{aligned}&\sum_{j\in U_{\mathrm{k}}^i}\Delta_{ij}(E_jX_jE_j^{\mathrm{T}}-E_lX_lE_l^{\mathrm{T}})\\&=\sum_{j\in U_{\mathrm{k}}^i}[\frac{1}{2}\Delta_{ij}\sum_{j\in U_{\mathrm{k}}^i}\Delta_{ij}(E_jX_jE_j^{\mathrm{T}}-E_lX_lE_l^{\mathrm{T}})\\&\quad+\frac{1}{2}\Delta_{ij}\sum_{j\in U_{\mathrm{k}}^i}\Delta_{ij}(E_jX_jE_j^{\mathrm{T}}-E_lX_lE_l^{\mathrm{T}})]\\&\le\sum_{j\in U_{\mathrm{k}}^i}[\frac{1}{4}\delta_{ij}^2T_{ijl}+(E_jX_jE_j^{\mathrm{T}}-E_lX_lE_l^{\mathrm{T}})T_{ijl}^{-1}\cdot\\&\quad(E_jX_jE_j^{\mathrm{T}}-E_lX_lE_l^{\mathrm{T}})].\end{aligned} \tag{21}$$

Substituting (21) into (20), there is

$$\begin{aligned}\Phi_i \triangleq & (E_iX_i+V_i\phi_i^{\mathrm{T}})A_i^{\mathrm{T}}+A_i(E_iX_i+V_i\phi_i^{\mathrm{T}}) \\ & -H_i^{\mathrm{T}}B_i^{\mathrm{T}}-B_iH_i+\frac{1}{\sigma_i}H_i^{\mathrm{T}}N_i^{\mathrm{T}}N_iH_i \\ & +\sigma_iB_iM_iM_i^{\mathrm{T}}B_i^{\mathrm{T}}+\varepsilon_iD_iD_i^{\mathrm{T}}+\frac{1}{\varepsilon_i}(E_iX_i \\ & +V_i\phi_i^{\mathrm{T}})F_i^{\mathrm{T}}F_i(E_iX_i+V_i\phi_i^{\mathrm{T}})+\sum_{j\in U_{\mathrm{k}}^i}\hat{\pi}_{ij}(E_jX_jE_j^{\mathrm{T}} \\ & -E_lX_lE_l^{\mathrm{T}})+\sum_{j\in U_{\mathrm{k}}^i}\frac{1}{4}\delta_{ij}^2T_{ijl}+\sum_{j\in U_{\mathrm{k}}^i}(E_jX_jE_j^{\mathrm{T}} \\ & -E_lX_lE_l^{\mathrm{T}})T_{ijl}^{-1}(E_jX_jE_j^{\mathrm{T}}-E_lX_lE_l^{\mathrm{T}})<0. \end{aligned} \tag{22}$$

It can be shown that if all the inequalities of (9) are satisfied, the condition $\Phi_i<0$ holds by Lemma 4. Thus, the inequality (20) is guaranteed.

Case II: $U_{\mathrm{k}}^i=\mathbb{S}$. By Assumption 2 and Lemma 3, there must be parameter matrices $W_{ij}>0$ $(j\neq i)$ such that

$$\begin{aligned}\sum_{j=1}^{s}\pi_{ij}E_jX_jE_j^{\mathrm{T}} = & \sum_{j=1,j\neq i}^{s}\pi_{ij}E_jX_jE_j^{\mathrm{T}}+\pi_{ii}E_iX_iE_i^{\mathrm{T}} \\ \leq & \sum_{j=1,j\neq i}^{s}\hat{\pi}_{ij}(E_jX_jE_j^{\mathrm{T}}-E_iX_iE_i^{\mathrm{T}})+\sum_{j=1,j\neq i}^{s}\frac{1}{4}\delta_{ij}^2W_{ij} \\ & +\sum_{j=1,j\neq i}^{s}(E_jX_jE_j^{\mathrm{T}}-E_iX_iE_i^{\mathrm{T}})W_{ij}^{-1}(E_jX_jE_j^{\mathrm{T}}-E_iX_iE_i^{\mathrm{T}}).\end{aligned}$$

Similarly, substituting the above inequality into (20), let

$$\begin{aligned}\bar{\Phi}_i \triangleq & (E_iX_i+V_i\phi_i^{\mathrm{T}})A_i^{\mathrm{T}}+A_i(E_iX_i+V_i\phi_i^{\mathrm{T}}) \\ & -H_i^{\mathrm{T}}B_i^{\mathrm{T}}-B_iH_i+\frac{1}{\sigma_i}H_i^{\mathrm{T}}N_i^{\mathrm{T}}N_iH_i \\ & +\sigma_iB_iM_iM_i^{\mathrm{T}}B_i^{\mathrm{T}}+\varepsilon_iD_iD_i^{\mathrm{T}}+\frac{1}{\varepsilon_i}(E_iX_i \\ & +V_i\phi_i^{\mathrm{T}})F_i^{\mathrm{T}}F_i(E_iX_i+V_i\phi_i^{\mathrm{T}}) \\ & +\sum_{j=1,j\neq i}^{s}\hat{\pi}_{ij}(E_jX_jE_j^{\mathrm{T}}-E_iX_iE_i^{\mathrm{T}}) \\ & +\sum_{j=1,j\neq i}^{s}\frac{1}{4}\delta_{ij}^2W_{ij}+\sum_{j=1,j\neq i}^{s}(E_jX_jE_j^{\mathrm{T}} \\ & -E_iX_iE_i^{\mathrm{T}})W_{ij}^{-1}(E_jX_jE_j^{\mathrm{T}}-E_iX_iE_i^{\mathrm{T}})<0.\end{aligned} \tag{23}$$

It also can be shown that all the inequalities of (10) are satisfied, and then $\bar{\Phi}_i<0$ holds by Lemma 4. Thus (20) is also guaranteed.

Case III: $U_{\mathrm{k}}^i\neq\mathbb{S}$ and $i\notin U_{\mathrm{k}}^i$. If the condition $E_iX_iE_i^{\mathrm{T}}\leq E_jX_jE_j^{\mathrm{T}}$ $(\forall j\in U_{\mathrm{uk}}^i)$ holds in Theorem 1, the following inequality can be derived with parameter matrices $Q_{ij}>0$ $(j\in U_{\mathrm{k}}^i)$ by Assumption (3) and Lemma 3

$$\begin{aligned}\sum_{j=1}^{s}\pi_{ij}E_jX_jE_j^{\mathrm{T}} = & \sum_{j\in U_{\mathrm{k}}^i}\pi_{ij}E_jX_jE_j^{\mathrm{T}}+\sum_{j\in U_{\mathrm{uk}}^i,j\neq i}\pi_{ij}E_jX_jE_j^{\mathrm{T}} \\ & +(-\sum_{j\in U_{\mathrm{k}}^i}\pi_{ij}-\sum_{j\in U_{\mathrm{uk}}^i,j\neq i}\pi_{ij})E_iX_iE_i^{\mathrm{T}} \\ \leq & \sum_{j\in U_{\mathrm{k}}^i}\pi_{ij}(E_jX_jE_j^{\mathrm{T}}-E_iX_iE_i^{\mathrm{T}}) \\ \leq & \sum_{j\in U_{\mathrm{k}}^i}\hat{\pi}_{ij}(E_jX_jE_j^{\mathrm{T}}-E_iX_iE_i^{\mathrm{T}})+\sum_{j\in U_{\mathrm{k}}^i}\frac{1}{4}\delta_{ij}^2Q_{ij} \\ & +\sum_{j\in U_{\mathrm{k}}^i}(E_jX_jE_j^{\mathrm{T}}-E_iX_iE_i^{\mathrm{T}})Q_{ij}^{-1}(E_jX_jE_j^{\mathrm{T}}-E_iX_iE_i^{\mathrm{T}}).\end{aligned}$$

Substituting it into (20), let

$$\begin{aligned}\tilde{\Phi}_i \triangleq & (E_iX_i+V_i\phi_i^{\mathrm{T}})A_i^{\mathrm{T}}+A_i(E_iX_i+V_i\phi_i^{\mathrm{T}}) \\ & -H_i^{\mathrm{T}}B_i^{\mathrm{T}}-B_iH_i+\frac{1}{\sigma_i}H_i^{\mathrm{T}}N_i^{\mathrm{T}}N_iH_i \\ & +\sigma_iB_iM_iM_i^{\mathrm{T}}B_i^{\mathrm{T}}+\varepsilon_iD_iD_i^{\mathrm{T}}+\frac{1}{\varepsilon_i}(E_iX_i \\ & +V_i\phi_i^{\mathrm{T}})F_i^{\mathrm{T}}F_i(E_iX_i+V_i\phi_i^{\mathrm{T}})+\sum_{j\in U_{\mathrm{k}}^i}\hat{\pi}_{ij}(E_jX_jE_j^{\mathrm{T}} \\ & -E_iX_iE_i^{\mathrm{T}})+\sum_{j\in U_{\mathrm{k}}^i}\frac{1}{4}\delta_{ij}^2Q_{ij}+\sum_{j\in U_{\mathrm{k}}^i}(E_jX_jE_j^{\mathrm{T}} \\ & -E_iX_iE_i^{\mathrm{T}})Q_{ij}^{-1}(E_jX_jE_j^{\mathrm{T}}-E_iX_iE_i^{\mathrm{T}})<0.\end{aligned} \tag{24}$$

According to all the inequalities of (11), $\tilde{\Phi}_i<0$ holds by Lemma 4. Thus (20) is also guaranteed.

Above all, if all the inequalities (9)-(11) hold, we conclude that CUSMJLSs (6) with GUTR (3) and input quantizer (14) is regular and stochastically stable by Lemma 2. □

Remark 5: Different from the description of SMJSs in [39], the singular matrix $E(r_t)$ in the system (1) is also dependent on the mode jumping process $\{r_t,t\geq 0\}$. Thus SMJS (1) is also called as the mode-dependent SMJS. Compared with the controller design of [38] and [39], we give a new-type design of the gain matrix K_i as (7). And the parameter β_i of K_i could be easily obtained when solving matrix inequalities (11)-(13), which is less conservative.

4. NUMERICAL EXAMPLE

In this section, a numerical example demonstrates the effectiveness of the method mentioned above. Consider a CUSMJLSs as (6) with 3 modes $\mathbb{S}=\{1,2,3\}$ and the GUTR as

$$\Pi=\begin{bmatrix}-3.2+\Delta_{11} & ? & ? \\ 2.4+\Delta_{21} & -4+\Delta_{22} & 1.6+\Delta_{23} \\ ? & 1.7+\Delta_{32} & ?\end{bmatrix},$$

where $\Delta_{11},\Delta_{21}\in[-0.02,0.02]$; $\Delta_{22},\Delta_{32}\in[-0.1,0.1]$; $\Delta_{23}\in[-0.08,0.08]$, and $\Delta=0.2$. The initial condition is $x_0^{\mathrm{T}}=\begin{bmatrix}-1.5 & 1 & -2\end{bmatrix}$. The parameters are as follows.

When $r_t=i=1$, there are

$$E_1=\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix},\ A_1=\begin{bmatrix}6&0&-3\\1&5&12\\-1&-2.1&5\end{bmatrix},$$

$$B_1=\begin{bmatrix}-2.5 & 9\\ 1.5 & 0\\ 0 & -3.6\end{bmatrix}, D_1=\begin{bmatrix}-0.12\\ 0.18\\ -0.5\end{bmatrix}, M_1=\begin{bmatrix}0.2\\ -1\end{bmatrix},$$
$$F_1=\begin{bmatrix}0.45 & -0.8 & 0.4\end{bmatrix},\ N_1=\begin{bmatrix}0.8 & 1.6\end{bmatrix},$$

and $\Lambda_1(t)=0.75\sin(t)$, $\Xi_1(t)=0.5\cos(2t)$.

When $r_t=i=2$, there are

$$E_2=\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix},\ A_2=\begin{bmatrix}-5&3&9\\2&8&0\\-9&1&-36\end{bmatrix},$$
$$B_2=\begin{bmatrix}-9&9\\-1&2\\1&-5\end{bmatrix}, D_2=\begin{bmatrix}-0.1\\0.2\\0.5\end{bmatrix}, M_2=\begin{bmatrix}-3\\-1.2\end{bmatrix},$$
$$F_2=\begin{bmatrix}0.15 & -2.1 & 0.1\end{bmatrix},\ N_2=\begin{bmatrix}0.2 & 0.6\end{bmatrix},$$

and $\Lambda_2(t)=0.25$, $\Xi_2(t)=0.6$.

When $r_t=i=3$, there are

$$E_3=\begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix},\ A_3=\begin{bmatrix}6&-12&-6\\3&2.4&0\\-6&-9&-15\end{bmatrix},$$
$$B_3=\begin{bmatrix}-1&6\\5&0.2\\10&-21\end{bmatrix}, D_3=\begin{bmatrix}-0.3\\-0.1\\-0.5\end{bmatrix}, M_3=\begin{bmatrix}1.2\\1\end{bmatrix},$$
$$F_3=\begin{bmatrix}0.24 & -0.2 & 0.2\end{bmatrix},\ N_3=\begin{bmatrix}-0.5 & 0.5\end{bmatrix},$$

and $\Lambda_3(t)=0.8$, $\Xi_2(t)=0.4$.

Firstly, assume the positive scalar $\mu=0.64$ such that $|e_\mu|_\infty\le\frac{\mu}{2}$. And it is easy to know that $\psi=0.55$ by above parameter uncertainties. Then, according to the condition $E_i\phi_i=0,\ i=1,2,3$, we could choose that

$$\psi_1=\psi_2=\begin{bmatrix}0\\0\\1\end{bmatrix},\ \ \psi_3=\begin{bmatrix}0&0\\1&0\\0&1\end{bmatrix},\ \ V_1=V_2=\begin{bmatrix}0\\0\\0.2\end{bmatrix}$$

and

$$V_3=\begin{bmatrix}0&0\\2&0\\0&2\end{bmatrix}.$$

Secondly, we testify that the CUSMJLS (6) with (14) is regular and stochastically stable based on Theorem 1. By the condition $X_iE_i^{\mathrm{T}}=E_iX_i\ge 0\ (i=1,2,3)$ in Theorem 1, it is easy to know that

$$X_1=\begin{bmatrix}X_{111}&X_{112}&0\\X_{112}&X_{122}&0\\0&0&X_{133}\end{bmatrix},\ \ X_2=\begin{bmatrix}X_{211}&X_{212}&0\\X_{212}&X_{222}&0\\0&0&X_{233}\end{bmatrix}$$

and

$$X_3=\begin{bmatrix}X_{311}&0&0\\0&X_{322}&X_{323}\\0&X_{323}&X_{333}\end{bmatrix}.$$

Now let $\varepsilon_1=0.24$, $\varepsilon_2=1.5$, $\varepsilon_3=3$, $\sigma_1=16$, $\sigma_2=0.6$ and $\sigma_3=2$. Utilizing the LMI toolbox of Matlab, we could solve the matrix inequalities (11)-(13) and obtain the following results

$$X_1=\begin{bmatrix}1.6930&-1.3822&0\\-1.3822&9.9809&0\\0&0&25.3911\end{bmatrix},$$
$$X_2=\begin{bmatrix}15.7035&1.2075&0\\1.2075&1.9262&0\\0&0&0.9796\end{bmatrix},$$
$$X_3=\begin{bmatrix}5.4190&0&0\\0&2.1716&0.3899\\0&0.3899&0.1394\end{bmatrix},$$
$$H_1=10^3\cdot\begin{bmatrix}-1.0943&0.2345&0.2814\\0.5472&-0.1173&-0.1407\end{bmatrix},$$
$$H_2=10^3\cdot\begin{bmatrix}-9.5141&-1.3214&2.1142\\3.1714&0.4405&-0.7047\end{bmatrix},$$
$$H_3=10^3\cdot\begin{bmatrix}0.5319&0.5532&-1.1700\\0.5319&0.5532&-1.1701\end{bmatrix},$$
$$T_{113}=10^4\cdot\begin{bmatrix}2.8444&-0.6095&-0.7314\\-0.6095&0.1307&0.1567\\-0.7314&0.1567&0.1881\end{bmatrix},$$
$$W_{21}=10^4\cdot\begin{bmatrix}2.9464&0.4092&-0.6548\\0.4092&0.6568&-0.0909\\-0.6548&-0.0909&3.1455\end{bmatrix},$$
$$W_{23}=10^4\cdot\begin{bmatrix}2.9457&0.4091&-0.6546\\0.4091&0.6568&-0.0909\\-0.6546&-0.0909&0.1455\end{bmatrix},$$
$$Q_{32}=10^4\cdot\begin{bmatrix}0.5047&0.6774&-0.9097\\0.6774&1.4185&-0.5520\\-0.9097&-0.5520&2.5194\end{bmatrix},$$

where $l=3$ satisfying $E_3X_3E_3^{\mathrm{T}}-E_1X_1E_1^{\mathrm{T}}\ge 0$ and $E_3X_3E_3^{\mathrm{T}}-E_2X_2E_2^{\mathrm{T}}\ge 0$.

Lastly, we design the quantizer $q(u(t))$ in Theorem 1 as follows

$$\begin{cases} q(u(t))=10^3\cdot\begin{bmatrix}0.7071&0.0744&-1.4070\\-0.3536&-0.0372&0.7035\end{bmatrix}\\ \quad\cdot x(t)-1.1022\mathrm{sgn}(B_1^{\mathrm{T}}(E_1X_1+V_1\phi_1^{\mathrm{T}})^{-1}x(t))+e_\mu;\\ q(u(t))=10^3\cdot\begin{bmatrix}0.0581&0.0322&-1.0571\\-0.0194&-0.0107&0.3524\end{bmatrix}\\ \quad\cdot x(t)-1.1022\mathrm{sgn}(B_2^{\mathrm{T}}(E_2X_2+V_2\phi_2^{\mathrm{T}})^{-1}x(t))+e_\mu;\\ q(u(t))=\begin{bmatrix}-98.1546&-276.6000&585.0000\\-98.1546&-276.6000&585.0500\end{bmatrix}\\ \quad\cdot x(t)-1.1022\mathrm{sgn}(B_3^{\mathrm{T}}(E_3X_3+V_3\phi_3^{\mathrm{T}})^{-1}x(t))+e_\mu. \end{cases}$$

Above all, we know that there exist appropriate parameter matrices $X_i,H_i\ (i=1,2,3)$, T_{113}, W_{21}, W_{23} and V_{32} such

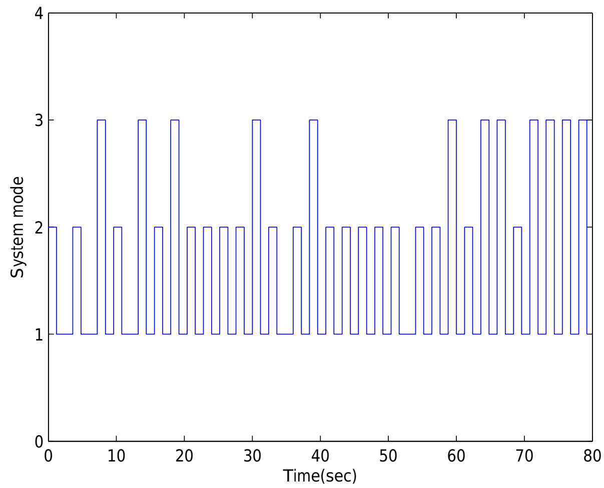


Fig. 1. System switching modes.

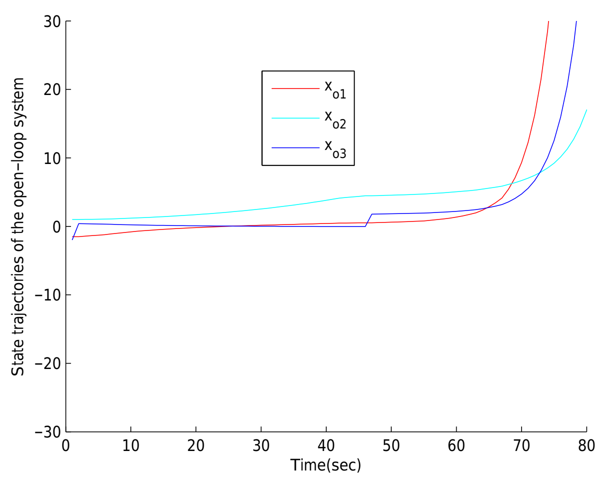


Fig. 2. Open-loop system state trajectories.

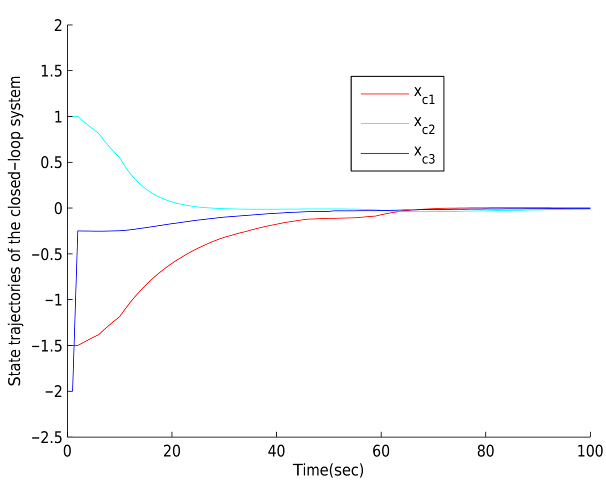


Fig. 3. Closed-loop system state trajectories with stabilising input quantizer.

that all the inequalities in (11)-(13) hold in Theorem 1. Therefore, this numerical example is stochastically stable according to the above result.

The switching mode, open-loop system state trajectories and closed-loop system state trajectories are presented in Figs. 1-3. Moreover, Fig. 1 shows a possible system modes evolution which meets the GUTR given in this example. Fig. 2 shows that the open-loop system states are not stochastically stable. However, with the quantizer controller designed as (14), Fig. 3 depicts the state response curves of the closed-loop system. It can be seen that each state trajectory of the closed-loop system is stochastic stable in spite of system uncertainty and GUTR.

5. CONCLUSIONS

In this paper, the regularity and stochastic stability problem for a class of continuous-time uncertain singular Markovian jump linear systems with generally uncertain transition rates and input quantization have been investigated. The input quantization controller is constructed by two parts. The nonlinear part is proposed to eliminate the effect of input quantization. The linear part is designed by solving several inequality conditions against model uncertainties and generally uncertain transition rates. In comparison with the existing result in the literature, less conservativeness has been derived by introducing new relaxed inequality conditions. Finally, a numerical example is provided to show the effectiveness of the proposed results.

REFERENCES

[1] S. Xu and J. Lam, *Robust Control and Filtering of Singular Systems*, Springer-Verlag, New York, 2006.

[2] E. K. Boukas, *Stochastic Switching Systems: Analysis and Design*, Birkhauser, 2005.

[3] X. Liu and H. Xi, "Quasi-synchronization of Markovian jump complex heterogeneous networks with partly unknown transition rates," *Int. J. Control Autom.*, vol. 12, pp. 1336-1344, December 2014. [click]

[4] H. R. Karimi, "A sliding mode approach to H_∞ synchronization of master-slave time-delay systems with Markovian jumping parameters and nonlinear uncertainties," *J. Franklin Inst.*, vol. 349, no. 4, pp. 1480-1496, May 2012.

[5] Y. Kao, C. Wang, and L. Zhang, "Delay-dependent robust exponential stability of impulsive Markovian jumping reaction-diffusion Cohen-Grossberg neural networks," *Neural Process. Lett.*, vol. 38, no. 3, pp. 321-346, December 2013. [click]

[6] Y. Kao, J. Guo, C. Wang, and X. Sun, "Delay-dependent robust exponential stability of Markovian jumping reaction-diffusion Cohen-Grossberg neural networks with mixed delays," *J. Franklin Inst.*, vol. 349, no. 6, pp. 1972-1988, August 2012.

[7] Y. Kao, C. Wang, F. Zha, and H. Cao, "Stability in mean of partial variables for stochastic reaction-diffusion systems with Markovian switching," *J. Franklin Inst.*, vol. 351, no. 1, pp. 500-512, January 2014. [click]

[8] Y. Li and Y. Kao, "Stability of stochastic reaction-diffusion systems with Markovian switching and impulsive perturbations," *Math. Probl. Eng.*, vol. 2012, pp. 1-13, September 2012.

[9] Z. Chen and Q. Huang, "Exponential L_2-L_∞ filtering for a class of stochastic system with Markovian jump parameters and mixed mode-dependent time-delays," *Int. J. Control Autom.*, vol. 12, no. 3, pp. 552-563, February 2012.

[10] E. K. Boukas, *Control of Singular Systems with Random Abrupt Changes*, Springer, Berlin, 2008.

[11] Y. Ding, H. Zhu, S. Zhong and Y. Zhang, "L_2-L_∞ filtering for Markovian jump systems with time-varying delays and partly unknown transition probabilities," *Commun. Nonlinear Sci. Numer. Simulat.*, vol. 17, no. 7, pp. 3070-3081, July 2012. [click]

[12] S. Ma and E. K. Boukas, "A singular system approach to robust sliding mode control for uncertain Markov jump systems," *Automatica*, vol. 45, no. 11, pp. 2707-2713, November 2009. [click]

[13] G. Wang and Q. Zhang, "Robust control of uncertain singular stochastic systems with Markovian switching via proportional-derivative state feedback," *IET Control Theory Appl.*, vol. 8, no. 8, pp. 1089-1096, May 2011.

[14] J. Wang, H. Wang, A. Xue, and R. Lu, "Delay-dependent H_∞ control for singular Markovian jump systems with time delay," *Nonlinear Anal. Hybrid Syst.*, vol. 8, pp. 1-12, May 2013. [click]

[15] L. Wu, P. Shi, and H. Gao, "State estimation and sliding-mode control of Markovian jump singular systems," *IEEE Trans. Automat. Control*, vol. 55, no. 5, pp. 1213-1219, May 2010.

[16] Z. Wu, J. H. Park, H. Su, and J. Chu, "Stochastic stability analysis for discrete-time singular Markov jump systems with time-varying delay and piecewise-constant transition probabilities," *J. Franklin Inst.*, vol. 349, no. 9, pp. 2889-2902, November 2012. [click]

[17] L. Zhang and E. K. Boukas, "Stability and stabilization of Markovian jump linear systems with partly unknown transition probabilities," *Automatica*, vol. 45, no. 2, pp. 463-468, February 2009. [click]

[18] M. K. Song, J. B. Park, and Y. H. Joo, "Stability and stabilization for discrete-time Markovian jump fuzzy systems with time-varying delays: partially unknown transition probabilities case," *Int. J. Control Autom.*, vol. 11, no. 1, pp. 136-146, February 2013. [click]

[19] M. Karan, P. Shi, and C. Kaya, "Transition probability bounds for the stochastic stability robustness of continuous- and discrete-time Markovian jump linear systems," *Automatica*, vol. 42, no. 12, pp. 2159-2168, December 2006. [click]

[20] L. Zhang and J. Lam, "Necessary and sufficient cnditions for analysis and synthesis of Markov jump linear systems with incomplete transition descriptions," *IEEE Tran. Autom. Contr.*, vol. 55, no. 7, pp. 1695-1701, July 2010.

[21] L. Zhang, E. K. Boukas and J. Lam, "Analysis and synthesis of Markov jump linear Sstems with time-varying delays and partially known transition probabilities," *IEEE Tran. Autom. Contr.*, vol. 53, no. 10, pp. 2458-2464, November 2008.

[22] E. Tian, D. Yue, and G. Wei, "Robust control for Markovian jump systems with partially known transition probabilities and nonlinearities," *J. Franklin Inst*, vol. 350, no. 8, pp. 2069-2083, October 2013.

[23] Y. Wei, J. Qiu, H. R. Karimi, and M. Wang, "A new design H_∞ filtering for continuous-time Markovian jump systems with time-varying delay and partially accessible mode information," *Signal Process.*, vol. 93, no. 9, pp. 2392-2407, September 2013. [click]

[24] L. Zhang and E. K. Boukas, "Mode-dependent H_∞ filtering for discrete-time Markovian jump linear systems with partly unknown transition probability," *Automatica*, vol. 45, no. 6, pp. 1462-1467, June 2009. [click]

[25] Y. Ding, H. Zhu, S. Zhong, and Y. Zhang, "Exponential mean-square stability of time-delay singular systems with Markovian switching and nonlinear perturbations," *Appl. Math. Comput.*, vol. 219, no. 4, pp. 2350-2359, November 2012. [click]

[26] Q. Ma, S. Xu, and Y. Zou, "Stability and synchronization for Markovian jump neural networks with partly unknown transition probabilities," *Neurocomputing*, vol. 74, no. 17, pp. 3403-3411, October 2012.

[27] J. Tian, Y. Li, J. Zhao, and S. Zhong, "Delay-dependent stochastic stability criteria for Markovian jumping neural networks with mode-dependent time-varying delays and partially known transition rates," *Appl. Math. Comput.*, vol. 218, no. 9, pp. 5769-5781, January 2012. [click]

[28] J. Lin, S. Fei, and J. Shen, "Delay-dependent H_∞ filtering for discrete-time singular Markovian jump systems with time-varying delay and partially unknown transition probabilities," *Signal Process.*, vol. 91, no. 2, pp. 277-289, February 2011.

[29] Y. Guo and Z. Wang, "Stability of Markovian jump systems with generally uncertain transition rates," *J. Franklin Inst.*, vol. 350, no. 9, pp. 2826-2836, November 2013. [click]

[30] R. W. Brockett and D. Liberzon, "Quantized feedback stabilization of linear system," *IEEE Trans. Automat. Control*, vol. 45, no. 7, pp. 1279-1289, July 2000.

[31] B. Zheng and G. Yang, "Decentralized sliding mode quantized feedback control for a class of uncertain large-scale systems with dead-zone input," *Nonlinear Dynam.*, vol. 71, no. 3, pp. 417-427, February 2013. [click]

[32] M. Fu and L. Xie, "The sector bound approach to quantized feedback control," *IEEE Trans. Automat. Control*, vol. 50, no. 11, pp. 1698-1711, November 2005.

[33] E. Tian, D. Yue, and X. Zhao, "Quantised control design for networked systems," *IET Control Theory A.*, vol. 1, no. 6, pp. 1693-1699, November 2007.

[34] S. Yun, Y. Choi, and P. Park, "H_2 control of continus-time uncertain linear with input quantization and matched disturbances," *Automatica*, vol. 45, no. 10, pp. 2435-2439, October 2009. [click]

[35] W. Che and G. Yang, “Discrete-time quantized H_∞ filtering with quantizer ranges consideration,” *Proc. of 2009 American Control Conference*, pp. 5659-5664, December 2009.

[36] C. Peng and Y. Tian, “Networked H_∞ control of linear systems with state quantization,” *Inform. Sciences*, vol. 177, no. 24, pp. 5763-5774, December 2007. [click]

[37] N. Xiao, L. Xie, and M. Fu, “Stabilization of Markov jump linear systems using quantized state feedback,” *Automatica*, vol. 46, no. 10, pp. 1696-1702, October 2010. [click]

[38] Y. Li, P. Zhang, Y. Kao, and H. R. Karimi, “Quantized state-feedback stabilization for delayed Markovian jump linear systems with generally incomplete transition rates,” *Abstract and Applied Analysis*, vol. 2014, Article-ID 961925, May 2014.

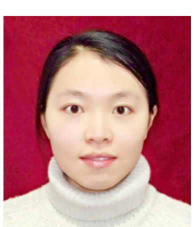
[39] Y. Kao, J. Xie, and C. Wang, “Stabilization of singular Markovian jump systems with generally uncertain transition rates,” *IEEE Transactions on Automatic Control*, vol. 59, no. 9, pp. 2604-2610, September 2014.

[40] J. Xiong and J. Lam, “Robust H_2 control of Markovian jump systems with uncertain switching probabilities,” *Internat. J. Systems Sci*, vol. 40, no. 3, pp. 255-265, March 2009. [click]

[41] L. Xie, M. Fu, and C.E.de Souza, “H_∞ control and quadratic stabilization of systems with parameter uncertainty via output feedback,” *IEEE T. Automat. Contr.*, vol. 37, no. 8, pp. 1253-1256, August 1992.

[42] Z. Wu, H. Su, and J. Chu, “H_∞ filtering for singular Markovian jump systems with time delay,” *Int. J. Robust Nonlinear Control*, vol. 20, no. 8, pp. 939-957, May 2010. [click]

[43] S. Xu, P. V. Dooren, R. Stefan, and J. Lam, “Robust stability and stabilization for singular systems with state delay and parameter uncertainty,” *IEEE T. Automat. Contr.*, vol. 47, no. 7, pp. 1122-1128, July 2002.



Jing Xie received B.S., M.S. and Ph.D. degrees from Ocean University of China in 2009, 2012 and 2015, respectively. She is a Lecturer at the College of Automation Engineering in Qingdao University of Technology, Qingdao, China. Her research interests include singular Markovian jumping systems, sliding mode control and so on.

Yong-Gui Kao received the B.E. degree from Beijing Jiaotong University in 1996. He received his M.E. and Ph.D. degrees from Ocean University of China, in 2005 and 2008, respectively. He is now an Associate Professor at Department of Mathematics, Harbin Institute of Technology (Weihai). His research interest covers stochastic systems, impulsive systems, singular systems, Markovian jumping systems, artificial intelligence, neural networks, stability theory and sliding mode control.

Cai-Hong Zhang received her B.S. degree from Ludong University, Yantai, China in 2004, her M.S. and Ph.D. degrees from Ocean University of China, in 2007 and 2011, respectively. From September 2009 to September 2010, she was a visiting scholar at University of Minnesota Duluth, USA. She is a Lecturer at the College of Automation and Electrical Engineering of Qingdao University, Qingdao, China. Her research interests include nonlinear control and model theory.

Hamid Reza Karimi received the B.Sc. (First Hons.) degree in power systems from the Sharif University of Technology, Tehran, Iran, in 1998, and the M.Sc. and Ph.D. (First Hons.) degrees in control systems engineering from the University of Tehran, Tehran, in 2001 and 2005, respectively. He is currently a professor of Applied Mechanics with the Department of Mechanical Engineering, Politecnico di Milano, Milan, Italy. His current research interests include control systems and mechatronics. He is currently the Editor-in-Chief of the DESIGNS (MDPI Switzerland) and an Editorial Board Member for some international journals, such as the IEEE TRANSACTIONS ON INDUSTRIAL ELECTRONICS, the IEEE TRANSACTIONS ON CIRCUIT AND SYSTEMS I: REGULAR PAPERS, the IEEE/ASME TRANSACTIONS ON MECHATRONICS, IEEE TRANSACTIONS ON SYSTEMS, MAN, AND CYBERNETICS: SYSTEMS, Information Sciences, the IEEE ACCESS, IFAC-Mechatronics, Neurocomputing, the Asian Journal of Control, the Journal of The Franklin Institute, the International Journal of Control, Automation, and Systems, the International Journal of Fuzzy Systems, the International Journal of e-Navigation and Maritime Economy, and the Journal of Systems and Control Engineering. He is also a member of the IEEE Technical Committee on Systems with Uncertainty, the Committee on Industrial Cyber-Physical Systems, the IFAC Technical Committee on Mechatronic Systems, the Committee on Robust Control, and the Committee on Automotive Control. He is a Senior Member of IEEE and awarded as the 2016 Web of Science Highly Cited Researcher in Engineering.